

# Comments on “enhancement of an efficient liveness-enforcing supervisor for flexible manufacture systems”

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**Abstract** Region theory can synthesize maximally permissive supervisors by solving a set of inequalities based on the marking/transition-separation instances (MTSIs). It is infeasible to solve these inequalities for either a sizable net or a small net with a sizable initial marking. Huang et al. [1] propose novel crucial MTSIs to reduce the number of MTSIs. Experimental results show that the proposed control policy is the most efficient algorithm among the closely related approaches. One example shows that it not only reaches all live states but also employs fewer control arcs than that by Li et al. Huang et al. offer no hints on why it employs fewer control arcs. This paper develops theory to explain the physics behind.

**Keywords** Petri nets · Flexible manufacturing system · Deadlock prevention

## 1 Introduction

In an FMS, a set of processes share a set of costly resources. To be competitive and save the cost, these resources must be utilized as much as possible. This leads to strong competition for resources among process. Consequently, deadlocks occur due to processes holding resources mutually waiting for others to release their resources. Deadlock prevention [1–16] adds monitors to problematic siphons to prevent them from becoming insufficiently marked.

Uzam and Zhou [2, 3] apply region analysis to a well-known  $S^3PR$  (systems of simple sequential processes with resources) in Fig. 1. The benchmark reaches 26,750 states, where 21,581 are legal, i.e., either good or dangerous states

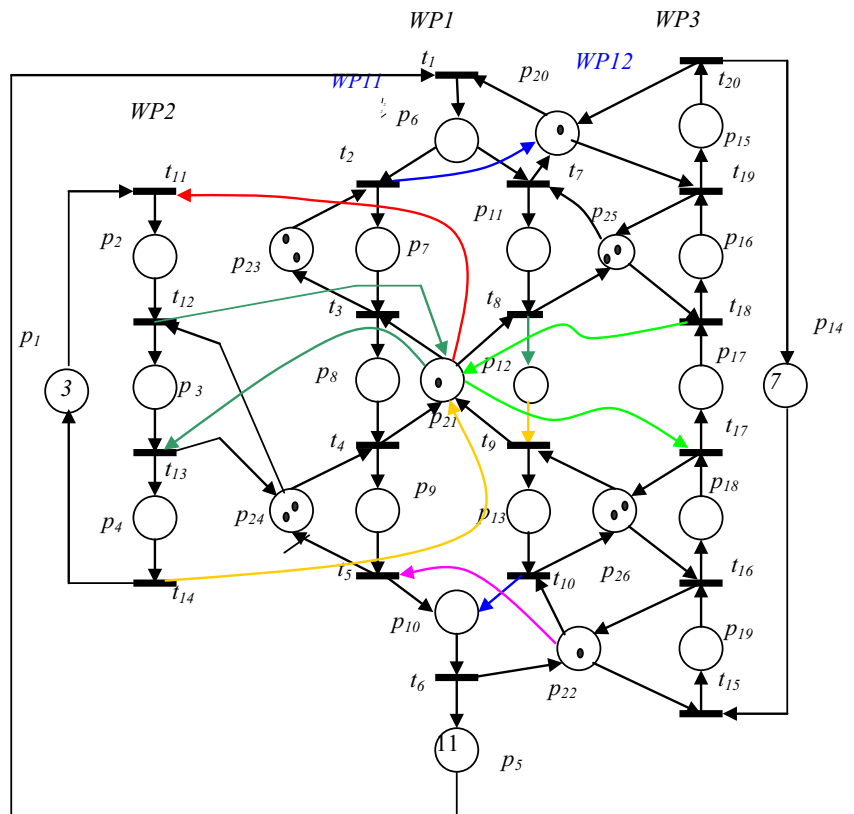
(i.e., boundary states to reach forbidden regions). There are 5,299 elements in the set of marking/transition-separation instances (MTSI) denoted by  $\Omega$ . This implies that 5,299 LPPs (linear programming problems) have to be solved to find an optimal liveness-enforcing supervisor with 21,581 reachable states in the controlled system. However,  $|\Omega|$  (cardinality of  $\Omega$ ) grows exponentially with respect to the size of a plant model. It is clearly infeasible to solve  $|\Omega|$  LPPs for either a sizable net or a small net with a large initial marking.

