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Taiwanese teachers' implementation of a new 'constructivist mathematics curriculum': How cognitive and affective issues are addressed

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ABSTRACT

This paper aims to investigate the ways in which four Grade 5 teachers perceived and implemented a new constructivist mathematics curriculum, after all their past experience of traditional mathematics in Taiwan. The meaning and indicators of constructivist and traditional mathematics were explored and developed based on reviews of three countries' mathematics curricula and studies on mathematics teaching. Through interviews and classroom observations, teachers' practices were analyzed in these terms, separated into cognitive and affective aspects. The teachers were found to meet the new curriculum halfway, to address cognitive issues more effectively than affective ones, and to implement a common curriculum differently. The results support the need to analyze the teaching of mathematics in relation to affective as well as cognitive elements, and are discussed in relation to mathematics teaching internationally.

1. Introduction

Mathematics has been traditionally viewed as a school subject focusing on achievements, strict rules, efficient procedures and right answers (Schoenfeld, 1989; Turner et al., 1998), often accompanying more teacher-centred teaching methods. With the influence of individual and social constructivism on pedagogies, the mathematics classroom involves an increasing introduction of non-routine, open-ended and project-based mathematical problems, often accompanying more student-centred teaching methods (e.g., Boaler, 1998; Burton, 1994; Riordan and Noyce, 2001). While constructivist mathematics constitutes a dramatic reform in the mathematics curricula of nations, such as Taiwan and the US, there still remains a tension with traditional, transmission-oriented teaching methods (Hamm and Perry, 2002; Manouchehri and Goodman, 2000; McCaffery et al., 2001). The present study was conducted in the first academic year (August 2001–July 2002) that the four teachers studied taught a textbook based on a curriculum of constructivist mathematics mandated by the government in Taiwan in 1993 and introduced gradually since then. The four teachers studied had all learnt mathematics themselves based on a 'traditional' mathematics curriculum; they were now required by the government and scholars to teach according to the *principles* and *pedagogies* of constructivist mathematics. This study focused, therefore, on the management by these teachers of the transition from a traditional to the new constructivist mathematics curricula. The purpose of the present study was to document the process of teachers' adaptation to a distinctly new curriculum, which might serve as valuable historical experiences for curricular reforms in Taiwan and other countries in the future.

The results supported other current research indicating that constructivist mathematics places additional cognitive and affective demands on teachers and their students. A model is presented within the present study, which attempts to describe the key elements in a range of teaching styles comprising both cognitive and affective elements. These elements were elicited from international comparison of national curricula, from previous research concerning the tension between old and new curricula, and research concerning mathematical pedagogies.

1.1. Comparison of national mathematics curricula in Taiwan, the US, and England

Constructivism was officially introduced into the mathematics curriculum in Taiwan in 1993 by the Curriculum Standard for Primary Schools (Ministry of Education in Taiwan, 1993). The goals of mathematics education are to help students acquire mathematical knowledge from daily life and to cultivate students' attitudes and abilities to use mathematical methods efficiently to solve practical problems by encouraging children to communicate with members in the learning community and discovering patterns. Teachers are also encouraged to help students think actively and learn independently. In order to achieve these goals, the national

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curriculum gives teachers more opportunities to develop their pedagogy and put their educational beliefs into practice. The responsibility for publishing textbooks has also been gradually transferred from the government to private publishers. Given the current trend, students are likely to have more diverse problem-solving experiences than before. This move mirrors similar developments in a number of countries.

In the US, for example, the Curriculum and Evaluation Standards for School Mathematics (NCTM, 1995) articulates five goals: children learn to 'value mathematics; become confident in their ability to do mathematics; become mathematical problem-solvers; learn to communicate mathematically; learn to reason mathematically.' In the mathematics classroom students have to be exposed to numerous and varied interrelated experiences in which they are encouraged to value the mathematical enterprise, develop mathematical habits of mind, and understand and appreciate the role of mathematics in human affairs. In addition, children need to be encouraged to create, explore, guess and even make and correct errors so that they gain confidence in their ability to solve complex problems. They also need to be supported by teachers in reading, writing and discussing mathematics and in conjecturing, testing and building arguments about the validity of a conjecture. Similar issues are also raised in the recent Principles and Standards for School Mathematics by NCTM (2000) (Rousseau, 2004).

The mathematics curriculum policy of England also reveals a similar trend, with special emphasis on using and applying mathematics (Campbell and Kyriakides, 2000). In the National Curriculum for England (QCA, 1999), there are four attainment targets in mathematics: using and applying mathematics; number and algebra; shape, space and measures; handling data. 'Using and applying mathematics' is the most important of the four targets as this target is incorporated into each of the other three. The curriculum emphasizes the application of thinking and practical skills in real-life situations; using multiple tools such as mental, written and calculator methods to solve problems; and involving information and communication technologies in learning factual or declarative knowledge. After comparing the roles of applying mathematics in several versions of the National Curriculum and National Numeracy Strategy, Hughes et al. (2000) proposed that the target of using and applying mathematics has three characteristics: decision-making; communication; and reasoning and proof. A later development in the national curriculum has been made for secondary education (QCA, 2007a,b). Two of the three major competences in mathematics for secondary students are applying and communicating mathematics, which are partly consistent with the notion of individual and social constructivism. (The third competence is selecting mathematical tools and methods.)

This review of national curricular documentation reveals that constructivist principles underpin some of the policies of mathematics curricula in England, the US and Taiwan. These curricula of constructivist mathematics comprise four characteristics: (1) meaningful learning or understanding, which emphasizes providing students rich experiences or connections between concepts and application of mathematics; (2) creative thinking, reasoning, or exploration, which pertains to emphasizing diverse solution methods and providing students an experimenting learning environment, in which students are encouraged to guess, conjecture and test hypotheses; (3) independent learning, which consists of being sensitive to students' needs and giving students autonomy to activate their mathematical minds and self-reflections on the mathematics enterprise; (4) social interaction, which focuses on providing more and diverse opportunities for dialogues between teachers and students and between students on individual, group and class levels.

1.2. Tension between implementation of old and new curricula of mathematics in Taiwan

What is an 'old' Taiwanese mathematics classroom like? Stigler and Perry's (1990) study, which focused on the comparison between mathematics classrooms in four cities of three nations: Sendai, Japan; Taipei, Taiwan; Minneapolis and Chicago, USA, 1979–1980 and 1985–1986, indicated some good qualities of mathematics teachings in Taiwan. Compared with mathematics classrooms in the US, Taiwan and Japan classrooms had less off-task behaviour of students. Asian children had more opportunities to have their work assessed, and to observe the evaluation of other students' performance than American students. Both Taiwanese and Japanese teachers used far more manipulatives and real-world problems than did the American teachers. There are however some controversial characteristics: classrooms in Taipei and Japan were centrally organized, with the teacher as the leader of the children's activities 90% of the time (compared to 50% in the US); children spent the vast majority of time working, watching and listening together as a class and were rarely divided into smaller groups. Taiwanese classrooms were more performance-oriented, while the Japanese classrooms were more reflective; in other words, there was more verbal discussion of mathematical concepts and procedures. Taiwanese teachers emphasized fast and accurate performance, or getting the right answer quickly. They were also devoted to practicing rapid mental calculation, an activity that was never observed in Japanese and American classrooms.

