An Agent-Based Skeleton of the Network-Based Trust Games

Shu-Heng Chen
Department of Economics
National Chengchi University
Taipei, Taiwan 11605
chen.shuheng@gmail.com

Tong Zhang
Research Institute of Economics and Management
Southwestern University of Finance and Economics
Chengdu, China
tongzhang@swufe.edu.cn

Abstract

This paper presents an agent-based model of the network-based trust game by combing two essential ingredients, namely new economic geography and cooperative games. Through simulating this model, we would like to show the co-evolution of trusts, networks, economic growth and income distribution. Variations of the proposed design allow us to examine the role of social trust in economic growth and can allow us to further explore the effect of culture and personality in the trust-based growth theory.

1. MOTIVATION AND BACKGROUND

It has been well-acknowledged by economists that social trust, as well as social capital and social networks, is considered as part of nature and a cause of the wealth of nations. The empirical research conducted so far largely supports the hypothesis that *trust has a positive effect on economic growth*, even if its relative importance compared to other factors is an issue still debated (Knack and Keefer, 1997; Zak and Knack, 2001; Beugelsdijk, de Groot, and van Schaik, 2004; Dearmona and Grier, 2009; Algan and Cahuc, 2010). Nonetheless, trust is not a simple static concept. Neither is it an isolated component, which can be studied by disentangling it from other social entities. On the contrary, it is constantly evolving with social embeddedness and complexities, acting and reacting upon it. Therefore, a formal model to harness the essences of trust dynamics can be useful, whereas a simple analytical model may not be sufficient enough to accommodate its rich surroundings, such as social networks, social norms, mechanism designs and cultures.

1.1. Trust Games

Over the last fifteen years, economists' pursuits for the very fundamental understanding of trust in an economic context have been carried out in the form known as the *trust game experiments* (Berg, Dickhaut, and McCabe, 1995). In the *trust game* or *investment game*, a first mover, the Truster, decides how much of a given endowment to send to a second mover, the Trustee. Any amount sent is multiplied by a positive integer before it reaches the second-mover. The Trustee, then decides how much to send back. In theory, self-interested Trustees will keep everything and self-interested Trusters who anticipate this will transfer nothing in the first place.

Advancements in the study of trust game experiments have indicated that *trust is not a simple idea even in such a simple purified situation*. It is a complex realization of many other elements, which were generally well observed in other behavioral experiments, such as *reciprocity*, *altruism*, *other-regarding preference*, *risk attitude*, *belief*, and *gender*. It can involve emotion elements, but it can also be based on deliberate calculations. Probably the most exciting scientific progresses which enable us to see that there are indeed several different modules governing trust behavior in the trust games is the neuroscientific study of trust, dubbed *neurotrust game* (Zak and Knack, 2001; Zak and Kugler, 2011).

The current experimental economics do shed some light on a number of fundamental elements of trust and trustworthiness. Yet, those experiments are normally conducted using a protocol of one-shot two-person games, within which both *dynamics (repeated games)* and *social networks (multi-person games)* are left out. Nevertheless, if we cast ourselves into the research line of inquiring the relation between trust and prosperity as in the above-mentioned literature, then these two, trust dynamics and social networks, seem to be essential. In addition, the productivity (investment return) characterized by the multiplier in the conventional trust games is normally exogenously given, but the multiplier, regardless of being three or higher, would be less arbitrarily if it can be endogenously related to the scale of network production. Therefore, leaving the social or economic networks aside cannot capture the intertwined relation between trust and the magnitude of the multiplier.

