On the Role of Risk Preference in Survivability

Shu-Heng Chen[1] and Ya-Chi Huang[2]

[1] AI-ECON Research Center, Department of Economics, National Chengchi University, Taipei, Taiwan
chchen@nccu.edu.tw

[2] AI-ECON Research Center, Department of Economics, National Chengchi University, Taipei, Taiwan
yachi@aiecon.org

Abstract. Using an agent-based multi-asset artificial stock market, we simulate the survival dynamics of investors with different risk preferences. It is found that the survivability of investors is closely related to their risk preferences. Among the eight types of investors considered in this paper, only the CRRA investors with RRA coefficients close to one can survive in the long run. Other types of agents are eventually driven out of the market, including the famous CARA agents and agents who base their decision on the capital asset pricing model.

1 Introduction

The paper is concerned with a part of the debate on the *market selection hypothesis*. The debate, if we trace its origin, started with the establishment of what become known as the *Kelly criterion* ([8]), which basically says that a rational long-run investor *should* maximize the expected growth rate of his wealth share and, therefore, should behave as if he were endowed with a logarithmic utility function. Alternatively speaking, the Kelly criterion suggests that there is an optimal preference (rational preference) which a competitive market will select and that is logarithmic utility. The debate on the Kelly criterion has a long history, so not surprisingly, there is a long list of both pros and cons standing alongside the developments in the literature.[1]

The Kelly criterion may further imply that an agent who maximizes his expected utility under the *correct* belief may be driven out by an agent who maximizes his expected utility under an *incorrect* belief, simply because the former does not maximize a logarithmic utility function, whereas the latter does. [1] were the first to show this implication of the Kelly criterion in a standard asset pricing model. As a result, the market selection hypothesis fails because agents with accurate beliefs are not selected. A consequence of this failure is that asset prices may not eventually reflect the beliefs of agents who make accurate predictions, and hence may persistently deviate from the *rational expectations equilibrium* and violate the *efficient market hypothesis*.

[1] See [11] for a quite extensive review.

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However, a series of recent studies indicates that the early analysis of [1] is not complete. [10] shows that, if the saving behavior is endogenously determined, then the market selection hypothesis is rescued, and in the long run, only those optimizing investors with *correct beliefs* survive. The surviving agents do not have to be log-utility maximizers, and they can have diverse risk preferences. [10]'s analysis is further confirmed by [2] in a connection of the market selection hypothesis to the *first theorem of welfare economics*. [2] show that in a dynamic and complete market *Pareto optimality* is the key to understanding selection either for or against traders with correct beliefs: in any optimal allocation the survival or disappearance of a trader is determined entirely by beliefs, and not by risk preferences.

Despite the rigorousness of these theoretical studies, there exists a fundamental limitation, which may make it difficult to grasp their empirical counterparts, namely, they are *non-constructive*.[2] Take [10] as an example. First, the analysis crucially depends on the appearance of agents who *eventually make accurate predictions* or *eventually make accurate next period predictions*. Nevertheless, the process that shows the emergence of these sages is unknown. It is, therefore, not clear how these agents emerge, or whether they will ever emerge.[3] Second, maximizing expected utility is equivalent to assuming that agents are able to solve any infinite-time stochastic dynamic optimization problem implied by their utility function. However, current dynamic optimization techniques, regardless of whether they include stochastic optimal control or stochastic dynamic programming, can only help us solve a very limited subset of the whole problem space. As for the rest of them, it is necessary to rely on numerical approximations, and their effectiveness to a large extent is also unknown.

Given these practical limitations, we are motivated to re-examine the issue from a more realistic perspective or, technically speaking, a computational perspective. By remaining in the general equilibrium analysis framework, we replace the rational agents with bounded-rational agents. More precisely, these agents are constructed in terms of what is known as *autonomous agents* in agent-based computational economics ([12]). Basically, these agents are able to learn to optimize and to forecast in an autonomous manner. So, they are not necessarily utility-maximizers. Instead, they use adaptive computing techniques to approximate the optimal solution. In this sense, they are Herbert Simon's *satisfying* agents. Similarly, they base their decisions upon beliefs which may not be and may never be correct, but are reviewed and revised continuously ([9]).