The Taiwanese mathematics classrooms described in Stigler and Perry's (1990) study were based on the previous mathematics curriculum officially introduced in 1975, which emphasised knowledge acquisition. The major teaching method was knowledge transmission by teachers. Students learned mathematics by memorising and spending much time practicing calculation skills. As such, students were unable to explain the reasons for calculation procedures and lost their interest in learning mathematics (Liu and Shu, 1995).

With the influence of constructivist philosophies on the curricula in different parts of the world, the teaching methods and materials of traditional mathematics in Taiwan have been criticized and discouraged as they do not employ mathematical thinking, but rote learning, as the following statement reveals:

> The past mathematics teaching, especially those based on the textbook published in 1975, placed too much emphasis on students' calculation ability and ignored reasoning and understanding. Students became calculation machines. This is not a correct direction for mathematics teaching. Based on this rationale, the intellectuals of education reform introduce and advocate constructivist mathematics, wishing to put right the wrong teaching methods... (electronic newspaper by the Humanistic Education Foundation, 17 September 2003).

A mathematics curriculum based on the principle of constructivism, 'knowledge is not acquired by transmission but by learner active construction (Liu and Shu, 1995),' therefore, was developed through the process of experimentation, pilot and implementation and was officially introduced in 1994 (Liu, 2004).

The dramatic difference between the old and new mathematics curricula inevitably raised public arguments in Taiwan society. For example, it has been argued that rote learning does not necessarily inhibit children's mathematical thinking; that there is also a strong possibility that the teaching methods of constructivist mathematics may hinder culturally deprived children from learning valuable cultural heritages at school about how to solve problems in a formally effective way; and that a decline in children's mathematical ability will reduce their capacity to learn advanced mathematics. This will lead, it is further argued, to the collapse of the competitive ability of the whole country, as the following report indicates:

> **Save mathematics education. Scholars' plea pays**. . .A scholar and professor in mathematics (Professor C-S Lin). . . and a group of scholars who cannot stand the drop in Taiwanese students' mathematical ability actively contacted S-B Chen (the President), Y-J Lee (the Chair of Academic Sinica) and J-T Huang (the Minister of Education). They urged the Ministry of Education to remedy the lagging behind of the past ten years, after which it was possible to talk about raising standards. . .. He found his son had bad 'habit' in doing mathematics; for example, no clear understanding of the place value system when doing basic calculation. . . 'Every year, 30,000 students are being influenced. If we do not take action now, the problem will become much more serious in the future.' Professor Lin said that now many Year 7 students (aged 12–13) could not even do multiplication of fractions. . .There would be a difficulty in linking to the senior high school. Students would fail in science subjects at universities. 'The competitive ability of the whole country will collapse.' For more than half a year, he had almost stopped other jobs and devoted himself to primary and secondary mathematics curricula. . .He said that it was important. Primary mathematics was the most important but our primary mathematics had been seriously damaged for the past seven to eight years. . . (Chen, R. Y., China Times, 19 May 2003).

Parents have also given their voices on this issue, as a father's statement shows:

> My son is going to graduate from the primary school one month later. He's 'not born at the right time,' coming across the education reform. In these six years my son and I had a negative relationship because of the so called 'constructivist mathematics.' Accompanying your children during their doing homework becomes my heavy burden. There were frequent conflicts between the teacher's and parents' calculation procedures. My son said, 'The teacher told us we definitely could not use the "old" calculation procedures, otherwise even we got the right answer we still could not get the points.' But we just don't understand 'what is the new style' and are used to the methods we learnt before. I sometimes argued, 'Why bother to make simple procedures complicated?' My son didn't appreciate but just argued, an unhappy ending. . ..I gave up my principle ('don't send my son to the after-school cram class') and decided to hand over the responsibility for helping my son learn mathematics to the after-school cram class, in order to avoid unhappy experiences and the possible lagging behind of my son's ability at the starting stage. . .. (Tsai, M.-T., United Daily News, 23 May 2002)

Some support for this positive view of traditional methods is supplied by the fact that, historically, Taiwan students have shown high achievement compared with their international counterparts, e.g., the Trends in International Math and Science Study (TIMSS) of 2003 (Mullis et al., 2004).

Obviously, the picture of Taiwanese mathematics classrooms described in Stigler and Perry's (1990) research and the pedagogy that the public are familiar with are based on the old mathematics curriculum, which is significantly different from what is advocated in the new constructivist mathematics. Implementing this significantly distinct new curriculum from the past is likely to be a major challenge for teachers in Taiwan, as in other parts of the world.

1.3. Tension in the cognitive and affective aspects of pedagogies

Cognitively, the difference between traditional and constructivist mathematics lies in the different interpretation of the source of knowledge: knowledge transmission as opposed to knowledge construction. If the source of knowledge is by means of teachers' transmission, the teachers' mission is to ensure that the most precious knowledge from the former generations and culture has been taught to the children. In line with this view, traditional mathematics in Taiwan directly taught students 'the compressing representations' of formal mathematics (Nunes, 1997, p. 37). On the other hand, if the source of knowledge is based on the concept of knowledge construction or that 'knowledge is co-produced in settings' (Boaler, 2000, p. 3), teachers are the coordinators or activators; children are the host in the mathematics classroom, building 'their own' knowledge of mathematics through personal and social construction or communication. Constructivist mathematics encourages multiple representations of mathematics, especially children's informal mathematics, in other words, 'incorporating students' inadequate solutions into instruction' (Stipek et al., 1998, p. 467).

The distinct characteristics of constructivist and traditional mathematics have been also reflected in mathematics teaching practice in other cultures. Askew et al.'s study (1997) categorized effective teachers of numeracy in England into three orientations: connectionist, transmission and discovery. The teaching methods of the 'transmission orientation' are congruent with those of traditional mathematics in Taiwan, in which mathematical concepts and routine procedures are directly introduced by teachers in 'discrete packages' (p. 32). Constructivist mathematics in the new Taiwanese curriculum, on the other hand, seems to advocate teaching methods with some of the characteristics of both connectionist and discovery orientations. As with the connectionist orientation, Taiwanese constructivist mathematics encourages a teaching method of interaction between teachers and children through 'dialogue' and 'reasoning'; the connectionist orientation establishes the most effective teachers of numeracy (p. 34). Similar to the discovery orientation, constructivist mathematics urges teachers to support students to 'discover' their own solution methods, although this is by 'practical activities' (discovery orientation) and by 'dialogue' (connectionist orientation).