1.2. Network Games

The seminal work by Jackson and Wolinsky (1996) and Bala and Goyal (2000), who pioneered a game-theoretic approach to study the formation of social and economic networks, led to a surging interest in *network game experiments*. Through these experiments, we can examine whether some specific kinds of networks will be formed as the network game theory suggests and the formation processes. These networked "societies", through their formation and growing, can

have implications for wealth distribution, measured by the total payoffs of the society; nevertheless, wealth distribution or the allocation rule is normally treated exogenously. Hence, in this setup, it leaves no role for social trust to play in the wealth distribution process. As a result, most of these network game experiments has neither to do with trust nor wealth creation. If one, however, can replace the exogenously given allocation rule with endogenously generated trust-based distribution, then a basic framework for studying trust and wealth creation, through social network (social capital), using human-subject experiments or artificial agent simulations seems to be feasible.

1.3. Network-Based Trust Game

In this paper, we will then extend the current experimental research on trust games into a dynamic (repeated) protocol with an embedded social network which is co-evolving with the trust dynamics. We call this novel design the *network-based trust game*. From there, we will study the relation among trust formation, social network, and wealth creation, using agent-based simulation. This novel trust game has the potential to bridge different kinds of studies on trust, from experimental economics, social networks, social capital, to economic growth, and shed light on the formation of a good society in terms of its achieved efficiency and equality.

Our design on the production function, particularly, the investment multiplier, implies a fully-connected social network as the wealthiest state (highest social efficiency) (Section 2.1.). Hence, it is interesting to know whether this fully-connected network will emerge after subjects constantly changing their investment portfolios and reciprocities. Equally interesting is the wealth distribution associated with the emergent social network.

2. NETWORK-BASED TRUST GAMES
2.1. The Model Sketch

The network-based trust game is a hybridization of both the repeated trust games and the network games. It is outlined as follows.

N agents engage in a repeated trust game with T repetitions (T rounds). In each round, each agent has to make a two-stage decisions.

1. (*Partner Selection*) In the first stage, which is called the *network formation stage* or the *partner selection stage*, the subject i ($i = 1, 2, ..., N$), acting as a *trustor*, has to decide with whom he would like to choose to be his *trustees*, say j ($j \neq i$). Denote this decision by δ_{ij}.

$$\delta_{ij} = \begin{cases} 1, & if\ i\ choose\ j, \\ 0, & otherwise. \end{cases} \quad (1)$$

A link between i and j is said to be formed only if either $\delta_{ij} = 1$ or $\delta_{ji} = 1$. That is all links are *directed*.

2. Based on the first-stage decisions, a network topology is determined by a set of links, g,

$$g = \{\bar{ij} : \delta_{ij} = 1\ or\ \delta_{ji} = 1, 1 \leq i < j \leq N\}. \quad (2)$$

The *neighbors* of agent i, denoted by N_i, is defined as follows.

$$N_i = \{j : \delta_{ij} = 1\ or\ \delta_{ji} = 1, j = 1, 2, ..., N \\ and\ j \neq i\}. \quad (3)$$

3. In the second stage, a standard trust game is implemented on each pair connecting by a link $\bar{ij}$. This will separate agent i's neighbors into two sets: the trustees of i (to whom agent i will send money, $\delta_{ij} = 1$) and the trustors of i (from whom agent i will receive money, $\delta_{ji} = 1$), denoted as $N_{i,S}$ and $N_{i,R}$, respectively. Obviously, $N_i = N_{i,S} \cup N_{i,R}$, but $N_{i,S} \cap N_{i,R}$ may be nonempty.

4. (*Investment*) Then, for all those links that the agent i plays the role as an trustor ($\delta_{ij} = 1$), he/she has to make a decision on the investment on each of this link, $k_{i,j}$ ($j \in N_{i,S}$) constrained by his/her endowment or wealth.

$$\sum_{j \in N_{i,S}} k_{i,j} \leq K_i, \quad (4)$$

and send the money. In the meantime, agent i's trustors js ($j \in N_{i,R}$) also make decisions on their investment on i and send the money. As to the investment decision per se or how $k_{i,j}$ is determined will be discussed in Section 2.2..