By introducing autonomous agents, we are getting closer to the world of flesh and blood, and enhancing the study of the empirical relevance of risk preference to survival dynamics.

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[2] This kind of issue is generally shared in many general equilibrium analyses.

[3] Back to the real world, we have not been convinced that these agents have ever appeared in human history.

2 A Simple Multi-asset Model

The simulations presented in this paper are based on an agent-based version of the multi-asset market as per the studies of [1] and [10]. The market is complete in the sense that the number of states is equal to the number of assets, say M. At each date t, the outstanding volume of each asset is exogenously fixed at one unit. There are I investors in the market, with each being indexed by i. At time t asset m will pay dividends w_m if the corresponding state m occurs, and 0 otherwise. The behavior of these states follows a finite-state stochastic process, which does not have to be stationary. The dividends w_m will be distributed among the I investors proportionately according to their owned shares of the respective asset. The dividends can only be either re-invested or consumed. Hoarding is prohibited. If agent i chooses to consume c, her satisfaction is measured by her utility function $u(c)$. This simple multi-asset market clearly defines an optimization problem for each individual as follows:

$$\max_{\{\{\delta^i_{t+r}\}^\infty_{r=0},\{\alpha^i_{t+r}\}^\infty_{r=0}\}} E\{\sum_{r=0}^{\infty}(\beta^i)^r u^i(c^i_{t+r}) \mid B^i_{t-1}\} \tag{1}$$

subject to

$$c^i_{t+r} + \sum_{m=1}^{M} \alpha^{i,*}_{m,t+r} \cdot \delta^{i,*}_{t+r} \cdot W^i_{t+r-1} \leq W^i_{t+r-1} \quad \forall r \geq 0, \tag{2}$$

$$\sum_{m=1}^{M} \alpha^i_{m,t+r} = 1, \quad \alpha^i_{m,t+r} \geq 0 \quad \forall r \geq 0. \tag{3}$$

In equation (1), u^i is agent i's temporal utility function, and β^i, also called the discount factor, reveals agent i's time preference. The expectation $E(\ \ )$ is taken with respect to the most recent belief B^i_t, which is a probabilistic model used to represent agent i's subjective belief regarding the stochastic nature of the state. The maximization problem asks for two sequences of decisions, one related to saving, and the other to the portfolios, denoted by

$$\{\{\delta^i_{t+r}\}^\infty_{r=0}, \{\alpha^i_{t+r}\}^\infty_{r=0}\},$$

where δ^i_t is the saving rate at time t, and

$$\alpha^i_t = (\alpha^i_{1,t}, \alpha^i_{2,t}, ..., \alpha^i_{M,t})$$

is the portfolio comprising the M assets.

Equations (2) and (3) are the budget constraints. W^i_t is the wealth of agent i at time t, which is earned from the dividends paid at time t. Notice that these budget constraints do not allow agents to consume or invest by borrowing.

The equilibrium price $\rho_{m,t}$ is determined by equating the demand for asset m to the supply of asset m, i.e.

$$\sum_{i=1}^{I} \frac{\alpha^{i,*}_{m,t} \cdot \delta^{i,*}_t \cdot W^i_{t-1}}{\rho_{m,t}} = 1, \quad m = 1, 2, ..., M. \tag{4}$$

By rearranging Equation (4), we obtain the market equilibrium price of asset m:

$$\rho_{m,t} = \sum_{i=1}^{I} \alpha_{m,t}^{i,*} \cdot \delta_t^{i,*} \cdot W_{t-1}^i. \quad (5)$$

3 The Agent-Based Multi-asset Artificial Stock Market

An agent-based version of the Blume-Easley-Sandroni standard multi-asset model is developed in [4]. There they ([4]) propose a sliding-window adaptation scheme to approximate the original infinite-time horizon optimization problem (Equations (1) – (3)) by a finite-time horizon optimization problem. The stochastic optimization problem (1) has two mainstays: first, finding an appropriate belief, and second, under that belief, searching for the best decisions regarding saving and portfolios. To distinguish the two, [3] calls the former "*learning how to forecast*," and the latter *learning how to optimize*. Genetic algorithms are then applied to evolve both beliefs and investment strategies.[4]

To simulate this agent-based multi-asset artificial stock market, a software called *AIE-ASM Version 5.0* is written using *Delphi, Version 7.0*. In each single run, we generate a series of artificial data.