McCaffery et al. (2001) compared two kinds of teaching practice in high schools in the US. The teaching materials based on new curriculum reform in the US are called 'integrated mathematics', while there are still some schools who use 'traditional sequence' mathematics textbooks. The teaching methods and materials of 'integrated mathematics' are similar to those of constructivist mathematics, and 'traditional sequence' mathematics is similar to traditional mathematics. Manouchehri and Goodman (2000) compared two American Grade 7 teachers' implementation of the 'standard-based textbook'. The traditional textbooks in the US, according to Manouchehri and Goodman's descriptions, were similar to the traditional textbooks in Taiwan: 'the teacher disseminates bits and pieces of knowledge among students' (p. 1). The standard-based textbook, like 'constructivist mathematics' textbooks in Taiwan, places emphasis on student-centred teaching and students' investigations of mathematical ideas.

There are fewer studies considering affective aspects of these different pedagogies, where the teacher adopts strategies designed to improve the emotional engagement or experience of the students, or a more student-centred approach. However, this is clearly a significant issue, and one which poses significant challenges for teachers being required to make the transition from a traditional to a constructivist pedagogy, and for their students. Focusing on teachers' authority in six American Grade 1 mathematics classrooms, Hamm and Perry (2002), for example, found that, given the high percentages of 'answer-known questions', 'teacher verifying' and 'acknowledge by restating student ideas' in the Grade 1 mathematics classrooms, it is obvious that the mathematics authority in these classrooms still tends to be the teachers, rather than the pupils or 'classroom communities' (p. 135), which are advocated by the official mathematics curriculum in the US.

Three broad conclusions emerge from the above review of the literature on international comparison of curricula and pedagogies in relation to constructivist and traditional mathematics. Firstly, although researchers use different systems to categorise teaching methods, these systems appear to be consistent with the dichotomy between constructivist and traditional mathematics in Taiwan. Secondly, the agreement between these research results is that constructivist mathematics is more progressive, beneficial and effective than traditional mathematics. For example, constructivist mathematics is effective teaching in terms of students' achievement (Askew et al., 1997; McCaffery et al., 2001) and positive affective responses (Stipek et al., 1998). Thirdly, however, it is clear that the transition from traditional to constructivist pedagogies presents significant cognitive and affective challenges to both teachers and students.

2. Methodology

Research attempting to investigate pedagogies within mathematical education has used both quantitative and qualitative methodologies. McCaffery et al. (2001) and Hamm and Perry (2002) used elements to identify different patterns of teaching methods; in other words, they quantified the quality of teaching methods; Askew et al. (1997) and Manouchehri and Goodman (2000), on the other hand, described different teaching methods in a holistic way; these studies used qualitative research methods.

The present study aimed to use both research methods to investigate Taiwanese teachers' perceptions and implementations of a new constructivist mathematics curriculum, in the cognitive and affective aspects, after their long experiences of traditional mathematics.

2.1. Participants

The participants were four mathematics, also class, teachers (Tina, Alice, Mark and Frieda) and their respective Grade 5 students (other pseudonyms are used), aged nine to ten, in a public primary school in Taipei, Taiwan. There were approximately the same numbers of boys and girls in each class. Each class had 29 children. There were no significant differences between the four classes' mathematics achievements prior to and during the study period, according to the results from their regular school tests.

2.2. Measures

Three measures were used to understand how the teachers implemented the new curriculum: a general interview with the four teachers concerning their conceptions of mathematics teaching, classroom observations of a series of lessons relating to a 'fractions' topic, and teacher interviews at the end concerning their teaching of this topic. All interviews and classroom observations were conducted by the first author. All interviews were audio-recorded and then transcribed; all observations were video-recorded.

Measure 1: questions about teaching conceptions. Ten interview questions were used to elicit the teachers' general conceptions of mathematics teaching, which included aspects of teaching methods, teaching styles, learning strategies, learning motives and learning affect. Each interview lasted about 50 min.

Measure 2: The classroom observation checklist. This checklist contained two parts: structured observation and narrative descriptions. The coding rules for structured observation comprised seven dimensions, which were developed based on the review of international comparisons of curricula and pedagogies between traditional and constructivist mathematics, as follows.

- Dimension (1) *Rich connections*: Teachers' interventions, explanations or questions richly connect to related mathematical concepts, and deeply clarify students' misconceptions.
- Dimension (2) *Ill-structured problems* (vs. well-structured problems): *Ill-structured problems* have a number of correct answers. Most of them are authentic or contextualized problems with multiple or open-ended solutions (Nitko, 1996). Examples of ill-structured problems are mathematics projects (Boaler, 1998), construction problems (Schoenfeld, 1989), and explanation problems (asking reasons for solutions) (English, 1997). In contrast, well-structured problems are tasks that are clearly laid out, give students all the information they need, and usually have one correct answer that students can obtain by applying a procedure or algorithm taught in class. Examples of well-structured problems include calculation problems, routine word problems, and fact problems (asking declarative knowledge in mathematics). In the present study, the observer recorded either ill-structured or well-structured mathematics problems that the teachers posed for students to solve.
- Dimension (3) *Diverse solutions* (vs. single solutions): Teachers invite or give diverse answers or solution methods. In contrast, they might focus on providing a single correct answer or solution method; other possible answers and solution methods seem not to be considered.
- Dimension (4) *Sensitivity*: Teachers give students affection, respect and encouragement, taking account of students' interests, needs and concerns.
- Dimension (5) *Autonomy*: Teachers give students opportunities to develop, negotiate and support their choices and rules.
- Dimension (6) *Student talk* (vs. teacher talk): *Students talk*, model solution methods, or demonstrate their abilities. In contrast, there are times in class when teachers are the focus, talking, modeling how to solve problems, and showing teaching materials.
- Dimension (7) *Private interactions* (vs. whole-class interaction): Teachers conduct *private* teacher–student or teacher-group *interactions*, the attention of the whole class is not on the interaction (Hart, 1989). In whole-class teaching, on the other hand, teachers have all students in class focus on one single episode of presentations or dialogues.

Dimensions 1–3 focus on the cognitive aspects of constructivist mathematics, i.e. deep and creative teaching; Dimensions 4–7 emphasize the affective/social aspects, i.e. student-centered teaching. Dimensions 1, 4 and 5 were adapted from the conceptions of the three domains of stimulation, sensitivity and autonomy in the Adult Engagement Scale (Pascal and Bertram, 2001a), which fitted the focus of the present study on both cognitive (Dimension 1: rich connections or stimulation) and affective issues (Dimensions 4–5: sensitivity and autonomy) in relation to real mathematics teaching. The scale was developed to evaluate the quality of an adult's interaction with children, which was viewed as a critical factor in shaping successful and effective learning experiences of children (Pascal and Bertram, 2001b). Dimensions 1, 4 and 5 were coded on a five-point Likert scale, *5 = totally engaging* to *1 = totally non-engaging*. Other dimensions were coded in a dichotomous fashion; missing data were allowed when the dichotomous definitions could not be applied to the situation.