They further propose an iterative approach [4]. At each iteration, a first-met bad marking (FBM) is singled out from the reachability graph of a given Petri net model. The objective is to prevent this marking from being reached via a place invariant of the Petri net. A well-established invariant-based control method is used to derive a control place. This process is carried out until the net model becomes live. The proposed method is generally applicable, easy to use, effective, and straightforward although its off-line computation is of exponential complexity. Huang et al. apply the FBM method to a well-known  $S^3PR$  to achieve a near maximumally permissive control policy (21,562 states). The more FBM, the more monitors and control arcs will be added. The benchmark above requires more monitors and arcs than the one in [5], where several monitors using the FBM method are lumped into one.

Li et al. [5] propose a two-stage policy as described below. In the first stage, a monitor is added for each elementary siphon based on the method in [6, 13]. Next, the controlled net is still with some dead marking, and they utilize the theory of regions to control the deadlock problem. An MTSI is associated with each illegal or forbidden marking. It states the MTSIs play a guard role to prevent all possible markings from being entered into the dead zone. However, quite a few of the equations must be solved by the MTSI stage [5]. The resulting model is maximally permissive.

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Fig. 1 Benchmark 1



Huang et al. [1] propose novel crucial MTSIs (CMTSIs) to reduce the number of MTSIs and hence the computation burden. In addition, the number of control arcs is fewer. Experimental results show that the proposed control policy is the most efficient algorithm among the closely related approaches. Furthermore, the resulting model remains maximally permissive with no redundant monitors. It is quite time-consuming to remove redundant monitors using the method in [10]. However, Huang et al. offer no hints on why it employs fewer control arcs.

Intuitively fewer MTSI corresponds to fewer forbidden markings and hence, fewer control arcs are employed as mentioned earlier. However, both models (Huang et al. [1] and Li et al. [5]) employ the same number of monitors and are maximally permissive. This paper develops a new theory to explain the physics behind.

To shorten the paper, we assume the reader is familiar with basic terminologies of Petri nets in [6].

## 2 Formal derivation

We first give an intuition as to why fewer control arcs are employed in [1] followed by an example and a theory to explain it in more detail. In the sequel, we restrict marking to the set of operation places as in [4] using the FBM approach.

### 2.1 Intuitive explanation

The FBM method adds a monitor for each FBM. There may be a number of FBM for a siphon, and each one needs a monitor. If one lumps these monitors into a single one, fewer control places and arcs can be employed. However, the modified (called controller) region  $[V]$  ( $[V]$  plus  $V$  forms a  $P$ -invariant [17]) is larger (than that of marked operation places) and affects the uncontrolled model more; hence it reaches fewer states. The MTSI method considers all FBM while the CMTSI considers only a few equivalently lumping several FBM into the critical one. This roughly explains why the CTMSI method employs fewer arcs.

For the benchmark in Fig. 1, the FBM method by Uzam et al. results in the controlled model in Table 1. If one lumps  $\{V_6^1, V_6^2, V_6^3, V_6^4\}$  (resp.  $\{V_9^1, V_9^2\}$ ,  $\{V_{10}^1, V_{10}^2\}$ ,  $\{V_{11}, V_{12}, V_{12}\}$ ) into a single  $V_6$  (resp.  $V_9$ ,  $V_{11}$ ), the resulting model reaches fewer (21,363) [16] states than the FBM one (21,562) in [4], but with 11 monitors and 54 control arcs fewer than 19 monitors and 120 control arcs reported in [8].

