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Strong diagnosability of regular networks under the comparison model

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Abstract

Diagnosability has played an important role in the reliability of multiprocessor systems. The strongly t-diagnosable system is $(t + 1)$ diagnosable except when all of the neighbors of a node are simultaneously faulty. In this paper, we discuss the in-depth properties of diagnosability for t-regular and t-connected networks under the comparison model. We show that a t-regular and t-connected multiprocessor system with at least $2t + 6$ nodes, for $t \geqslant 4$, is strongly t-diagnosable under the comparison model if the following two conditions hold: (1) the system is triangle free, and (2) there are at most $t - 2$ common neighbors for each pair of distinct nodes in the system.

Keywords: Diagnosability; Connectivity; PMC model; Comparison model; t-diagnosable; Strongly t-diagnosable

1. Introduction

Fault tolerance is a fundamental consideration in the design of a multiprocessor system. The degree of fault tolerance of a multiprocessor system is closely bound up with the reliability of this system. As the number of processors in a multiprocessor system increases, the complexity of the system can adversely affect its reliability. To maintain reliability, the system should be able to identify faulty nodes and replace them with fault-free ones. The process of finding faulty nodes is called the *diagnosis* of the system, and the maximum number of faulty nodes the system can identify is called its *diagnosability*.

Several different models of fault diagnosis have been proposed in the literature [9–11]. In 1967, the Preparata, Metze, and Chien (PMC) model was introduced for system-level diagnosis in multiprocessor systems [11]. The PMC model was also used in [2,3,5,6,8,13]. Malek and Maeng proposed the comparison model, also known as the MM model [9,10]. The MM model sends the same input (or task) from a node w to a pair of distinct neighbors, u and v, and then compares their responses. The node w is called the *comparator* of nodes u and v. In a special version of the MM-model called the MM*-

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model, a comparison is performed by each processor for each pair of distinct neighbors. Different comparators can examine the same pair of nodes, and either the two responses will be consistent, or they will disagree. The goal of the comparison is to identify the faulty/fault-free status of the nodes in the system. Using a comparison model, Sengupta and Dahbura described a diagnosable system and presented a polynomial algorithm to determine the set of all faulty nodes under the MM*-model [12]. The MM*-model was employed in [1,2,4,7,14,15], and this type of comparison model is studied here.

Identifying the degree of diagnosability for a system is an important subject of the reliability of multiprocessor systems. In a given system, it is impossible to determine whether a processor is fault-free or faulty if all of its neighbors are faulty. Therefore, Lai et al. [8] proposed a new measure of diagnosability in multiprocessor systems, which is *strongly t-diagnosable*. If all of the neighbors of a processor in a t-diagnosable system are not simultaneously faulty, the $t+1$ faulty node can be identified under the PMC model. This may prove practical in real systems. In this paper, we discuss the properties of strongly t-diagnosable for t-regular and t-connected networks of N nodes with $N \geqslant 2t+6$ and $t \geqslant 4$ under the comparison model, which will contribute to the more precise identification of diagnosability for multiprocessor systems.

The next section provides background and explains the notations used in this paper. In Section 3, we derive the strongly t-diagnosable network of a t-regular and t-connected network under the comparison model. Section 4 contains concluding remarks.

2. Preliminaries and notations

A multiprocessor system can be described as an undirected graph $G=(V,E)$ whose the vertex set V represents the set of processors and the edge set E represents the set of communication links. Assume that V' is a subset of V, the subgraph of G induced by $V-V'$ is denoted as $G-V'$. The neighbor set of a node v is defined as $N(v)=\{u \in V \mid (u,v) \in E\}$, moreover, the neighbor set of a node v in V' is denoted by $N_{V'}(v)$. The connectivity of G is defined as $\kappa(G)=\min\{|V'| \mid V' \subseteq V$, and $G-V'$ is not connected$\}$. A graph G is t-connected if $\kappa(G) \geqslant t$. It follows from Menger's theorem that there exist t *internally node-disjoint* (abbreviated as *disjoint*) paths between any two distinct nodes in a given t-connected graph.