Both 'molecular' and 'molar' approaches were used to design the structured observation (Wilkinson, 1995, p. 217). The molecular approach helped to make easy the definition of behaviours and increase reliability by specifying particular behaviours (Dimensions 2–3 and 6–7). For example, the observer indicated the behaviour of ill-structured problems on the checklist if the teacher and students worked on a mathematics problem that asked students to create a word problem using the calculation of '7 ÷ 5 = 1 2/5.' The 'molar' approach helped in the understanding of the meaning of behaviours by taking large behavioural wholes as units of behaviour (Dimensions 1, 4 and 5). For instance, the observer indicated the degree of engagement in rich connections on the checklist if the teacher explained the meaning of the fraction 7/5 by linking it to student prior knowledge of division (7 ÷ 5) or by relating it to daily life through scaffolding questions, e.g., there are seven cakes but five people wish to share them; how can we deal with it? May we draw seven cakes first and see how we can deal with it fairly?

'Time-sampling' (Hayes, 2000, p. 63) was used over the period of a lesson to ensure that a reasonably accurate picture was established of typical activities in the classroom. Every two minutes, indicated by the timer on the observer's computer, the observer recorded the ongoing flow of teachers' and students' behaviour that lasted for at least five seconds within a ten second period. Based on the narrative records, the observer recorded behaviours on Dimensions 2–3 and 6–7; every eight minutes the observer recorded on all seven dimensions. For other times, in order to yield dense narrative descriptions, the observer recorded as much as she could about what was happening during class. The observer's insights were also added into the narrative descriptions as far as possible immediately after each observation.

The classroom observation checklist was developed and piloted by the observer (the first author) and another observer through nine hours of classroom observation. Intense discussion was made after each observation. The reliability of coding between two observers for each dimension was computed by dividing *the total number of coding records that the two observers agree with each other* by *the total number of coding records* (Miles and Huberman, 1994). The reliability coefficients of the seven dimensions were .73, .76, .88, .82, .73, .98, and .82 respectively. Since the reliability of the classroom observation checklist was established, a single observer used the checklist to carry out the remaining observation. In addition, the training in the pilot study helped the observers clearly understand the meaning of each dimension and the procedure for coding, which facilitate more correct and efficient coding behaviour in the remaining observation.

Measure 3: Questions after teaching. The teachers were interviewed after their teaching of the fractions topic in relation to five questions, concerning their opinions about the topic, teaching designs, goals, teaching strategies and effective teaching. Each interview lasted around 1 h.

2.3. Context of the four Taiwanese teachers' implementation of the new constructivist mathematics curriculum

Teachers in Taiwan have to teach according to the national curriculum mandated by the government. Teachers also had to teach according to the textbooks designed by the government before the 1994 curricular reform. After the 1994 reform, teachers in a school can choose one textbook in a particular school subject (e.g., mathematics) for their students at the same grade from several textbooks provided by publishers, who design their textbooks based on the national curriculum. The authority of textbooks has slightly decreased since the 1994 curricular reform, which emphasizes what students can do (competence indicators) rather than what teachers teach (instructional goals) no matter which textbooks teachers choose to use.

It is a prevalent phenomenon in Taiwan that parents and after school cram-class teachers intervene in student learning in addition to formal schooling. Parents teach their primary-school children after school if they are able and have time to do so. Parents send their children to after school cram classes either because they do not have time to supervise their children's learning after school or because they believe that after school cram-class teachers can better help their children than themselves in improving their children's achievements, especially in mathematics and English.

Parents and after school cram-class teachers, however, have not experienced 'in-service' training on teaching based on the new constructivist mathematics curriculum although the government did provide parents with books, free of charge, about the introduction of the new curriculum, suggesting guidelines for helping their children learn mathematics based on constructivist mathematics, as a consequence of the significant 1994 curricular reform. Despite the efforts, the fact that all the past experiences of parents and after school cram-class teachers were related to traditional mathematics appeared to have created stresses and challenges for the four teachers in their implementation of the new curriculum.

> Alice: Parents, like us, all learned from the old curriculum. They cannot understand why their children do mathematics in this way (writing down the thinking process in detail). They (parents) force their children to accept their effective ways to directly obtain answers (for mathematics problems).

> Frieda: Sometimes the children have had prior fixed impressions from their parents or after-school-class teachers.. . . They (parents and after-school-class teachers) use old teaching methods. There are strong rejections from parents and after school cram-class teachers, 'Why does the mathematics now make easy things complicated?'

> Mark: Parents and after school cram-class teachers sometimes teach faster than school. . .When I guide students to use their own methods to solve a mathematics problem (based on the new curriculum), I find students use old methods (taught by parents or after school cram-class teachers).

> Tina: When I find my students use a fixed way of solving a mathematics problem learned from parents and after school cram-class teachers, I'll ask them a different question. . ..For example, after school cram-class teachers generally only teach students the definition for the centre of a circle, I'll ask students: I just don't know where the centre of a circle is. How can I know it?

The four teachers, as class teachers, taught several school subjects for their respective classes, e.g., Chinese, mathematics, social sciences and physical education. They taught their respective 5-grade classes until the students graduated from the primary school stage. This is a normal practice in Taiwan primary schools. Each class teacher continuously teaches a class for the two years of the lower grade level (1–2 grades), middle grade level (3–4 grades), or higher grade level (5–6 grades). Class teachers are not expected to change classes before completing the teaching of a particular class within a certain grade level.

The four teachers, Tina, Alice, Frieda and Mark, had around seven, four, one and twelve years of teaching experiences, respectively, before they participated in the present study. Tina was studying for her master degree for in-service teachers on mathematics education and therefore was experiencing graduate education on mathematics teaching based on the new constructivist mathematics curriculum when she participated in the present study. Alice and Frieda were relatively new teachers and had experienced some pre-service teacher education on constructivist mathematics. Mark was the most experienced teacher and did not experience any pre-service education on constructivist mathematics; as such, he attended workshops for preparing in-service teachers on how to implement the new constructivist mathematics curriculum. Alice, Frieda and Mark were not mathematics or mathematics-education majors.

Teachers in Taiwan have to attend several hours of in-service teacher training, normally funded by the government, each year but they are generally allowed to choose the workshops on topics in which they feel interest and relevance to their work. In addition, the government published additional books to prepare in-service teachers for the significant new constructivist mathematics curriculum. The instruction guidebooks accompanying the textbooks, which are a normal practice in Taiwan, also provide help in implementing a new curriculum. Despite the several sources of help, Mark confessed that it was hard for him to change from a teaching habit based on traditional mathematics to that based on constructivist mathematics but he perceived that he had tried his best.

> I have to completely transform my old-fashioned thoughts about mathematics teaching. In fact, I am quite nervous and will read the instruction guidebook (before teaching mathematics)...Compared with my previous teaching, now the most significant differences are that I try best, when possible, to let students talk and write more. I say ‘try best’ because it is hard to change habits. I am also worried whether students can really talk and express what they think about mathematics because in the past most students did not talk in class.

