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Segment theory to compute elementary siphons in Petri nets for deadlock control

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Unlike other techniques, Li and Zhou add control nodes and arcs for only elementary siphons greatly reducing the number of control nodes and arcs (implemented by costly hardware of I/O devices and memory) required for deadlock control in Petri net supervisors. Li and Zhou propose that the number of elementary siphons is linear to the size of the net. An elementary siphon can be synthesized from a resource circuit consisting of a set of connected segments. We show that the total number of elementary siphons, $|\Pi_E|$, is upper bounded by the total number of resource places $|P_R|$ lower than that min($|P|$, $|T|$) by Li and Zhou where $|P|$ ($|T|$) is the number of places (transitions) in the net. Also, we claim that the number of elementary siphons $|\Pi_E|$ equals that of independent segments (simple paths) in the resource subnet of an S^3PR (systems of simple sequential processes with resources). Resource circuits for the elementary siphons can be traced out based on a graph-traversal algorithm. During the traversal process, we can also identify independent segments (i.e. their characteristic *T*-vectors are independent) along with those segments for elementary siphons. This offers us an alternative and yet deeper understanding of the computation of elementary siphons. Also, it allows us to adapt the algorithm to compute elementary siphons in [2] for a subclass of S^3PR (called S^4PR) to more complicated S^3PR that contains weakly dependent siphons.

Keywords: deadlock control; elementary siphons; flexible manufacturing systems; Petri nets; siphons; S^3PR

1. Introduction

Flexible Manufacturing System (FMS) has emerged over the past 20 years as a new type of the manufacturing system. An FMS is a computer controlled configuration to produce different products automatically. Typically an FMS consists of several machines to process concurrently different types of raw parts under a preestablished production sequence sharing a limited number of resources such as machines, AGVs, robots, buffers, and fixtures. The main tasks in designing an FMS include process routing, the selection of a sequence of operations, scheduling, and the assignment of time and resources. To effectively operate an FMS and meet its production objectives, the use of limited resources among various competing jobs has to be carefully controlled or coordinated. Deadlocks may occur in an FMS during its operation, which are undesirable phenomena in a highly automated FMS [20].

There are three approaches [1] to control deadlocks (1) deadlock detection and recovery, (2) deadlock avoidance, and (3) deadlock prevention. Recovery permits the occurrence of deadlocks, and when deadlocks detected, the system can restore to a normal state by simply reallocating the resources [7,19]. Avoidance [10,21] determines possible system evolutions at each system state using an on-line control policy and chooses the correct ones to proceed. Prevention establishes the control policy in a static way [9,14,16] by building freely a Petri net model first and then adding necessary control to it such that the controlled model is deadlock-free. Control places and related arcs are often used to achieve such purpose.

Prevention is preferred to avoidance because the computational effort is carried out once and off-line. Hence, it runs much faster in real-time cases compared with deadlock avoidance algorithms where much time is consumed by doing on-line each time the system ought to change the state. Deadlock prevention control policy is essential when it is unacceptable to have deadlocks and real time response time is critical. Although the number of minimum siphons grows exponentially with the size of the *PN*, in practical cases, as indicated by [8], it is not exponential.

A Petri net model is constructed for an FMS. The analysis of this *PN* model is conducted and system properties are claimed. The *PN* must satisfy three properties: boundedness, liveness, and reversibility [12]. These properties are critical for an FMS to operate in a stable, deadlock-free, and cyclic way.

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Liveness in FMS modeled by ordinary Petri nets (OPN) is closely related to emptiable siphons [9]. A siphon (trap, respectively) is a set of places where tokens can leak out (inject in, respectively). Once an emptiable siphon is found, output transitions of places in the siphon can never be fired. Hence, the net is not live.

Ezpeleta et al. proposed a class of *PN* called systems of simple sequential processes with resources (S^3PR) [9]. Liveness can be enforced by adding a control place – and associated arcs – to each emptiable siphon S to prevent S from becoming empty of tokens. However, this method generally requires adding too many control places and arcs to the original Petri net model. Iterative control methods in [11] find all emptiable siphons in each iteration step and add control places. The method becomes very difficult and complex even for a moderate-size model due to the fact that there are too many emptiable siphons.

Li and Zhou [13–17] proposed simpler Petri net controllers based on the concept of elementary siphons (generally much smaller than the set of all emptiable siphons in large Petri nets) to minimize the new addition of places, which incurs costly hardware of I/O devices and memory [17]. Emptiable siphons can be divided into two groups: elementary and dependent; characteristic T-vectors of the latter are linear combinations of that of the former.

They added a control place for each elementary siphon S_e without generating new emptiable siphons by the method developed in [9], while controlling all dependent emptiable siphons S too so that there is no need to add a control place for S. This leads to much fewer control places so that the method is suitable for large-scale Petri nets.

They [13] proved that the total number of elementary siphons is upper bounded by min($|P|$, $|T|$) where $|P|$ ($|T|$) is the total number of places (transitions) in the net. An elementary siphon can be synthesized from a resource circuit consisting of a set of connected segments. We show that the total number of elementary siphons, $|\Pi_E|$, is upper bounded by the total number of resource places $|P_R|$ lower than that min($|P|$, $|T|$) by Li and Zhou. We claim that the number of elementary siphons $|\Pi_E|$ equals that of independent segments (simple paths) in the resource subnet of an S^3PR. Resource circuits for the elementary siphons can be traced out based on a graph-traversal algorithm. During the traversal process, we can also identify independent segments (i.e. their characteristic T-vectors are independent) along with those segments for elementary siphons.

This offers us an alternative and yet deeper understanding of the computation of elementary siphons. Also it allows us to adapt the algorithm to compute elementary siphons in [2] for a subclass of S^3PR (called S^4PR) to more complicated S^3PR, where the dependent siphons may be weakly rather than strongly in [2] for S^4PR. Please refer to [3,4] for the authors' other works on weakly dependent siphons.

The rest of this paper is organized as follows. Section 2 presents the preliminaries about *PN* and S^3PR nets. Moreover, we show that a siphon can be synthesized by constructing handles upon a strongly connected component of the resource subnet (called sub-SCRC). In this section, we also define characteristic T-vectors, elementary, and dependent siphons. Section 3 presents segment theory as the basis to understand the approach described in Section 4 under some assumption to make it easier to understand. Section 5 removes the assumption by considering some twists. Section 6 applies the theory to a well-known S^{3PR}. Section 7 concludes the paper. To improve the readability, some proofs are moved to Appendix I.

2. Preliminaries

In this paper, we assume that the reader is familiar with the *PN* basis [7]. Here we present only the definitions that are used in this paper.

Definition 1: An OPN is a 4-tuple $PN = (N, M_0) = (P, T, F, M_0)$, where $N = (P, T, F)$ is a net, $P = \{p_1, p_2, \ldots, p_a\}$ a set of places, $T = \{t_1, t_2, \ldots, t_b\}$ a set of transitions, with $P \cup T \neq \emptyset$ and $P \cap T = \emptyset$, and F: $(P \times T) \cup (T \times P) \rightarrow \{0, 1\}$ is the flow relation. A node x in $N = (P, T, F)$ is either *a* $p \in P$ or *a* $t \in T$. The post-set of node x is $x^\bullet = \{y \in P \cup T | F(x, y) > 0\}$, and its pre-set $^\bullet x = \{y \in P \cup T | F(y, x) > 0\}$. An OPN is called a state machine (SM) if $\forall t \in T$, $|t^\bullet| = |^\bullet t| = 1$.

Definition 2: A P-vector (denoted by $\Sigma L(p)p$) is a column vector L: $P \rightarrow Z$ indexed by P and a T-vector ($\Sigma J(t)t$) is a column vector J: $T \rightarrow Z$ indexed by T, where Z is the set of integers. The incidence matrix of N is a matrix A: $P \times T \rightarrow Z$ indexed by P and T such that $A(p, t) = -1$, if $p \in {}^\bullet t \backslash t^\bullet$; $A(p, t) = 1$, if $p \in t^\bullet \backslash {}^\bullet t$; and otherwise $A(p, t) = 0$ for all $p \in P$ and $t \in T$. $N' = (P', T', F')$ is called a subnet of N where $P' \subseteq P, T' \subseteq T$, and $F' = F \cap ((P' \times T') \cup (T' \times P'))$. A net N is strongly connected iff for every node pair (n_i, n_j), n_i, $n_j \in P \cup T$, there exists a directed path from n_i to n_j. Let $N_1 = (P_1, T_1, F_1)$ and $N_2 = (P_2, T_2, F_2)$, then $N_1 \cup N_2 = (P_1 \cup P_2, T_1 \cup T_2, F_1 \cup F_2)$ and $N_1 \cap N_2 = (P_1 \cap P_2, T_1 \cap T_2, F_1 \cap F_2)$. If $N_3 = N_1 \cup N_2$, then $N_1 = N_3 \backslash N_2$ denotes the removal of N_2 from N_3.