5. All the investment $k_{i,j}$ will then be associated with a multiplier $\tau_{i,j}$, which depends on the network topology. This is the major novelty and the key contribution of the paper. The idea is to fully acknowledge the significance of the network size or the scale effect on productivity. A similar idea has been found in many places in the economic literature, such as the knowledge externality or spillover in the endogenous growth theory, the agglomeration or scale effect in economic geography, etc. There are several possibilities to operate this scale effect in our model. For an illustration, one exemplar case given here is *the largest fully connected subnetworks*, $g^*_{\bar{ij}}$, that $\bar{ij}$ or, equivalently, $\bar{ji}$ belongs to.

$$g^*_{\bar{ij}} = \{(i', j') : \delta_{i'j'} + \delta_{j'i'} \neq 0,\ if \\ \delta_{uv} + \delta_{vu} \neq 0, u = \{i, j\}, v = \{i', j'\}\} \quad (5)$$

By Equation (5), we are searching for the maximally fully connected subnetworks within which the business relation between i and j is embedded. Intuitively, if the business between i and j is run within a well-connected society instead of a fragmentally isolated small group, then we expect a larger scale effect.

6. Consequently, we assume that the multiplier $\tau_{i,j}$ is monotonically increasing in the size (the number of nodes) of the fully connected network implied by the set of links $g^*_{\bar{ij}}$. Denote the latter by $N_{\bar{ij}}$. Then one example of the scale-based multiplier is given as a follows.

$$\tau_{i,j} = 1 + \alpha(\frac{N_{\bar{ij}}}{N}), \qquad (6)$$

where α is a constant. By setting $\alpha = 2$ and removing the scale effect characterized by $N_{\bar{ij}}/N$, we then have the usual setting of having a multiplier of three, frequently used in experimental economics. The production function and the total return received by the trustee is

$$y_{i,j} = \tau_{i,j}k_{i,j}. \qquad (7)$$

By Equation (6), $\tau_{i,j} = \tau_{j,i}$; hence, $y_{j,i} = \tau_{j,i}k_{j,i}$.

7. (*Kickbacks*) Then, as the usual second stage of the trust game, agent *i* have to make his/her decision on the share of the yield $y_{j,i}$ ($j \in N_{i,R}$) that he/she would like to return to his/her trusters *j*. We denote his/her own reserve by $y^i_{j,i}$ and hence his/her trustworthiness by $y^j_{j,i}$.

$$y^i_{j,i} + y^j_{j,i} = y_{j,i} \qquad (8)$$

In the meantime, he/she also receives money from his/her own trustees, $y^i_{i,j}$ ($j \in N_{i,S}$). The details of the decision on kickbacks will be fully developed in Section 2.2..

8. This finish one round of the network-based trust game. An end-result is the net income earned by agent *i*:

$$K_i(t+1) = K_i(t) +$$
$$\sum_{j\in N_{i,S}} (y^i_{i,j}(t) - k_{i,j}(t)) + \sum_{j\in N_{i,R}} y^i_{j,i}(t). \qquad (9)$$

9. We then back to step (1). Each subject renews the network formation decisions, and they together form a (possibly) new network topology. The trust game, step (3) to (8) is then played with this renewed social network. It will be interesting to study what are the additional links that these subjects add or delete.

10. The cycle from step (1) to (8), as described in (9), will continue until T is achieved.

2.2. Behavioral Aspects of the Model

Section 2.1. gives a general description of the network-based trust game model. However, unlike most studies on the trust game, which is done through human-subject experiments, this study is based on agent-based simulation. Hence, we need a separate section to address the behavioral aspects of the model. That is we need to formulate the possible interesting behaviors of artificial agents in this model, which, of course, can be further verified using the lab experiments. Based on the description in Section 2.1., there are three major behavioral aspects required to be addressed, namely, decision on *trustee selection* (Setp 1), *investment and portfolio* (Step 4), and *kickbacks* (Step 7).