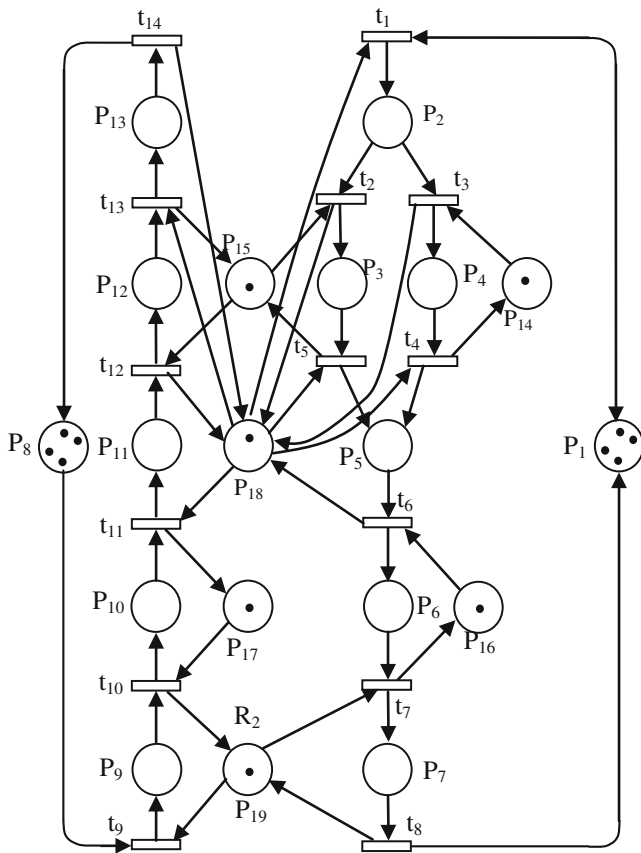
### 2.2 Example from [1]

The second example from [1] is shown in Fig. 2. Using the two-stage policy by Li et al., add monitor (places  $V_{S2}$ ,  $V_{S3}$ , and  $V_{S4}$ ) in the first stage for each elementary siphon. The partially controlled net is called (N2L1, M0).

**Table 1** Controlled model by Uzam et al. [8] for the benchmark in Fig. 1

$S$	$V(M_0)$	$V_S^*$	$\bullet V_S$	$[V_S]$
$S_1$	$V_1(2)$	$t_9, t_{15}$	$t_{10}, t_{16}$	$p_{13}, p_{19}$
$S_{10}$	$V_2(2)$	$t_3, t_{11}$	$t_4, t_{13}$	$p_2, p_3, p_8$
$S_{18}$	$V_3(2)$	$t_7, t_{17}$	$t_8, t_{18}$	$p_{11}, p_{17}$
$S_{16}$	$V_4(2)$	$t_8, t_{16}$	$t_9, t_{17}$	$p_{12}, p_{18}$
$S_{15}$	$V_5(3)$	$t_8, t_{15}$	$t_{10}, t_{17}$	$p_{12}, p_{13}, p_{18}, p_{19}$
$S_4$	$V_6^1(5)$	$t_3, t_{12}, t_{15}$	$t_5, t_{13}, t_{17}$	$p_3, p_8, p_9, p_{12}, p_{13}, p_{18}, p_{19}$
$S_4$	$V_6^2(5)$	$t_4, t_8, t_{11}, t_{15}$	$t_5, t_9, t_{13}, t_{17}$	$p_2, p_3, p_9, p_{12}, p_{18}, p_{19}$
$S_4$	$V_6^3(5)$	$t_4, t_9, t_{11}, t_{15}$	$t_5, t_{10}, t_{13}, t_{17}$	$p_2, p_3, p_9, p_{13}, p_{18}, p_{19}$
$S_4$	$V_6^4(5)$	$t_4, t_{11}, t_{15}$	$t_5, t_{13}, t_{17}$	$p_2, p_3, p_9, p_{18}, p_{19}$
$S_{17}$	$V_7(5)$	$t_1, t_{17}$	$t_3, t_8, t_{19}$	$p_6, p_7, p_{11}, p_{16}, p_{17}$
$S_{19}$	$V_8(3)$	$t_7, t_{16}$	$t_8, t_{17}$	$p_{11}, p_{18}$
$S_{20}$	$V_9^1(4)$	$t_7, t_{15}$	$t_9, t_{17}$	$p_{11}, p_{12}, p_{18}, p_{19}$
$S_{20}$	$V_9^2(4)$	$t_7, t_9, t_{15}$	$t_8, t_{10}, t_{17}$	$p_{11}, p_{13}, p_{18}, p_{19}$
$S_{21}$	$V_{10}^1(6)$	$t_3, t_7, t_{12}, t_{15}$	$t_5, t_8, t_{13}, t_{17}$	$p_3, p_8, p_9, p_{11}, p_{18}, p_{19}$
$S_{21}$	$V_{10}^2(6)$	$t_4, t_7, t_{11}, t_{15}$	$t_5, t_8, t_{13}, t_{17}$	$p_2, p_3, p_9, p_{11}, p_{18}, p_{19}$
$S_{22}$	$V_{11}(9)$	$t_1, t_9, t_{15}, t_{18}$	$t_5, t_8, t_{10}, t_{17}, t_{19}$	$p_6, p_7, p_8, p_9, p_{11}, p_{13}, p_{16}, p_{18}, p_{19}$
$S_{22}$	$V_{12}(9)$	$t_1, t_4, t_9, t_{15}$	$t_3, t_5, t_8, t_{10}, t_{18}$	$p_6, p_7, p_9, p_{11}, p_{13}, p_{17}, p_{18}, p_{19}$
$S_{22}$	$V_{13}(9)$	$t_1, t_4, t_{15}, t_{18}$	$t_3, t_5, t_9, t_{17}, t_{19}$	$p_6, p_7, p_9, p_{11}, p_{12}, p_{16}, p_{18}, p_{19}$