Definition 3: $\Gamma = [n_1\ n_2 \ldots n_k] \in (P \cup T)^*$ is a path, iff $n_{i+1} \in n_i^\bullet$; $I = 1, \ldots, k-1$. Furthermore, a path Γ

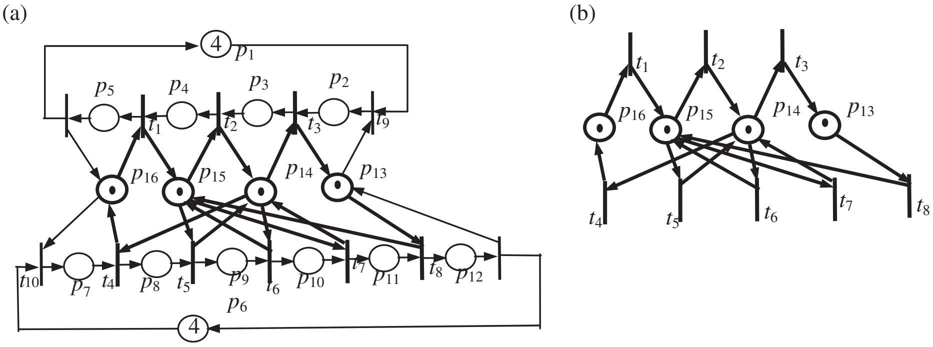

Figure 1. (a) Example of an S^3PR [16], where $c_1=[p_{15}t_2p_{14}t_6p_{15}]$ is an elementary circuit and $[p_{15}t_5p_{14}]$ and $[p_{15}t_7p_{14}]$ are PP'-handles of c_1. (b) Maximal resource subnet N_u.

is characterized as a circuit iff $x_1=x_n$. Γ is an elementary directed path in N if $\forall(i,j)$, $1\le i<j<k$, $n_i\ne n_j$. Γ is (non) virtual if it is elementary and $k=2$ ($k>2$); i.e. it contains only (more than) two nodes. $\forall i$ defined above, $|n_i^\bullet|=|{}^\bullet n_i|=1$ in N, iff $1<I<k$, (i.e. n_i is an interior node of Γ), then Γ is called a simple path or a segment with terminal nodes n_1 and n_k. $\Gamma=\Gamma_1\Gamma_2$ indicates that Γ_1 adjoins Γ_2 to form Γ; i.e. $\Gamma=\Gamma_1\cup\Gamma_2$ and Γ_1 intersects Γ_2 at a single node.

Definition 4: A place vector Y (with components $Y(p)$, $p\in P$) is called a P-invariant iff $Y\ne 0$ and $Y^T\bullet A=0$, where A is the incidence matrix. $\|Y\|=\{p\in P|Y(p)\ne 0\}$ is the support of Y. A P-invariant is minimal if there does not exist a P-invariant Y' such that $\|Y'\|\subseteq\|Y\|$. A siphon (trap) D (τ) is a non-empty subset of places such that ${}^\bullet D\subseteq D^\bullet$ ($\tau^\bullet\subseteq{}^\bullet\tau$), that is, every transition having an output (input) place in D (τ) has an input (output) place in D (τ). A minimal siphon does not contain a siphon as a proper subset. A minimal siphon is called a strict minimal siphon (*SMS*), denoted by S, if it does not contain a trap.

Definition 5: A subnet $N_i=(P_i,T_i,F_i)$ of N is generated by $X=P_i\cup T_i$, if $F_i=F\cap(X\times X)$. It is an I–subnet, denoted by I, of N if $T_i={}^\bullet P_i$. I_S is the I–subnet (the subnet derived from (S, ${}^\bullet S$)) of an SMS S. Note that $S=P(I_S)$; S is the set of places in I_S.

2.1 *S³PR*

We add *bold texts* for new terms to the following definitions [9]. The reader is referred to [9] for more details of the S^3PR model.

Definition 6 [9]: A simple sequential process (S^2P) is a net $N=(P\cup\{p^0\},T,F)$ where: (1) $P\ne\varnothing$, $p^0\notin P$ (p^0 is called the process idle or initial or final operation place); (2) N is strongly connected SM, and (3) every circuit of N contains the place p^0.

Definition 7 [9]: A simple sequential process with resources (S^2PR), also called a working processes (WP), is a net $N=(P\cup\{p^0\}\cup P_R,T,F)$ so that (1) the subnet generated by $X=P\cup\{p^0\}\cup T$ is an S^2P; (2) $P_R\ne\varnothing$ and $P\cup\{p^0\}\cap P_R=\varnothing$; (3) $\forall p\in P$, $\forall t\in{}^\bullet p$, $\forall t'\in p^\bullet$, $\exists r_p\in P_R$, ${}^\bullet t\cap P_R=t'^\bullet\cap P_R=\{r_p\}$; (4) The two following statements are verified: $\forall r\in P_R$, *a)* ${}^{\bullet\bullet}r\cap P=r^{\bullet\bullet}\cap P\ne\varnothing$; *b)* ${}^\bullet r\cap r^\bullet=\varnothing$. (5) ${}^{\bullet\bullet}(p^0)\cap P_R=(p^0)^{\bullet\bullet}\cap P_R=\varnothing$. $\forall p\in P$, p is called an operation place. $\forall r\in P_R$, r is called a resource place. $H(r)={}^{\bullet\bullet}r\cap P$ denotes the set of holders of r (operation places that use r). Any resource r is associated with a minimal P-invariant whose support is denoted by $\rho(r)=\{r\}\cup H(r)$.

Definition 8 [9]: A system of S^2PR (S^3PR) is defined recursively as follows: (1) An S^2PR is defined as an S^3PR; (2) let $N_i=(P_i\cup P_i^0\cup P_{Ri},T_i,F_i)$, $i\in\{1,2\}$ be two S^3PR so that $(P_1\cup P_1^0)\cap(P_2\cup P_2^0)=\varnothing$. $P_{R_1}\cap P_{R_2}=P_C(\ne\varnothing)$ and $T_1\cap T_2=\varnothing$. The net $N=(P\cup P^0\cup P_R,T,F)$ resulting from the composition of N_1 and N_2 via P_C (denoted by $N_1\,\mathrm{o}\,N_2$) defined as follows: (1) $P=P_1\cup P_2$; (2) $P^0=P_1^0\cup P_2^0$; (3) $P_R=P_{R_1}\cup P_{R_2}$; (4) $T=T_1\cup T_2$, and (5) $F=F_1\cup F_2$ is also an S^3PR. A path (circuit, subnet) Γ (c, N') in N is called a resource path (circuit, subnet) if $\forall p\in\Gamma$ (c,N'), $p\in P_R$. A resource subnet N_u is maximal if $\nexists$ a resource subnet N' such that N_u is a proper subnet of N'. A strongly connected resource subnet of N is briefed as SCRS. An SCRS is called a basic circuit if it is an elementary circuit.

An example of S^3PR and its maximal resource subnet N_u are shown in Figure 1(a) and (b), respectively. Note that a resource circuit is also an SCRS. We construct an SMS based on the concept of handles.

Definition 9 [5]: Let $N=(P,T,F)$ be a net. $H_1=[n_s n_1 n_2\ldots n_k n_e]$ and $H_2=[n_s n_1' n_2'\ldots n_h' n_e]$ are elementary directed paths, n_i, $n_j'\in P\cup T$, $I=1,2,\ldots,k$, $j=1,2,\ldots,h$. H_1 and H_2 are said to be mutually complementary since H_1 and H_2 have the same terminal nodes: start node n_s and end node n_e. Each of H_1 and H_2 is called a handle in N if $n_i\ne n_j'$, $\forall i,j$ defined above; n_s (n_e) is called a terminal node or the start (the end node) of H_1 and H_2. n_i and n_j' ($1 \le I \le k$, $1 \le j \le h$) are called the interior nodes of H_1 and H_2, respectively. $P_{\text{in}}(H_1) = \{p | p \in P, p \in H_1, p \ne n_s, p \ne n_e\}$ is the set of interior (not terminal) places of H_1. Note that n_s and n_e may be identical. H^{xy} is a XY-handle where X and Y can be T or P. X is T (P) if $n_s \in T$ ($n_s \in P$). Y is T (P) if $n_e \in T$ ($n_e \in P$). H_1 is a resource handle if all places in H_1 are resource places. The handle H to a subnet N' (similar to the handle of a tea pot) is an elementary directed path from n_s in N' to another node n_e in N'; any other node in H is not in N'. H is said to be a handle in $N'' = N' \cup H$. A (non) PP'-handle is a PP-handle (not) of the form $[r\,t\,r']$, r, $r' \in P_R$. N_i^e denotes an expanded subnet N_i by adding all PP'-handles to N_i.

In Figure 1, resource handles H_1^{PP} $[p_{14}\ t_4\ p_{16}\ t_1\ p_{15}]$ and H_2^{PP} $[p_{14}\ t_6\ p_{15}]$ are mutually complementary; p_{14} and p_{15} are the start node and the end node, respectively; p_{16} is the interior node of H_1^{PP}. In Figure 1(b), $N_u = N' \cup H$, where $H = [p_{15} t_5 p_{14}]$ is a PP'-handle. In [2], we construct an SMS of an arbitrary S^{3}PR by building handles upon an SCRS υ. Two different SCRS with the set of resource places correspond to the same SMS [2]. PP'-handles does not contain interior places and hence we have

Property 1: N_i and N_i^e (Definition 9) have the same set of resource places and correspond to the same SMS.