A multigraph $M=(V,C)$ is usually applied to the comparison scheme of the system, where V represents the node set and C represents the labeled-edge set. The following definitions [1,7,12] are useful in this paper. Let $(u,v)_w$ denote an edge labeled by w. In M, each $(u,v)_w \in C$ expresses that the nodes u and v are compared by w. The same pair of nodes may be compared by distinct comparators, thus M is a multigraph. For each $(u,v)_w \in C$, the result of comparing nodes u and v by w is denoted as $r((u,v)_w)$ such that $r((u,v)_w)=0$ if the outputs of u and v agree, and $r((u,v)_w)=1$ otherwise. If $r((u,v)_w)=0$ and w is fault free, it is apparent that both u and v are fault free. Moreover, if $r((u,v)_w)=1$, there must exist at least one faulty node among u, v and w. If w is faulty, the results of the comparison would be unreliable because the exact status of u and v are unknown. The complete result of all comparisons, defined as a function $s: C \rightarrow \{0,1\}$, is called the *syndrome* of the diagnosis.

Given a subset $F \subseteq V$, if a syndrome s can arise when all the nodes in F are faulty and all the nodes in $V-F$ are fault free, then F is said to be *consistent* with s. A system is said to be *diagnosable* if there is a unique $F \subseteq V$ that is consistent with s for every syndrome s. A system is called a *t-diagnosable* system if the system is diagnosable as long as its number of faulty nodes is no greater than t. Let F be the set of faulty nodes, then $\sigma(F)$ denote the set of syndromes that could be produced under the occurrence of F. Two distinct sets, $S_1, S_2 \subset V$, are said to be *indistinguishable* if and only if $\sigma(S_1) \cap \sigma(S_2) \neq \emptyset$; otherwise, S_1 and S_2 are said to be *distinguishable*. The symmetric difference between S_1 and S_2 is defined as $S_1 \Delta S_2=(S_1-S_2) \cup (S_2-S_1)$.

The purpose of this study is to discuss the properties of diagnosability for t-regular networks. The following lemma and definition are useful for distinguishing two sets in the comparison model, and will be used later to prove a strongly t-diagnosable system under a comparison model.

Lemma 1. *(See [12].) For any S_1, S_2 where $S_1, S_2 \subset V$ and $S_1 \neq S_2$, (S_1,S_2) is a distinguishable pair if and only if at least one of the following conditions is satisfied (as illustrated in Fig. 1):*

(1) *$\exists u, w \in V-S_1-S_2$, and $\exists v \in S_1 \Delta S_2$ such that $(u,v)_w \in C$,*
(2) *$\exists u, v \in S_1-S_2$, and $\exists w \in V-S_1-S_2$, such that $(u,v)_w \in C$,*
(3) *$\exists u, v \in S_2-S_1$, and $\exists w \in V-S_1-S_2$, such that $(u,v)_w \in C$.*

Definition 1. (See [8].) A system G is strongly t-diagnosable if the following two conditions hold:

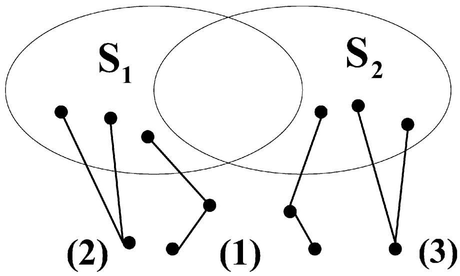

Fig. 1. Three distinguishable conditions under the comparison model.

(1) G is t-diagnosable and
(2) For any two distinct subsets $S_1, S_2 \subset V$, with $|S_i| \leqslant t+1$, $i = 1, 2$, either
  (a) (S_1, S_2) is a distinguishable pair or
  (b) (S_1, S_2) is an indistinguishable pair and there exists a node $v \in V$ such that $N(v) \subseteq S_1$ and $N(v) \subseteq S_2$.

According to the above definition, it is apparent that a $(t+1)$-diagnosable system is strongly t-diagnosable and it is naturally "stronger" than a t-diagnosable system. The strongly t-diagnosable system is almost $(t+1)$-diagnosable under the PMC model. It is only not $(t+1)$-diagnosable when all of the neighbors of a node v are simultaneously faulty [1,8]. However, we are interested in systems that are t-diagnosable rather than $(t+1)$-diagnosable. In the remainder of this paper, we will focus our attention on the properties of strongly t-diagnosable for the t-regular and t-connected networks.

3. Strongly t-diagnosable on regular networks

In this section, we will show that a t-regular and t-connected network with at least $2t+6$ nodes is strongly t-diagnosable under the comparison model if the following two conditions are both satisfied: (1) the system is triangle free, and (2) each pair of distinct nodes have at least two neighbors that are not in common. The content of Lemma 2 describes a sufficient condition for a t-diagnosable system.