4 Experimental Design

Since the main focus of this paper is to examine the relevance of risk preference to survivability, we shall assume that the autonomous agents are identical in all aspects except in terms of their preferences over risk. With this assumption, we run two series of experiments. These two experiments differ in their constituent agent types. In Experiment 1, the market is composed of eight types of agents, and they are distributed evenly among 40 market participants, i.e. five agents for each type. These eight types of agents are agents with the seven utility functions specified in Table 1 plus the CAPM (capital asset pricing model) believers.

The type-one agent has the logarithmic utility function. We are very much interested in knowing whether this type of agent has any advantage over others in the long-run wealth share. As to types two to six, they are also frequently used in economic analysis.[5] Among them, type four has the well-known *CARA* (constant absolute risk aversion) utility function. In addition to these six familiar types of utility functions, we also consider any arbitrary utility function. By using Taylor's expansion, an arbitrary analytical utility function can be approximated by a finite-order polynomial function. Here, we consider the approximation only up to the sixth order.

Notice that types 3 to 7 refer to a class of parametric utility functions. Parameters of these types of utility functions, namely, $\alpha_1, ..., \alpha_4$, $\beta_1, ..., \beta_3$, and $a_0, a_1, ..., a_6$, can in principle be randomly or manually generated as long as they

[4] Details can be found in [4].

[5] See, for example, [6], pp. 27-33.

Table 1. Types of the Utility Function $u(c)$: Experiment 1

	Utility Type	Relative Risk Aversion (RRA)
Type 1	$u(c) = \log(c)$	1
Type 2	$u(c) = \sqrt{c}$	0.5
Type 3	$u(c) = \alpha_1 + \beta_1 c$	0
Type 4	$u(c) = \frac{\alpha_2}{\beta_2} \exp\{\beta_2 c\}$	$-\beta_2 c$
Type 5	$u(c) = \frac{1}{(\gamma_3+1)\beta_3}(\alpha_3 + \beta_3 c)^{\gamma_3+1}$	$-\frac{\beta_3\gamma_3}{\frac{\alpha_3}{c}+\beta_3}$
Type 6	$u(c) = c - \frac{\alpha_4}{2}c^2$	$\frac{\alpha_4}{\frac{1}{c}-\alpha_4}$
Type 7	$u(c) = a_0 + \sum_{i=1}^{6} a_i c^i$	$-\frac{2a_2c+6a_3c^2+12a_4c^3+20a_5c^4+30a_6c^5}{a_1+2a_2c+3a_3c^2+4a_4c^3+5a_5c^4+6a_6c^5}$

satisfy the regular first- and second-order conditions: $u^{'} > 0$ and $u^{''} < 0$. Since each type of utility function is assigned to five agents, parameter values are generated for each agent for each type separately. So, type 3 agents may have different values of (α_1, β_1), type 4 agents have different values of (α_2, β_2), and so on and so forth.

In Experiment 2, all agents are restricted to the family of the CRRA (constant relative risk aversion) utility functions,

$$u(c) = \begin{cases} c^\rho/\rho, & if \quad -\infty < \rho < \infty, \\ \ln^c, & if \quad \rho = 0. \end{cases} \tag{6}$$

They, however, differ in terms of their RRA coefficients, i.e. $1 - \rho$. The smaller the ρ, the larger the risk aversion coefficient. Eleven different ρs, starting from 0, 0.1., 0.2.,..., to 0.9, and 1.0, are distributed evenly to all 55 agents, with five agents for each ρ.