Handle-construction procedure [2]: Given an SCRS υ, (1) add all PP'-handles; that is, of the form $[r_1\ t'\ r_2]$, $r_1 \in \upsilon$ and $r_2 \in \upsilon$, to υ. The resulting SCRS is called an **expanded** SCRS υ'; (2) add all PP-, TP- and, virtual PT-handles (that are part of an $I(\rho(\text{r}))$ to υ' to form υ''. (3) $P(\upsilon'')$ is an SMS if it does not contain an $\rho(r)$, $r \in P(\upsilon)$.

Example: For the net in Figure 1, first find resource circuit $c_1 = [p_{15} t_2 p_{14} t_6 p_{15}]$ (a circuit is strongly connected; hence, it is an SCRS) and construct S_2 on expanded c_1 (by adding PP'-handles $[p_{15} t_5 p_{14}]$ and $[p_{15} t_7 p_{14}]$). Second add TP-handles $[t_2 p_4 t_1 p_{15}]$ and $[t_7 p_{11} t_8 p_{15}]$ to get I_{S_1} and $S_1 = P(I_{S_1}) = \{p_4, p_{11}, p_{14}, p_{15}\}$. The rest SMS of $S_2 = \{p_4, p_{12}, p_{13}, p_{14}, p_{15}\}$, $S_3 = \{p_5, p_{11}, p_{14}, p_{15}, p_{16}\}$, and $S_4 = \{p_5, p_{12}, p_{13}, p_{14}, p_{15}, p_{16}\}$ can be constructed similarly.

2.2 *Elementary siphons and characteristic T-vectors*

This section defines elementary, dependent siphons and characteristic T-vectors.

Definition 10 [15]: Let $\Omega \subseteq P$ be a subset of places of N. P–vector λ_Ω is called the characteristic P-vector of Ω iff $\forall p \in \Omega$, $\lambda_\Omega(p) = 1$; otherwise $\lambda_\Omega(p) = 0$. η is called the characteristic T-vector of Ω, if $\eta^T = \lambda_\Omega^T \bullet A$, where A is the incidence matrix.

Table 1. Four SMS in Figure 1 and there η. $\eta_4 = \eta_2 + \eta_3 - \eta_1$.

SMS	η	Set of places
S_1	$[+t_2 - t_3 - t_4 + t_7]$	$\{p_4, p_{11}, p_{14}, p_{15}\}$
S_2	$[+t_2 - t_4 + t_8 - t_9]$	$\{p_4, p_{12}, p_{13}, p_{14}, p_{15}\}$
S_3	$[+t_1 - t_3 + t_7 - t_{10}]$	$\{p_5, p_{11}, p_{14}, p_{15}, p_{16}\}$
S_4	$[+t_1 + t_8 - t_9 - t_{10}]$	$\{p_5, p_{12}, p_{13}, p_{14}, p_{15}, p_{16}\}$

Physically, the firing of a transition t where ($\eta(t) > 0$, $\eta(t) = 0$, and $\eta(t) < 0$) increases, maintains, and decreases the number of tokens in S.

Definition 11 [13]: Let $N = (P, T, F)$ be a net with $|P| = m$, which has k SMS $S_1, S_2, \ldots, S_k$, $m, k \in \text{IN}$, where $\text{IN} = \{0, 1, 2, \ldots\}$. Define $[\lambda]_{k \times m} = [\lambda_1 | \lambda_2 | \cdots | \lambda_k]^T$ and $[\eta]_{k \times n} = [\eta_1 | \eta_2 | \cdots | \eta_k]^T$. $[\lambda]$ ($[\eta]$) is called the characteristic $P(T)$-vector matrix $[\lambda]$ ($[\eta]$) of the siphons in N. Let $\eta_{S\alpha}$, $\eta_{S\beta}, \ldots,$ and $\eta_{S\gamma}$ ($\{\alpha, \beta, \ldots, \gamma\} \subseteq \{1, 2, \ldots, k\}$) be a linear independent maximal set of matrix $[\eta]$. Then $\Pi_\text{E} = \{S_\alpha, S_\beta, \ldots, S_\gamma\}$ is called a set of elementary siphons. $S \notin \Pi_\text{E}$ is called a strongly dependent siphon if $\eta_S = \sum_{S_i \in \Pi_\text{E}} a_i \eta_{S_i}$ where $a_i \ge 0$. $S \notin \Pi_\text{E}$ is called a weakly dependent siphon if $\exists$ non-empty $A, B \subset \Pi_\text{E}$, such that $A \cap B = \varnothing$ and $\eta_S = \Sigma_{S_i \in A}\ a_i \eta_{S_i} - \Sigma_{S_j \in B} b_j \eta_{S_j}$ where a_i, $b_j > 0$.

In Figure 1, there are three elementary siphons $S_1 - S_3$ and 1 weakly dependent siphon S_4; their characteristic T-vectors η are shown in Table 1.

Li and Zhou [15] proposed to find elementary siphons based on all SMS. First, they construct the characteristic $P(T)$-vector matrix $[\lambda]$ ($[\eta]$) of the siphons in N followed by finding linearly independent vectors in $[\lambda]$ ($[\eta]$). Note that $[\lambda] = [\lambda_1 | \lambda_2 | \cdots | \lambda_k]^T$ and $[\eta] = [\eta_1 | \eta_2 | \cdots | \eta_k]^T$, $k = |\Pi|$. Finally, the siphons corresponding to these independent vectors are the elementary siphons in the net system. But they have to look for all SMS, the number of which grows exponentially.

Similarly, we propose ζ_R based on the set of resource places in S.

Definition 12 [2]: Let $R \subseteq P$ be a subset of resource places of N, and λ_R the characteristic P-vector of R. ζ_R is called the characteristic T-vector for R, if $\zeta_R^T = \lambda_R^T \bullet A$.

Physically, the sets where $\zeta_R > 0$, $\zeta_R = 0$, and $\zeta_R < 0$, are the sets of transitions whose firings increase, maintain and decrease the number of tokens in R. Note that ζ_R is a linear sum of all ζ_r. In the sequel, all η referred will be for SMS in contrast to ζ for the set of resource places in the SMS.

Lemma 1 [2]: $\zeta_R = \Sigma_{r \in R}\ \zeta_r$ *where R is the set of all resource places in a resource subnet.*

Table 2. The segments, $\delta(\Gamma)$, segment η, and the paths for the net in Figure 1.

Sements	$\delta(\Gamma)$	η	Paths
Γ_1	$p_4, p_8, p_9, p_{10}, p_{11}, p_{14}, p_{15}$	$[+t_2 - t_3]$	$[p_{15}\ t_2\ p_{14}]$
Γ_2	$p_3, p_4, p_8, p_{10}, p_{11}, p_{14}, p_{15}$	$[-t_5 + t_6]$	$[p_{14}\ t_6\ p_{15}]$
Γ_3	$p_3, p_4, p_9, p_{10}, p_{11}, p_{14}, p_{15}$	$[-t_4 + t_5]$	$[p_{15}\ t_5\ p_{14}]$
Γ_4	$p_3, p_4, p_8, p_9, p_{10}, p_{12}, p_{13}, p_{14}, p_{15}$	$[+t_3 - t_7 + t_8 - t_9]$	$[p_{14}\ t_3\ p_{13}\ t_8\ p_{15}]$
Γ_5	$p_3, p_5, p_8, p_9, p_{10}, p_{11}, p_{14}, p_{15}, p_{16}$	$[+t_1 - t_2 + t_4 - t_{10}]$	$[p_{14}\ t_4\ p_{16}\ t_1\ p_{15}]$
Γ_6	$p_3, p_4, p_8, p_9, p_{11}, p_{14}, p_{15}$	$[-t_6 + t_7]$	$[p_{15}\ t_7\ p_{14}]$

Thus, it is easy to compute ζ in an algebraic fashion given a graphic SCRS since ζ_R is a linear sum of all ζ_r. In [2], we proposed an algorithm based on Lemma 1 to compute SMS from ζ_R. It seems that

Observation 1: Given an S^3PR, the number of elementary siphons is upper bounded by $|P_R|$, the number of resource places in the net.

This is better than that by Li and Zhou as shown in the following. Note that the computed siphon from ζ_R may not be minimal. To simplify the presentation, we ignore such a twist and assume that each siphon synthesized is minimal and an SMS. Afterwards, we will come back to discuss it.

Lemma 2 [13]: *The number of elements in any set of elementary siphons in net N equals rank([η]), where rank([η]) is the rank of [η].*

Theorem 1 [13]: *Let $|\Pi_E|$ be the number of elementary siphons in net $N=(P, T, F)$. Then we have $|\Pi_E| \le min\{|P|, |T|\}$.*

In the sequel, we will show that $|\Pi_E|$ is closely related to the number of segments.

3. Segment theory

Let S_0 be a strongly dependent siphon, $S_1, \ldots, S_n$ be elementary siphons, and $\eta_{S_0} = \eta_{S_1} + \eta_{S_2} + \cdots + \eta_{S_n}$. In [13], we show that the SCRS for S_0, N_{S_0}, is a compound resource circuit containing those SCRS for $N_{S_1}, \ldots, N_{S_n}$ and the intersection between any two N_{S_i} and N_{S_j}, $I \neq j$ is at most a resource place. This, as will be shown later (Theorem 3), can be generalized to resource paths, called *segments*: $\eta_{\Gamma_0} = \eta_{\Gamma_1} + \eta_{\Gamma_2} + \cdots + \eta_{\Gamma_n}$ where Γ_0 is a compound segment containing $\Gamma_1, \ldots, \Gamma_n$ and the intersection between any two Γ_i and Γ_j, $i \neq j$ is at most a resource place. η of a segment is defined similarly to that for an SMS. Similar to what we build I_S by constructing handles upon an SCRS, we can build I_Γ by constructing handles upon Γ_I and find $P(I_\Gamma) = \delta(\Gamma)$, called a partial siphon since $P(I_S) = S$ is a siphon. $P(I_\Gamma)$ is a set of places; we find its characteristic P-vector and T-vector $\eta_{\Gamma 0}$ based on Def. 10.

