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Valuation of Catastrophe Equity Puts With Markov-Modulated Poisson Processes

Chia-Chien Chang
Shih-Kuei Lin
Min-Teh Yu

Abstract

We derive the pricing formula for catastrophe equity put options (CatEPuts) by assuming catastrophic events follow a Markov Modulated Poisson process (MMPP) whose intensity varies according to the change of the Atlantic Multidecadal Oscillation (AMO) signal. U.S. hurricanes events from 1960 to 2007 show that the CatEPuts pricing errors under the MMPP(2) are smaller than the PP by 30 percent to 66 percent. The scenario analysis indicates that the MMPP outperforms the exponential growth pattern (EG) if the hurricane intensity is the AMO signal, whereas the EG may outperform the MMPP if the future climate is warming rapidly.

Introduction

The increasing number of catastrophe (CAT) events, particularly hurricane activity in the early 1990s, has created large fluctuations in the price and availability of reinsurance and several CAT-linked instruments (e.g., CAT Bonds, CatEPuts, etc). Increases in Atlantic hurricane activity over recent decades are believed to reflect simultaneous increases in tropical Atlantic warmth (e.g., Emanuel, 2005). Some recent studies (e.g., Goldenberg et al., 2001; Landsea, 2005) attribute these increases to a natural climate cycle termed the Atlantic Multidecadal Oscillation (AMO), which is a climate signal measuring the change in the sea surface temperature (SST, and salinity) of the North Atlantic,[1] whereas the other studies suggest that climate change is instead playing the

Chia-Chien Chang is Assistant Professor of Finance, National Kaohsiung University of Applied Science, Kaohsiung 415, Taiwan. Shih-Kuei Lin is Associate Professor of Finance, National Chengchi University, Taipei 116, Taiwan. Min-Teh Yu is Professor of Finance, National Taiwan University and Providence University, Taipei 10617, Taiwan. The authors can be contacted via e-mail: cchiac@cc.kuas.edu.tw, square@nuk.edu.tw, and mtyu@pu.edu.tw.

[1] Aside from the AMO signal, AIR Worldwide Corporation in 2006 argues that there are three key climate mechanisms that affect hurricane activity: El Nino/Southern Oscillation (ENSO), Quasi-Biennial Oscillation (QBO), and North Atlantic Oscillation (NAO). ENSO measures temperature anomalies in the Pacific Ocean off the coast of Peru. La Niña years are typically characterized by increased hurricane activity, while such activity is lower in El Niño years. However, the period of ENSO is too irregular to make it very useful for forecasting hurricane

dominant role (Emanuel, 2005; Webster et al., 2005). Therefore, it is crucial to model the dynamic process of hurricane activity properly and price CAT-linked instruments (e.g., CatEPuts) corresponding to the global climate change or the change of the AMO signal for the development of the (re)insurance market.

CatEPuts are a form of options that give the owner the right to sell a specified amount of its stock to investors at a predetermined price if CAT losses surpass a specified trigger. Thus, CatEPuts can provide insurers with additional equity capital precisely when they need funds to cover CAT losses. The first CatEPut was issued on behalf of RLI Corporation in October 1996, giving RLI the right to issue up to $50 million of cumulative convertible preferred shares. In 1997, Horace Mann Educators Corporation and LaSalle Re Educators Corporation also entered into a multi-year $100 million CatEPut, respectively. In 2001, the Trenwick Group contracted the right to issue up to $55 million of cumulative convertible preferred shares to European Reinsurance Company of Zurich, a subsidiary of Swiss Re. The CatEPut was exercised in the next year to add equity to Trenwick's balance sheet. Hence, in practice CatEPuts have provided insurance firms with a useful channel to raise additional capital to hedge against CAT losses.

When pricing CatEPuts, it is prudent to develop a model that depicts the joint dynamics of the share value and losses process. Cox, Fairchild, and Pedersen (2004) assume that the share price process is driven by a geometric Brownian motion with additional downward jumps of a specific size in a CAT event. Their model assumes that only a CAT event affects the stock price, whereas the size of the CAT is irrelevant. Jaimungal and Wang (2006) extend the results of Cox, Fairchild, and Pedersen (2004) to analyze the pricing of CatEPuts under stochastic interest rates with losses generated by a compound Poisson process (PP). In addition to Cox, Fairchild, and Pedersen (2004) and Jaimungal and Wang (2006), others, for example, Cummins and Geman (1995) and Chang, Chang, and Yu (1996) look at CAT futures options, Louberge, Kellezi, and Gilli (1999) investigate a CAT bond with a pure PP, and Vaugirard (2003a, 2003b) and Lee and Yu (2002, 2007) research a CAT bond with a compound PP.[2]

activity over a 5-year time horizon. QBO is a climate signal that tracks the direction of the equatorial winds in the stratosphere. While the QBO is the easiest signal to forecast, it has the weakest correlation with hurricane activity. NAO is a low-pressure ridge that, in a positive phase, typically forms off of the coast of Greenland, allowing a high-pressure ridge to form in the Northeastern Atlantic ("Bermuda High"). The position of the NAO and the Bermuda High are very important in steering tropical storm tracks and the risk of "land-falling" hurricanes. However, the predictability of the NAO decays quickly, rendering it virtually useless for forecasting hurricane activity 5 years out. Therefore, we focus on the relationship between hurricane activity and the change in the AMO signal.

[2]The recent development of CAT risk also focuses on the convergence of the financial services industry and (re)insurance sector, which is driven by the increase in frequency and severity of CAT risk. These developments have led to the development of hybrid insurance/financial instruments. For example, Klein and Wang (2009) illustrate the regulation of CAT risk and financing in the United States and the EU. Cummins and Weiss (2009) and Barrieu and Loubergé (2009) provide an overview of hybrid and pure financial markets instruments. Finken and Laux (2009) show that CAT bonds can play an important role in the pricing of reinsurance contracts when there is asymmetric information between inside and outside reinsurers about an insurer's risk.

FIGURE 1
Frequency of Hurricanes in the United States from 1960 to 2007 Relative to the AMO Index

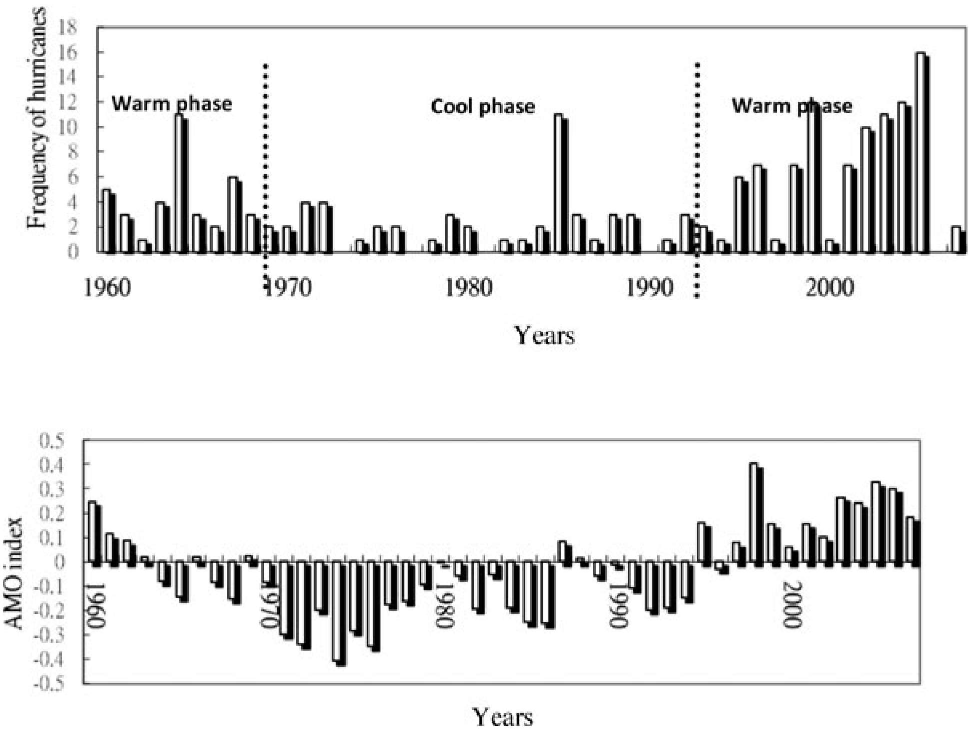


Figure 1 shows the annual frequency of U.S. hurricanes between 1960 and 2007 relative to the AMO index.[3] If the intensity process of hurricane events stands for a PP, then the frequency of hurricane events should stay at the same level over the years. However, the top of Figure 1 apparently exhibits different frequencies for hurricane events through time. Specifically, it indicates that there are larger hurricane activities since 1995. Meteorologists (e.g., Landsea and Gray, 1992; Kevin, 2005) have recognized for some time that SSTs and water vapor play a critical role in energy of tropical cyclones (typhoons and hurricanes), with hurricane formation generally limited to regions where SSTs exceed 26°C. Specifically, higher SSTs are positively correlated with hurricane activities, but there is still an argument of whether the increased trend in SSTs since 1995 reflects a continuous increase brought on by global warming or the result of cyclical trends in AMO.