5 Simulation Results

5.1 Wealth Share Dynamics

Figure 1 shows the wealth-share dynamics of the eight types of investors in Experiment 1. Notice that each line is based on the average of 100 simulations. The results clearly indicate the strong dominance of the type-one investors, i.e. the agents who have a log utility function. While in some cases type-two investors are still hanging in there for the first 100 periods, their shares eventually decline toward zero. Maybe the most striking result is the extinction of the CARA type of agents (type-4 agents). It is striking because the CARA utility function has been used so extensively in the finance literature that one can hardly cast any doubt on its appropriateness.[6] Equally surprising is the finding that CAPM believers also fail to survive. This result is consistent with an earlier finding by [11], who shows that a sufficient condition to drive CAPM traders to extinction is that an investor endowed with a logarithmic utility function enters the market.

[6] For example, it was used to develop the standard asset pricing model ([5]), and was also used in agent-based artificial stock market simulations ([7]).

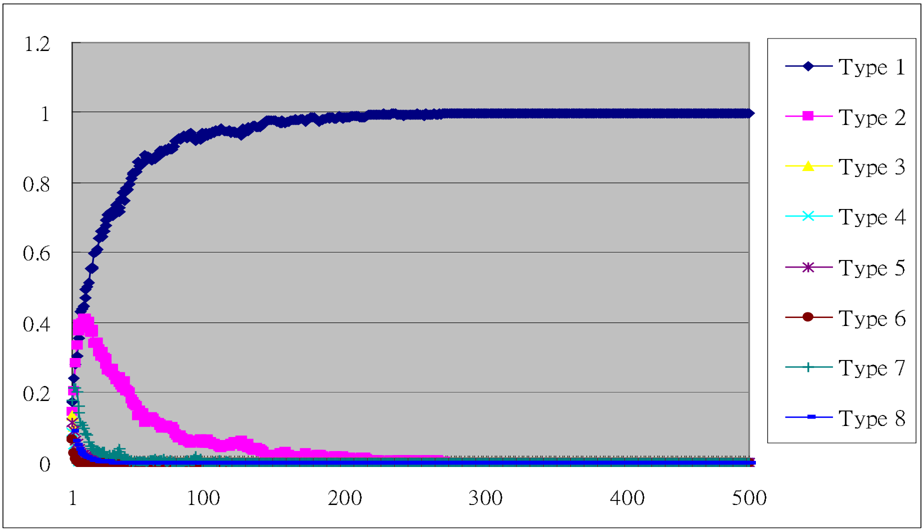

Fig. 1. Time Series Plot of the Wealth Share of Eight Types of Investors: Experiment 1

Since the type-one investors have a constant relative risk aversion coefficient that is one, our experimental results also lend support to Blume and Easley's main argument: *the market selects those investors whose coefficient of relative risk aversion is nearly one.*[7] To further examine this claim, the wealth share dynamics of Experiment 2 is depicted in Figure 2.

As can be seen from Figure 2, the wealth share seems to be positively correlated with the RRA coefficient. Agents with very low values for their the RRA coefficients are driven out of the market at different speeds. The lower the RRA, the faster the evaporation. Towards the end of this 100-period simulation, all agents with RRA values of less than 0.6 are driven out of the market. However, when the RRA coefficient increases to 0.9, the respective agents perform equally well, and sometimes even better, in terms of their wealth shares, as compared with the log-utility agents.

5.2 Saving Rates

Since we assume that the autonomous agents are identical in all aspects except in terms of their preferences over risk, there are only two decision variables left for us to trace the reason why *the market selects those investors whose coefficient of relative risk aversion is nearly one*, namly *saving* and *portfolio*.

Figure 3 is the box-whisker plots of the saving rates. Each plot shows the lifetime distribution of the saving rate δ_t associated with a specific RRA coefficient.

[7] See [1], Theorem 5.4, pp. 23-24. The words in italics shown in the main text are not quoted exactly from that theorem, which was originally made by controlling saving rates. Since saving rates are treated endogenously in our paper, our finding suggests that the theorem can still hold true even if the assumption of saving rates is relaxed.