Table 3. The SMS and their η for the net in Figure 1.

SMS	$\eta_c = \Sigma \eta_\Gamma$	Segments
S_1	$[+t_2 - t_3 - t_4 + t_7]$	$\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_6$
S_2	$[+t_2 - t_4 + t_8 - t_9]$	$\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_6$
S_3	$[+t_1 - t_3 + t_7 - t_{10}]$	$\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_5, \Gamma_6$
S_4^*	$[+t_1 + t_8 - t_9 - t_{10}]$	$\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5, \Gamma_6$

Table 2 shows the segments, $\delta(\Gamma)$, segment η, and the paths for the net in Figure 1. We do not add PP'-handles (of the form $[r\ t\ r']$) to Γ above because they do not add new places to $\delta(\Gamma)$.

Note that the algorithm in [2] would not work here since the net in Figure 1 is not an S^4PR defined in [2] due to the presence of weakly dependent siphons. The segment theory developed here helps to compute elementary siphons for S^3PR that contains weakly dependent siphons.

Formally, we generalize the definition of the characteristic T-vector to a partial siphon defined in the following.

Definition 13: Let Γ be a segment; that is resource path. A partial siphon, $\delta(\Gamma)$ built on Γ, is defined to be set of places in Γ plus those in the TP- and PP-handles to Γ; i.e. $\delta(\Gamma) = P(\Gamma) \cup \{p | p \in \rho(r),\ r \in P(\Gamma),\ p \in H^{PP} \cup H^{TP}\} = P(I_\delta)$, where I_δ is the I-subnet of $\delta(\Gamma)$. $\eta^T(\Gamma) = \lambda_S^T \bullet A$ is the characteristic T-vector of $\delta(\Gamma)$, λ_δ is the characteristic P-vector of $\delta(\Gamma)$, and A is the incidence matrix. $\eta(\Gamma)$ is called a segment η. In contrast, those corresponding to connected-component as well as that by the sub-SCRS are called connected-component η (CC η) and SCRS (or SMS) η, respectively. The set of all SMS (segment) η is called the SMS (segment) space.

In Figure 1, each resource circuit c is a compound segment containing $\Gamma_1, \ldots, \Gamma_n$ and the intersection between any two Γ_i and Γ_j, $i \neq j$ is at most a resource place. We have as shown in Table 3, $\eta_c = \eta_{\Gamma_1} + \eta_{\Gamma_2} + \cdots + \eta_{\Gamma_n}$.

Definition 14: $\eta_1, \eta_2, \ldots, \eta_q$ are mutually independent if none can be expressed as linear combinations of others. The maximum set of mutually independent η is called a basis. Γ_1, $\Gamma_2, \ldots, \Gamma_q$ are mutually independent iff their η_1, $\eta_2, \ldots, \eta_q$ are.

It is unclear how to find new independent segments at each synthesis step. Based on segment η, we can compute connected-component η (CC η) and SCRS η using the following theorem.

Theorem 3: *For an* S^3PR, $\eta_3 = \eta_1 + \eta_2$ *if* $\exists r \in R$, $\Gamma_1 \cap \Gamma_2 = \{r_1\}$, *where* Γ_1 *and* Γ_2 *are two directed resource paths,* η_1, η_2, *and* η_3 *are the characteristic T-vectors of* Γ_1, Γ_2, *and* Γ_3, *respectively, and* $\Gamma_3 = \Gamma_1\Gamma_2$.

The above results can be generalized to cases where the intersection contains more than one place, but no common arcs.

Corollary 1: *For an* S^3PR, $\eta_3 = \eta_1 + \eta_2$ *if* $\exists r \in R$, $\Gamma_1 \cap \Gamma_2 = \beta \subseteq P_R$ *and* Γ_1 *and* Γ_2 *do not have any common arcs, where* Γ_1 *and* Γ_2 *are two directed resource paths,* η_1, η_2 *and* η_3 *are the characteristic T-vectors of* Γ_1, Γ_2, *and* Γ_3, *respectively.*

Lemma 3: *The* η *of any segment* Γ_1 (*which is not a* PP'*-handle*) *in an* S^3PR *is independent w.r.t. those of all other segments.*

***Corollary 2*:** *If a new SCRS* υ_1 *contains new segment* Γ *not in the SCRS of all old elementary siphons, then* η *of the SMS built on* υ_1, η_1, *is independent w.r.t those of all old elementary siphons.*

4. The approach

We may find the exact number of elementary siphons by finding the set of independent characteristic T-vectors from that of all SMS. But the time complexity is exponential as mentioned earlier. Alternatively, we propose a knitting-technique approach to incrementally find all SCRS for elementary siphons. Thus, it is more efficient than that in [13] where the computation of elementary siphons cannot be started until all SMS have been found.

We will find elementary siphons in an incremental fashion. At each step, we show that the number of new elementary siphons equals that of new independent segments. It holds till the end and we thus prove that the exact number of elementary siphons equals that of independent segments, which grows linearly w.r.t. $|P_R|$ – the number of resource places in N.

We first find a circuit $c_1 = [p_{15} t_2 p_{14} t_6 p_{15}]$ (Figure 1) and construct elementary siphon S_1 on expanded c_1 (by adding PP'-handles $[p_{15}\ t_5\ p_{14}]$ and $[p_{15}\ t_7\ p_{14}]$), denoted by c_1^e. Next we add $H_1 = [p_{14} t_3 p_{13} t_8 p_{15}]$ (p_{13} is the only interior node.) with complementary handle (Definition 9)

Table 4. Synthesis process for the net in Figure 1 and η of the new elementary siphon.

Step	Characteristic T-vectors η	η
1 Expanded c_1	$c_1(=\Gamma_1\Gamma_2)+$ PP'-handles (Γ_3, Γ_6)	η_{S_1}
2 H_1	Γ_4	$\eta_{S_2} = \eta_{S_1} + \eta_{H_1}$
3 H_2	Γ_5	$\eta_{S_3} = \eta_{S_1} + \eta_{H_2}$

$H_1^c = [p_{14}\ t_6\ p_{15}]$ which is a PP'-handle with no interior node (Definition 9). Now we have a new SCRS $c_2 = [p_{15}\ t_2\ p_{14}\ t_3\ p_{13}\ t_8\ p_{15}] = (c_1 \cup H_1)\backslash H_1^c$ corresponding to new elementary siphon S_2. The new SCRS $\upsilon^* = c_1^e \cup c_2$ is the net traced (N^t) so far.

Next we add $H_2 = [p_{14}\ t_4\ p_{16}\ t_1\ p_{15}]$ with $H_2^c = H_1^c = [p_{14}\ t_6\ p_{15}]$ that is a PP'-handle. Now we have a new SCRS $c_3 = [p_{14}\ t_4\ p_{16}\ t_1\ p_{15}\ t_2\ p_{14}] = (c_1 \cup H_2)\backslash H_2^c$ and construct elementary siphon S_3 on expanded c_3 (by adding PP'-handles $[p_{15}\ t_5\ p_{14}]$, $[p_{15}\ t_6\ p_{14}]$ and $[p_{15}\ t_7\ p_{14}]$), denoted by c_3^e. No more handles can be added and the only SCRS left is the whole traced net corresponding to dependent siphon S_4. Table 4 summarizes the above process.

In short, an SCRS can be initiated from a circuit followed by continuously adding PP-handles H_i. Each time we add a new H_i, we find new elementary siphons. Note that H_i cannot be a PP'-handle since a PP'-handle does not add a new resource place to the traced resource subnet and based on Property 1, no new SMS is generated.

H_i added at Step I is a new independent segment. We discuss how new segments can be generated in addition to H_i. Upon each new handle H_i ($[r_7\ t_{11}\ r_{10}\ t_{12}\ r_5]$ in Figure 2(b)) with at least one new place (r_{10}), we build an elementary siphon based on an elementary circuit ($[r_1 t_7 r_7 t_{11} r_{10} t_{12} r_5 t_5 r_6 t_6 r_1]$) that contains H_i.

If both $n_s(H_i)$ and $n_e(H_i)$ fall on terminal nodes of segments, based on Corollary 2, η of the new SMS constructed on an elementary circuit c^o containing H_i (a new segment) is independent w.r.t existing ones. Besides H_i, c^o contains other existing segments that can be expressed as linear combinations of characteristic T-vectors of old elementary siphons. Thus, there is only one new independent segment corresponding to one new elementary siphon.

Lemma 4: *If both* $n_s(H_i)$ *and* $n_e(H_i)$ *are on terminal nodes of segments other than* H_i *itself, then* H_i *is the only one new independent segment.*

Proof: Obvious from the above discussion.