Lemma 2. *(See [1].) Let G be a t-regular and t-connected network with N nodes and $t > 2$. G is t-diagnosable under the comparison model if $N \geqslant 2t+3$.*

Based on Lemma 2, now we do not need to consider the t-regular and t-connected networks with $N < 2t+3$ nodes. Moreover, we study only the networks of t-regular and t-connected with $t \geqslant 4$ in the following discussion.

Then, we first discuss the networks of 4-regular and 4-connected with N nodes such that $2t+3 \leqslant N \leqslant 2t+5$. We classify the networks of 4-regular and 4-connected with N nodes into three categories: (1) $N = 2t+3 = 11$, (2) $N = 2t+4 = 12$, and (3) $N = 2t+5 = 13$, and give an example for each category to illustrate that this network is not a strongly t-diagnosable system.

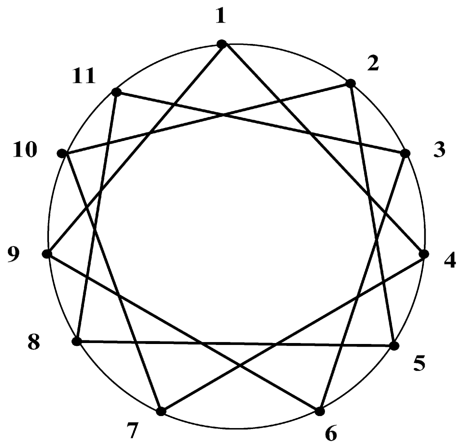

Fig. 2. An example of 4-connected and not strongly 4-diagnosable, with 11 nodes.

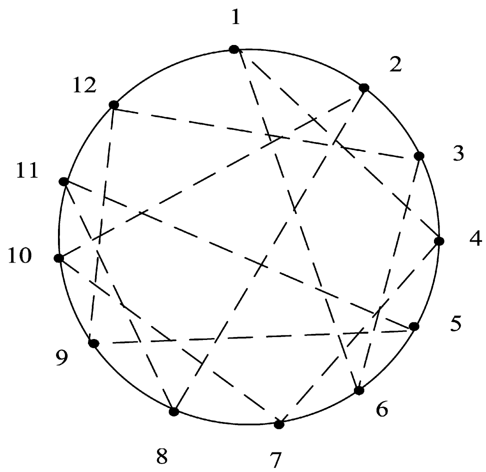

Fig. 3. An example of 4-connected and not strongly 4-diagnosable, with 12 nodes.

The graph shown in Fig. 2 is a 4-regular and 4-connected network with $N = 2t+3 = 11$ nodes. It is a triangle free network. Let $S_1 = \{2, 3, 5, 9, 11\}$ and $S_2 = \{4, 6, 7, 9, 11\}$. According to Lemma 1 and Definition 1, it can be simply determined that the pair (S_1, S_2) is indistinguishable, moreover, and this network is not strongly 4-diagnosable under the comparison model.

Then we consider the second category of 4-regular and 4-connected networks, which have $N = 2t+4 = 12$ nodes. Fig. 3 is a 4-regular and 4-connected network with 12 nodes that is triangle free. Let $S_1 =$

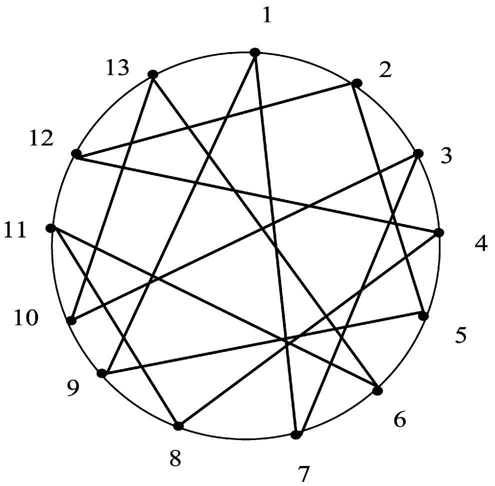

Fig. 4. An example of 4-connected and not strongly 4-diagnosable network, with 13 nodes.