2.2.1. Trustee Selection

First, trustee selection. The starting question is how to characterize an appropriate set of alternatives for agents. We can make no restriction on the set of the candidates, i.e., the agent can always consider every one in the society except himself $\{1,2,...,N\} \setminus \{i\}$, but how many trustees he can choose at each run of the game. One obvious setting is as many as he wants. However, considering all costs associated with communication, search, computation, or simply, transaction costs, it seems to be reasonable to assume an incremental process for the upper limit of the number of trustees that an agent can choose. This upper limit is primary restricted by the cost affordability of the agent. Here, without making these costs explicitly, we indirectly assume that the affordability depends on the wealth of the agent, i.e., K_i. Hence, in a technical way, we assume that the additional number of trustee (link) can be available if the growth of the wealth is increase up to a certain threshold. For example, an additional link becomes possible if he has positive growth in wealth, and likewise if his wealth decrease.

$$l_{max}(t) = \begin{cases} l_{max}(t-1)+1, \text{if } \dot{K}_i(t) > 0, \\ \text{unless } l_{max}(t-1) = N-1 \\ l_{max}(t-1)-1, \text{if } \dot{K}_i(t) \leq 0, \\ \text{unless } l_{max}(t-1) = l_{min} \end{cases} \qquad (10)$$

where

$$\dot{K}_i(t) = \ln \frac{K_i(t)}{K_i(t-1)} \qquad (11)$$

Note that Equation (10) serves only as a beginning for many possible variants, but the idea is essential the same: each agent starts with a minimum number of links, say $l_{min} = 1$, and gradually increases the number of links associated with his good investment performance, and vice versa. One can certainly consider different measure of investment performance, but we shall leave this issue for the further study. The rule (10) leaves two possibilities for the agent to change at each point in time: either adding one link (if he has not come to the maximum)or or deleting one link (if he has not come to the minimum). For the former case, he will choose one from the unconnected links, $\mathbf{S} - N_{i,S}(t-1)$; for the later case, he will choose one from the connected links, $N_{i,S}(t-1)$. Let us assume for the both case, his main concern for this one-step change is performance-based or trust-based. Call this the

trust-based selection mechanism, which basically says that the agent tend to add the most trustworthy agent and delete the least trustworthiness agent. To do so, let us define the *effective rate* of return of the investment from agent i to j, measured in terms of its kickbacks, as

$$\kappa_{i,j} = \begin{cases} \frac{y_{i,j}^j(t-1)}{k_{i,j}(t-1)}, & \text{if } k_{i,j}(t-1) > 0 \\ 0, & \text{if } k_{i,j}(t-1) = 0. \end{cases} \quad (12)$$

Then the frequently used logit distribution can be used to substantiate the trust-based selection mechanism as follows.

$$Prob(j|j \in (\mathbf{S} - N_{i,S}(t-1))) = \frac{\exp(\lambda_1 k_{j,i}(t-1))}{\sum_{j \in \mathbf{S} - N_{i,S}(t-1)} \exp(\lambda_1 k_{j,i}(t-1))} \quad (13)$$

$$Prob(j|j \in N_{i,S}(t-1)) = 1 - \frac{\exp(\lambda_1 \kappa_{i,j}(t-1))}{\sum_{j \in N_{i,S}(t-1)} \exp(\lambda_1 \kappa_{i,j}(t-1))} \quad (14)$$

Above, Equation (13) applies to the situation when agent i can add links, whereas Equation (14) applies the situation when agent i needs to delete a link. By Equation (13), agent i tends to favor more on those agents who have trust for him and investing in him, i.e., $j \in N_{i,R}(t-1)$, than those who did not $j \notin N_{i,R}(t-1)$. By Equation (14), agent i most likely will cut off the investment to the agent who offers him the least favorable return rate, the lowest κ.