In summary, Tina appeared to have the most confidence in the understanding of constructivist mathematics and in the implementation of the new constructivist mathematics curriculum, Frieda and Alice the second, and Mark the least in terms of training on the new constructivist mathematics. They, as professional teachers, perceived a necessary change and were prepared by the government for a transition to the teaching of a new mathematics curriculum. The four teachers also perceived that the significant others of their students, e.g., parents and after school cram-class teachers, were not well-prepared for the new curriculum and intervened in their students’ learning using traditional mathematics, which was at odds with the new curriculum approach.

3. Results

3.1. Taiwanese teachers’ views on constructivist and traditional mathematics

This study was carried out in the second half of the first academic year, 2001–2002, when the four teachers and their Grade 5 students first experienced ‘constructivist mathematics.’ As implementers of the new mathematics curriculum, the four teachers accepted the advantages of the new mathematics but interpreted it in slightly different ways. Tina made a clear differentiation between constructivist and traditional mathematics.

> Now the trend is, I don’t tell you directly ‘to do like this and then it will be equal’, but to for the students to find something to let them discover the rule and law... The new mathematics ... emphasizes [that] ... discussion can help students to clarify their problems ... they (students) can then regulate their own ideas because they can listen to what their peers say and find it seems useful ...Through discussion, different ideas are stimulated or gradually become clear....Some children can see the laws but some can’t. Of course, what they see is incomplete laws... Some children can’t see it and some have different points. They will develop these points during discussion. The teacher is a guide. When you see they seem to find something, you can enforce it. Like this, it is to let children find the law... (On the other hand, if taught) by traditional methods, students can use (the formula) but don’t understand the reasons ... they can all memorize ... ‘for any fraction, if its numerator and denominator are divided or multiplied by the same numbers, the new fraction will have the same value as the original one’ ... but when I asked them ‘why?’ ... no one could answer ...they are stuck to a method that they don’t really know the reasons.

In old mathematics, teachers directly transmitted knowledge to children. For example, ‘What is equivalent fractions? It is numerator and denominator are the same if they are divided or multiplied by the same number (to use Tina’s examples).’ In new mathematics, children construct knowledge with teachers through the teacher posing problems, children discussing and the teacher guiding children into a deeper understanding of mathematics. Children are the centre of learning. Teachers are mentors and have to avoid any direct ‘instruction.’ This difference was also indicated by the other three teachers.

Alice approved of the new mathematics curriculum where more emphasis was placed on reasoning step by step before conclusions were obtained.

> The children can clearly speak out every step ... they can write out their thinking processes, letting others know ... In the past, mathematics was to solve out the answers by calculation ... The new curriculum will guide you gradually with a low speed. In the past, I feel, if you give students new problems, they would be stuck.

Frieda considered that the aim of the new constructivist mathematics was ‘nice’: ‘apply mathematics to life,’ and ‘every one can learn mathematics happily;’ therefore, it was ‘*more humanistic*.’

> It strengthens the base, focusing on children’s thinking process. It emphasizes thinking, not just memorizing formula....Teachers had to ask important key questions, provide positive feedback and give more time for discussion in class. The teaching materials were active, humanistic and easier.

Mark described his feelings about constructivist mathematics.

> When I first experienced the new math, I felt I hated it. It seemed just like that we accumulate additions to calculate multiplication ... I felt it was strange, which is also the response of most parents. Gradually I accepted it after teaching, studying and attending in-service training courses.

He finally supported one idea in the new curriculum when talking of constructivist methods, that for the children they place fewer demands on the brain, instead of ‘adults’ methods, which although effective, place more burden on the brain.’ For example, the adults’ method: $3\ 1/5 \times 3 = 48/5$; the children’s methods: $3\ 1/5 \times 3 = (3 + 3 + 3) + (1/5 + 1/5 + 1/5)$ or $= 3 \times 3 + 1/5 \times 3$ (according to Mark’s explanation).

On the other hand, the four teachers were also concerned with the impact of these new teaching methods on children. These new teaching methods were viewed as less effective than the old teaching methods, which were still used by parents and their after school cram-class teachers, as has been described in Section 2.3.

The other point was to what extent children could learn from these new teaching methods, which were viewed as more student-centred, more open for discussion and more understanding/thinking-based than the old teaching methods. The answer was that mathematics seemed to be a special school subject that always favoured the reasoning mind, and underachievers were 'guests in the classroom' (to use Tina's term). 'In mathematics, if you can, you just can, and, if you can't, you just can't', as Frieda stated. The four teachers were all aware of this problem and dealt with it by encouraging or forcing every child to take a part in discussion. Alice's observation provided a significant example.

> In group discussion, there is a problem, that is, children with high ability teach children with low ability. Sometimes able children teach in bad manners. They say, "Why are you so stupid! For you, our group did not get the point!" They (children with low ability) feel frustrated and gradually they will be afraid of speaking out their thoughts. They just listen to the strong children's and have a deep sense of inferiority. Top children are top and low children are low.... In class, I choose, for example, Number 4 in each group, and then the Number 4s have to explain for their groups. Therefore, they (children with high ability) have to teach the weak children.

Mathematics is like an unequal battle, in which some children seem to be always the winners and some the losers. Based on this viewpoint, in mathematics Alice created opportunities for and encouraged the 'strong children' to help the 'weak children.'

In summary, the four teachers shared many common perspectives on the 'constructivist curriculum.' They acknowledged the new curriculum's strengths in basing mathematics pedagogies on children's developmental needs and on the epistemology of social and individual constructivism. They were also concerned about the curriculum's perceived weakness ignoring salient ability differences in mathematics learning between students and its dramatic opposition to the traditional mathematics pedagogy prevalent in the society, supported especially by parents and after school cram-class teachers. However, as we shall see, this apparently shared perspective was translated into practices with significantly different elements.

3.2. Taiwanese teachers implementing constructivist or traditional curricula

3.2.1. Results from structured observation

Table 1 reports the scores for each of the 4 teachers on each of the dimensions identified in the structured observations of their mathematics teaching.

Dimensions 1, 4 and 5, based on mean scores derived from the Likert scales, Kruskal–Wallis tests (an alternative non-parametric technique to one-way between-group ANOVA) were used to compare the mean ranks between the teachers because some of the sample sizes in the cells (i.e. the number of observations) were less than 30 (Pallant, 2001) (Tina: $N = 31$; Alice: $N = 22$; Mark: $N = 13$; Frieda: $N = 13$). As can be seen in Table 1, there were significant differences between the four teachers in the degrees of rich connections (D1) ($\chi^2 = 10.86$, $p = .01$), sensitivity (D4) ($\chi^2 = 30.18$, $p < .0005$) and autonomy (D5) ($\chi^2 = 17.36$, $p = .001$). In order to determine where the differences lay between the teachers, for each dimension, six Mann–Whitney U tests were used to compare mean ranks between each pair of teachers, each sharing the significance level of .00833 (Howitt and Cramer, 1997). Based on these statistical test results, the classes were assigned ranks for each dimension as follows: The class ranked H (high) is significantly higher than the class ranked L (low); the class ranked A (average) is not significantly different from either classes H (high) or L (low).

