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MULTIVARIATE METHODS IN ASSESSING THE ACCURACY OF PREDICTION MARKETS EX ANTE BASED ON THE HIGHEST-PRICE CRITERION

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1 ABSTRACT

This study successfully establishes the principal component analysis with discriminant analysis (PCA-DA) model to assess the accuracy of contracts in the prediction markets *ex ante* based on the highest-price criterion. Trained by the xFuture data (7,274 contracts of future events) from 2006-2011, the PCA-DA model shows learning effects and provides 97.72% confidence to predict the outcome of any contract discriminated to the correct prediction group in the Exchange of Future Events. However, we need to greatly improve the low confidence of 19.58% for the PCA-DA model to predict the result of any contract discriminated to the incorrect prediction group.

Keywords: Principal component analysis, discriminant analysis, PCA-DA model, prediction markets, Exchange of Future Events, degree of market consensus

JEL codes: C38, C53

2 INTRODUCTION

A prediction market, operating like a futures market, can be used as a mechanism to integrate information from different sources to predict the outcomes of future events. Prediction markets have two major features: 'provide appropriate incentives and punishment mechanisms' and 'perform continuous corrections'. Traders place orders for predictions of future events based on public or private information. The actual results of the future events determine the rewards for the trader, whereas the price of the future event

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contract represents the entire market's prediction of the result of the future event.

As suggested in Arrow *et al*. (2008), the ability of groups of people to make predictions is a potent research tool. Prediction markets have been used for forecasting future trends and social change as well as managing risks of persons, enterprises, societies and states. This instrument has been widely applied to predictions for elections, entertainments, sports, technology, government and enterprise projects, economic and financial indicators, epidemics, and even climate changes.[1]

In recent two decades, prediction markets have been proven empirically to be remarkably accurate in forecasting future events with a lower prediction error than conventional forecasting methods *ex post*. (e.g. Arrow et al., 2008; Ortner, 1998; Pennock, 2001; Wolfers and Leigh, 2002; Brüggelambert, 2004; Servan-Schreiber, 2004; Wolfers and Zitzewitz, 2004; Gürkaynak and Wolfers, 2005; Leigh and Wolfers, 2006; Manski, 2006; Berg et al., 2008). For instance, Berg et al. (2008) compared the predictions of the 'Iowa Electronic Markets'(IEM) [2] with those of 964 surveys conducted by mainstream media institutions, such as Gallup, Harris and the New York Times. They analyzed the performances of the two groups' predictions regarding US presidential elections in the period 1988 to 2004. The results showed that the predictions of prediction markets were closer to the eventual outcome than traditional polls 74% of the time.

Yet, little empirical research has been done on assessing the accuracy of prediction markets *ex ante*. The question is: how confident we are with that prediction before the final result is revealed? For instance, if there are only couple trades for a particular contract or a particular contract is trading for just several days, are we confident in the prediction result of this particular future event based upon the previous accuracy record of prediction markets?

Berg et al. (1997) conducted two out-of-sample predictions with the 1996 and 2000 US presidential election markets from the IEM. Using a regression model, they found that the out-of-sample prediction results were not good. Berg et al. (2003) examined the 1992, 1996 and 2000 US presidential elections, used a nonlinear estimation method in addition to the difference from the assumed vote percentage in the first-order autoregressive process, fitted it with the results estimated by implied volatility, and estimated the standard deviation of the forecast. They believed that the potential predictions in a winner-take-all (WTA) market can be regarded as a reasonable criterion for the confidence levels of the traders' predictions of the total vote count. Nevertheless, they did not propose an actual operational method.

Furthermore, we need to determine a criterion to assess the accuracy of prediction markets. According to Agrawal *et a*l. (2010), the equilibrium price

[1] See Intrade (intrade.com) and the Exchange of Future Events (xfuture.org), and Iowa Electronic Markets (http://tippie.uiowa.edu/iem) for application cases in details.
[2] Established at the University of Iowa in the US in 1988.

on prediction markets can be regarded as the probability of outcome for WTA contracts. Consider a contract that pays $1 if an X event occurs. If the market price of this X event contract is traded at 53 cents, essentially the market predicts a 53% chance of the event occurring. Therefore, if a threshold of 50 cents is used for assessing the occurrence of this event, we will say that the market predicts this X event *ex ante*. Nevertheless, the outcome of this contract is either occurrence or non-occurrence *ex post*. The prediction will be accurate if X indeed comes true, and vice versa.