See Tables 1 and 2 in [15] for  $S_1$ – $S_{18}$ ,  $S_{19} = \{V_3, V_4, p_{12}, p_{17}\}$ ,  $S_{20} = \{V_4, V_1, p_{13}, p_{18}\}$ ,  $S_{21} = \{V_8, p_{21}, p_{22}, p_{24}, p_{17}, p_{10}, p_{12}\}$ , and  $S_{22} = \{V_6, V_7, p_2, p_3, p_{10}, p_{15}, p_{20}, p_{22}, p_{23}, p_{25}, p_{26}\}$ .

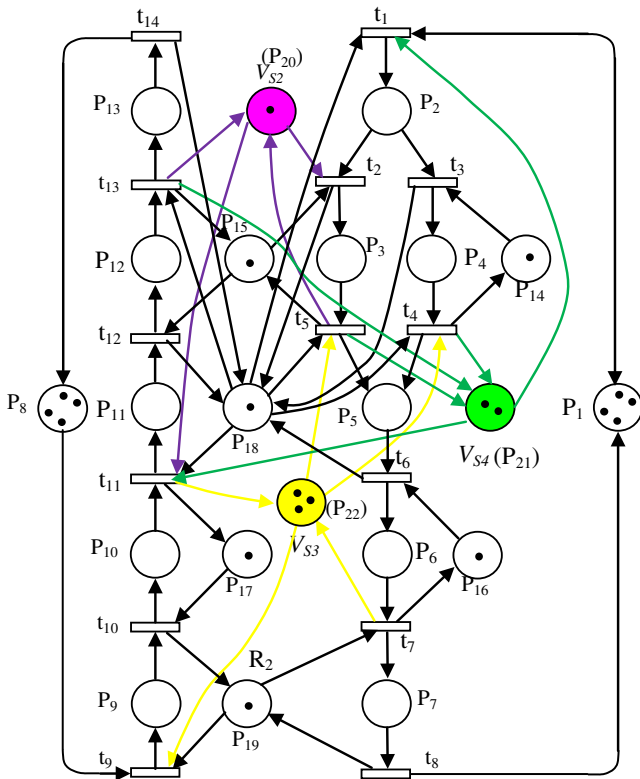


**Fig. 2** Benchmark 2 from [5]

It is worth to mention that eight MTSIs are found. It states the MTSIs play a guard role to prevent all possible markings from being entered into the dead zone. However, quite a few of the equations must be solved by the MTSI stage [5]. Huang et al. [1], however, only employs one CMTSI to find the final controlled model, which is much better than the above eight sets of MTSIs required by Li et al. More MTSI results in more refined controlled model and hence requires more control arcs. Theoretically, the CMTSI should employ fewer monitors than the MTSI method. But both methods employ six monitors. The next subsection will uncover this mystery.

### 2.3 Theory

All siphons for the net in Fig. 3 (synthesized from control circuits  $c_1 = [V_{S2} t_{11} V_{S3} t_5]$  and  $c_2 = [V_{S4} t_{11} V_{S3} t_5]$  based on the theory in [15] in the partial controlled model in Fig. 3) cannot be emptied based on the theory in [17]. Siphon  $S_8 = \{p_7, p_{11}, p_{12}, p_{17}, p_{19}, V_{S2}, V_{S3}\}$  (see Table 1 in [17]) is synthesized from the core subnet obtained by adding TP-handle  $H^{TP1}[t_{11} p_{17} t_{10} p_{19} t_7 V_{S3}]$  with resource place  $r=p_{19}$  upon  $c_3$ .  $S_8$  is emptiable as explained below.  $[S_8] = \{p_3, p_5, p_6, p_9, p_{10}\}$ ,  $M_0(p_{19})=1$ ,  $M_0(V_{S3})=3$ , and  $M_0(V_{S2})=1$ .  $\max M([S]) = M_0(C_S) = 3 + 1 = 4$  as explained below ( $C_S = \{V_{S2}, V_{S3}\}$  is the set of control places in  $S$ ). Firing  $t_9$  once grabs one token from each of  $p_{19}$  and  $V_3$ . The other two tokens in  $V_3$  go to  $p_{10}$  and  $p_6$ ,