The recent report of the Intergovernmental Panel on Climate Change (IPCC, 2007) indicates that climate change in the future could be more serious, owing to global warming, with more unanticipated tropical cyclone events. Hence, global climate

[3]The AMO index value represents a temporal reconstruction of SST anomalies averaged over the region of the North Atlantic bounded by 45°–65° North and 60°–20° West.

change is likely to exert a gradual shift in hurricane intensity from one state to another. Since 1995, the Atlantic basin has been in an active, or warm, phase with characteristics leading to higher SSTs and cyclical increases in tropical cyclone activity in the Atlantic. Some scientists indicate that increases in tropical cyclone activity are associated with a warm phase of the AMO. In 2006, leading CAT risk modeling firms, such as the Swiss Re Company, argued that the increasing hurricane activities in the early 1990s are most closely correlated with the AMO signal, and thus they updated their projections of hurricane activities in the Atlantic for the next 10 years based on the expectation of increased hurricane frequency due to a positive AMO index.

According to the change of AMO signal in Figure 1, the periods of 1960–1970 and 1995–2007 are defined as the warm phase (or positive index) of the AMO and can be regarded as state one. The period 1971–1994 is defined as the cool phase (or negative index) of the AMO and can be viewed as another state. This figure shows that the mean frequency of the entire period (1960–2007) is 3.85, where the mean frequencies of the warm phase and cool phase are 5.58 and 2.13, respectively. Warm phases in the AMO therefore are theorized as leading to higher SSTs and being above the long-term average hurricane activity in the Atlantic. Cool phases in the AMO are theorized to lead to lower SSTs and being below the long-term average hurricane activity. As shown in Figure 1, the period 1960–1970 is in state 1, but transits to state 2 in the period 1971–1994, and then in 1995–2007 the state transits back to state 1. Therefore, the intensity process of hurricane events is different at different phases (states) of the AMO signal, and the transition of the two states seems to follow a homogenous Markov process.

This study intends to contribute to the literature in three threads. First, we propose a more general Markov jump diffusion model, which advances the PP used in the jump diffusion model, to a Markov modulated PP (MMPP). The MMPP stands for a doubly stochastic PP where the underlying state is governed by a homogenous Markov chain (see Last and Brandt, 1995).[4] More precisely, instead of a constant (average) intensity rate under the PP and compound PP, the intensity rates of hurricane events are different at different states of the AMO signal under the MMPP with two states: MMPP(2). Second, we derive the closed-form solutions for CatEPuts under the Markov jump diffusion model and show that the derived formula can be reduced to the pricing formulas of Cox, Fairchild, and Pedersen (2004) and Jaimungal and Wang (2006). Third, we use the data of U.S. hurricane events from 1960 to 2007 to investigate the pricing performance of the valuation of the CatEPuts. The results show that the pricing errors under the MMPP(2) are smaller than the PP in pricing the CatEPut and the pricing errors can be reduced by 30 percent to 66 percent depending on the measurement methods.

We further discuss the measurement errors that would result from a misdiagnosis of the true driver of different climatic patterns. The result indicates that the MMPP outperforms the EG if the intensity of hurricane events is driven by the AMO signal, whereas the EG may be superior to the MMPP if the future climate is in the process of

[4]The MMPPs have been applied in many fields, such as hydrology (e.g., Stern and Coe, 1984; Ramesh, 1998; Davison and Ramesh, 1996), queuing theory (e.g., Olivier and Walrand, 1994; Du, 1995), etc.

rapid warming. Sensitivity analysis also indicates that hurricane intensity dominates the mean and variance of hurricane losses when determining the CatEPut prices.

The remainder of the article is organized as follows. The "CatEPut Contract and the Model" section illustrates the CatEPut contract and the model. The "Valuation of CatEPuts" section derives the change measure and the pricing formulas of CatEPut. Results and discussions are present in the next section. The "Conclusions" section summarizes the study and provides the conclusions. For ease of exposition, most proofs are in the appendices.

CATEPUT CONTRACT AND THE MODEL

CatEPut Contract

The CatEPut option in general allows the owner to issue shares at a predetermined price, much like a regular put option. However, that right is only exercisable in the event that the accumulated losses of the purchaser of the CatEPut exceed a critical coverage limit during the option's lifetime. Hence, such a contract can be viewed as a special form of a double trigger put option, where the option's payoff is a function of share price and level of insured losses.

Cox, Fairchild, and Pedersen (2004) introduce a model that jointly describes the dynamics of the share value and losses process for a CatEPut. They model the CAT arrival process in terms of a PP and assume that only a CAT event decreases the share value whereas the CAT size is irrelevant. Jaimungal and Wang (2006) generalize the results of Cox, Fairchild, and Pedersen (2004) to consider the stochastic interest rate and propose that the loss sizes should affect share value. They assume that the losses follow a compound PP. Thus, the payoffs of CatEPuts, $V(T)$, at maturity T can be described as follows:

$$V(T) = 1_{\{L(T)-L(t_0)>L\}}(K - S(T))1_{\{S(T)<K\}}, \tag{1}$$

where $S(T)$ is the share value and K represents the strike price at which the issuer is obligated to purchase unit shares in the event that the total loss level exceeds the specified losses, L. Furthermore, $L(T) - L(t_0)$ denotes the total losses of the insured over the time-period $[t_0, T]$.

Model Setup

Let uncertainty in the economy be described by the filtered probability space $(\Omega, F, P, (F_t)_{t=0}^{T^*})$. We assume the existence and uniqueness of P, such that markets are complete. Let $\{S(t) : t > 0\}$ denote the share value process, $\{L(t) : t > 0\}$ denotes the total loss process of the insured, and $\{r(t) : t > 0\}$ denotes the risk-free short rate process. Hence, the natural filtration $F = \{F_t : t > 0\}$ is generated by these three processes. We follow Jaimungal and Wang (2006) and assume that the interest rate is independent of the loss, and the dynamics of the share price and the interest rate can be written as

$$S(t) = S(0)\exp\left\{\int_0^t \left(\mu(s) - \frac{1}{2}\sigma_S^2\right)ds + \sigma_S W_S(t) - \alpha L(t)\right\}, \tag{2}$$

$$L(t) = \sum_{n=1}^{\Phi(t)} Y_n, \tag{3}$$

$$d\,r(t) = \tilde{k}\,[\tilde{\theta} - r(t)]\,dt + \sigma_r\,dW_r(t),$$

where $S(0)$ is the initial share price and drift μ is the instantaneous return of $S(t)$ at time t.

Terms $\tilde{k}$ and $\tilde{\theta}$ are, respectively, the constant reversion rate and long-term mean in the drift coefficient of the Vasicek dynamics of $r(t)$. Terms σ_S and σ_r are the volatilities of returns on share price and spot rate, respectively, whereas $\{W_S(t) : t > 0\}$ and $\{W_r(t) : t > 0\}$ are correlated Brownian motions of the returns of share price and the interest rate with respect to F_t, and they satisfy $E(W_S(t)\ W_r(t)) = \rho\,dt$, where ρ is the correlation coefficient of these two terms. Term α denotes the downward percentage in the share price per specific CAT loss. Here, let $\{Y_n : n = 1, 2, \ldots\}$ be the sequence of independent and identically distributed nonnegative random variables representing the size of the nth loss with the density function $f_Y(y)$.

Markov Modulated Poisson Process

The empirical data of Figure 1 display that the frequency of hurricane events can be significantly different under different phases of the AMO signal. In view of such an observation, this study proposes a model whereby the intensity process of hurricane events follows the MMPP and where the climate state is governed by a homogeneous Markov chain. We consider that the sample path for the share price is continuous except on finite points in time, and the intensity of hurricane events depends on the state of the climate environment.

An MMPP, $\Phi(t)$, is a Poisson process whose intensity, $\lambda_{X(t)}$, varies according to a homogenous Markov process, $X(t)$, with transition function, $P_{ij}(t)$, for the finite state space X $=\{1, 2, \ldots, I\}$. In other words, a Poisson process $\Phi(t)$ is called MMPP if the conditional distribution $P(\Phi \mid X)$ is equal to the distribution of a PP with the intensity function $\lambda_{X(t)}$. Particularly, the distribution of the MMPP can be denoted by

$$P(\Phi(t) = m \mid X(t), t > 0) = \frac{\left(\int_0^t \lambda_{X(s)}\,ds\right)^m}{m!} \exp\left[-\int_0^t \lambda_{X(s)}\,ds\right] \quad P - \text{a.s.}$$

Let transition rate $\Psi(i, j)$ be denoted as

$$\Psi(i, j) = \begin{cases} \nu(i, j), i \neq j, \\ -\sum_{j \neq i} \nu(i, j), & \text{otherwise} \end{cases}$$

where $i, j \in$ X. Since the Markov chain has a finite number of states, the Poisson intensity rate takes discrete values corresponding to each state. Last and Brandt (1995) give the moment generating function for the joint distribution function as follows:

$$P^*(z,t) = \sum_{m=0}^{\infty} P(m,t)\, z^m, \quad 0 < z < 1, \tag{4}$$

where $P(m,t) :\equiv (P_{ij}(m,t))$ represents the $I \times I$ transition probability matrix and $P_{ij}(m,t) = P_i(X(t) = j,\ \Phi(t) = m) = P_i(X(0) = j,\ \Phi(t) = m)$ denotes the transition probability with m jump times from state $X(0) = i$ to state $X(t) = j$. Notation $\Psi :\equiv (\Psi(i, j))$ represents the $I \times I$ matrix of the transition rate, and Λ denotes the $I \times I$ diagonal matrix with diagonal elements λ_i. Here, $P(m,0) :\equiv (1_{\{m=0\}} D_{ij})$, where $D_{ij} = 1,$ if $i = j$; 0, otherwise.