$\{1,5,7,9,11\}$ and $S_2=\{1,2,3,7,11\}$. Similarly, the pair (S_1,S_2) is indistinguishable, and this network is not strongly 4-diagnosable under the comparison model. Next, we consider the third category. Fig. 4 is a 4-regular and 4-connected network with $N=2t+5=13$ nodes and is triangle free. Let $S_1=\{1,5,6,8,12\}$ and $S_2=\{1,3,8,10,12\}$. Similarly, the pair (S_1,S_2) is indistinguishable, and this network is not strongly 4-diagnosable under the comparison model.

Thus, we have showed that the networks of 4-regular and 4-connected with N nodes, for $2t+3\leqslant N\leqslant 2t+5$, are not strongly 4-diagnosable under the comparison model. Then, in the remaining discussion of this section, we will focus our attention on the properties of strongly t-diagnosable for the t-regular and t-connected networks with N nodes, where $N\geqslant 2t+6$ and $t\geqslant 4$. To reduce the complexity of proof, we distinguish the following two lemmas:

Lemma 3. *Given a t-regular and t-connected network G with $N\geqslant 2t+6$ nodes for $t\geqslant 4$. Let with $0\leqslant |S_1\cap S_2|\leqslant t-1$ and $|S_i|=t+1$ for $i=1,2$. (S_1,S_2) is a distinguishable pair under the comparison model if G is triangle free.*

Proof. Let $V''=S_1\cup S_2$ and $V'=V-V''$, where V' may not be connected. And we can observe that $|V''|=2(t+1)-|S_1\cap S_2|\geqslant t+3$ and $|S_1\Delta S_2|=|V''-S_1\cap S_2|\geqslant 4$. We first consider that the subgraph induced by V' contains a connected component R with cardinality of at least 2. We thus let w be any node in R and v be any node in $S_1\Delta S_2$. Because G is t-connected, it follows that there exist t disjoint paths from w to v. However, there exist at most $t-1$ disjoint paths from w to v via the nodes in $S_1\cap S_2$ because $|S_1\cap S_2|\leqslant t-1$. Therefore, there exists at least one path from w to v such that no node of this path belongs to $S_1\cap S_2$. Because w is contained in nontrivial component R, we can find another node u in R, which is adjacent to w. Hence, there exist $u,w\in V-S_1-S_2$ and $v\in S_1\Delta S_2$ such that both u and v can be compared by w. Then, the condition (1) in Lemma 1 is satisfied and we conclude that (S_1,S_2) is a distinguishable pair.

Next, we consider the condition that all the connected components of the subgraph induced by V' are isolated nodes. Thus, all the t neighbors of each node of V' are contained in $V''=S_1\cup S_2$. In this condition, (S_1,S_2) is a distinguishable pair if and only if condition (2) or (3) in Lemma 1 is satisfied. Considering the cardinality of $S_1\cap S_2$, we deal with the following three cases:

Case 1. $0\leqslant |S_1\cap S_2|\leqslant t-3$.

Because $0\leqslant |S_1\cap S_2|\leqslant t-3$ and $t\geqslant 4$, each node of V' must have at least three neighbors in $S_1\Delta S_2$ such that two of them are concurrently contained in either S_1-S_2 or S_2-S_1. Thus, either condition (2) or (3) in Lemma 1 is satisfied, and (S_1,S_2) is a distinguishable pair.

Case 2. $|S_1\cap S_2|=t-2$.

In this case, $|V''|=2(t+1)-|S_1\cap S_2|=t+4$ and $|V'|=N-|V''|\geqslant t+2$. Assume that (S_1,S_2) is an indistinguishable pair. Therefore, conditions (2) and (3) of Lemma 1 cannot be satisfied, implying that each node of V' has at most one neighbor in S_1-S_2 and at most one neighbor in S_2-S_1, respectively. Thus, each node of V' must be connected to at least $t-2$ neighbors in $S_1\cap S_2$ because the degree of node is t. Therefore, we can find at least $(t+2)(t-2)=t^2-4$ edges which are incident from V' to $S_1\cap S_2$. However, the number of nodes in $S_1\cap S_2$ is $t-2$ and the degree of node is t, thus, there must be no more than t^2-2t edges incident to $S_1\cap S_2$. For $t\geqslant 4$, we have $t^2-4>t^2-2t$, which is a contradiction. Consequently, (S_1,S_2) is a distinguishable pair.

Case 3. $|S_1\cap S_2|=t-1$.