On the other hand, if $n_s(H_i)$ ($n_e(H_i)$) is in the middle of an existing segment Γ^s (Γ^e), the presence of $n_s(H_i)$ ($n_e(H_i)$) may break Γ^s (Γ^e) into two

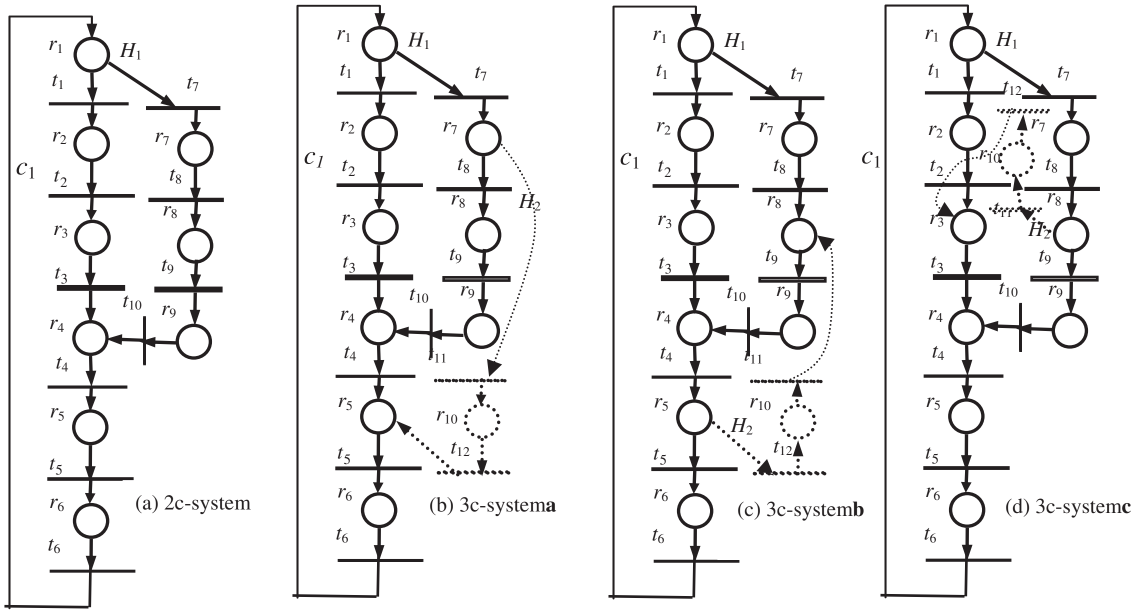

Figure 2. Examples of 2c-system and 3c-systems.

new ones. In Figure 2(c), $n_s(H_i) = r_5$ ($n_e(H_i) = r_8$) breaks the segment $[r_4 t_4 r_5 t_5 r_6 t_6 r_1]$ ($[r_1 t_7 r_7 t_8 r_8 t_9 r_9 t_{10} r_4]$) into segments $\Gamma_1^s = [r_4 t_4 r_5]$ and $\Gamma_2^s = [r_5 t_5 r_6 t_6 r_1]$ ($\Gamma_1^e = [r_1 t_7 r_7 t_8 r_8]$ and $\Gamma_2^e = [r_8 t_9 r_9 t_{10} r_4]$). Formally, we have

Definition 15: Let H_i be the new handle added at the ith synthesis step. Γ^s (Γ^e) is called the source (end) segment of H_i if $n_s(H_i)$ ($n_e(H_i)$) is an interior node of Γ^s (Γ^e); i.e. $n_s(H_i) \in P_{\text{in}}(\Gamma^s)$ ($n_e(H_i) \in P_{\text{in}}(\Gamma^e)$). The presence of $n_s(H_i)$ ($n_e(H_i)$) breaks Γ^s (Γ^e) into Γ_1^s and Γ_2^s (Γ_1^e and Γ_2^e) such that $\Gamma^s = \Gamma_1^s \Gamma_2^s$ ($\Gamma^e = \Gamma_1^e \Gamma_2^e$) and $n_e(\Gamma_1^s) = n_s(H_i) = n_s(\Gamma_2^s)$ ($n_e(\Gamma_1^e) = n_e(H_i) = n_s(\Gamma_2^e)$). See Definition 3 for the meaning of $\Gamma^s = \Gamma_1^s \Gamma_2^s$.

Lemma 5: *If $n_s(H_i)$ ($n_e(H_i)$) is an nonterminal node of any segment other than H_i itself, then* (1) $\eta_s = \eta_s^1 + \eta_s^2$ ($\eta_e = \eta_e^1 + \eta_e^2$). (2) *There is only one new independent segment out of two new segments Γ_1^s and Γ_2^s (Γ_1^e and Γ_2^e).*

This lemma implies that one new elementary siphon S arises due to nonterminal node of $n_s(H_i)$ ($n_e(H_i)$). Thus, there are at most three new elementary siphons for the new H_i.

The above cases assume that $n_s(H_i)$ and $n_e(H_i)$ are on **different** segments. In summary, we have the following observation.

Observation 2:

Case I: $n_s(H_i)$ and $n_e(H_i)$ are on the **same** segment

(a) H_i^c is a non PP'-handle
(b) H_i^c is a PP'-handle

Case II: $n_s(H_i)$ and $n_e(H_i)$ are on **different** segments

(a) Both $n_s(H_i)$ and $n_e(H_i)$ are on terminal nodes
(b) Exactly one of $n_s(H_i)$ and $n_e(H_i)$ is on a terminal node
(c) Neither $n_s(H_i)$ nor $n_e(H_i)$ is on terminal nodes

We will show that Case I can be reduced to Case II.

Clearly, the set of η generated by the segment η covers that generated by the CC (connected-component) η as well as that by the SCRS η. Also the basis to generate that corresponding to the SCRS η may be smaller than that corresponding to the CC η.

Consider the case where Γ_1 always appear together with $\Gamma_2, \ldots, \Gamma_q$ (all together called a *composite segment*), and they may be disjoint. Thus, there is only one independent segment out of q segments.

Definition 16: The set γ of $\Gamma_1, \Gamma_2, \ldots, \Gamma_q$ is called a composite segment if for every SCRS υ such that $\Gamma_g \in \upsilon$, $g \in \{1, 2, \ldots, q\}$, $\gamma \subset \upsilon$. $\Gamma_1, \Gamma_2, \ldots, \Gamma_q$ are said to be mutually dependent.

For instance, in Figure 3(a), Γ_1 ($[r_3 t_3 r_4]$) and Γ_2 ($[r_6 t_6 r_1]$) appear in every SCRS. Further, if H_2^c ($[r_4 t_4 r_5 t_5 r_6]$) is reduced to a PP'-path $[r_4 t_5 r_6]$ (Figure 3(c)), then H_2^c, Γ_1 and Γ_2 always appear together and they are a composite segment. Note that the SCRS in Figure 3(a)–(c) belongs to Case I.a (I.b) and can be reduced to that in Figure 3(b)–(d) which conforms to (*equivalent* η) Case II.a (Case II.b) in that both have the same SMS η in terms of the same segment η. This is true in general and we will only deal with Case II (i.e. assuming the absence of composite segments) in the sequel to simplify the discussion.

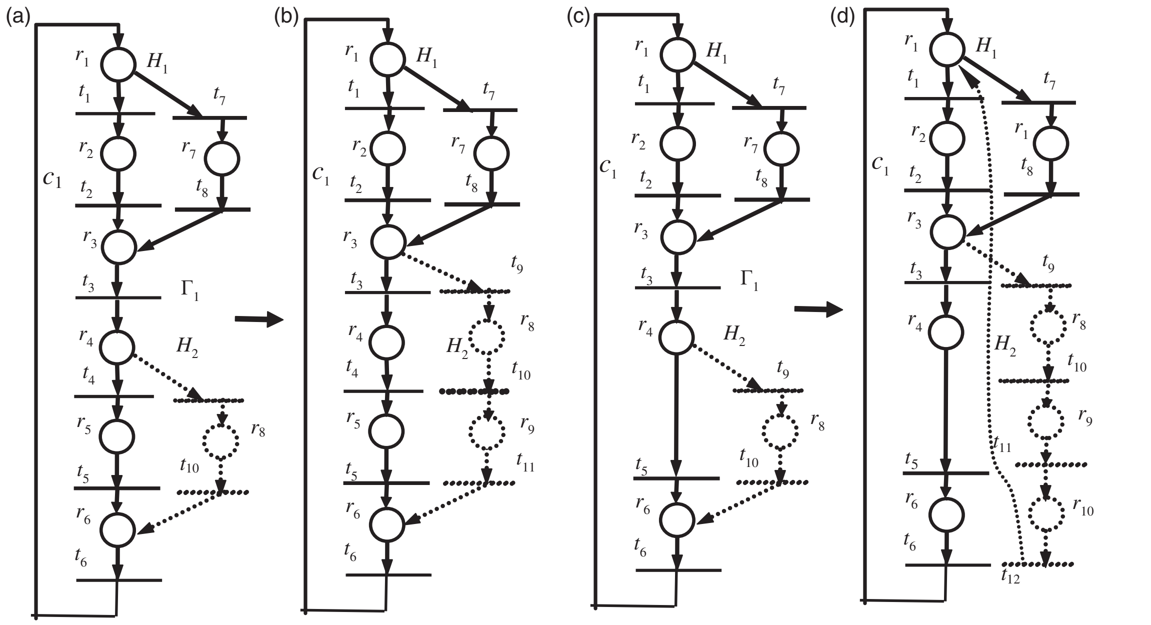

Figure 3. Case I: $n_s(H_2)$ *and* $n_e(H_2)$ are on the same segment. (a) Case I.a., (b) η–equivalent Case II.b., (c) Case I.b., (d) η–equivalent Case II.a.