The intensity of the MMPP has the property:

$$-\alpha \sum_{n=1}^{\Phi(t)} Y_n + \Lambda \kappa_1\, t,$$

which is a martingale in t. The last term $\Lambda\kappa_1\, t$ denotes the compensation for the occurrence of the downward jumps in stock price due to the total CAT losses until time t, and $\kappa_1 = E[(1 - \exp(-\alpha Y))]$. Appendix A presents the proof.

The MMPP reduces to the PP under certain assumptions. In a general setting with I states, if $\nu_k \to 0,\ \ \nu_i \to \infty,\ \ k \in X,\ \ i \in X,\ \ k \neq i$, for every $i \to \infty$, then it implies that the Markov chain will not leave state k and thus the MMPP reduces to the PP with intensity λ_k. However, , if $\lambda_1 = \lambda_2 = \cdots = \lambda_I = \lambda$, then the MMPP reduces to the PP with intensity λ.

VALUATION OF CATEPUTS

Note as the underlying stochastic process for the claim arrival process is the MMPP, that there is no unique equivalent martingale probability measure. In other words, we have several choices of equivalent martingale probability measures to price a CatEPut when the market is incomplete. First, this section illustrates the selected process of an equivalent martingale probability measure. Next, when the martingale probability measure is chosen, the valuation of a CatEPut can be achieved by using an equivalent martingale probability measure.

Equivalent Martingale Probability Measure

We follow Cox, Fairchild, and Pedersen (2004) and Jaimungal and Wang (2006) and make use of Merton's (1976) assumption that the jumps are diversifiable, and therefore the jump intensity rate and distribution are not altered from the original physical probability measure P to the risk-neutral probability measure Q. The risk-neutral process for the joint share price, interest rate, and loss dynamics under probability measure Q can be written as:[5]

[5]See Jaimungal and Wang (2006) for a similar derivation.

$$S(t) = S(0)\exp\left\{\int_0^t \left(r(s) - \frac{1}{2}\sigma_S^2\right) ds + \sigma_S\, W_S^Q(t) - \alpha L(t) + \Lambda^Q \kappa_1\, t\right\}, \tag{5}$$

$$L(t) = \sum_{n=1}^{\Phi(t)} Y_n, \tag{6}$$

$$d\,r(t) = k\,[\theta - r(t)]\,dt + \sigma_r\, dW_r^Q(t),$$

where

$$W_S^Q(t) = W_S(t) + \int_0^t \frac{\mu - r(u)}{\sigma_S} du,$$

$$W_r^Q(t) = W_r(t) - \int_0^t \frac{1}{\sigma_r}[k\theta - \tilde{k}\tilde{\theta} + (\tilde{k} - k)\, r(u)]\, du.$$

Hence, we obtain

$$P^Q(\Phi_t = m) = E[\beta_1(t)\, 1_{\{\Phi_t = m\}}] = P^P(\Phi_t = m),$$

which means that the investors receive a zero premium for the jump risk, and thus the jump intensity and transition probability are unaffected by the measure change, that is, $\Lambda^Q = \Lambda$, $Q(m,t) = P(m,t)$.

In the presence of a stochastic interest rate, a similar factorization can be obtained by performing a measure change to the forward-neutral measure Q^T. This study assumes that the stochastic interest rates follow the Vasicek model, and thus the price of a zero-coupon bond with maturity T, $B(t,T)$, is given by

$$B(t,T) = \exp(A(t,T) - U(t,T)\, r(t)),$$

where

$$A(t,T) = \left(\theta - \frac{\sigma_r^2}{2k^2}\right)(B(t,T) - (T-t)) - \frac{\sigma_r^2}{4k} B^2(t,T),\ U(t,T)$$
$$= \frac{1}{k}[1 - \exp(-k(T-t))].$$

The forward price of the share price, under Q^T, is thus calculated as

$$\frac{S(T)}{B(T,T)} = \frac{S(t)}{B(t,T)} \exp\left[-\frac{1}{2}\int_t^T \tilde{\sigma}^2(u,T)du + \int_t^T \sigma_S d\tilde{W}_S(u) + \int_t^T \sigma_r B(u,T)\, d\tilde{W}_r(u) - (\alpha(L(T) - L(t)) - \Lambda\kappa_1(T-t))\right], \tag{7}$$

where

$$\tilde{W}_S(t) = W_S^Q(t) + \int_0^t \rho\,\sigma_r\, U(u,T)\, du,\ \ \tilde{W}_r(t) = W_r^Q(t) + \int_0^t \sigma_r\, U(u,T)\, du,$$

$$d(\tilde{W}_S(t)\ \tilde{W}_r(t)) = \rho\, dt,$$

and

$$\tilde{\sigma}^2(u,T) = \sigma_r^2 B^2(u,t) + 2\rho\sigma_S\sigma_r B(u,t) + \sigma_S^2.$$

Proposition 1: *Let $\beta_1(t)$ denote the Radon–Nikodym processof transition probability*

$$\beta_1(t) \equiv \frac{dQ'(m,t)}{dQ(m,t)} = (1-\kappa_1)^m \exp(\Lambda\kappa_1 t).$$

For any $B \in F_T$, there then exists a new risk-neutral probability measure Q' with $Q'(B) = E^Q(1_{\{B\}}\beta_1(T))$ on F_t. Under the original risk-neutral measure Q, the transition probability matrix is $Q(m,t)$, with transition rate matrix Ψ and jump intensity matrix Λ. Through the change of measure, the new risk-neutral transition probability becomes $Q'(m,t)$ with transition rate matrix Ψ and jump intensity matrix $\Lambda(1-\kappa_1)$.

Proposition 2: *Let $\beta_2(t)$ denote the Radon–Nikodym process of the hurricane loss size*

$$f_Y^{Q'}(y_1),\quad f_Y^{Q'}(y_2),\ldots,f_Y^{Q'}(y_m) = \left[\frac{e^{-\alpha y_1} f_Y(y_1)}{(1-\kappa_1)} \frac{e^{-\alpha y_2} f_Y(y_2)}{(1-\kappa_1)} \cdots \frac{e^{-\alpha y_m} f_Y(y_m)}{(1-\kappa_1)}\right].$$

For any $B \in F_T$, there then exists a new risk-neutral probability measure Q' with $Q'(B) = E^Q(1_{\{B\}}\beta_2(T))$. Hence, under the original risk-neutral probability measure Q, the density function of each specific hurricane loss is $f_Y(y)$. Through the change of measure, under the new risk-neutral probability measure Q', the new density function of each specific CAT loss is

$$f_Y^{Q'}(y) = \frac{e^{-\alpha y} f_Y(y)}{(1-\kappa_1)}.$$

Appendix B sketches a detailed proof of Propositions 1 and 2.

The Pricing Formula

Under the assumption that there are no arbitrage opportunities in the market, we price the CatEPuts by using an equivalent martingale probability measure, Q^T. Let $V(t;t_0)$ represent the value of the option at time t, which was signed at time $t_0 < t$ and matures at time T. Therefore, we have

$$V(t;t_0) = B(t,T)E^{Q^T}[1_{\{L(T)>L+L(t_0)\}}(K - S(T))\ 1_{\{S(T)<K\}}].$$

As the joint share price and loss dynamics stand for the Markov jump diffusion model, by using Equation (7) and an equivalent martingale probability measure, Q^T, the formula of a CatEPut can be obtained as Theorem 1.

Theorem 1: *The value of CatEPut contracts is given as*

$$\begin{aligned} V(t;t_0) = \quad & \sum_{m=1}^{\infty} Q(m, T-t) \int_{\tilde{L}}^{\infty} f_Y^m(y^m) \big[K\,\phi\big(-d_{2m}^{MM}(y^m)\big) \\ & - S(t)\exp[-\alpha y^m + \Lambda\,\kappa_1(T-t)]\phi\big(-d_{1m}^{MM}(y^m)\big)\big]\, dy^m, \end{aligned} \tag{8}$$

where $y^m = \sum_{n=1}^{m} Y_n$ represents the total losses of m insured CAT claims under the original risk-neutral probability measure Q with density function $f_Y^m(y^m)$, and $\phi(\cdot)$ denotes the cumulative distribution function for a standard normal random variable:

$$d_{1m,2m}^{MM} = \frac{\ln(S(t)/K\ B(t,T)) \pm \frac{1}{2}\tilde{\sigma}^2(t,T) - \alpha y^m + \Lambda\,\kappa_1(T-t)}{\tilde{\sigma}(t,T)},$$

$$\tilde{\sigma}^2(t,T) = \sigma_S^2(T-t) + \frac{2k\rho\sigma_r\sigma_S + \sigma_r^2}{k^2}[(T-t) - U(t,T)] - \frac{\sigma_r^2}{2k}U^2(t,T),$$

$$\tilde{L} = L + L(t_0) - L(t).$$

Equation (8) can be viewed as the transition probability sum of the expectation of a put option with a limitation on the total losses of the insured exceeding specified losses $\tilde{L}$. Appendix C sketches a detailed proof.

To further illustrate the property of the CatEPut solution, we consider several special cases and show their specific formulas in the following remarks and corollary.