It follows that $|V''|=2(t+1)-|S_1\cap S_2|=t+3$ and $|V'|=N-|V''|\geqslant t+3$. Assume that (S_1,S_2) is an indistinguishable pair. Thus, conditions (2) and (3) of Lemma 2 cannot be satisfied. Using the same argument as in Case 2, we can find at least $(t+3)(t-2)=t^2+t-6$ edges incident from V' to $S_1\cap S_2$. Similarly, because $|S_1\cap S_2|=t-1$ and the degree of node is t, there must be no more than t^2-t edges incident to $S_1\cap S_2$. Thus, we have $t^2+t-6>t^2-t$ for $t\geqslant 4$, which is a contradiction. Hence, (S_1,S_2) is a distinguishable pair, completing the proof of the lemma. □

Lemma 4. *Given a t-regular and t-connected network G with $N \geqslant 2t+6$ nodes for $t \geqslant 4$. Let $S_1, S_2 \subset V$ with $|S_1 \cap S_2| = t$ and $|S_i| = t+1$ for $i = 1, 2$. (S_1, S_2) is a distinguishable pair under the comparison model if the following two conditions hold*:

(1) *G is triangle free.*
(2) *Each pair of distinct nodes u and v of G has $|N(u) \cap N(v)| \leqslant t-2$.*

Proof. Let $V'' = S_1 \cup S_2$ and $V' = V - V''$. According to the assumption of $|S_1 \cap S_2| = t$ and $|S_1| = |S_2| = t+1$, we have $|V''| = t+2$. Moreover, there are just two nodes, denoted as $v1$ and $v2$, contained in $S_1 \Delta S_2$ such that $v1 \in S_1 - S_2$ and $v2 \in S_2 - S_1$. Considering the three conditions of Lemma 1, it can be observed that (S_1, S_2) is a distinguishable pair if and only if the condition (1) in Lemma 1 holds. More precisely, (S_1, S_2) is a distinguishable pair if and only if there exists one nontrivial component R with $|R| \geqslant 2$ in the subgraph induced by V' such that $v1$ or $v2$ is adjacent to some node in R. We assume that all nodes in the subgraph induced by V' are trivial components, and then we will show that (S_1, S_2) is a distinguishable pair through proof by contradiction.

If both $N(v1)$ and $N(v2)$ are contained in V'' (i.e., $|N_{V'}(v1)| = |N_{V'}(v2)| = 0$), combining with the fact of $|N(v2)| = t = |N(v2)|$, $|S_1 \cap S_2| = t$, and $t \geqslant 4$, it indicates the following two conditions: (1) a triangle will be produced if $v2$ is a neighbor of $v1$; and (2) if $v2$ is not a neighbor of $v1$, all nodes of $S_1 \cap S_2$ would be contained in $N(v1) \cap N(v2)$. Therefore, it generates a contradiction that there exists either $|N(v1) \cap N(v2)| > t-2$ or a triangle. Thus, $N_{V'}(v1)$ and $N_{V'}(v2)$ cannot simultaneously be empty. The following cases are considered according to the cardinality of $N_{V'}(v1)$, where the discussion for $|N_{V'}(v2)|$ can be treated similarly.

Case 1. There are t nodes in $N_{V'}(v1)$, that is, $|N_{V'}(v1)| = t$.

According to the assumption that two distinct nodes have at most $t-2$ common neighbors, i.e., $|N(v1) \cap N(v2)| \leqslant t-2$, we can observe that there exist at least two nodes, say $u1$ and $u2$, in $N_{V'}(v1) - (N_{V'}(v1) \cap N_{V'}(v2))$. By the assumption that each node in the subgraph induced by V' is a trivial component and $|S_1 \cap S_2| = t$, each of $u1$ and $u2$ has $t-1$ neighbors (except for $v1$) in $S_1 \cap S_2$, moreover, there exist at least $t-2$ common neighbors of $u1$ and $u2$ in $S_1 \cap S_2$. Together with the common neighbor $v1$, the total number of common neighbors of $u1$ and $u2$ is at least $t-1$, which is a contradiction with $|N(u1) \cap N(u2)| \leqslant t-2$. Thus, there exists one nontrivial component in the subgraph induced by V', which is adjacent to $v1$ or $v2$ such that (S_1, S_2) is a distinguishable pair.