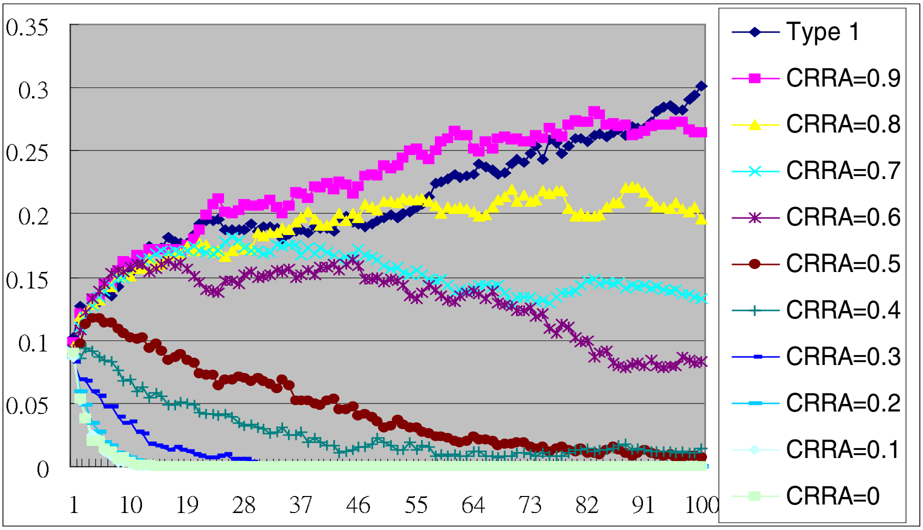

Fig. 2. Time Series Plot of the Wealth Shares of Eleven Types of Investors: Experiment 2

To generate each plot, we first take an average of the saving rate of the five agents for the same RRA coefficient. This is done period by period. A single history of δ_t $(t = 1, 2, ..., 100)$ is then derived by further taking an average over the entire 100 simulation runs. So, in the end, we have a single time history of δ_t for each RRA coefficient. The eleven boxes are drawn accordingly.

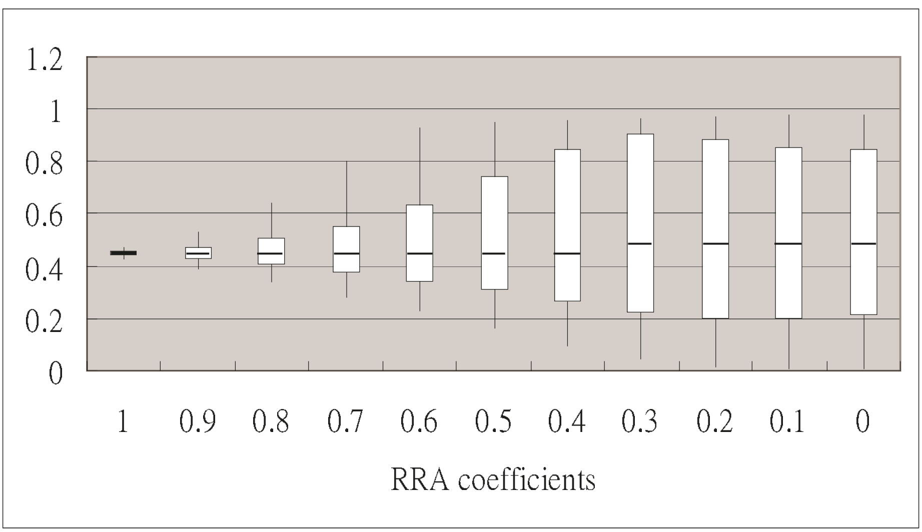

Fig. 3. Distribution of Saving Rates

The line appearing in the middle of the box indicates the median of the 100 observed saving rates for a specific RRA coefficient. While higher saving rates, as what Blume and Easley suggested, will place agents in an advantageous position to survive, we find that the saving rate of the log-utility agents (the case where the RRA coefficient is one) are not significantly higher than other types of agents. This is evidenced by the very close medians observed from the eleven types of agents. Thus, even though the level of saving rate may contribute to survival to a certain degree, our medians simply vary too little to give us a chance to test it.