Furthermore, a threshold of 50 cents is only appropriate when there are two competing event contracts in a contract group. Should there be more than two competing candidates in a contract group, the highest-price criterion is probably better to evaluate whether the prediction market is correct because there might be no price above 50 for all contracts. For instance, no price of 32 football team contracts in the Exchange of Future Events (xFuture, http://xfuture.org) predicting the championship of the 2010 FIFA World Cup was above 50 until July 7, 2010, three days before the final. These contract prices in this group were correlated since only one team would win the championship. Before the final, these prices would be hardly above 50 due to fierce competition among 32 teams.

This paper uses multivariate methods to assess the accuracy of prediction markets *ex ante* based on the highest-price criterion. It constructs a prediction evaluation model combining principle component analysis (PCA) with discriminant analysis (DA) to analyze the data of the xFuture. We contribute to the literature by proposing such a PCA-DA model to evaluate prediction market's forecasting capability *ex ante*, thereby making it possible for public and private sectors to manage economic risks and social changes more efficiently.

3 LITERATURE

The prediction accuracy of 'prediction markets' is affected by the overall market performances and complex behaviors of market participants. As a result, the prediction accuracy can be hardly determined by just a few explanatory variables. In the past, multiple regression analyses could always determine the representative key factors or variables with significant coefficients, but the fitted value from the multiple regression approach could not be used to calculate the accuracy rates.

Under this presumption, we take advantage of PCA and completely integrate comprehensive and largely usable variables into our model system. At the same time, this model avoids the collinearity problem and maximized the information collected by us. Complete information allows more accurate prediction work. Moreover, after incorporating PCA, we introduce DA to solve the shortcomings of the regression analysis, which cannot help us categorize samples or calculate accuracy rates.

In the literature, many papers used PCA as the application method. For example, Bei and Cheng (2013) built a Brand Power Index by the PCA scores. Bodo et al. (1993) adopted multistep PCA to capture the differences between Italian regional labour markets. Nellis (1982) employed principal component analysis to measure the extent of international financial integration under fixed and floating exchange rate regimes. Perignon et al. (2007) used PCA to search for common factors and tendencies regarding the bond returns in the US, Japan and Germany. Chantziara and Skiadopoulos (2008) found that PCA can be used to forecast the changes in the futures prices of energy futures and financial commodities, such as crude oil and natural gas. Kessler and Scherer (2011) applied PCA and state space model (SSM) to independently study the common components that affect the liquidity of hedge funds. Wang (2012) used PCA to determine the relationship among integrated technologies, specific technology fields and patentees.

Blaskowitz and Herwartzb (2011) used PCA to forecast and determine the positive or negative tendencies of economic variables. Their article was the first to contain the concept of grouping in the PCA literature. However, Anzanello and Fogliatto (2011) and Kessler and Scherer (2011) have criticized PCA because in the forecast grouping, it is not as clear as DA. In this study, to predict whether a candidate will win, we had to distinguish between the two forecasting results (i.e. "discriminated correct prediction" and "discriminated incorrect prediction"). The detailed statistical theories can be found in Loh and Vanichsetakul (1988).

Charbaji et al. (1993) employed DA to classify the rescheduling and non-rescheduling countries. Fuller and Seninger (1984) adopted DA to find the important factors to affect the youth labour market in urban. Thomas and James (1968) forecasted whether consumer behavior would change in the future by using DA to accurately divide consumers into innovators and non-innovators. Crask and Perreault (1977) discussed the validity of DA in the small sample condition. Blin and Whinston (1975) applied DA in an election study of decisions based on a majority vote. Mitchneck (1995) applied the classificatory power of DA to economic geography topics. Kim (2001) found major determinants of wage performance of occupational groups with a combination of cluster and discriminant analysis. Kim (2001) used the same methods to determine the major determinant affecting global economic integration. Mitra et al. (2002) used DA to discuss the effects of trade protection measures on small open countries.