Remark 1: If $L(t) = \sum_{i=1}^{I} \sum_{n=1}^{N_i(t)} Y_n$, where $N_i(t)$ are independent PPs with jump intensity λ_i, and there are I independent PPs, then Equation (7) reduces to the mixed PP (MPP) and becomes the following equation:

$$\frac{S(T)}{B(T,T)} = \frac{S(t)}{B(t,T)} \exp\left[-\frac{1}{2}\int_t^T \tilde{\sigma}^2(u,T)\,du + \int_t^T \sigma_S\, d\tilde{W}_S(u) + \int_t^T \sigma_r\, B(u,T)\, d\tilde{W}_r(u) \right.$$
$$\left. - \left(\alpha \sum_{n=1}^{N_i(T-t)} Y_n - \lambda_i \kappa_1\, t \right) \right], \quad \text{with probability} \quad p_i,\ i = 1, 2, \ldots, I. \tag{9}$$

The pricing formula of a CatEPut is then given by

$$V(t;t_0) = \sum_{m=1}^{\infty} \sum_{i=1}^{I} p_i\ \frac{e^{-\lambda_i(T-t)}[\lambda_i(T-t)]^m}{m!} \int_{\tilde{L}}^{\infty} f_Y^m(y^m) \left[K\,\phi\left(-d_{2m}^{MP}(y^m) \right) \right.$$
$$\left. - S(t)\exp(-\alpha y^m + \lambda_i \kappa_1 (T-t))\phi\left(-d_{1m}^{MP}(y^m) \right) \right] dy^m, \tag{10}$$

where p_i is the probability occurring at state i, and $\sum_{i=1}^{I} p_i = 1$.

$$d_{1m,2m}^{MP} = \frac{\ln(S(t)/K\,B(t,T)) \pm \frac{1}{2}\tilde{\sigma}^2(t,T) - \alpha y^m + \lambda_i\,\kappa_1\,(T-t)}{\tilde{\sigma}(t,T)}, \qquad i = 1, 2, \ldots, I.$$

Remark 2: If $\lambda_1 = \lambda_2 = \ldots \lambda_I = \lambda$, then the MMPP, $\Phi(t)$, simplifies to a single PP, $N(t)$, with an intensity rate of CAT events, λ, and Equation (7) reduces to

$$\frac{S(T)}{B(T,T)} = \frac{S(t)}{B(t,T)} \exp\left[-\frac{1}{2}\int_t^T \tilde{\sigma}^2(u,T)\,du + \int_t^T \sigma_S d\tilde{W}_S(u) + \int_t^T \sigma_r\, B(u,T)\, d\tilde{W}_r(u) \right.$$
$$\left. - \left(\alpha \sum_{n=1}^{N(T-t)} Y_n - \lambda \kappa_1\, t \right) \right], \tag{11}$$

which implies a single jump component with magnitude Y and intensity λ. This equation is also the dynamic process setting for the model of Jaimungal and Wang (2006), and the pricing formula of CatEPut can be derived as the following.

$$V(t;t_0) = \sum_{m=1}^{\infty} \frac{e^{-\lambda(T-t)}[\lambda(T-t)]^m}{m!} \int_{\tilde{L}}^{\infty} f_Y^m(y^m)\big[K\,\phi\big(-d_{2m}^{PP}(y^m)\big) \\ -S(t)\exp(-\alpha y^m + \lambda\,\kappa_1(T-t))\big)\phi\big(-d_{1m}^{PP}(y^m)\big)\big]\,dy, \tag{12}$$

where

$$d_{1m,2m}^{PP} = \frac{\ln(S(t)/K\;B(t,T)) \pm \frac{1}{2}\tilde{\sigma}^2(t,T) - \alpha y^m + \lambda\,\kappa_1\,(T-t)}{\tilde{\sigma}(t,T)}.$$

Note that when $Y = 1$ and under the assumption of a constant interest rate, the closed-form formula of a CatEPut reduces to that of Cox, Fairchild, and Pedersen (2004) as follows:

$$V(t;t_0) = \sum_{m=n}^{\infty} \frac{e^{-\lambda(T-t)}[\lambda(T-t)]^m}{m!} \\ \times\big[K\,e^{-r(T-t)}\phi\left(-d_2^{PP}\right) - S(t)\exp\left[-\alpha m + \lambda\,(1-e^{-\alpha})(T-t)\right]\phi\left(-d_1^{PP}\right)\big],$$

where

$$d_{1,2}^{PP} = \frac{\ln(S(t)/K\,) + \left(r \pm \frac{1}{2}\sigma_S^2\right)(T-t) - \alpha m + \lambda\,(1-e^{-\alpha})(T-t)}{\sigma_S\sqrt{(T-t)}}.$$

Corollary 1: *If $L(t) = \sum_{n=1}^{\Phi(t)} Y_n$ and Y_n is a sequence of nonnegative independent identical distributed random variables with exponential density $f_Y(y) = \eta e^{-\eta y}$, $y > 0, \eta > 0$, then based on Propositions 1 and 2, the pricing formula of the CatEPut is given by*

$$V(t;t_0) = \sum_{m=1}^{\infty} Q'(m, T-t)\left[K\,B(t,T)\frac{\exp(-\Lambda\kappa_1\,(T-t))}{(1-\kappa_1)^m}\,g_1(\tilde{L}, a_1) - S(t)g_2(\tilde{L}, a_2)\right], \tag{13}$$

where

$$g_1(\tilde{L}, a_1) = \int_{-\infty}^{a_1}\int_{\tilde{L}}^{\infty} \frac{\exp\left[-(\mu_y/\sigma_y^2)v\right]v^{(\mu_y^2/\sigma_y^2)-1}}{\Gamma\left(\mu_y^2/\sigma_y^2\right)\left(\mu_y/\sigma_y^2\right)^{(\sigma_y^2/\mu_y^2)}}\frac{1}{\sqrt{2\pi\tilde{\sigma}^2(t,T)}}\exp\left[-\frac{(u+\alpha v)^2}{2\tilde{\sigma}^2(t,T)}\right]dv\,du,$$

$$g_2(\tilde{L},a_2)=\int_{-\infty}^{a_2}\int_{\tilde{L}}^{\infty}\frac{\exp\left[-(\mu_y/\sigma_y^2+\alpha)v\right]v^{(\mu_y^2/\sigma_y^2)-1}}{\Gamma(\mu_y^2/\sigma_y^2)(\mu_y/\sigma_y^2+\alpha)^{(\sigma_y^2/\mu_y^2)}}\frac{1}{\sqrt{2\pi\tilde{\sigma}^2(t,T)}}$$
$$\times\exp\left[-\frac{(u+\alpha v)^2}{2\tilde{\sigma}^2(t,T)}\right]dv\,du,$$

$$a_{1,2}=\pm\frac{1}{2}\tilde{\sigma}^2(t,T)-\Lambda\kappa_{1,2}(T-t)+\ln(KB(t,T)/S(t)),\ \kappa_1=\frac{\alpha}{\eta+\alpha},\quad \kappa_2=\frac{\alpha}{\eta+2\alpha},$$

and $\mu_y=\frac{m}{\eta}$ *and* $\sigma_y^2=\frac{m}{\eta^2}$ *are the mean and variance of the gamma distribution for the total losses, respectively. Appendix D sketches a detailed proof.*

Note that if the increase in unanticipated hurricane events with time is caused by global warming (e.g., see Emanuel, 2005; Mann and Emanuel, 2006; IPCC, 2007; Lin, Chang, and Powers, 2009), then the intensity of hurricane events appear to display a directional long-term upward drift, which is assumed to follow the exponential growth pattern (EG):$\lambda(t)=\lambda(0)\exp[\mu_\lambda t]$, where μ_λ represents the growth rate of the hurricane events. The pricing formula of the CatEPut is then given by

$$V(t;t_0)=\sum_{m=1}^{\infty}\frac{e^{-\tilde{\lambda}(T)(1-\kappa_1)}(\tilde{\lambda}(T)(1-\kappa_1))^m}{m!}$$
$$\times\left[KB(t,T)\frac{\exp(-\tilde{\lambda}(T)\kappa_1(T-t))}{(1-\kappa_1)^m}g_1(\tilde{L},a_3)-S(t)g_2(\tilde{L},a_4)\right],\qquad(14)$$

where

$$a_{3,4}=\pm\frac{1}{2}\tilde{\sigma}^2(t,T)-\tilde{\lambda}(T)\kappa_{1,2}(T-t)+\ln(KB(t,T)/S(t)),$$

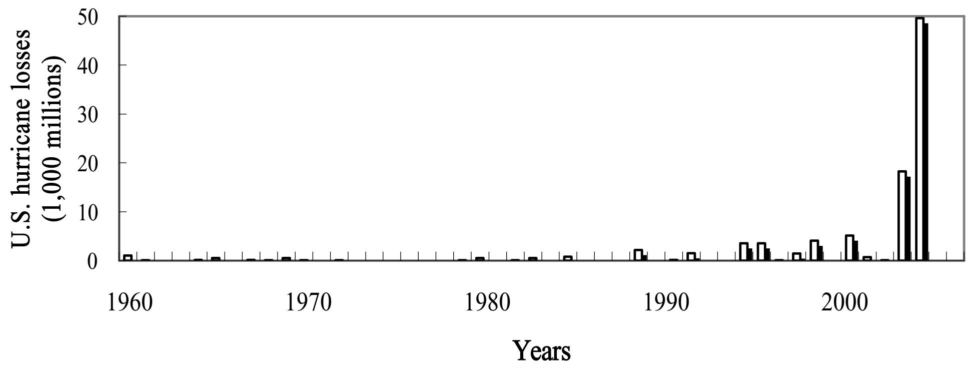
$$\tilde{\lambda}(T)=\int_t^T\lambda(t)e^{\mu_\lambda(u-t)}du=\frac{\lambda(t)}{\mu_\lambda}[e^{\mu_\lambda(T-t)}-1].$$

RESULTS AND DISCUSSION

Data Description

This section evaluates CatEPuts whose total losses are linked to hurricane losses available from the U.S. database on spatial hazard events and losses. Prior to estimating the parameters of the intensity and severity distributions of hurricane activities, two adjustments affecting the intensity and severity of CATs are made to the data on hurricane losses. We follow Cummins, Lewis, and Phillips (1999) and adjust hurricane losses by changes in *Houses under Construction Fixed-Weighted Index* and *Population Index* from the U.S. Census Bureau. Figures 1 and 2, respectively, display the adjusted number of hurricane events and the adjusted hurricane losses during 1960 to 2007.