**Fig. 3** Partial controlled model obtained at the first stage in [1, 5] for the net in Fig. 1

respectively. Now,  $M(p_{17}) = M(p_{19}) = M(V_3) = 0$ . The token in  $V_{S2}$  goes to  $p_3$  to make  $M(V_{S2})=0$ . The total number of tokens trapped in  $[S]$  is  $4 = M_0(C_S) = \sum_{V \in S} M_0(V)$  while  $M(S)=0$ . Siphon  $S_{10} = \{p_7, p_{11}, p_{12}, p_{17}, p_{19}, V_{S4}, V_{S3}\}$  (see Table 1 in [17]) is synthesized from the core subnet obtained by adding TP-handle  $H^{TP2}[t_{11} p_{17} t_{10} p_{19} t_7 V_{S3}]$  with resource place  $r=p_{19}$  upon  $c_3$ .  $S_{10}([S_{10}] = \{p_2, p_3, p_4, p_5, p_6, p_9, p_{10}\})$  is emptiable for a similar reason.

For  $S_8$ , two monitors are added as explained below for the two FBM:  $M_{\Psi_1} = p_3 + p_6 + p_9 + p_{10}$  and  $M_{\Psi_2} = p_3 + p_5 + p_9 + p_{10}$ , where  $\Psi^1 = \{p_3, p_6, p_9, p_{10}\}$  and  $\Psi^2 = \{p_3, p_5, p_9, p_{10}\}$ . Two monitors are added using the FBM method:  $V_{S8}^1$  (with  $[V_{S8}^1] = \Psi^1, M_0(V) = 4 - 1 = 3, \bullet V_{S8}^1 = \{t_5, t_7, t_{11}\}$ , and  $V_{S8}^{1\bullet} = \{t_2, t_6, t_9\}$ ) and  $V_{S8}^2$  (with  $[V_{S8}^2] = \Psi^2, M_0(V_{S8}^2) = 3, \bullet V_{S8}^2 = \{t_6, t_{11}\}$ , and  $V_{S8}^{2\bullet} = \{t_2, t_4, t_9\}$ ). They implement the following inequalities:

$$M(\Psi^1) = M(p_3) + M(p_6) + M(p_9) + M(p_{10}) < 4 = M_{\max}(\Psi^1) \tag{1}$$

and

$$M(\Psi^2) = M(p_3) + M(p_5) + M(p_9) + M(p_{10}) < 4 = M_{\max}(\Psi^2) \tag{2}$$