The methodology of combining PCA with DA is widely used in the scientific literature. For example, Wilton and Pessemier (1981) distinguished the influence of information on the consumer's willingness to accept new products. Rosen and Grandbois (1983) used DA to verify and indicate whether the factors extracted by PCA can determine the distribution of family income. Newman and Sheth (1985) studied data on US presidential elections. They extracted and used the principal component of a 'partial eigenvalue greater than one' and then introduced weighted average PCA into DA to distinguish and predict the possible votes for the two candidates. Zhao, Krishnaswamy and Chellappa (1998) used it to study automatic face recognition. Do and Kirk (1999) used it in a biostatistics study, and Jombart et al. (2010) used a PCA-DA model in a discrimination study on gene arrangement structure.

4 METHODOLOGY AND DATA

4.1 PCA-DA model

We use the properties of PCA to highlight the important characteristics of a large number of variables with highly correlation. The number of principal components used should be less than the number of the original variables. It reduces the dimensions of many variables to a few principal components to describe the overall data types and properties of the variables. Every principal component is a linear combination of all of the original variables, and each principal component is considered to be a new independent variable.

$$X_1^{*}=W_{11}X_1+W_{12}X2+\ldots+W_{1p}X_p \qquad (1)$$

where X_1^{*} is the first principal component score, W_{1i} is the factor loadings of the *i*-th factor of the first principal component, and X_j is the standardized value of the *j*-th original variable. The principal components can explain the proportion of the original variables, which will decrease as fewer principal components are being used. Therefore, 'dimension reduction' and 'explanation proportion' have a trade-off correlation.

General textbooks and studies (e.g. Sharma (1996); Stevens (2002)) have described three rules for screening the number of principal components in PCA. First, the eigenvalue is greater than one. Second, scholars should observe the steep slope of a graph. Cattell (1996) suggested that the number of principal components at the turning point on the steep slope of a graph represents the optimal number of principal components. Third, scholars should determine the degree to which the variation in the independent variable is explained by the principal components. Hair Jr. *et al.* (1998) suggested that new principal components should be able to explain more than 60% of the total variation in the original variables, whereas Everitt and Dunn (2001) suggested that this value should reach more than 70%. We follow the eigenvalue-greater-than-one rule, which is often used in literature.

After the PCA process, we introduce DA to solve the shortcomings of the linear regression method, which cannot help us categorize samples or calculate accuracy level. The major advantage of DA is to create a classification rule of discrimination function or Fisher function. For the DA operation in SPSS, we need to input independent and dependent variables. The dependent variable must be a dummy and the value of dummy is one if the highest-price criterion satisfies, and zero otherwise.

Then, we combine PCA with DA by taking the principal component variables computed by PCA as the independent variables of DA. The Fisher function of the PCA-DA model is to classify every single contract into the "group of discriminated correct prediction" or the "group of discriminated incorrect prediction" at the eve of the outcome. Based on the price ranking of the contracts in a particular contract group and the PCA-DA discrimination results, we are able to assess the likelihood that a future event will occur if this event belongs to the group of discriminated correct prediction.

We define the notations: T, F, O and X and two accuracy rates of discrimination. With the highest-price forecasting criterion and the *ex post* result of the future events in the xFuture, "T" denotes a contract correctly predicted and "F" denotes a contract incorrectly predicted. Before the final outcome is revealed, "O" denotes a contract classified by the PCA-DA model to the "T" group and "X" denotes a contract classified by the PCA-DA model to the "F" group. (See Table 1.)

Table 1: The Matrix of DA results

		Grouping of discriminant analysis	
Prediction grouping	Number in group	O	X
Correct prediction group	T	*A*	*B*
Incorrect prediction group	F	*C*	*D*

The accuracy rate of discrimination for (in)-correct prediction ARDCP (ARDIP) is defined as the share of the contracts belonging to the "T" ("F") group divided by the contracts discriminated by the PCA-DA model to the "O" ("X") group. These two accuracy rates are expressed in the following two equations based on the classification of Table 1:

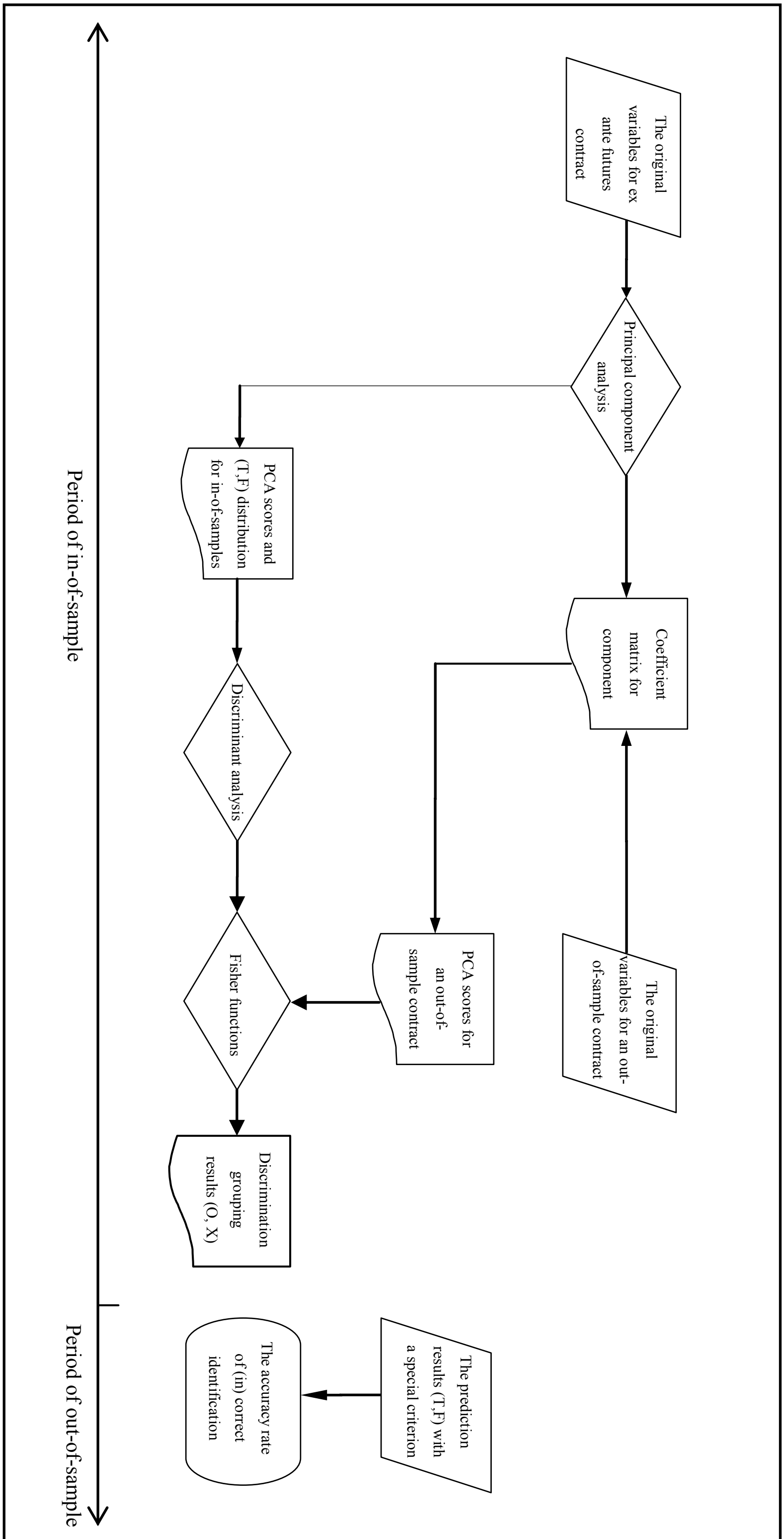
$$ARDCP = A / (A+C) \tag{2}$$

$$ARDIP = D / (B+D) \tag{3}$$

After using the highest-price criterion to obtain a distribution of T and F of in-sample data, which are dependent variables of DA, the PCA-DA model will be trained by these in-sample data to construct a prediction mechanism for out-of-sample data. Following the above steps, we obtain a group of *ex ante* discrimination functions. We employ the coefficient matrix and then convert the original variables for every out-of-sample contract to obtain a group of PCA scores.[3] Next, we input these PCA scores into discrimination

[3] We converted all of the original independent variables into the Z-value matrix and referred to it as Z. We used SPSS multivariate software to calculate and obtain the matrix of the component scores, which we referred to as symbol P. The out-of-sample PCA score was S=ZP.