FIGURE 2
Hurricane Losses in the United States During 1960–2007



The mean μ_y and the standard deviation σ_y of the adjusted hurricane losses are estimated to be 1.98 and 7.56, respectively. The average intensity over all sample years is 3.85 and we take it as the intensity of a single PP to reflect the frequencies of hurricane events per year. Figure 1 seems to reveal that a smaller intensity of hurricane events is found in the cool phase of the AMO signal (1971–1994) and a larger hurricane intensity in the warm phase of the AMO signal (1960–1970 and 1995–2007). Hence, during the sample period we divide the AMO signal into two states: the warm phase is regarded as state 1 and the cool phase is reviewed as state 2. The intensities of MMPP(2) are calculated as 5.58 and 2.13, respectively. On the other hand, since 1995 the hurricane frequencies also have an increasing pattern possibly owing to global warming. Thus, we assume that the intensities of hurricane events stand for the exponential growth pattern (EG): $\lambda(t) = \lambda(0)\exp[\mu_\lambda t], t = 0, 1, \ldots 47$. The initial values of the intensity of hurricane events $\lambda(0)$ and the parameter μ_λ are estimated to be 5 and 0.048, respectively.

Pricing Errors

This section evaluates the pricing performance of CatEPuts under the alternative intensity processes: MMPP(2), mixed PP, EG, and PP. In order to compute the values of CatEPuts, we follow Jaimungal and Wang (2006) and assume the following parameter values: equity price of insurance company, $S = 25$; exercise price, $K = 80$; trigger level of losses ratio, $\tilde{L} = 5$; parameters of the interest rate model are: $r(0) = 2$ percent, $k = 0.3$, $\theta = 5$ percent, $\rho = -0.1$, $\sigma_r = 15$ percent; equity volatility, $\sigma_S = 0.2$; option term, $T = 4$; the percentage drop per loss, $\alpha = 0.01$. The infinite summation of hurricane events, m, is truncated at level $m = 200$ such that the respective cumulative Poisson probabilities are very close to 1.

From the data of hurricane events shown in Figure 1 and for the case of two independent PP (mixed PP), we obtain that the probability occurring at state 1 is $p_1 = 24/.48$ and then $p_2 = 24/.48$. On the other hand, for the Markov modulated PP with two states, MMPP(2), the transition probability at hurricane frequencies m from state $X(t) = 1$ to state $X(T) = 2$ is $P_{12}(m, T - t) = 1/.11$ and then $P_{11}(m, T - t) =$

TABLE 1
Pricing Errors

	MMPP(2)	Mixed PP	EG	PP
APE	8.465	13.440	14.455	16.635
AAE	31.572	54.413	50.942	55.715
ARPE	0.662	1.360	1.400	1.977
RMSE	33.753	47.299	55.731	58.423

Note: The parameter values for base valuation are $S = 25$, $K = 80$, $\tilde{L} = 5$, $r(0) = 2\%$, $k = 0.3$, $\theta = 5\%$, $\rho = -0.1$, $\sigma_S = 0.2$, $\alpha = 0.01$, $T = 4$, $m = 200$, $N = 48$, $\mu_y = 1.98$, and $\sigma_y = 7.56$. The intensity of single PP is 3.85. The parameters of the mixed PP are: $p_1 = 24/48$ and $p_2 = 24/48$. The parameters of the MMPP(2) are: $P_{12}(m, T - t) = 1/.11$, $P_{21}(m, T - t) = 1/24$, $\nu_1 = 0.098$, $\nu_2 = 0.045$, $\lambda_1 = 5.58$, and $\lambda_2 = 2.13$.

10/.11. Furthermore, the probability from state $X(t) = 2$ to state $X(T) = 1$ is set as $P_{21}(m, T - t) = 1/24$, and $P_{22}(m, T - t) = 23/24$. Through the transition probability, the transition rate of two states can be obtained: $\nu_1 = 0.098$, $\nu_2 = 0.045$, in order to capture the leaving length for intensity at a different state.

We obtain the real value (P_R) of CatEPuts using the frequency and loss data of hurricane events and the theoretical value (P_T) of CatEPuts under the MMPP(2), mixed PP, EG, and PP using parameter values generated from the hurricane data and other values provided above. For comparison purposes, we compute four measurements of pricing errors: average percentage error (APE), average absolute error (AAE), average relative percentage error (ARPE), and relative measure square error (RMSE):

$$\text{APE} = \frac{1}{E(P_R)} \sum_{n=1}^{N} \frac{|P_R - P_T|}{N}, \quad \text{AAE} = \sum_{n=1}^{N} \frac{|P_R - P_T|}{N},$$

$$\text{ARPE} = \frac{1}{N} \sum_{n=1}^{N} \frac{|P_R - P_T|}{P_R}, \quad \text{RMSE} = \sqrt{\sum_{n=1}^{N} \frac{(P_R - P_T)^2}{N}}.$$

where $E(P_R)$ is mean of the real CatEPut value and N is the total number of observations.

Table 1 gives an overview of these four measurements of pricing errors. It indicates that the pricing errors under the MMPP(2) are all smaller than those under the mixed PP, EG, and PP in all four measurements. Taking APE as an example, the improvement rate of pricing errors using the EG over the PP is only 13.10 percent [(16.635 − 14.455)/16.635]. The improvement rate rises to 19.20 percent [(16.635 − 13.440)/16.635] if we use the mixed PP. Furthermore, if the frequency of hurricane activity is assumed to follow the MMPP(2) according to the change in the AMO signal, the improvement rate increases further to 49.11 percent [(16.635 − 8.465)/16.635]. Therefore, when hurricane activities are the underlying CAT events, our results show

TABLE 2
Scenario Analysis of Measurement Errors

True Hurricane Frequency Predicting Model	Scenario Analysis I MMPP-AMO Signal EG-Global Warming	Scenario Analysis II EG-Global Warming MMPP–AMO Signal
APE	6.458	2.721
AAE	19.271	6.031
ARPE	0.728	0.091
RMSE	21.982	7.832

Note: This table describes the four measurement errors resulting from the misdiagnosis of the true driver of climatic pattern. Based on scenario analysis I, we use the EG model, assuming the global climate change, to predict the pattern for the frequency of hurricanes, whereas the true frequency of hurricanes follows the MMPP driven by the AMO signal. On the other hand, scenario analysis II investigates the opposite case of scenario analysis I, that is, we use the MMPP to predict the pattern for the frequency of hurricanes, assuming the driver of climatic pattern is the AMO signal, whereas the true frequency of hurricanes follows the EG pattern driven by the global climate change. The parameter values for base valuation are $S = 25$, $K = 80$, $\tilde{L} = 5$, $r(0) = 2\%$, $k = 0.3$, $\theta = 5\%$, $\rho = -0.1$, $\sigma_S = 0.2$, $\alpha = 0.01$, $T = 4$, $m = 200$, $N = 48$, $\mu_y = 1.98$, and $\sigma_y = 7.56$.

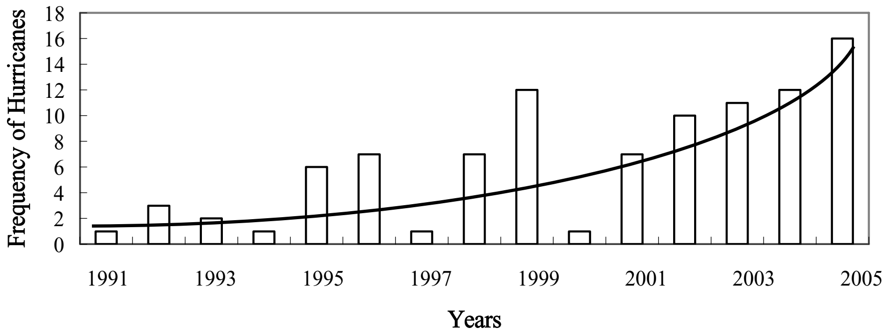
that the MMPP(2) can reduce the pricing errors by 30–66 percent depending on the measurement methods, and the MMPP dominates the PP in pricing the CatEPuts.

Scenario Analysis

We next conduct a scenario analysis to examine the measurement errors of CatEPuts from a misdiagnosis of the true driver of climatic patterns, and Table 2 presents the results. First, as shown in Figure 1, compared to EG, the MMPP(2) seems to capture more precisely the dynamic process of U.S. annual hurricane frequency between 1960 and 2007. Thus, based on the sample data of 1960–2007, scenario analysis I presents the measurement errors of CatEPuts from adopting a long-term upward pattern of the EG model in hurricane intensity due to the global climate change, whereas the true frequency of hurricane events follows the MMPP(2) driven by the AMO signal of multidecadal fluctuations. The measurement errors of CatEPuts between the real value (P_{R}) (using Equation (13)) and theoretical value (P_T) (using Equation (14)) can hence be obtained. The result of scenario analysis I indicates that using the EG model to describe hurricane intensity could produce a range of 0.728-21.982 in measurement errors according to the four measurement methods when the true driver of a hurricane event is the AMO signal.