2.2.2. Investment and Portfolio

Once the new set of trustees ($N_{i,S}(t)$) is formed, the truster then have to decide the investment portfolio applied to them, i.e., how to distribute a total of wealth, $K_i(t)$ over $N_{i,S}(t) \cup \{i\}$. We assume again that this decision will also be trust-based. The idea is that agent i tends to invest a higher proportion of his wealth to those who look more promising or trustworthy, and less to the contrary. Technically, very similar to the decision on the trustee deletion (Equation 14), let us assume that agent i will base his portfolio decision on the effect rate of return $\kappa_{i,j}(t-1)$. Those who have reciprocate agent i handsomely in the previous period will be assigned a larger fund and vice versa. Then a trust-based portfolio manifested by the logit distribution is as follows.

$$w_{i,j}(t) = \frac{\exp(\lambda_2 \kappa_{i,j}(t-1))}{\sum_{j \in N_{i,S}(t) \cup \{i\}} \exp(\lambda_2 \kappa_{i,j}(t-1))}, \quad \forall j, j \in N_{i,S}(t) \cup \{i\} \quad (15)$$

where $w_{i,j}(t)$ is the proportion of the wealth to be invested in agent j; consequently,

$$k_{i,j}(t) = w_{i,j}(t)K_i(t). \quad (16)$$

Two remarks need to be made here. First, part of Equation (15) is self-investment, i.e., $w_{i,i}(t)$.

$$w_{i,i}(t) = \frac{\exp(\lambda_2 \kappa_{i,i}(t-1))}{\sum_{j \in N_{i,S}(t) \cup \{i\}} \exp(\lambda_2 \kappa_{i,j}(t-1))} \quad (17)$$

Like the typical trust game, agent i certainly can hoard a proportion of wealth for himself; however, by the rule of the trust game, this capital will have no productivity and its effective rate of return is always 1, $\kappa_{i,i}(t) = 1, \forall t$. Therefore, by Equation (15), hoarding become more favorable when agents suffer general losses in his investment, namely, $\kappa_{i,j}(t-1) < 1$ for most j. Of course, when that happens, the social trustworthiness observed by agent i is lower and he then take a more cautionary step in external investment.

Second, for the new trustee ($j \notin N_{i,S}(t-1)$), $\kappa_{i,j}(t-1)$ is not available. We shall then assume that it is $\kappa_{i,0}$, which can be taken as a parameter of agent i's general trust for *strangers*. The culture or the personality which tends to have little trust for strangers, being afraid that they will take all money away, has a lower κ_0 and zero to an extreme. The culture or the personality which tends to be more friendly toward strangers has a relatively higher κ_0. The introduction of this parameter then leaves us a room to examine how this initial trust may impact the later network formation.

2.2.3. Kickbacks

Finally, it is the decision related to kickbacks. When investing to others, agent i also plays the role of a trustee and receives the money from others $k_{j,i}$ ($j \in N_{i,R}$). At the end, the total revenues generated by these investments are

$$Y_i(t) = \sum_{j \in N_{i,R}(t)} y_{j,i}(t) = \sum_{j \in N_{i,R}(t)} \tau_{j,i}(t)k_{j,i}(t) \quad (18)$$

Let us assume that the total fund available to be distributed over agent i himself and his all trusters is simply this sum, $Y_i(t)$. That is agent i will not make additional contribution from his *private* wealth to this distribution.[1] Furthermore, we assume again that the decision on kickbacks is again trust-based. We assume that agent i tends to reciprocate more to those who *seem* to have a high degree of trust on him and less to those who *seem* less. This subjective judgement is determined by the received size of investment, $k_{j,i}(t)$.[2] Hence,

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[1] For either altruistic reason or other strategic reasons, violations of this assumption is possible, but in this paper we shall not deal with this more thoughtful design.