Figure 1: Flow chart of the out-of-sample prediction made by PCA combined with DA



functions and produce discrimination results (either "O" or "X"). We illustrate the flow charts of the PCA-DA model in Figure 1:

4.2 Variables

According to the literature and the characteristics of the future events in the xFuture, we summarize six types of variables: marginal traders, degree of market consensus, properties of market transactions, difficulty of predicting a topic, speculation and manipulations (avatars) and properties of a predicted event. According to Forsythe et al. (1992), Forsythe et al. (1999) and Oliven and Rietz (2004), marginal traders are an important factor that affects the prediction accuracy of the market. However, there is still no consensus on the operational definition of a marginal trader, and obtaining the related data is not easy. Based on Luckner et al. (2006) method, our paper defined marginal traders as the proportion of traders with better transaction results. Similar operational definition is made for "avatars". Six types of variables and original variables are summarized as follows:

Table 2: Variable types and description of the original variables

Variable type	Variable name	Variable description
Marginal traders	*GP_share_lyc_R*	One day before expire for this contract, the ratio of the number of people involved in this contract from the previous-year's top-performing *R* people in the all-category versus the total number of people involved in this contract, *R*=100, 200, 300.
	GP_share_lyc_S%	One day before expire for this contract, the ratio of the number of people involved in this contract from the previous-year's top-performing *S*% of people in the all-category versus the total number of people involved in this contract, *S*=1, 5, 10.
	GP_share_365d_T	One day before expire for this contract, the ratio of the number of people involved in this contract from 365 days ago top-performing *T* people in the all-category versus the total number of people involved in this contract, *T*=100, 200, 300.
	GP_share_365d_U%	One day before expire for this contract, the ratio of the number of people involved in this contract from 365 days ago top-performing *U*% of people in the all-category versus the total number of people involved in this contract, *U*=1, 5, 10.
	GP_share_30d_V	One day before expire for this contract, the ratio of the number of people involved in this contract from 30 days ago top-performing *V* people in the all-category versus the total number of people involved in this contract, *V*=100, 200, 300.
	GP_share_30d_W%	One day before expire for this contract, the ratio of the number of people involved in this contract from 30 days ago top-performing *W*% of people in the all-category versus the total number of people involved in this contract, *W*=1, 5, 10.

	Limit_ratio_volume	Ratio of the volume of transacted positions with limited prices versus the volume of transacted positions with no-limited prices.
Degree of market consensus	*WBAS_all*	The definition is as follows: $WBAS2 = \frac{\sum bid\ of\ sell\ \times\ volume\ to\ sell - \sum bid\ of\ buy}{\sum volume\ to\ sell + \sum volume\ to\ b}$
	Buy_sell	Ratio of shares to buy to shares to sell.
Properties of market transactions	*Trades*	Volume involved in this contract.
	Traders	Number of traders involved in this contract.
	Days	Number of days between the day that the contract is first traded and the day before the liquidation of the contract.
	Volume	Traded volume of this contract.
	Two_way	Ratio of two-way traders to total traders.
	IP_share	Ratio of traders in Taiwan to total traders. We define traders in Taiwan as those traders who use IP addresses registered in Taiwan to trade.
	Traded_order_ratio	Ratio of the total number of traded orders to the total number of orders.
Difficulty of predicting a topic	*NC*	Contract number of a contract group.
	Price_gap	Price gap between the first highest-price contract and the second highest-price contract in a contract group.
Speculation and manipulations (avatars)	*Avatar_ratio_3*	Assume an avatar exists if 3 members share the same password. This variable is the ratio of the number of members who are regard as the avatars to the total number of traders of this contract.
	Avatar_Xd_ratio_3	Assume an avatar exists if 3 members share the same password. This variable is the ratio of the number of members who are regard as the avatars to the total number of traders of this contract. The above numbers of traders are calculated on prior to expire *X* days , *X*= 15, 30, 365
	Avatar_volume_ratio_3	Assume an avatar exists if 3 members share the same password. This variable is the ratio of the number of long or short positions by the avatars versus total the number of long or short positions by all traders.
	Avatar_volume_Yd_ratio_3	Assume an avatar exists if 3 members share the same password. This variable is the ratio of the number of long or short positions by the avatars versus total the number of long or short positions by all traders. The above numbers of positions are calculated on prior to expire *Y* days, *Y*=15, 30, 365.
Properties of a predicted event	P^w	The “weighted average price” on prior to expire 1 days.
	Category	There are a total of 4 dummy variables: politics, economics, sports, entertainments.