Since the set of SCRS η is a subset of that of CC η, the number of elementary siphons ($|\Pi_E|$) is no greater than that of segments, denoted by ω. Thus, we have

Lemma 6: $|\Pi_E| \leq \omega$.

We further show that $|\Pi_E| = \omega$ based on the following property.

Property 2: For an SCRS υ, every segment η can be expressed as a linear combination of that of elementary siphons in υ.

To prove this property, we first find the number of new segments followed by showing that a new segment η can be expressed as linear combinations of characteristic *T*-vectors of new and old elementary siphons.

The following lemma computes the number of new segments for each case.

Lemma 7: *Let u be the number of new segments generated by adding* H_i. (1) $u = 1$ *for Case II.a.* (2) $u = 3$ *for Case II.b.* (3) $u = 5$ *for Case II.c.*

Note that when n_s and n_e are the same place, there is still only one new segment and it remains true that $u = 1$. It does not change the conclusion. Upon the first c_1 ($[r_1 t_1 r_2 t_2 r_3 t_3 r_4 t_4 r_5 t_5 r_6 t_6 r_1]$ in Figure 2(a)) found, we build the first elementary siphon. We then add a handle H_1 ($[r_1 t_7 r_7 t_8 r_3]$ in Figure 2(a)) with at least one new place. This results in three segments ($[r_1 t_1 r_2 t_2 r_3]$, $[r_1 t_7 r_7 t_8 r_3]$, and $[r_3 t_3 r_4 t_4 r_5 t_5 r_6 t_6 r_1]$) and two elementary circuits c_1 and $c_2 = [r_1 t_7 r_7 t_8 r_3 t_3 r_4 t_4 r_5 t_5 r_6 t_6 r_1]$ (one elementary siphon for each circuit). Thus, the third elementary siphon corresponds to the SCRS (the whole net, called 2c-system in Definition 17) containing the two elementary circuits.

Definition 17: A 1c-system is a SCRS with only one elementary circuit c_1. A 2c-system forms by adding a handle H_1 upon c_1. A 3c-system forms by adding a handle H_2 upon a 2c-system with neither $n_s(H_2)$ nor $n_e(H_2)$ being on terminal nodes.

Figure 2(b)–(d) shows three possible 3c-systems. The following two lemmas compute new segment η^n in terms of elementary siphon η_e in the new traced net $N^{t'}$. Note that if a new segment is part of a composite one, there is no need to compute the new segment η^n. Hence, we assume in the sequel that no new segment is part of a composite one.

Lemma 8: *For a* 2c-*system defined above containing* c_1 *and* c_2 *or* 3 *segments* Γ_1, Γ_2, *and* Γ_3 *(with three characteristic T-vectors* $\eta_1 - \eta_3$, *respectively) such that* $\Gamma_1 \cup \Gamma_2 = c_1$, $\Gamma_1 \cup \Gamma_3 = c_2$. *There are totally three elementary siphons (with three characteristic T-vectors* $\eta'_1 - \eta'_3$, *respectively) constructed from* c_1, c_2, *and* $c_1 \cup c_2$, *respectively.* (1) $\eta'_1 = \eta_1 + \eta_2$, $\eta'_2 = \eta_1 + \eta_3$, *and* $\eta'_3 = \eta_1 + \eta_2 + \eta_3$, (2) $\eta_1 = \eta'_1 + \eta'_2 - \eta'_3$, $\eta_2 = \eta'_3 - \eta'_2$, *and* $\eta_3 = \eta'_3 - \eta'_1$.

Note that if Γ_3 is a *PP'*-handle, $c_1 \cup c_2 s = c_1^e = c_1 \cup \Gamma_3$ implying $\eta'_3 = \eta'_1$. And we have only two new elementary siphon η (η'_3 and η'_2) and two new independent segments Γ_1 and Γ_2. We can still solve $\eta_2 = \eta'_3 - \eta'_2$. Note that Γ_1 and Γ_3 together form a composite segment. However, we assumed earlier the absence of composite segments and need not be concerned with such a twist. In the next lemma, we will apply lemma 8 when we hit a

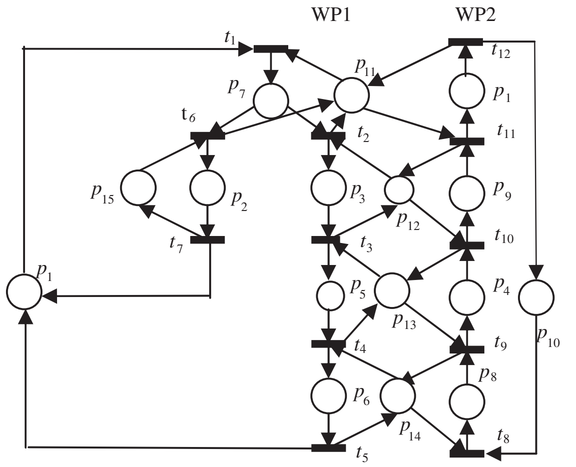

Figure 4. An example of nonminimal siphon. $[p_{11}\, t_{11}\, p_{12}]$ and $[p_{12}\, t_2\, p_{11}]$ form a circuit c and a composite segment. The siphon $\{p_3, p_7, p_{16}, p_{11}, p_{12}\}$, unlike others, synthesized from c is not minimal since it contains siphon $\{p_3, p_{16}, p_{11}\}$ as a proper subset.

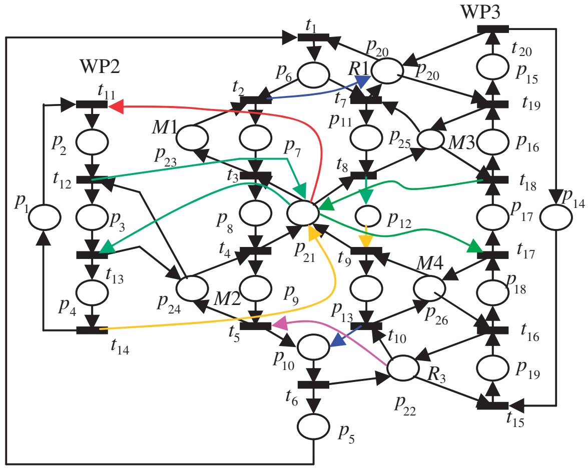

Figure 5. An example of S^3PR [9].

3c-system by decomposing the 3c-system into some 2c-systems. Again, we assume the absence of composite segments.

We are now ready to prove Property 2.

Lemma 9: *Assuming Property 2 holds for the old traced net N^t, so does it for the new traced net $N^{t'}$ where N^t ($N^{t'}$) is the net traced right before (after) the addition of H_i.*

Theorem 4: *For an SCRS υ, every segment η can be expressed as a linear combination of that of elementary siphons in υ.*

We now prove $|\Pi_E| = |\omega|$.

Theorem 5: *For an SCRS υ, $|\Pi_E| = |\omega|$.*

We now formally prove Observation 1.

Theorem 6: *Given an S^3PR, the number of elementary siphons is upper bounded by $|PR|$, the number of resource places in the net.*

One may argue that it is not correct. For example, if there exists four resource places, and any two of them can form a resource circuit, then there may be six elementary siphons. This, however, ignores the fact that some of the corresponding segments must be PP′-handles. Thus, the theorem remains valid.

We will find elementary siphons in an incremental fashion. At each step, we show that the number of new elementary siphons equals that of new independent segments. It holds till the end and we thus prove that $|\Pi_E| = |\omega|$ for an arbitrary SCRS.

5. Some twists

If all segments in the resource subnet are mutually independent, then it is easy to find the total number of elementary siphons. Unfortunately, some are dependent segments, some are part of a composite segment and some produce nonminimal siphons as shown in Figure 4, where $[p_{11}\, t_{11}\, p_{12}]$ and $[p_{12}\, t_2\, p_{11}]$ form a circuit c and a composite segment. The siphon $\{p_3, p_7, p_{16}, p_{11}, p_{12}\}$, unlike others, synthesized from c is not minimal since it contains siphon $\{p_3, p_{16}, p_{11}\}$ as a proper subset. The distinct feature of this segment is that a holder place (p_7) of a resource (p_{11}) on the segment, unlike others, has an output transition (t_6) not on the segment. Without (resp. With) t_6, the holder place p_7 would be on the sole *PT*-handle $[p_{11}\ t_1\ p_7\ t_2]$ (resp. *PP*-handle $[p_{11}\, t_1\, p_7\, t_6\, p_{11}]$) to the segment. The rest handles containing p_{11} are either TP- or *PP*-handles. Thus, the synthesized siphon contains p_{11} plus all its holder places $H(p_{11})$; that is $\rho(p_{11})$ (see Definition 7). In summary, we have the following.

Observation 3: The synthesized siphon from a segment Γ is not minimal iff there exists a place p on Γ without a *PT*-handle H to Γ with $n_s(H) = p$.