Case 2. $N_{V'}(v1)$ is an empty set, that is, $|N_{V'}(v1)| = 0$.

Because $|N_{V'}(v1)| = 0$, in this case all neighbors of $v1$ are contained in $V'' = S_1 \cup S_2$. We first prove that $v2$ is not a neighbor of $v1$. Assume that $v2$ is a neighbor of $v1$, then all of the remaining $t-1$ neighbors of $v1$ must be contained in $S_1 \cap S_2$. Moreover, in order to keep triangle free, all those $t-1$ neighbors of $v1$ in $S_1 \cap S_2$ would be not adjacent to $v2$, and there exists at most one feasible neighbor of $v2$ in $S_1 \cap S_2$. Therefore, at least $t-2$ neighbors of $v2$ will be contained in V', i.e., $|N_{V'}(v2)| \geqslant t-2$. Let v^+ be an arbitrary node in $N_{V'}(v2)$, then v^+ has $t-1$ neighbors in $S_1 \cap S_2$ by our assumption that all nodes in the subgraph induced by V' are trivial components. Thus, there should exist at least $t-2$ common neighbors of v^+ and $v1$ in $S_1 \cap S_2$. Together with one another common neighbor $v2$, there exist at least $t-1$ common neighbors of v^+ and $v1$, which contradicts with the assumption that two nodes have at most $t-2$ common neighbors. Therefore, $v2$ is not a neighbor of $v1$.

Thus, all the t neighbors of $v1$ are contained in $S_1 \cap S_2$ such that $N(v1) = S_1 \cap S_2$. By the assumption of $|N(v1) \cap N(v2)| \leqslant t-2$, $v2$ should have at most $t-2$ neighbors in $S_1 \cap S_2$, in other words, there will exist at least two neighbors of $v2$ in V', i.e., $|N_{V'}(v2)| \geqslant 2$. Considering any two nodes in $N_{V'}(v2)$, these two nodes are not adjacent to $v1$ since $v1$ does not has any neighbor in V'. Moreover, the two nodes are trivial components in the subgraph induced by V' according to our assumption, thus, each of them has $t-1$ neighbors in $S_1 \cap S_2$. Because $N(v1) = S_1 \cap S_2$, there exist $t-1$ common neighbors of $v1$ and each of the two nodes in $S_1 \cap S_2$, which contradicts the assumption that two distinct nodes have at most $t-2$ common neighbors. Therefore, there exists one nontrivial component in the subgraph induced by V', which is adjacent to $v1$ or $v2$ such that (S_1, S_2) is a distinguishable pair.

Case 3. The number of neighbors of $v1$ in V' is neither t nor 0, that is, $1 \leqslant |N_{V'}(v1)| \leqslant t-1$.

Let v^+ be an arbitrary node in $N_{V'}(v1)$, all the $t-1$ neighbors (except for $v1$) of v^+ are contained in V'' because v^+ is assumed to be a trivial component in the subgraph induced by V'. Moreover, for keeping triangle free, all these $t-1$ neighbors of v^+ are not adjacent to $v1$. Hence, the feasible neighbors of $v1$ in V'' are those nodes which are not adjacent to v^+.

Now we discuss the following two cases, which are distinguished by the number of common neighbors of $v1$ and $v2$ in V'.

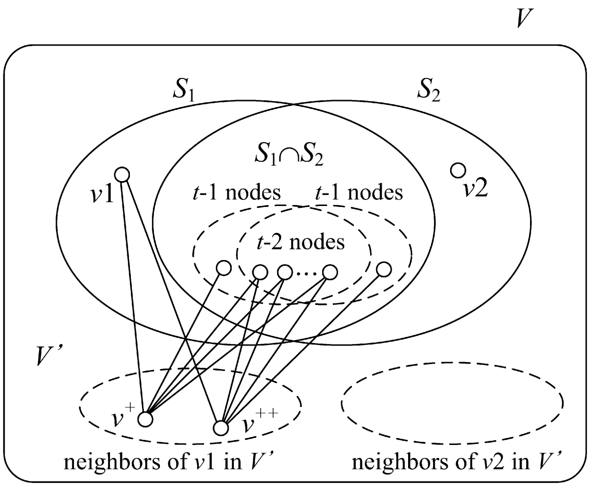

Fig. 5. The illustration for subcase 3.1 in Lemma 3.