For Dimensions 2–3 and 6–7, based on percentage scores derived from the dichotomous categorical data, results of chi-square tests revealed that the four teachers were significantly different in the dimensions of ill-structured problems (D2) ($\chi^2 = 45.10$, $p < .0005$, $N = 339$), student talk D(6) ($\chi^2 = 19.70$, $p < .0005$, $N = 312$) and private interaction (D7) ($\chi^2 = 8.48$, $p = .04$, $N = 331$), but not in the dimension of diverse solutions D(3) ($\chi^2 = 4.28$, $p = .23$, $N = 339$). In order to identify where the differences lay between the four teachers, the 4×2 chi-square for each dimension was partitioned into six 2×2 chi-squares, each comparing two of the four teachers. As before, the normal significance level of .05 was shared between the six chi-square tests, each sharing the significance level of .0083. On this basis, ranks were again assigned of H (high), A (average) or L (low), to each of the classes in the same way as for the earlier dimensions.

In order to summarize the above analysis and to facilitate the link between the data of structured observation, narrative observation and interviews, two number lines were developed to locate the four teachers in the cognitive and affective/social aspects respectively using mean class ranks, as shown in Fig. 1.

A mean class rank was obtained by calculating the average of a teacher's ranks, Rank High as '1', Rank Low as '0' and Rank Average as '0.5', for the cognitive aspect in the direction of divergent intervention and for the emotional aspect in the direction of student-centred interaction. The relative positions of the four teachers in Fig. 1 reveal their varied degrees of implementing the constructivist mathematics curriculum in the cognitive and affective/social aspects. Tina was interested in deeply creative teaching (cognitive aspects), but placed little concern on student-centred teaching (affective aspects). In contrast to Tina, Alice revealed a strong commitment to affective aspects, but little to cognitive aspects. Similar to Alice, Mark placed little emphasis on

Table 1
Ranks (R), percentages (%) and means (M) of the 7 observation dimensions for the four teachers.

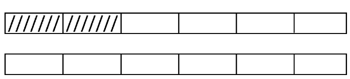
Dimensions	Cognitive aspects				Affective/social aspects		
	D(1) Rich connections	D(2) Ill-structured problems	D(3) Diverse solutions	D(4) Sensitivity	D(5) Autonomy	D(6) Student talk	D(7) Private interaction
	R(M)	R(%)	R(%)	R(M)	R(M)	R(%)	R(%)
Tina	H(4.32)	H(60)	A(70)	L(3.10)	L(3.32)	L(24)	A(6)
Alice	A(4.14)	L(19)	A(78)	H(4.14)	H(4.27)	H(51)	H(13)
Mark	A(3.85)	L(31)	A(80)	A(3.54)	A(3.85)	L(27)	L(2)
Frieda	L(3.54)	H(56)	A(67)	L(2.92)	L(3.38)	L(26)	A(4)
χ^2	10.86	45.10	4.28	30.18	17.36	19.70	8.48
p	.01	<.0005	.23	<.0005	.001	<.0005	.04

creatively connectionist teaching, but was moderate on student-centred teaching. On the other hand, Frieda had a moderate degree of deeply creative teaching, but a low degree of student-centred teaching.

3.2.2. Results from interviews and narrative classroom observations

3.2.2.1. Tina. In cognitive aspects, Tina tended to change well-structured problems to ill-structured ones, in order to help children "*climb up the slope*" of a deep and clear understanding of mathematics. As she stated in the interview, "*I feel mathematics is reasoning ability. If they can clarify the concepts, then they can generalize.*" For example, after she found that children typically used "2 ÷ 6 = 2/6 = 1/3" to solve the Problem, '*Two ribbons (of equal length) are equally divided between six people. How much ribbon will one person get?*', Tina posed an ill-structured problem, "*How do you divide these two ribbons between six people?*". The following shows how Tina revised Sam's drawing and stimulated thinking by rich intervention that clarified Sam's misconceptions.

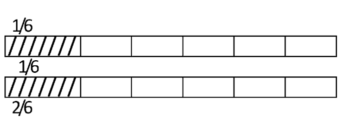
(Sam draws as shown below.)



Tina: Sam, why did you separate this ribbon into six parts?. . .. . .

Tina: You said, because there are six people, the first ribbon is divided into six parts firstly. The second ribbon is divided into six parts as well. If this is what Sam means, then one ribbon is divided into six parts and one person gets one part. Then the second is also divided into six parts and the person also gets one part. Is this what you mean?. . .

(After several queries to Sam and the whole class to try to ensure that every pupil understood, Tina revised Sam's drawing as follows:)



As Tina stated, "*The problems in the textbook are routine. I talk using different objects or directions.*" She worked on a single mathematical problem in depth mainly by using ill-structured assistance questions to stimulate children's rich connections. Tina appeared to be a challenging teacher, challenging her students' mathematical minds by never-ending high-order questions using diverse approaches. The analysis of structured observation in Table 1 revealed her significant use of ill-structured problems (60%).

In the affective aspects, Tina tended to exclude irrelevant and childish behaviours from her mathematics classroom. Walking around group tables during group discussion, Tina said:

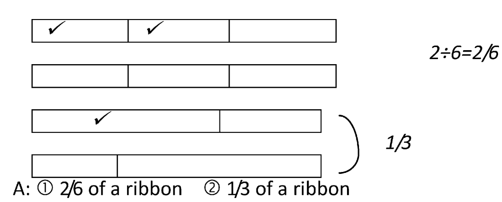
I find that one group is not very good. The group has finished work and begun to draw something irrelevant. I want to give a minus point to the group.

Using punishers to suppress 'irrelevant' behaviours appeared to be a reasonable practice for Tina, who placed much emphasis on cognitively rich connections and correct concepts in teaching 'mathematics' as a school subject. As a consequence, there were relatively little expression by the children of childish behaviour, personal messages, and both positive and negative emotions in Tina's mathematics classrooms.

3.2.2.2. Alice. In cognitive aspects, Alice is a contrast to Tina. Her teaching normally followed the textbook and teacher's manual. She focused on the main well-structured problems in the textbook and let the ill-structured sub-problems be developed by group discussion and classmate questioning. As most problems in the textbook were well-structured and Alice rarely posed additional problems, her class therefore spent only 19% of the time working on ill-structured problems.

The significance of Alice's teaching lay in affective aspects. She "*hoped that the strong (children) could help the weak*" and claimed that "*personality is more important than ability*". In her class children excitedly enjoyed their special discussion culture of group discussion, peer modeling and classmate challenging, as shown in the following extract:

(Tim and Tom show their group discussion results on a whiteboard to the whole class on the platform.)