Only one monitor (as explained below) is added to the last emptiable siphon  $S_{10}$  with  $[S_{10}] = \{p_2, p_3, p_4, p_5, p_6, p_9, p_{10}\}$  even though it is a mixture siphon with a non-sharing place  $p_{16}$ . The only possible sets of marked operation places for the unmarked  $S_{10}$  are  $\Psi^1 = \{p_3, p_4, p_5, p_9, p_{10}\} \subset [S_{10}]$ ,  $\Psi^2 = \{p_3, p_4, p_6, p_9, p_{10}\} \subset [S_{10}]$ ,  $\Psi^3 = \{p_2, p_3, p_6, p_9, p_{10}\} \subset [S_{10}]$ , and  $\Psi^4 = \{p_2, p_4, p_6, p_9, p_{10}\} \subset [S_{10}]$  with markings  $M_{\Psi^1} = p_3 + p_4 + p_5 + p_9 + p_{10}$ ,  $M_{\Psi^2} = p_3 + p_4 + p_6 + p_9 + p_{10}$ ,  $M_{\Psi^3} = p_2 + p_3 + p_6 + p_9 + p_{10}$ , and  $M_{\Psi^4} = p_2 + p_4 + p_6 + p_9 + p_{10}$ , respectively; the number of tokens in  $\Psi^1$  or  $\Psi^2$  or  $\Psi^3$  is five.

By theory, four monitors are needed; one for each of above  $\Psi$ . However, only one monitor is needed as explained in [1] by exploring marking inequalities at the expense of extra steps. The following theorem helps to eliminate these extra steps.

*Theorem 1* Let  $S$  (resp.  $S'$ ) be a mixture siphon with the set of marked operation places  $\Psi$  (resp.  $\Psi'$ ) when  $S$  (resp.  $S'$ ) is unmarked under  $M_a$ .  $V_S$  (resp.  $V_{S'}$ ) is the monitor added to  $S$  (resp.  $S'$ ) with  $M_0(V_S) = M_{\max}(\Psi) - 1$  (resp.  $M_0(V_{S'}) = M_{\max}([\Psi']) - 1$ ) and  $H(V_S) = \Psi$  (resp.  $H(V_{S'}) = \Psi'$ ).  $\Psi \subset \Psi'$  and  $\forall M \in R(N, M_0)$ , such that  $M(\Psi') = M_{\max}(\Psi') \Rightarrow M(\Psi) = M_{\max}(\Psi)$ . Then  $V_{S'}$  is redundant.

*Proof* After adding  $V_S$ ,  $M(\Psi') = M(\Psi) + M(\Psi' \setminus \Psi) < M_{\max}(\Psi) - 1 + M_0(R(\Psi' \setminus \Psi)) = M_{\max}(\Psi') - 1$ ; hence,  $S'$  is controlled.

Now apply the developed theory to the example in Fig. 2. Set  $\Psi_{S_{10}}$  of marked operation places for the unmarked  $S_{10}$  may be a superset of those  $\Psi_{S_8}$  of  $S_8$  and not emptiable by theorem 1 since  $M(\Psi_{S_{10}}) = M(\Psi_{S_8}) + M(p_4) < M_{\max}(\Psi_{S_8}) + M_0(p_{14}) = M_{\max}(\Psi_{S_{10}})$  (by Eqs. 1 and 2), and  $S_{10}$  cannot become unmarked. For instance,  $\Psi_{S_8} = \{p_3, p_6, p_9, p_{10}\}$  and  $\Psi_{S_{10}} = \{p_2, p_3, p_6, p_9, p_{10}\}$ .