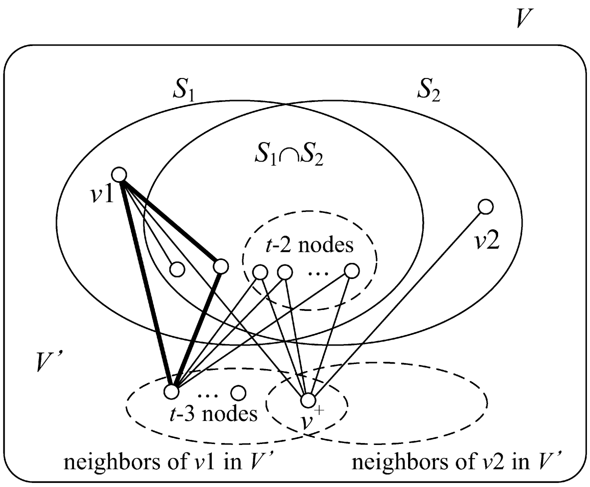

Fig. 6. An illegal condition for subcase 3.2 in Lemma 3.

Subcase 3.1. There exists no common neighbor of $v1$ and $v2$ in V'.

In this condition, v^+ is not adjacent to $v2$, thus all the $t-1$ neighbors (except for $v1$) of v^+ are contained in $S_1 \cap S_2$, and $v1$ has at most one feasible neighbor in $S_1 \cap S_2$. Note that $v1$ would have one more neighbor in V'' if $v2$ is adjacent to $v1$. Thus, $v1$ has at most two neighbors in V'' and has at least $t-2$ neighbors in V'. Because $t \geqslant 4$, $v1$ must have one neighbor, denoted as v^{++}, other than v^+ in V'. By assumption, v^{++} is also a trivial component in the subgraph induced by V'. As the same discussion about v^+, v^{++} has $t-1$ neighbors (except for $v1$) in $S_1 \cap S_2$, too. Because $|S_1 \cap S_2| = t$, there obviously exist at least $t-2$ common neighbors of v^+ and v^{++} in $S_1 \cap S_2$, which is illustrated in Fig. 5. Together with another common neighbor $v1$, we have $|N(v^+) \cap N(v^{++})| \geqslant t-1$, which is a contradiction for the assumption that two nodes have at most $t-2$ common neighbors.

Subcase 3.2. There exists at least one common neighbor of $v1$ and $v2$ in V'.

Because there exists one common neighbor, in order to keep triangle free, $v1$ cannot be adjacent to $v2$. Then we consider the following two conditions.

(1) $|N_{V'}(v1) \cap N_{V'}(v2)| = 1$.

Let v^+ be the unique common neighbor of $v1$ and $v2$ in V'. The other $t-2$ neighbors (except for $v1$ and $v2$) of v^+ are all contained in $S_1 \cap S_2$ because v^+ is a trivial component in the subgraph induced by V'. Thus, the number of feasible neighbors of $v1$ in $S_1 \cap S_2$ is at most two. We first discuss the case that $v1$ has two neighbors in $S_1 \cap S_2$. Then, all the other $t-3$ neighbors (except for v^+) of $v1$ are contained in V', which are all assumed to be trivial in the subgraph induced by V'. Therefore, each of those $t-3$ nodes must have $t-1$ neighbors in $S_1 \cap S_2$. For keeping triangle free, all the $t-1$ nodes in $S_1 \cap S_2$ cannot be adjacent to $v1$ (otherwise, a 3-cycle as described in Fig. 5 would appear). However, there exist at most $t-2$ suchlike nodes in $S_1 \cap S_2$. Thus, we get a contradiction.

Next, we discuss the case that $v1$ has exactly one neighbor, say $v1^+$, in $S_1 \cap S_2$. Since $v1$ is not adjacent to $v2$, the remaining $t-1$ neighbors of $v1$ should be all contained in V'. Then, except for v^+, $v1$ has $t-2$ neighbors in V', which are assumed to be trivial in the subgraph induced by V' and not adjacent to $v2$ because v^+ is the unique common neighbor of $v1$ and $v2$ in V'. Thus, for keeping triangle free, each of these $t-2$ nodes must have $t-1$ neighbors in $(S_1 \cap S_2) - \{v1^+\}$, i.e., all of the $t-1$ nodes in $(S_1 \cap S_2) - \{v1^+\}$ are their common neighbors, which is a contradiction to the assumption that two distinct nodes have at most $t-2$ common neighbors.