Figure 3 shows, on the other hand, the annual frequency of U.S. hurricane events between 1991 and 2005, and the hurricanes' intensity seems to show an upward pattern, especially since 2001. Hence, scenario analysis II investigates the opposite case of scenario analysis I to show the measurement errors of CatEPuts using the MMPP(2) to predict the frequency of hurricane events, whereas the true frequency of hurricane events follows the EG pattern. Under the EG model, corresponding to global climate change, the initial values of the intensity of hurricane events $\lambda(0)$ and the parameter μ_λ are estimated to be 1 and 0.185, respectively. Under the MMPP(2),

FIGURE 3
Frequency of Hurricanes in the United States from 1991 to 2005



corresponding to the AMO signal, the average hurricane event intensity of state 1 (cool phase; 1991–1994) is 1.75, and the average hurricane intensity of the other state (warm phase; 1995–2005) is 8.18. Similarly, we can obtain the measurement errors of CatEPuts between the real value (P_R) (using Equation (14)) and theoretical value (P_T) (using Equation (13)). The result shows that the MMPP(2) model produces smaller measurement errors of 0.091–7.832 depending on the four measurement methods.

Based on the trend of hurricane frequency in a long-term period (1960–2007), the MMPP(2) outperforms the EG model. Furthermore, in a short-term period (1991–2005), the MMPP(2) also seems to outperform the EG model, owing to the smaller measurement errors of CatEPuts. Hence, we conclude that it is more appropriate to use the MMPP(2) than the EG model if the intensity of hurricane events is truly forced by the AMO signal. However, if the future climate is in the process of more rapid warming, caused in part by human activities, including emissions of greenhouse gases and aerosols, and changes in land use, then the hurricane frequency could be in a directional long-term upward trend and instead of the MMPP(2), the EG model may be appropriate to describe the process of hurricane activities.

Sensitivity Analysis

This section presents a sensitivity analysis for CatEPut prices under alternative parameter values using the MMPP(2) and the PP. Table 3 reports the CatEPut prices under the PP with alternative values of hurricane event parameters. We note that CatEPut prices increase with hurricane intensity as well as the mean and variance of hurricane losses. It also shows that hurricane intensity dominates the mean and variance of the hurricane losses in determining the CatEPut prices under the PP. This implies that it is more critical to model hurricane intensity than identify the loss distribution of hurricane events when pricing the CatEPuts.

Table 4 presents how transition probability affects CatEPut prices. When ν_1 increases, then the transition probability of leaving state 1 to state 2 is higher, hence decreasing the CatEPut prices due to the lower intensity at state 2. Similarly, if ν_2 increases, then the transition probability of leaving state 2 to state 1 is higher, and hence the higher intensity at state 1 increases the CatEPut prices.

TABLE 3
CatEPut Prices Under the PP

		λ			
σ_y	μ_y	2	4	6	8
6	1	5.082	9.289	13.914	19.640
	3	7.053	12.820	17.472	24.428
8	1	5.502	10.494	14.414	22.243
	3	7.923	13.987	20.688	27.932

Note: Other parameter values are: $S = 25$, $K = 80$, $\tilde{L} = 5$, $r(0) = 2\%$, $k = 0.3$, $\theta = 5\%$, $\rho = -0.1$, $\sigma_S = 0.2$, $\alpha = 0.01$, $T = 4$, and $m = 200$.

TABLE 4
CatEPut Prices Under the MMPP(2)

	(λ_1, λ_2)			
Model	(4,1)	(6,1)	(4,3)	(6,3)
$(\nu_1, \nu_2) = (0.09{,}0.04)$	11.720	18.052	13.530	19.212
$(\nu_1, \nu_2) = (0.1{,}0.04)$	7.189	10.494	8.999	11.654
$(\nu_1, \nu_2) = (0.095{,}0.035)$	8.394	11.994	10.204	13.154
$(\nu_1, \nu_2) = (0.095{,}0.045)$	12.887	18.668	14.697	19.828

Note: Other parameter values are: $S = 25$, $K = 80$, $\tilde{L} = 5$, $r(0) = 2\%$, $k = 0.3$, $\theta = 5\%$, $\rho = -0.1$, $\sigma_S = 0.2$, $\alpha = 0.01$, $T = 4$, $m = 200$, $\mu_y = 1.98$, and $\sigma_y = 7.56$.

TABLE 5
CatEPut Prices and Transition Rates

$(\nu_1 \to 0, \nu_2 \to \infty)$			
(ν_1, ν_2)	(0.1,1)	(0.01,10)	(0.001,100)
MMPP(2)	14.5635	14.5639	14.5642
PP ($\lambda_1 = 6$)	14.5642	14.5642	14.5642
$(\nu_1 \to \infty, \nu_2 \to 0)$			
(ν_1, ν_2)	(1,0.1)	(10,0.01)	(100,0.001)
MMPP(2)	9.5960	9.5951	9.5943
PP ($\lambda_2 = 3$)	9.5943	9.5943	9.5943

Note: Other parameter values are: $S = 25$, $K = 80$, $\tilde{L} = 5$, $r(0) = 2\%$, $k = 0.3$, $\theta = 5\%$, $\rho = -0.1$, $\sigma_S = 0.2$, $\alpha = 0.01$, $T = 4$, $m = 200$, $\mu_y = 1.98$, $\sigma_y = 7.56$, $\lambda_1 = 5.58$, and $\lambda_2 = 2.13$.

Table 5 shows that, when the transition rate of state 1 converges to zero and the transition rate of state 2 converges to infinity, CatEPut prices are the same under the MMPP(2) and the PP with $\lambda_1 = 6$. Similarly, when the transition rate of state 1 converges to infinity and the transition rate of state 2 converges to zero, prices under the MMPP(2) are the same as the PP with $\lambda_2 = 3$.

CONCLUSIONS

The data of U.S. hurricanes events from 1960 to 2007 show that warm phases in the AMO signal are above the long-term average hurricane activity in the Atlantic, whereas cool phases in the AMO are below the long-term average hurricane activity. Therefore, this study applies the MMPP to model the intensity process of hurricane events corresponding to the change of the AMO signal. This study derives the generalized pricing formula for CatEPuts and demonstrates that the pricing formulas of Cox, Fairchild, and Pedersen (2004) and Jaimungal and Wang (2006) are special cases of the generalized pricing formula.

This study goes further to investigate the pricing performance of CatEPuts under the MMPP(2), mixed PP, EG, and PP in our sample data. The results show that the MMPP(2) generates lower pricing errors than the PP and pricing errors can be reduced by 30–66 percent depending on the measurement methods. This indicates that when the change in the AMO signal has significant different hurricane intensity rates, such as in our U.S. hurricane events sample, the MMPP(2) dominates the PP in pricing CatEPuts. Furthermore, the measurement errors that result from a misdiagnosis of the true driver of different climatic patterns indicate that the MMPP (2) is superior to the EG when the intensity of hurricane events is driven by the AMO signal. However, if the climate in the future is warming rapidly, caused in part by human activities (e.g., emission of greenhouse gases), then the EG may be appropriate to describe the process of hurricane activities. Sensitivity analysis also demonstrates that intensity plays a more important role than the mean and variance of the hurricane losses in determining the CatEPut prices. This provides additional evidence that it is more critical to model intensity than to identify the loss distribution of hurricane events when pricing CatEPuts. Finally, this study presents how transition rates affect the transition probabilities and CatEPut prices.

This study has several possible extensions and potential improvements. First, due to the discrete nature of jump risk, the market is incomplete and conventional riskless hedging is difficult to obtain. Thus, the issue of riskless hedging with jump risk remains an important challenge. Second, the model developed herein can be applied to other structured risk management products or alternative CAT risk transfer mechanisms, such as CAT bonds, contingent surplus notes, and hybrid insurance/financial products. Third, it is interesting to examine how CatEPuts can affect the default risk of reinsurance contracts and reinsures.

APPENDIX A

This appendix shows that $-\alpha \sum_{n=1}^{\Phi(t)} Y_n + \Lambda\kappa_1 t$ is a martingale in t. First, we know that

$$E\left\{\exp\left(-\alpha \sum_{n=1}^{\Phi(t)} Y_n\right)\right\} = P^P(\Phi_t = 0) + \sum_{m=1}^{\infty} E\left(\exp\left(-\alpha \sum_{n=1}^{m} Y_n\right)\middle| \Phi(t) = m\right) P^P(\Phi_t = m).$$

Because $(Y_1, Y_2, \ldots, Y_m)$ are independent identically distribution random variables, we have:

$$E\left\{\exp\left(-\alpha\sum_{n=1}^{\Phi(t)}Y_n\right)\right\}=\sum_{m=0}^{\infty}(E(\exp(-\alpha Y_n)))^m P(m,t).$$

Furthermore, we use the equation $P^*(z,t)=\sum_{m=0}^{\infty}P(m,t)\,z^m$ and its unique solution $P^*(z,t)=\exp[\,\Psi-(1-z)\,\Lambda]\,t$. Letting $Q(m,t)=(E(\exp(-\alpha Y_n)))^m P(m,t)\exp(\Lambda\kappa_1 t)$, we have

$$\begin{aligned}Q^*(z,t)&=\sum_{m=0}^{\infty}z^m(E(\exp(-\alpha Y_n)))^m P(m,t)\exp(\Lambda\kappa_1 t)\\&=\sum_{n=0}^{\infty}(z(1-\kappa_1))^m\,P(m,t)\,\exp(\Lambda\kappa_1 t)=\exp[\,\Psi-(1-z)(1-\kappa_1)\,\Lambda]\,t.\end{aligned}$$

Therefore, $-\alpha\sum_{n=1}^{\Phi(t)}Y_n+\Lambda\kappa_1\,t$ is a martingale in t.