The time complexity to detect segment with nonminimal siphons is linear to $|P(\Gamma)|$, the number of places in Γ. Now we deal with detecting composite segments. There are two types of composite segments. The resource subnet (no longer) stays strongly connected after the deletion of Type-I (II) segment. In addition, Type-II segment must be a *PP*′-segment. In Figure 3(a), Γ_1 ($[r_3\, t_3\, r_4]$) and Γ_2 ($[r_6\, t_6\, r_1]$) appear in every SCRS; they together form a composite segment. The net no longer stays strongly connected after the deletion of Type-II Γ_1. In Figure 3(c), *PP*′- segment $\Gamma_3 = [r_4\, t_5\, r_6]$, the above Γ_1 and Γ_2 always appear together and they form a composite segment. The net stays strongly connected after the deletion of Type-I Γ_3. The time complexity to test strongly connectedness is linear to the number of places in the resource subnet.

Table 5. Elementary siphons and their η for the net in Figure 5.

Elementary SMS	Places	η
S_1	p_{10}, p_{18}, p_{22}, p_{26}	$[-t_9+t_{10}-t_{15}+t_{16}]$
S_4	p_4, p_{10}, p_{17}, p_{21}, p_{22}, p_{24}, p_{26}	$[-t_3+t_5-t_8+t_{10}-t_{11}+t_{13}-t_{15}+t_{17}]$
S_{10}	p_4, p, p_{12}, p_{17}, p_{21}, p_{24}	$[-t_3+t_4-t_{11}+t_{13}]$
S_{16}	p_2, p_4, p_8, p_{13}, p_{17}, p_{21}, p_{26}	$[+t_9-t_8+t_{17}-t_{16}]$
S_{17}	p_2, p_4, p_8, p_{12}, p_{15}, p_{20}, p_{21}, p_{23}, p_{25}	$[-t_1+t_3+t_8-t_{17}+t_{19}]$
S_{18}	p_2, p_4, p_8, p_{12}, p_{16}, p_{21}, p_{25}	$[-t_7+t_8-t_{17}+t_{18}]$

Table 6. Dependent siphons and their η for the net in Figure 5.

Dependent siphons	Places	η relationship
S_2	p_4, p_{10}, p_{15}, p_{20}, p_{21}, p_{22}, p_{23}, p_{24}, p_{25}, p_{26}	$\eta_2=\eta_4+\eta_{17}$
S_3	p_4, p_{10}, p_{16}, p_{21}, p_{22}, p_{24}, p_{25}, p_{26}	$\eta_3=\eta_4+\eta_{18}$
S_5	p_4, p_9, p_{13}, p_{15}, p_{20}, p_{21}, p_{23}, p_{24}, p_{25}, p_{26}	$\eta_5=\eta_{10}+\eta_{16}+\eta_{17}$
S_6	p_4, p_9, p_{13}, p_{16}, p_{21}, p_{24}, p_{25}, p_{26}	$\eta_6=\eta_{10}+\eta_{16}+\eta_{18}$
S_7	p_4, p_9, p_{13}, p_{17}, p_{21}, p_{24}, p_{26}	$\eta_7=\eta_{10}+\eta_{16}$
S_8	p_4, p_9, p_{12}, p_{15}, p_{20}, p_{21}, p_{23}, p_{24}, p_{25}	$\eta_8=\eta_{10}+\eta_{17}$
S_9	p_4, p_9, p_{12}, p_{16}, p_{21}, p_{24}, p_{25}	$\eta_9=\eta_{10}+\eta_{18}$
S_{11}	p_2, p_4, p_8, p_{10}, p_{15}, p_{20}, p_{21}, p_{22}, p_{23}, p_{25}, p_{26}	$\eta_{11}=\eta_1+\eta_{16}+\eta_{17}$
S_{12}	p_2, p_4, p_8, p_{13}, p_{15}, p_{20}, p_{21}, p_{23}, p_{25}, p_{26}	$\eta_{12}=\eta_{16}+\eta_{17}$
S_{13}	p_2, p_4, p_8, p_{10}, p_{16}, p_{21}, p_{22}, p_{25}, p_{26}	$\eta_{13}=\eta_1+\eta_{16}+\eta_{18}$
S_{14}	p_2, p_4, p_8, p_{13}, p_{16}, p_{21}, p_{25}, p_{26}	$\eta_{14}=\eta_{16}+\eta_{18}$
S_{15}	p_2, p_4, p_8, p_{10}, p_{17}, p_{21}, p_{22}, p_{26}	$\eta_{15}=\eta_1+\eta_{16}$

Table 7. H_i (handle added at ith step) and new basic siphons (synthesized from basic circuits).

Handle	Basic circuits c_b	PP'-handles	S_b	Places
c_1	$[p_{22}\ t_{10}\ p_{26}\ t_{16}\ p_{22}]$		S_1	p_{10}, p_{18}, p_{22}, p_{26}
$H_1=c_2$	$[p_{21}\ t_{17}\ p_{26}\ t_9\ p_{21}]$		S_{16}	p_2, p_4, p_8, p_{13}, p_{17}, p_{21}, p_{26}
$H_2=c_3$	$[p_{21}\ t_{13}\ p_{24}\ t_4\ p_{21}]$		S_{10}	p_4, p_9, p_{12}, p_{17}, p_{21}, p_{24}
$H_3=c_4$	$[p_{21}\ t_8\ p_{25}\ t_{18}\ p_{21}]$		S_{18}	p_2, p_4, p_8, p_{12}, p_{16}, p_{21}, p_{25}
H_4	$c_5=[p_{21}\ t_{17}\ p_{26}\ t_{16}\ p_{22}\ t_5\ p_{24}\ t_4\ p_{21}]$	$[p_{21}\ t_{13}\ p_{24}]$, $[p_{26}\ t_9\ p_{21}]$, $[p_{22}\ t_{10}\ p_{26}]$	S_4	p_4, p_{10}, p_{17}, p_{21}, p_{22}, p_{24}, p_{26}
$H_5=c_7$	$[p_{20}\ t_{19}\ p_{25}\ t_7\ p_{20}]$		S^*	Non-minimal
H_6	$c_6=[p_{21}\ t_3\ p_{23}\ t_2\ p_{20}\ t_{19}\ p_{25}\ t_{18}\ p_{21}]$	$[p_{25}\ t_7\ p_{20}]$, $[p_{20}\ t_8\ p_{25}]$	S_{17}	p_2, p_4, p_8, p_{12}, p_{15}, p_{20}, p_{21}, p_{23}, p_{25}

$H_4=[p_{22}\ t_6\ p_{24}]$ and $H_6=[p_{21}\ t_3\ \mathrm{p}_{23}\ t_2\ p_{20}]$.

6. Application

The well-known S^3PR example in Figure 5 [9] consists of three robots (*R1*, *R2*, *R3*) and four machines (*M1–M4*). Tables 5 and 6 show all the elementary and dependent siphons and their η for the net in Figure 5, respectively. We add handles as in Table 7 (so that smaller basic circuits are traced prior to those larger ones), then all elementary siphons are obtained correctly. Note that for each H_i, both $n_s(H_i)$ and $n_e(H_i)$ are on terminal nodes of segments other than H_i itself. By Lemma 4, H_i is the only one new independent segment and its η is a new independent one. There are six elementary siphons, which is fewer than $|P_R|=7$. The time complexity is also linear to $|P_R|$ much better than that in [13,15], which takes an exponential amount of time.

7. Conclusion

In summary, this paper has the following contributions:

(1) We have shown that $|\Pi_\mathrm{E}|=|\omega|$; that is the number of new elementary siphons equals that of new independent segments. Thus, one can compute elementary siphons based on the set of independent segments from a set of elementary resource circuits (with little twist) for arbitrary S^3PR, S^3PMR [18], and weighted S^3PR [6]. Thus, it extends the elementary-siphons-computation algorithm in [2] for a simple subclass of S^3PR (called S^4PR) to more complicated S^3PR and nets more complicated than S^3PR that includes weakly dependent siphons.

(2) It takes polynomial amount of time to find independent segments from a linear number of initial segments and from which to compute

elementary siphons. This is better than the exponential amount of time required using the approach by Li et al. [4].

(3) We show (Theorem 6) that the total number of elementary siphons, $|\Pi_E|$, is upper bounded by the total number of resource places $|P_R|$ lower than that $\min(|P|, |T|)$ by Li and Zhou where $|P|$ ($|T|$) is the number of places (transitions) in the net.

Because only one resource is used in each job stage and the processes are modeled using state machines in S^3PR, its modeling power is limited. It cannot model iteration statements (loop) in each sequential process (SP) and the relationships of synchronization and communication among SP. At any operation place of a process, it cannot use multisets of resources. Future work shall extend the results to more complex nets than S^3PR and S^3PMR.

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Appendix I: Some Proofs

Proof of Theorem 3: There are 4 cases for t in $c' = [r_1, t_2, p_1, t_3, r_1]$ of $\rho(r_1)$. (a) t (t_2 and t_3) is on a *PP*-circuit c' to $\Gamma_1 \cup \Gamma_2$. $\eta_3(t) = \eta_1(t) + \eta_2(t) = 0 + 0$. (b) t (t_2 and t_3) is on a *TT*-handle to $\Gamma_1 \cup \Gamma_2$. Part of c' becomes a *TT*-handle $[t_2, p_1, t_3]$. $\eta_3(t_2) = 0 = \eta_1(t_2) + \eta_2(t_2) = -1 + 1$, $\eta_3(t_3) = \eta_1(t_3) + \eta_2(t_3) = \eta_1(t_3) + 0$. (c) t (t_2 and t_3) is on a *TP*-handle to $\Gamma_1 \cup \Gamma_2$. $\eta_3(t_2) = 1 = \eta_1(t_2) + \eta_2(t_2) = 0 + 1$, $\eta_3(t_3) = 0 = \eta_1(t_3) + \eta_2(t_3) = 0 + 0$. (d) t (t_2 and t_3) is on a *PT*-handle to $\Gamma_1 \cup \Gamma_2$. $\eta_3(t_2) = -1 = \eta_1(t_2) + \eta_2(t_2) = -1 + 0$, $\eta_3(t_3) = \eta_1(t_3) + \eta_2(t_3) = \eta_1(t_3) + 0$.