(2) $2 \leqslant |N_{V'}(v1) \cap N_{V'}(v2)| \leqslant t-2$.

Let nodes v^+ and v^{++} be two arbitrary nodes in $N_{V'}(v1) \cap N_{V'}(v2)$. Obviously, each of v^+ and v^{++} has $t-2$ neighbors (other than $v1$ and $v2$) in $S_1 \cap S_2$ since it is assumed to be trivial in the subgraph induced by V'. Suppose that the number of common neighbors of v^+ and v^{++} in $S_1 \cap S_2$ is at least $t-3$. Then, together with the other two common neighbors $v1$ and $v2$, there exist at least $t-1$ common neighbors of v^+ and v^{++}, which would violate the assumption that two nodes have at most $t-2$ common neighbors. Hence, we conclude that v^+ and v^{++} have at most $t-4$ common neighbors in $S_1 \cap S_2$.

Therefore, each node in $S_1 \cap S_2$ must be a neighbor of v^+ or a neighbor of v^{++}. To keep triangle free, all nodes of $S_1 \cap S_2$ must be not adjacent to $v1$. Because $v2$ is not adjacent to $v1$, all neighbors of $v1$ should be contained in V', i.e., $|N_{V'}(v1)| = t$, which is a contradiction for $1 \leqslant |N_{V'}(v1)| \leqslant t-1$. Hence, there exists a nontrivial component in the subgraph induced by V', which is adjacent to $v1$ or $v2$ such that (S_1, S_2) is a distinguishable pair, completing the proof of this lemma. □

By Lemmas 3 and 4, we derive the following theorem about the property of strongly t-diagnosable for a t-regular and t-connected network G with N nodes where $N \geqslant 2t+6$ and $t \geqslant 4$ under the comparison model.

Theorem 1. *For $t \geqslant 4$, given a t-regular and t-connected network G with $N \geqslant 2t+6$ nodes, G is strongly t-diagnosable under the comparison model if the following conditions are satisfied*:

(1) *G is triangle free.*
(2) *Each pair of distinct nodes u and v of G have $|N(u) \cap N(v)| \leqslant t-2$.*

Proof. We will show that both the two conditions of Definition 1 are satisfied to complete the proof of this theorem. Let $S_1, S_2 \subset V$ with $0 \leqslant |S_1 \cap S_2| \leqslant t$ and $|S_i| \leqslant t+1$ for $i = 1, 2$. It follows from Lemma 2 that G is t-diagnosable. Thus, condition (1) of Definition 1 holds.

Because G is t-diagnosable and not $(t+1)$-diagnosable, it is apparent that (S_1, S_2) is a distinguishable pair for $|S_i| \leqslant t$, $i = 1, 2$, and $0 \leqslant |S_1 \cap S_2| \leqslant t-1$. Moreover, it follows from Lemmas 3 and 4 that (S_1, S_2) is a distinguishable pair for $|S_i| = t+1$, $i = 1, 2$, and $0 \leqslant |S_1 \cap S_2| \leqslant t$. Hence, condition 2(a) of Definition 1 is satisfied.

Now, let v be an arbitrary node in V. Assume that $S_1 = N(v) \cup v$ and $S_2 = N(v)$, then we have $|S_1| = t+1$, $|S_2| = t$, and $|S_1 \cap S_2| = t$. Therefore, there exists one node $v \in V$ with $N(v) \subseteq S_1$ and $N(v) \subseteq S_2$ such that (S_1, S_2) is an indistinguishable pair. The condition 2(b) of Definition 1 is satisfied. Therefore, the theorem holds. □

4. Concluding remarks

In the field of diagnosability, the comparison model is a well-known and practical fault diagnosis model. Identifying the degree of diagnosability for a system under the comparison model is an important subject of the reliability of multiprocessor systems. In this paper, we studied the properties of the strongly t-diagnosable and proved that a t-regular and t-connected multiprocessor system with at least $2t+6$ nodes, for $t \geqslant 4$, is strongly t-diagnosable under the comparison model if the following two conditions hold: (1) the system is triangle free, and (2) there are at most $t-2$ common neighbors for each pair of distinct nodes in the system. Such results should be useful in the design of a fault-tolerant multiprocessor system and be contributive to the investigation of reliability of the systems.

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