APPENDIX B

This appendix illustrates the proof for Propositions 1 and 2, which describe the Radon–Nikodym processof transition probability and Radon–Nikodym processof a CAT loss, respectively.

First, we investigate the transition probability, $Q(m,t)$, for $0\le z\le 1$, and define

$$Q^*(z,t)=\sum_{m=0}^{\infty}Q(m,t)\,z^m, \tag{B1}$$

where $Q(m,0):=(1_{\{m=0\}}D_{ij})$ and $D_{ij}=1$, if $i=j$; and $=0$, otherwise.

By using Kolmogorov's forward equation, the derivative of $Q(m,t)$ becomes

$$\frac{d}{dt}Q(m,t)=(\Psi-\Lambda)Q(m,t)+1_{\{m\ge 1\}}\Lambda\;Q(m-1,\mathrm{t}),$$

where Λ denotes an $I\times I$ diagonal matrix with diagonal elements λ_i. Thus, its unique solution is

$$Q^*(z,t)=\exp[\,\Psi-(1-z)\,\Lambda]\,t, \tag{B2}$$

where $\Psi:=(\Psi(i,j))$ and $e^A:=\sum_{n=0}^{\infty}\frac{A^n}{n!}$, for any $(I\times I)$ matrix A.

By using Laplace inverse transform (B1) and the unique solution (B2), we obtain the joint distribution of X and $\Phi(t)$ at time t when

$$Q(m,t)=\frac{\partial^m}{\partial z^m m!}Q^*(z,t)\,|_{\,z=0}.$$

Let $Q'(m,t) = (1 - \kappa_1)^m \exp(\Lambda\kappa_1 t) Q(m,t)$, and thus

$$\begin{aligned} Q'^*(z,t) &= \sum_{m=0}^{\infty} Q'(m,t)\, z^m \\ &= \sum_{m=0}^{\infty} (1-\kappa_1)^m \exp(\Lambda\,\kappa_1\, t) Q(m,t)\, z^m \\ &= \sum_{m=0}^{\infty} (z(1-\kappa_1))^m \exp(\Lambda\,\kappa_1\, t) Q(m,t). \end{aligned}$$

The unique solution of $Q'^*(z,t)$ is

$$\begin{aligned} Q'^*(z,t) &= \exp[\Psi - (1 - (z\,(1-\kappa_1))\Lambda]\, t \times \exp[\,\Lambda\kappa_1\, t] \\ &= \exp[\Psi - ((1-z)\,(1-\kappa_1)\Lambda)]t. \end{aligned}$$

Therefore, the Radon–Nikodym derivative of the transition probability can be considered as

$$\frac{d\,Q'(m,t)}{d\,Q(m,t)} = (1-\kappa_1)^m \exp(\,\Lambda\kappa_1 t). \tag{B3}$$

Finally, we look into the size of the CAT loss, where $(Y_1, Y_2, \ldots, Y_m)$ are independent identically distribution random variables. Hence, the Radon–Nikodym derivative of the CAT loss can be

$$f_Y^{Q'}(y_1), f_Y^{Q'}(y_2), \ldots, f_Y^{Q'}(y_m) = \left[\frac{e^{-\alpha y_1} f_Y(y_1)}{(1-\kappa_1)} \frac{e^{-\alpha y_2} f_Y(y_2)}{(1-\kappa_1)} \cdots \frac{e^{-\alpha y_m} f_Y(y_m)}{(1-\kappa_1)}\right].$$

Thus, under new risk-neutral measure Q', the new density function of each specific CAT loss is

$$f_Y^{Q'}(y) = \frac{e^{-\alpha y} f_Y(y)}{(1-\kappa_1)}.$$

APPENDIX C

This appendix provides the proof for the derivation of the formula of CatEPut in Theorem 1. Let $V(t;t_0)$ represent the value of the option at time t, which was signed at time $t_0 < t$ and matures at time T. We have the following equation:

$$V(t;t_0) = E^{Q^T}[1_{\{L(T) > L + L(t_0)\}} B(t,T)(K - S(T))^+ \mid F_t]. \tag{C1}$$

Using the law of expected iteration, Equation (C1) can be rewritten as

$$E^{Q^T}[1_{\{L(T)>L+L(t_0)\}}\, E^{Q^T}(B(t,T)(K-S(T))^+ \mid L(T)) \mid F_t] = A - B,$$

where

$$\begin{aligned}
\mathrm{A} &= E^{Q^T}[1_{\{L(T)>L+L(t_0)\}} E^{Q^T}(K\, B(t,T)\, 1_{\{S(T)<K\}} \mid L(T)) \mid F_t] \\
&= E^{Q^T}\Bigg\{ \underset{\{L(T)>L+L(t_0)\}}{1} K\, B(t,T) P^{Q^T}\left(\frac{S(t)}{B(t,T)} \exp\left[-\frac{1}{2}\tilde{\sigma}^2(t,T) + \tilde{\sigma}(t,T)\tilde{W} \right.\right. \\
&\qquad \left.\left. -\alpha(L(T)-L(t)) + \Lambda\,\kappa_1(T-t)\frac{1}{2} \right] < K \right) \Bigg|\, F_t \Bigg\} \\
&= \sum_{m=1}^{\infty} Q(m, T-t)\; E^{Q^T}\{[\underset{\{L(T)>L+L(t_0)\}}{1} K\, B(t,T) \\
&\qquad \times \phi\big(-d_{2m}^{MM}(L(T)-L(t))\big) \big|\, \Phi(T-t)=m] \big|\, F_t\} \\
&= \sum_{m=1}^{\infty} Q(m, T-t)\; E^{Q^T}\left\{ \left[\underset{\{y^m>\tilde{L}\}}{1} K\, B(t,T)\, \phi\big(-d_{2m}^{MM}(y^m)\big) \big|\, \Phi(T-t)=m \right] \Bigg|\, F_t \right\},
\end{aligned} \tag{C2}$$

where

$$d_{2m}^{MM} = \frac{\ln\left(\dfrac{S(t)}{K\,B(t,T)}\right) - \frac{1}{2}\tilde{\sigma}^2(t,T) - \alpha y^m + \Lambda\,\kappa_1(T-t)}{\tilde{\sigma}(t,T)}, \quad \kappa_1 = E[1-\exp(-\alpha Y)].$$

$$\begin{aligned}
B &= E^{Q^T}\left[1_{\{L(T)>L+L(t_0)\}} E^{Q^T}\big(S(T)\, B(t,T)\, 1_{\{S(T)<K\}} \mid L(T)\big)\big|\, F_t\right] \\
&= E^{Q^T}\Bigg[1_{\{L(T)>L+L(t_0)\}} S(t) \exp\Bigg\{ -\frac{1}{2}\tilde{\sigma}^2(t,T) + \tilde{\sigma}(t,T)\tilde{W} \\
&\qquad -\alpha(L(T)-L(t)) + \Lambda\,\kappa_1(T-t) \Bigg\} 1_{\{S(T)<K\}} \Bigg|\, F_t \Bigg].
\end{aligned} \tag{C3}$$

Denote the Radon–Nikodym process for Brownian motion by the following formula

$$\left(\frac{dR}{dQ^T}\right)_t = \exp\left\{ \int_0^t \tilde{\sigma}(t,T)\, d\tilde{W}(u) - \frac{1}{2}\int_0^t \tilde{\sigma}^2(t,T)\, du \right\}. \tag{C4}$$

Hence, Equation (C3) can be rewritten as

$$\sum_{m=1}^{\infty} Q(m,T-t)\,E^{Q^T}\{1_{\{y^m>\tilde{L}\}}S(t)\exp[-\alpha y^m+\Lambda\kappa_1(T-t)]$$
$$\times\,\phi\big(-d_{1m}^{MM}(y^m)\big)\mid\Phi(T-t)=m\mid F_t\}$$
$$d_{1m}^{MM}=\frac{\ln\left(\dfrac{S(t)}{KB(t,T)}\right)+\dfrac{1}{2}\tilde{\sigma}^2(t,T)-\alpha y^m+\Lambda\,\kappa_1(T-t)}{\tilde{\sigma}(t,T)},\quad i=1,2,\ldots,I,$$
$$L(T)-L(t)=y^m,\quad \tilde{L}=L+L(t_0)-L(t).$$

APPENDIX D

This appendix provides a detailed proof of Corollary 1.