Tim: . . .It says two ribbons and six people, so two divided by six equals 2/6. And this (Tim and Tom point to the '1/3' on their whiteboard) is the answer after reducing the fractions to its lowest term. . ..

(Many children raise their hands to wait to be pointed at by Tim and Tom to ask questions.). . .

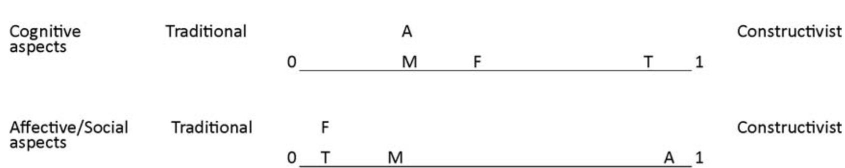


Fig. 1. Four teachers' mean class ranks[a] in cognitive and affective aspects. [a]The four teachers' mean class ranks are calculated as follows:

Cognitive aspects: creatively connectionist teaching

Tina: (1 + 1 + 0.5) ÷ 3 = 0.83
Alice: (0.5 + 0 + 0.5) ÷ 3 = 0.33
Mark: (0.5 + 0 + 0.5) ÷ 3 = 0.33
Frieda: (0 + 1 + 0.5) ÷ 3 = 0.5

Affective aspects: student-centered teaching

Tina: (0 + 0 + 0 + 0.5) ÷ 4 = 0.125
Alice: (1 + 1 + 1 + 1) ÷ 4 = 1
Mark: (0.5 + 0.5 + 0 + 0) ÷ 4 = 0.25
Frieda: (0 + 0 + 0 + 0.5) ÷ 4 = 0.125

Nick: How do you reduce it to the lowest term?

Tim: Six are divided by two, and two are also divided by two.

Nick: But you did not write it (in a strong and blaming voice.)

Tom: Ho! Ho! (Laugh.) OK, now we add it...(He choose a girl to ask questions...)

Alice (writes 2/6 and 1/3 on the blackboard): May I ask a question? Are 2/6 and 1/3 the same?

Whole class: Yes.

Alice (writes the equals sign '=' between 2/6 and 1/3): These are equivalent fractions. These are the same. We have talked about it last semester. Classmates might have some opinion about the style of what is written, not in a complete and clear way, just a picture and showing 1/3.

Tim: Teacher, but we have indicated clearly in the picture.

Alice: Good. ... This group at least pointed out another possible answer '1/3'. (Alice gives a point to Tom and Tim's group.)...

There was an intense and excited student-centred discussion culture in Alice's class, and Alice's interventions were carried out in a polite and gentle way. However, Tim and Tom's group drawing revealed a significant misconception between *1/3* of 'a ribbon' and *1/3* of 'two ribbons as a whole'. The successful discussion culture in Alice's class was based on her sensitive, encouraging and non-critical attitude, but at the expense of that clear understanding of mathematical process, which was emphasized by Tina.

3.2.2.3. Mark. In cognitive aspects, Mark, similar to Alice, taught normally according to the textbook and the teacher's manual. He valued "*children's methods*" and "*even sometimes ignored the methods that adults give and encourage the methods made by children themselves.*" In the affective aspect, unlike Alice, Mark let students solve problems on their own, though children did sit in groups and could discuss freely. He seldom involved himself in students' group discussion. Mark was trying to infuse affective issues into mathematics teaching, provided that the infusion of affective issues was related to the cognitive essence of mathematics. He spent a significant amount of time guiding children to diverse solutions of the problem, '*Please use the calculation procedure, $7 \div 5 = 1\ 2/5$, to make a mathematical problem.*' The whole class, under Mark's enthusiastic encouragement and guidance, produced diverse interesting "*childish answers*". In the case of other problems, Mark taught them efficiently. He taught slowly at the beginning and quickly afterwards. He also used 'self-learning' as a teaching method in class, by way of a change. Mark was also the only teacher who rarely used the method of 'giving or taking away points' for discipline or learning. He emphasized a "*relaxing*" learning environment, "*not giving children so much stress*". He let children know, "*It is not so hard. Think a while and you will be able to understand.*" It is because he believed that "*if the teacher is fierce, children will hate math after the teaching... though in class they really sit nicely and quietly.*" In addition, "*It is said in relaxing atmosphere children will learn and absorb better.*"

3.2.2.4. Frieda. She was the least experienced of the four teachers and managing the class was more difficult. This led to low scores in affective aspects. In cognitive aspects, Frieda, like Tina, would ask questions such as "*Why (or 'how') were the procedures produced (by children or herself)?*" to deepen children's understanding of the meaning of a mathematical concept. Unlike Tina, however, in the dimension of rich connections, Frieda paid a great deal of attention to children's noise and this seemed to distract the teaching focus. Sometimes, in affective aspect, this led to a negative atmosphere:

(Frieda is drawing 12 circles to represent apples on the blackboard for explaining 'a box of 12 apples, now only five apples left'.)

Alex: The apples are broken! (Some students: Ha! Ha! Ha! (Laughter.))

Egor: The apples were bitten by worms. (Some students: Ha! Ha! Ha! (Laughter.))

Frieda (draws the circle carefully): This is fine. Right?

David: Teach quickly! (in a loud and bored voice) (Frieda stops drawing and looks at David.)

Alex: We are being observed!

Frieda: What attitude should you have? No matter whether you are being observed or not?

David: I won't listen anymore in class!...Go on teaching! Don't listen to boring (irrelevant) questions! (in a loud and scolding voice)

Frieda: Go and stand outside to calm down for ten minutes. (David goes out.) Stand nicely otherwise the time will be longer. (David goes out and yells a few times.)

Frieda: 20 min.

After this, Frieda taught directly with little dialogue between her and the students; some students paid little attention to the lesson. This incident illustrates that when affective issues are not addressed in the mathematics classroom, it will have a direct impact upon the cognitive aspects of the lesson.

4. Discussion and conclusion

The present study identified two characteristics of the mathematics teaching emerging among teachers in Taiwan arising from the transition from traditional to constructivist curricula. These contribute to our understanding of how teachers adjust themselves to a new curriculum and new pedagogies and the ways that cultural constraints impact in the process.

4.1. Meet new curriculum 'Halfway', rather than 'All or None'

Having all the past experience of learning and teaching traditional mathematics, none of the teachers in the present study fully implemented the cognitive and affective requirements for constructivist mathematics, despite in-service training and curriculum demands for constructivist mathematics. Tina focused on the cognitive aspect in delving into a deep understanding of mathematics concepts, but ignored the affective aspect. Alice emphasized students' social interaction, which was effectively organized, but at the expense of a clear understanding of mathematics. Frieda had difficulty in managing classes and a negative emotional atmosphere was generated in the classroom, which distracted the students from the cognitive issues of learning mathematics, especially for constructivist mathematics that emphasizes children's active participation in constructing their knowledge. Mark had the most balanced pedagogy, integrating cognitive and affective concerns, combining individual and group learning, and distinguishing children's solutions from adults'. But he still failed to exercise these practices in any depth.