[2] Here, we use the term "seeming" or "subjective", because agent i cannot have a direct observation of agent j's portfolio,

$$\{w_{j,l}(t)\}_{l \in N_{j,S}(t) \cup \{j\}},$$

and then evaluate his received investment in light of this portfolio. For example, $k_{j,i}(t)$ can be big in the absolute size, but relatively small in its weight in the portfolio. In this case, agent j may not trust agent i as much as it *seems* to be.

a straightforward application of the logit model leads to the proportions of kickbacks allocated to each truster of agent i.

$$\omega_{i,j}(t) = \frac{\exp(\lambda_3 k_{i,j}(t))}{\sum_{j \in N_{i,R}(t) \cup \{i\}} \exp(\lambda_3 k_{i,j}(t))}, \qquad (19)$$
$$\forall j, \quad j \in N_{i,R}(t) \cup \{i\},$$

where $\omega_{i,j}(t)$ is the proportion of $Y_i(t)$ that will be returned to agent j as kickbacks. Hence,

$$y_{j,i}^{j}(t) = \omega_{i,j}(t) Y_i(t). \qquad (20)$$

Note that part of Equation (19) is the reserves that agents i keeps for himself. In fact,

$$\omega_{i,i}(t) = \frac{\exp(\lambda_3 k_{i,i}(t))}{\sum_{j \in N_{i,R}(t) \cup \{i\}} \exp(\lambda_3 k_{i,j}(t))}. \qquad (21)$$

By Equations (16) and (17), the self-investment is

$$k_{i,i}(t) = w_{i,i}(t) K_i(t), \qquad (22)$$

and the "retained earning" is

$$\sum_{j \in N_{i,R}(t)} y_{j,i}^{i}(t) = \omega_{i,i}(t) Y_i(t). \qquad (23)$$

Then the behavioral interpretation of Equation (21) is that agents who have a large hoarding size tends to be more selfish in the sense that he keeps a large proportion of the fund as "retained earnings", reserved for himself. These people invest a small share to others, but keep a large share to themselves. These people are, therefore, less social and less cooperative. The parameter which dictates this behavior is κ_0, introduced in Section 2.2.2..

3. PROSPECTS OF THE MODEL

Our main concern with efficiency and equality motivates a series of key observations, such as

$$\{g_t, \{K_i(t)\}_{i=1}^{N}\}_{t=1}^{T}, \qquad (24)$$

where g_t is the network topology at round t, as defined by Equation (2), and $\{K_i(t)\}_{i=1}^{N}$ is the wealth profile of all individuals at round t, as defined by Equation (9). An ideal situation which serves as a baseline for making comparison and defining deviation is:

$$g_T = g^*, K_{1,T} = K_{2,T} = \dots = K_{N,T}, \qquad (25)$$

where g^* is the network realizes the highest social efficiency; in the current case, g^* is the fully connected network.

One essential feature of the network-based trust game is that the productivity or the investment multiplier is no longer exogenously given; instead, it is endogenous determined by the production scale, depending on the connectivity of the network (Section 2.1., Equation (6)). However, this mapping (Equation (6)) is unknown to agents. This setting then makes sense, since it is in line with the economic intuition that trust facilitates *chance discovery* and helps form production network, and that in turn beefs up production. Through trust, if agents can smoothly work out their own allocation rules, then we may expect a full prosperity of the society; otherwise, if they don't, it can be rather primitive too.

Agents situated in this environment have to find out all possible rewarding investment opportunities through working with others based on their mutual trust. This, we believe, is the fundamental force behind the trust-growth relation found in those empirical studies (Section 1). While the relation between social trust and discovery or innovation has not been formally tested in economics, our simulation results may propel the formation of such a hypothesis for further study. Our setting simply provides us chances to observe how a group of "strangers" (presumably) can possibly find the most efficient way for production or fail to do that. Alternatively, is there a limit to growth as there is a limit to trust? What are the social networks that can be formed from these simulations? What would be the associated wealth distribution? We believe that the proposal made in this paper can generate lots of data which have good potentials for us to have a deep and fundamental study between trust, social network and wealth creation.

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