$$\begin{aligned}&E^{Q^T}\big[1_{\{L(T)>L+L(\mathrm{t}_0)\}}KB(t,T)1_{\{S(T)<K\}}\mid F_t\big]\\&=\sum_{m=1}^{\infty}Q(m,T-t)KB(t,T)P^{Q^T}\left\{\sum_{n=1}^{m}Y_n>\tilde{L},\tilde{\sigma}(t,T)\tilde{W}-\alpha\sum_{n=1}^{m}Y_n<\frac{1}{2}\tilde{\sigma}^2(t,T)\right.\\&\quad\left.-\Lambda\kappa_1(T-t)+\ln\left(\frac{KB(t)}{S(t)}\right)\mid\Phi(T-t)=m\mid F_t\right\}.\end{aligned}\tag{D1}$$

Because $Y_n\sim\exp(\eta)$, then $\sum_{n=1}^{m}Y_n\sim\Gamma(m,\frac{1}{\eta})$ with mean $\mu_y=\frac{m}{\eta}$, and variance $\sigma_y^2=\frac{m}{\eta^2}$. In addition, $\tilde{W}\overset{d}{\sim}N(0,1)$. Let $J_W=\tilde{\sigma}(t,T)\tilde{W}-\alpha\sum_{n=1}^{m}Y_n$, $J_Y=\sum_{n=1}^{m}Y_n$, and Equation (D1) can be rewritten as

$$\begin{aligned}&\sum_{m=1}^{\infty}Q(m,T-t)\,KB(t,T)\int_{-\infty}^{a_1}\int_{\tilde{L}}^{\infty}\frac{(\eta)^m\exp[-\eta v](v)^{m-1}}{\Gamma(m)}\frac{1}{\sqrt{2\pi\tilde{\sigma}^2(t,T)}}\\&\quad\times\exp\left[-\frac{(u+\alpha v)^2}{2\tilde{\sigma}^2(t,T)}\right]dv\,du\\&=\sum_{m=1}^{\infty}Q(m,T-t)\;KB(t,T)\int_{-\infty}^{a_1}\int_{\tilde{L}}^{\infty}\frac{\exp\big[-(\mu_y/\sigma_y^2)v\big]v^{(\mu_y^2/\sigma_y^2)-1}}{\Gamma\big(\mu_y^2/\sigma_y^2\big)\big(\mu_y/\sigma_y^2\big)^{(\sigma_y^2/\mu_y^2)}}\\&\quad\times\frac{1}{\sqrt{2\pi\tilde{\sigma}^2(t,T)}}\exp\left[-\frac{(u+\alpha v)^2}{2\tilde{\sigma}^2(t,T)}\right]dv\,du,\end{aligned}$$

where

$$a_1 = \frac{1}{2}\tilde{\sigma}^2(t,T) - \Lambda\kappa_1(T-t) + \ln(KB(t,T)/S(t)), \quad i = 1,2,\ldots,I,$$
$$\kappa_1 = \int_0^\infty (1-\exp(-\alpha y))\eta\exp(-\eta)y\,dy = \frac{\alpha}{\eta+\alpha}.$$

According to the change measure of transition probability matrix, $\frac{dQ'(m,t)}{dQ(m,t)} = (1-\kappa_1)^m \exp(\Lambda\kappa_1 t)$, we obtain $Q(m,t) = Q'(m,T-t)\frac{\exp[\Lambda\kappa_1(T-t)]}{(1-\kappa_1)^m}$ and hence

$$\begin{aligned}
&E^{Q^T}[1_{\{L(T)>L+L(t_0)\}}\,KB(t,T)\,1_{\{S(T)<K\}} \mid F_t] \\
&= \sum_{m=1}^{\infty} Q'(m,T-t)\,KB(t,T)\frac{\exp[\Lambda\kappa_1(T-t)]}{(1-\kappa_1)^m}\int_{-\infty}^{a_1}\int_{\tilde{L}}^{\infty}\frac{\exp\left[-(\mu_y/\sigma_y^2)v\right]v^{(\mu_y^2/\sigma_y^2)-1}}{\Gamma(\mu_y^2/\sigma_y^2)(\mu_y/\sigma_y^2)^{(\sigma_y^2/\mu_y^2)}} \\
&\quad\times\frac{1}{\sqrt{2\pi\tilde{\sigma}^2(t,T)}}\exp\left[-\frac{(u+\alpha v)^2}{2\tilde{\sigma}^2(t,T)}\right]dv\,du.
\end{aligned}$$

Furthermore,

$$\begin{aligned}
&E^{Q^T}[1_{\{L(T)>L+L(t_0)\}}S(T)\,1_{\{S(T)<K\}} \mid F_t] \\
&= \sum_{m=1}^{\infty} Q(m,T-t)\,E^{Q^T}\Bigg[1_{\{L(T)>L+L(t_0)\}}S(t)\exp\Bigg\{-\frac{1}{2}\tilde{\sigma}^2(t,T) + \tilde{\sigma}(t,T)\,\tilde{W} \\
&\quad -\alpha\sum_{n=1}^{m}Y_n + \Lambda\kappa_1(T-t)\Bigg\}1_{\{S(T)<K\}} \mid \Phi(T-t)=m \mid F_t\Bigg].
\end{aligned} \tag{D2}$$

In addition, due to $f_Y^{Q'}(y) = \frac{e^{-\alpha y}f_Y(y)}{(1-\kappa_1)}$, the new exponential density under Q' becomes

$$f_Y^{'}(y) = (\eta+\alpha)\exp[-(\eta+\alpha)y].$$

Hence, we have

$$\begin{aligned}
&S(t)\sum_{m=1}^{\infty} Q'(m,T-t)\,P^{Q}\Bigg\{\sum_{n=1}^{m}Y_n^{Q'} > \tilde{L}, \tilde{\sigma}(t,T)\tilde{W} - \alpha\sum_{n=1}^{m}Y_n^{Q'} < -\frac{1}{2}\tilde{\sigma}^2(t,T) \\
&\qquad -\Lambda\kappa_2(T-t) + \ln\left(\frac{KB(t)}{S(t)}\right) \mid \Phi(T-t)=m \mid F_t\Bigg\}.
\end{aligned} \tag{D3}$$

Since $Y^{Q'} \sim \exp(\eta+\alpha)$, then $\sum_{n=1}^{m} Y_n^{Q'} \sim \Gamma(m, \frac{1}{\eta+\alpha})$, and Equation (D3) becomes

$$S(t)\sum_{m=1}^{\infty} Q'(m,T-t)\int_{-\infty}^{a_2}\int_{\tilde{L}}^{\infty} \frac{(\eta+\alpha)^m \exp[-(\eta+\alpha)v](v)^{m-1}}{\Gamma(m)}$$

$$\times \frac{1}{\sqrt{2\pi\tilde{\sigma}^2(t,T)}}\exp\left[-\frac{(u+\alpha v)^2}{2\tilde{\sigma}^2(t,T)}\right] dv\, du$$

$$= S(t)\sum_{m=1}^{\infty} Q'(m,T-t)\int_{-\infty}^{a_2}\int_{\tilde{L}}^{\infty} \frac{\exp\left[-(\mu_y/\sigma_y^2+\alpha)v\right] v^{(\mu_y^2/\sigma_y^2)-1}}{\Gamma(\mu_y^2/\sigma_y^2)(\mu_y/\sigma_y^2+\alpha)^{(\sigma_y^2/\mu_y^2)}}$$

$$\times \frac{1}{\sqrt{2\pi\tilde{\sigma}^2(t,T)}}\exp\left[-\frac{(u+\alpha v)^2}{2\tilde{\sigma}^2(t,T)}\right] dv\, du,$$

where

$$a_2 = -\frac{1}{2}\tilde{\sigma}^2(t,T) - \Lambda\kappa_2(T-t) + \ln(KB(t,T)/S(t)), \quad i = 1,2,\ldots,I,$$

$$\kappa_2 = \int_0^{\infty} (1-\exp(-\alpha y))(\eta+\alpha)\exp(-(\eta+\alpha))y\, dy = \frac{\alpha}{\eta+2\alpha}.$$

APPENDIX E: ALGORITHM FOR THE SEQUENCE OF TRANSITION PROBABILITY

When calculating the CatEPut valuation under the MMPP, we need to evaluate the transition probability (Equation (8)). Abate and Whitt (1992) present a simple algorithm for numerically inverting probability generating functions based on the Fourier series method and obtain a simple computation with a convenient error bound from the discrete Poisson summation formula. A sequence of real numbers $\{P(m,T-t), m \geq 0\}$ with $P(m,T-t) \leq 1$ for all m uses the generating function, $P^*(z,t) = \sum_{m=0}^{\infty} P(m,t)\, z^m$, where z is a complex number. We assume that $P^*(z,T-t)$ can be evaluated for any given z, and our object is to obtain an approximation (with predetermined error bound) for $P(m,T-t)$ as a function of $P^*(z_1,T-t),\ldots,$ $P^*(z_m,T-t)$ for many finite complex numbers $z_1,\ldots,z_m$. Abate and Whitt provide a simple algorithm with an error bound. Let $i=\sqrt{-1}$ and let Re (z) be the real part of z. For $0<\varepsilon<1$ and $m \geq 1$:

$$|\, P(m,T-t) - \tilde{P}(m,T-t)\,| \leq \frac{\varepsilon^{2m}}{1-\varepsilon^{2m}},$$

where

$$\tilde{P}(m,T-t) = \frac{1}{2m\varepsilon^m}\Bigg[P^*(\varepsilon,T-t) + (-1)^m P^*(-\varepsilon,T-t)$$

$$+2\sum_{k=1}^{m-1}(-1)^k \mathrm{Re}\left(P^*\left(\varepsilon\exp\left(\frac{\pi k i}{m}\right),T-t\right)\right)\Bigg].$$

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