Let  $\Psi_S$  stand for  $\Psi$  of  $S$ . Then  $\Psi_{S_8}^1 \subset \Psi_{S_{10}}^1$ , and the condition in theorem 1 is satisfied; i.e.,  $\forall M \in R(N, M_0)$ , such that  $M(\Psi_{S_{10}}^1) = M_{\max}(\Psi_{S_{10}}^1) \Rightarrow M(\Psi_{S_8}^1) = M_{\max}(\Psi_{S_8}^1)$ . Thus, the monitor for  $\Psi_{S_{10}}^1$  is redundant. Similar conclusion

**Table 2** Siphon-based controlled model for the net in Fig. 2.

$S$	$V(M_0)$	$V_S^\bullet$	$\bullet V_S$	$[V_S]$
$S_2$	$V_{S2}(1)$	$t_2, t_{11}$	$t_5, t_{13}$	$p_3, p_{11}, p_{12}$
$S_3$	$V_{S3}(3)$	$t_4, t_5, t_9$	$t_7, t_{11}$	$p_5, p_6, p_9, p_{10}$
$S_4$	$V_{S4}(2)$	$t_1, t_{11}$	$t_4, t_5, t_{13}$	$p_2, p_3, p_4, p_{11}, p_{12}$
$S_8$	$V_{S8}^1(3)$	$t_2, t_6, t_9$	$t_5, t_7, t_{11}$	$p_3, p_6, p_9, p_{10}$
$S_8$	$V_{S8}^2(3)$	$t_2, t_4, t_9$	$t_6, t_{11}$	$p_3, p_5, p_9, p_{10}$
$S_{10}$	$V_{S10}^1(4)$	$t_1, t_6, t_9$	$t_2, t_4, t_7, t_{11}$	$p_2, p_4, p_6, p_9, p_{10}$

applies to  $\Psi_{S_{10}}^2$ .  $\Psi_{S_{10}}^3 = \{p_2, p_4, p_6, p_9, p_{10}\}$  is the only one such that  $\Psi_{S_8} \not\subset \Psi_{S_{10}}$  with marking  $M_{\Psi_{S_{10}}} = p_2 + p_4 + p_6 + p_9 + p_{10}$ ; the number of tokens in  $\Psi_{S_{10}}$  is five. Add monitor  $V_{S_{10}}(V_S \bullet = \bullet [\Psi^3] / [\Psi^3] \bullet = \{t_1, t_6, t_9\}$  and  $\bullet V_S = [\Psi^3] \bullet \setminus \bullet [\Psi^3] = \{t_2, t_4, t_7, t_{11}\}, S = S_{10}$ ). The final controlled model is shown in Table 2, the same as that in [1, 5]. The live controlled model can reach maximally permissive 205 states without weighted control arcs. Note that  $\Psi_{S_{10}}^3 \subset [S_{10}]$  and is more fragmented and hence uses more control arcs.

Furthermore, one can reduce the number of control arcs by setting  $[V]=[S]$  since only  $M_{\Psi_3}$  for  $S_{10}$  is reachable. The rest of unmarked-siphon states ( $M_{\Psi_1}$ ,  $M_{\Psi_2}$ , and  $M_{\Psi_4}$ ) have been prevented by the monitors for  $S_8$ . This explains why the controlled model by Hwang et al. employs fewer arcs.

### 3 Conclusion

Huang et al. [1] propose novel crucial MTSIs (CMTSIs) to reduce the number of MTSIs and hence the computation burden. In addition, the number of control arcs is fewer. Experimental results show that the proposed control policy is the most efficient algorithm among the closely related approaches while the resulting model remains maximally permissive. However, Huang et al. offer no hints on why it employs fewer control arcs. This paper thus develops a new theory to uncover the secret.

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