Table 3 Statistics of Five Categories Contracts in xFuture

	2006/07	2008	2009	2010	2011
All-contracts	282	2,433	1,656	2,231	672
Economics	133	1,060	671	392	213
Entertainments	10	208	453	676	219
Sports	85	712	312	981	208
Politics	76	463	164	128	24

Notes: Some contracts overlap on two subcategories.

5 RESULTS

We divide all samples into four subsets in order to conduct out-of-sample tests. For period 1, the in-sample contracts include contracts of year 2006-2007 and the out-of-sample contracts include contracts of year 2008. For period 2, the in-sample contracts include contracts of year 2006-2008 and the out-of-sample contracts include contracts of year 2009. For period 3, the in-sample contracts include contracts of year 2006-2009 and the out-of-sample contracts include contracts of year 2010. For period 4, the in-sample contracts include contracts of year 2006-2010 and the out-of-sample contracts include contracts of year 2011. (See Table 4)

Table 4: Periods of sample subsets for in-sample and out-of-sample contracts

Period	Calendar years for in-sample contracts	Calendar years for out-of-sample contracts
Period 1	2006-2007	2008
Period 2	2006-2008	2009
Period 3	2006-2009	2010
Period 4	2006-2010	2011

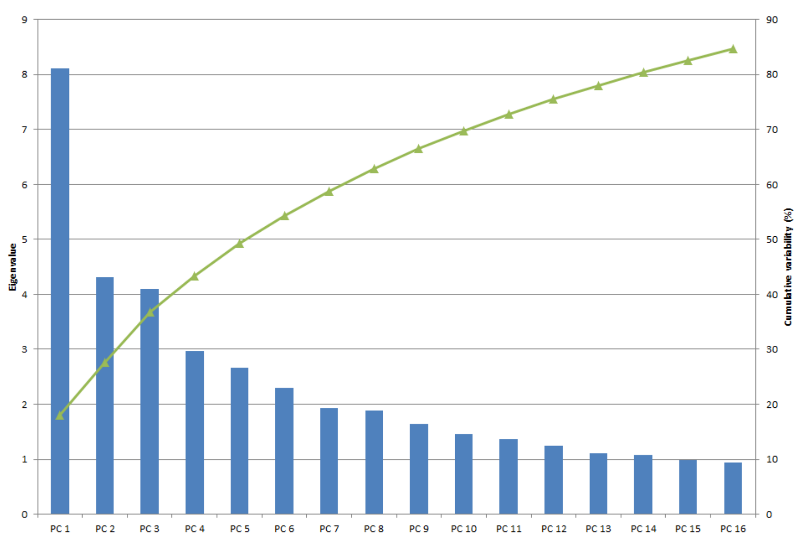
Take “period 1” as an example. We follow the eigenvalue-greater-than-one rule and derive 14 principle components through PCA process as the new independent variables for the PCA-DA model. The percentage of these principle components that could explain variability of the original 40 variables is 80.44%. The “scree plot” of the PCA results is shown in Figure 2.

Next, we put the scores of the 14 important principle component variables obtained from PCA and (T, F) distribution for in-sample contracts into the Fisher function. Table 5 presents the Fisher function structure for out-of-sample test of period 1.

Then, we put the PCA scores for out-of-sample contracts into the Fisher functions and get distribution of grouping results (O, X) for out-of-sample contracts. With the prediction results (T, F) with the highest-price criterion, we calculate ARDCP and ARDIP. In addition, we calculate the accuracy rate of prediction markets (ARPM) as the benchmark for ARDCP. ARPM is the ratio of contracts of correct prediction (“T” group) based on the highest-price criterion divided by all out-of-sample contracts.

The discrimination performance of the PCA-DA model refers to Figure 3. ARPM fluctuates from 90.79% in period 1, 89.37% in period 2, 88.75% in period 3 to 92.86% in period 4. Apparently, there is no learning effect for ARPM. In addition, there is no available mechanism to assess the prediction result of a contact based on detailed information of trading behavior or proprieties of a contract. That is, it will be very difficult to assess the prediction result of a particular contract of prediction markets *ex ante* based on the historical accuracy rate.