Proof of Corollary 1: This corollary holds by repeated applying the reasoning shown in the proof of Theorem 3.

Proof of Lemma 3: Let Γ_1 be of the form $[r_1 t_1 r_2 \cdots r_k]$ (e.g. H_2 $[r_7, t_{11}, r_{10}, t_{12}, r_5]$ in Figure 2(b)). Then $\eta_1(t_1) = 1$ (t_1 is on a *TP*-handle to Γ_1), yet $\eta(t_1) = 0$ (t_1 is on a *PP*-handle to other segments) for all other segment η. Thus, η_1 cannot be expressed in terms of (hence is independent to) those of all other segments.

Proof of Corollary 2: Assume contrarily that η_1 depends on those of all old elementary siphons, η_1 depends also on those of old segments, so does η_Γ since $\eta_1 = \eta_\Gamma + \cdots$ – contradiction to Lemma 3.

Proof of Lemma 5: (1) It follows from the fact that $\Gamma^s = \Gamma_1^s \Gamma_2^s$ ($\Gamma^e = \Gamma_1^e \Gamma_2^s$) and Theorem 3. (2) It follows from the expression in 1.

Proof of Lemma 7: Obvious by Definition 15.

Proof of Lemma 8: (1) It follows from Corollary 1. (2) It follows from algebra manipulations of equations in 1.

Proof of Lemma 9: There are 3 cases.
Case II.a: $u = 1$ with only one new elementary siphon S_{e_1} corresponding to the new circuit c_1 (1c-system) containing H_i. Hence, we can construct an elementary equation: $\eta_{H_i} + \eta^o = \eta_{e_1}$ where η_{H_i}, η^o and η_{e_1} are the characteristic *T*-vectors η for H_i, the directed path $c_1 \backslash H_i$ on c_1 and S_{e_1}, respectively. $c_1 \backslash H_i$ is in N^t; hence, by the assumption, η^o is a linear combination of that of elementary siphons in N^t, so is η_{H_i} in $N^{t'}$ since $\eta_{H_i} = \eta_{e_1} - \eta^o$.

Cases II.b: *u = 3* with two new elementary siphons S_{e_1} and S_{e_2} corresponding to the new circuit c_1 containing H_i and the new SCRS2 $\upsilon_2 = c_1 \cup c_2$ (2c-system) where c_2 contains H_i^c with only one new segment. By Lemma 8, η_{H_i} and $\eta_{H_i}^c$ can be expressed as linear combinations of that of η^o, $\eta_{e_1} = \eta_{c1}$ and $\eta_{e_2} = \eta_{c_1 \cup c_2}$, where η^o is the η for $\upsilon_2 \backslash (H_i \cup H_i^c)$ and is a linear combination of that of elementary siphons (denoted by η_e) in $N^{t,}$ so are η_{H_i} and $\eta_{H_i}^c$ (η for H_i^c) in $N^{t'}$. Now H_i^c contains only one new segment $\eta^n = \Gamma_1^s$ or Γ_2^s or Γ_1^e or Γ_2^e (other than H_i) and it may contain old segments; thus, η^n (as well as η_{H_i}) can be solved in terms of η_e in $N^{t'}$.

Case II.c: *u = 5* corresponds to a 3c-system. For the 3c-system**a** shown in Figure 2(b) containing c_1 ($= \Gamma_1 \cup \Gamma_2$; see Lemma 8), c_2 ($= \Gamma_1 \cup \Gamma_3$) and the new circuit c_3 containing H_i with 3 new elementary siphon η: η_{c_3}, $\eta_{c_2 \cup c_3}$, and $\eta_{c_1 \cup c_3}$. *$n_s(H_i)$* divides Γ_3 into two new segments Γ_3^1 and Γ_3^2; hence $\eta_3 = \eta_3^1 + \eta_3^2$. *$n_e(H_i)$* divides Γ_1 into two new segments Γ_1^1 and Γ_1^2; hence $\eta_1 = \eta_1^1 + \eta_1^2$. $c_3 = H_i \cup \Gamma_1^2 \cup \Gamma_3^1$. Consider the 2c-system formed by c_2 and c_3. By Lemma 8, we have (i) $\eta_{H_i} = \eta_{c_2 \cup c_3} - \eta_{c_2}$, and (ii) $\eta_3^2 + \eta_1^1 = \eta_{c_2 \cup c_3} - \eta_{c_3}$. Now, for the 2c-system formed by c_1 and c_3, We have (iii) $\eta_{H_i} + \eta_3^1 = \eta_{c_1 \cup c_3} - \eta_{c_1}$, and (iv) $\eta_1^1 + \eta_2 = \eta_{c_1 \cup c_3} - \eta_{c_3}$. Substituting Equation (1) into Equation (3), we have $\eta_3^1 = \eta_{c_1 \cup c_3} - \eta_{c_1} - \eta_{c_2 \cup c_3} + \eta_{c_2}$. From Equation (4), we have $\eta_1^1 = \eta_{c_1 \cup c_3} - \eta_{c_3} - \eta_2$, which in turn solves $\eta_3^2 = \eta_{c_2 \cup c_3} - \eta_{c_3} - \eta_{c_1 \cup c_3} + \eta_{c_3} + \eta_2$ from Equation (2) and $\eta_1^2 = \eta_1 - \eta_1^1 = \eta_1 - \eta_{c_1 \cup c_3} + \eta_{c_3} + \eta_2$.

Since η_{c_1}, η_{c_2}, η_1, η_2, and η_3 all can be expressed in terms of old elementary siphons, so do η for all new segments (H_i, Γ_1^1, Γ_1^2, Γ_3^1, and Γ_3^2). Similarly, for 3c-system**b** and 3c-system**c**, we can show that all new segment η^n can also be solved in terms of η_e in $N^{t'}$.

Proof of Theorem 4: Prove by induction. At first, we build c_1 with $\eta_{c_1} = \eta_{e_1}$. The thesis obviously holds. Next assume the assertion holds at $(i-1)$th step. By Lemma 9, so does it at ith step. Thus, it holds at the last step.

Proof of Theorem 5: By Theorem 4, η of every elementary siphon, denoted by η_e, can be expressed in terms of linear combinations of segment η; that is there is a matrix V such that $V \bullet \text{Л}_{\text{S}} = \text{Л}_{\text{E}}$ where Л_{S} (Л_{E}) is a vector with ω ($|\Pi_E|$) components of segment η (η_e). By Lemma 6, $|\Pi_E| \leq \omega$. Assume $|\Pi_E| < \omega$, then there are fewer equations than variables to solve; that is, it is impossible to solve each segment η in terms of linear combinations of elementary siphons. This implies that dependent siphons cannot either – contradiction. Hence, $|\Pi_{\text{E}}| = |\omega|$.

Proof of Theorem 6: Each new handle H_i must be a non-*PP*′ handle (each independent segment as well); hence, it must contain at least a resource place. Thus, the total number of iteration steps or independent segments) is at most $|P_R|$. By Theorem 5, $|\Pi_{\text{E}}| = |\omega|$ and the total number of elementary siphons is upper bounded by $|P_R|$.

運用分段理論計算基本虹吸在派翠網路的僵局控制

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摘要

不同於其他方法，李和周所提的僅為基本虹吸加入加控制節點與弧線之技術，明顯地減少了派翠網路所需的監控節點和弧線。李和周提出的虹吸技術 ，其基本虹吸的數量與網路結構的大小呈線性關係成長。基本虹吸可由基本資源迴圈合成 ，迴圈又以小區塊分段方式組成。 本文提出虹吸結構總數$|\Pi_E|$的上界為$|P_R|$(資源位置數) ， 其低於最小的($|P|$, $|T|$) ，其中$|P|$和$|T|$各是李和周所提出網路中的操作位置(place)和轉移(transitions) 的數量。 此外 ，我們聲稱基本虹吸的數量$|\Pi_E|$等於資源子網路獨立段的數量。 基本虹吸結構在簡單的循序過程資源系統(S^3PR)網路中，可視為獨立區段。一個圖的遍歷算法(graph-traversal)的基礎上，可以追溯到對應於一個網路中基本虹吸的資源迴圈。 在遍歷過程中， 我們也可以找出獨立的區段(即，其特徵T-向量都是獨立的)和對應於基本虹吸的區段。這提供了一個替代方案深入地瞭解基本虹吸結構。此外，也能夠運用這個演算法來為S^3PR的子類別(所謂S^4PR)，與包含弱依賴虹吸更複雜的S^3PR計算基本虹吸結構。

關鍵詞：彈性製造系統; 僵局控制; 派翠網路; 基本虹吸; 簡單的循序過程資源系統

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