In other words, none of the four teachers scored highly in both creatively connectionist teaching and student-centred teaching. There are several possible reasons for this. Firstly, there are very significant differences between constructivist and traditional mathematics in terms of epistemology and pedagogy. More time and in-service education is still needed to help teachers fully adapt to a new teaching paradigm. Secondly, traditional mathematics has been a long-lasting practice and culture in Taiwan. As these teachers stated in the interview, parents and after school cram-class teachers were still using traditional mathematics methods. Therefore, it seems likely that it will be very difficult to replace traditional mathematics completely in Taiwanese society. A similar cultural constraint resistant to curricular reform has been found elsewhere in the world, including, for example, India (Clarke, 2003). The final status is likely to be a compromise between or combination of constructivist and traditional mathematics in different ways in mathematics classrooms, as revealed by the results of the present study. Thirdly, there is some evidence to suggest that cognitive and affective aspects of constructivist mathematics teaching, in fact, are one dimension, but in the opposite direction. That is, when teachers are high in cognitive aspects, they appear to be low in affective aspects, and vice versa. The results from the study by Manouchehri and Goodman (2000) in the US also imply that cognitive and affective aspects are one dimension. Their study reports that the teacher with a positive evaluation of the constructivism-mathematics-based textbook emphasized cognitive variables in teaching, while the teacher with a negative evaluation focused on affective variables in teaching. A theme which emerges from the results of Manouchehri and Goodman's study and the present study is that the highly cognitive demands and challenges of constructivist mathematics pedagogy attract, encourage and stimulate teachers with a mathematicians' mind, though little of their energy can remain for affective issues. Other teachers, however, are likely to be discouraged by the high cognitive challenges of constructivist mathematics and divert their teaching focus to affective issues. A thorough conclusion on this point might be drawn by further studies with larger sample sizes.

4.2. Easier to address cognitive issues than affective issues

In line with previous literature on international comparisons between pedagogies, the four teachers participating in the present study identified significant cognitive and affective issues in implementing a constructivist mathematics curriculum.

However, they raised more cognitive than affective issues, elaborated upon them more in their interviews and tended, in their practice, to find it easier to address issues of a cognitive rather than affective nature. The dimensions with more than 50% of time spent on in the structured classroom observations (Table 1) for all the four teachers can be used as indicators of dominant teaching practices for present Taiwanese teachers. The results of this structured observation reveal that all the four teachers spent more than half of their time on providing or inviting diverse solutions to a single mathematical problem (67–80%). This is a significant achievement for the implementation of a constructivist mathematics curriculum, since traditional mathematics had long focused on teaching students the single best solutions for any problems. On the other hand, all of the four teachers spent very limited time on interaction with groups or individuals (private interactions, 2–13%), and a very significant amount of time on interaction with the whole class (87–98%). The result indicates that whole-class interaction is still typical of teaching practice in Taiwanese schools. Stigler and Perry's (1990) study indicated that Taiwanese teachers, in 1979–1980 and 1985–1986, invested 90% of time in activities where the teachers were the leaders. Even though constructivist mathematics emphasizes a more student-centered teaching, this emphasis was not revealed in the teachers' interaction styles in this study. However, whole-class interaction increases the opportunity to make children's mistakes (or 'mathematics abilities') public, which might have a negative influence on children's self-esteem and willingness to task risks. In their whole-class discussion, students seemed to experience a strong emotional impact, interwoven with the complexity of cognitive conflict from classmates' challenges. This suggests a controversial issue associated with the implementation of the new curriculum in Taiwan, in which social interaction or 'discussion culture' is strongly advocated (as in the USA and UK). Since mathematics is likely to be the most significant school subject that has strict right answers for plenty of well-structured problems and 'mistakes' can be very easily identified. In 'public interaction' (Hart, 1989), such as whole-class or group discussion, which is 'culturally' the predominant form of discussion favoured by Taiwanese teachers, therefore, individual students' mistakes inevitably become a 'public' affair. This leads to a direct conflict between cognitive and affective concerns in Taiwanese mathematics classrooms. Alice's avoiding or failing to indicate students' mistakes in whole-class discussion is likely to be based on this concern of maintaining students' self-esteem. Or, possibly, she chose to deal with mistakes later as her students' whole-class discussion was too exciting to control and was consuming too much time. Another issue which arises here is that the pedagogies of constructivist mathematics present high cognitive challenges in themselves and some teachers still need more time and training to manage these successfully.

4.3. Limitations of the present study and implications for future research

The present study has developed a structured procedure for analyzing different aspects of implementing a constructivist mathematics curriculum. The study, however, has only looked at four teachers from one culture, who shared many common characteristics, such as the predominance of whole-class teaching. It would, therefore, be beneficial for further studies to investigate a wider and more diverse range of teachers, in order to further clarify the points raised by the present results. Furthermore, there are certain limitations of the present study because of its qualitative methodology, e.g., few observers, judgements in coding, and selections of data, although inter-coder reliability, training for coding, multiple sources of data, and quantitative analyses were used to triangulate the findings.

With the trend toward constructivist mathematics in Taiwan, social interaction and complex mathematical problems are providing mathematics teachers in Taiwan with a new challenge. There is a need for training programs to develop teachers' appropriate pedagogies to adjust to the constructivist mathematics curriculum. There is also a need to understand the interaction between teaching practice and students' responses to the new constructivist curriculum.

Primary mathematics in Taiwan has been experiencing a relatively stable stage since the significant 1994 curricular reform. Nowadays, there is still a national curriculum focusing on student competences, based on which textbooks by private publishers or teaching materials by teachers are used in classrooms. Constructivist mathematics is now not a single pedagogy highly advocated by the government in classrooms but a daily practice together with traditional mathematics partly because of the strong and prevalent traditions in the society. Teachers' experiences during a significant transition from old to new curricula need documentation, as the present study shows, which becomes part of the history and the knowledge. There appears to be a need for further research that documents the implementation of the mixture of traditional, constructivist, and other pedagogies in mathematics classrooms in relation to teacher characteristics, e.g., teacher beliefs, teaching efficacy, and pre-service and in-service training at the stable stage of curricular development.

Taiwan students still have high mathematics achievement in recent international large-scale tests, such as the TIMSS 2007 (Mullis et al., 2008) and the Programme for International Student Assessment 2006 (OECD, 2007). This may relieve the worry of mathematics scholars and policy-makers in Taiwan who introduced the constructivist mathematics curriculum, which had created debates in the society. On the other hand, the results of these international tests revealed that Taiwan students, despite the high achievement, had more negative attitudes toward learning mathematics than their international counterparts. This suggest that more attention and further action need to be taken for the affective aspect of learning mathematics, which may be related to what happens in both the mathematics classroom and the society.

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