Figure 2: Eigenvalues of principle components and cumulative variability of original variables



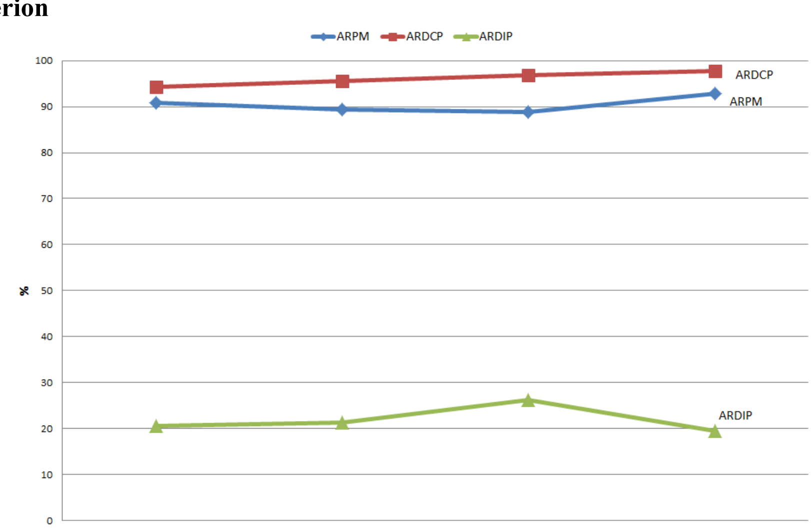
Obviously, ARDCPs are performing better than ARPMs for all four periods. ARPMs are between 88% and 93%, while ARDCPs are gradually improved from 94.27% in period 1 to 95.52% in period 2, 96.88% in period 3 and 97.72% in period 4. Compared to ARPM, ARDCP of the PCA-DA model shows very powerful discrimination capability to predict the result of out-of-sample contracts *ex ante*. In addition, the PCA-DA model shows learning effects throughout four periods. Therefore, with detailed trading information and proprieties of a contract, the PCA-DA model can provide more confidence (97.72% confidence for contracts after 2011) to predict the result of contracts in the xFuture *ex ante*.

Nevertheless, for contacts that the PCA-DA classifies as *incorrectly* predicted, many of them turn out to be *correctly* predicted by the xFuture participants with the highest-price criterion when the outcome is revealed. Accordingly, ARDIP is at the very low level of around 20%, which certainly leaves room for future improvements.

Table 5: Fisher function structure for out-of-sample test of period 1

PCA components	Coefficients of Fisher function	
	X	O
PC1	-.162	.007
PC2	.330	-.015
PC3	.332	-.015
PC4	-.376	.017
PC5	-.422	.019
PC6	-.005	.000
PC7	-.148	.007
PC8	-.681	.030
PC9	-1.173	.052
PC10	.447	-.020
PC11	.086	-.004
PC12	.067	-.003
PC13	-.047	.002
PC14	.372	-.017
Intercept	-4.411	-.046

Figure 3: Accuracy rate of PCA-DA prediction model with the highest-price criterion



Notes:

a. The in-sample contracts include: contracts in period 1 of year 2006-2007, contracts in period 2 of years 2006-2008, contracts in period 3 of years 2006-2009, contracts in period 4 of years 2006-2010.

b. The out-of-sample contracts include: contracts in period 1 of year 2008, contracts in period 2 of year 2009, contracts in period 3of year 2010, and contracts in period 4 of year 2011.

6 CONCLUSIONS

Prediction markets have been proving a very powerful mechanism to predict future events. However, other than past performance, there is no sufficient confidence and no mechanism to assess to the prediction accuracy of a particular future event contract in the prediction market *ex ante*. This study successfully establishes the PCA-DA model to assess the accuracy of contracts in the prediction markets *ex ante* based on the highest-price criterion.

Trained by the xFuture data from 2006-2011, the PCA-DA model shows learning effects and provides 97.72% confidence to predict the outcome of any contract discriminated to the correct prediction group in the xFuture. However, we need to greatly improve the very low confidence of 19.58% for the PCA-DA model to predict the result of any contract discriminated to the incorrect prediction group.

This model allows us to assess ahead of time the prediction accuracy in the prediction markets. In the future, we can simultaneously compute the confidence level with appropriate programing in the prediction markets to predict the outcome of any contract by using all available in-sample contracts in the xFuture.

Finally, we may try different criteria of price thresholds to assess the accuracy of prediction markets instead of the highest-price criterion. We can figure out a best-price criterion by computing the distribution of the discrimination performance of the PCA-DA models with different criteria of price thresholds. This might provide a better result to predict any contract classified to the incorrect prediction group.

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