However, that does not mean all agents have the saving behavior. This is revealed by comparing the boxes and whiskers. Compared to other types of agents, log-utility agents obviously have a very narrow box with a very short whisker, which indicates an unique feature of log-utility agents' saving behavior, namely, *a very stable saving behavior*.

From what we have seen in Figure 3, agents with lower RRA coefficient compared with the log-utility agents suffer from more unstable saving behavior, especially the lower *down-side* saving rates, which may contribute to the faster decline in their wealth share. This provides an significant evidence to explain why *the lower the RRA, the faster the evaporation.*

5.3 Portfolio Performance

In addition to the saving rate, portfolio performance may be another contributing factor to survivability. However, this possibility has already been excluded in [4], and is excluded here again. Table 2 gives the three basic performance measures: the mean return, the risk (variance), and the Sharpe ratio.[8] These statistics are averaged over the five agents of the identical type and are further averaged over the entire 100 simulation runs.

Table 2. Performance Measurements

RRA	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0
Mean	2.037	2.083	2.126	2.192	2.276	2.340	2.411	2.428	2.412	2.400	2.374
Variance	7.627	8.424	9.392	11.10	13.31	15.48	18.39	20.36	25.11	27.36	30.61
Sharpe Ratio	0.738	0.718	0.694	0.658	0.624	0.595	0.562	0.538	0.481	0.459	0.429

As we have seen in [4], the surviving agents do not have the highest rates of return. Nonetheless, the column "variance of return" indicates that these agents are under different exposures to risk. Agents with higher relative risk aversion coefficients choose to behave more prudently. Motivated by this finding, we go further to examine the *risk-adjusted return*, also known as the *Sharpe ratio*, and we find that, despite their low mean rate of return, the precautionary behavior of highly risk-averse agents actually helps them to earn a higher

[8] For the definition or calculation of these statistics, please see [4], Equations (21) and (22), for details.

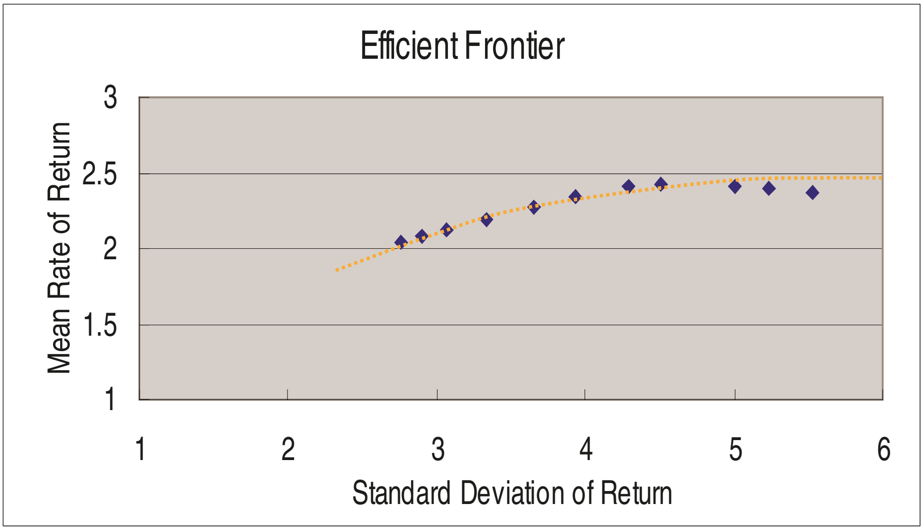

Fig. 4. Efficient Frontier

Sharpe ratio. However, there is no simple intuition to tell us why the agents with higher Sharpe ratios would survive. At least, one may suppose that every investor whose performance is situated at the *efficient frontier* has an equal chance to survive.[9] Therefore, we see no particular reason to attribute the survivability of agents with the RRA coefficient nearly one to their portfolio performance.

6 Concluding Remarks

The irrelevance of risk preference to the survivability of agents is dismissed in this paper. Our first experiment indicates that the only agents who survive in the long run (up to a 500-period simulation) are the log-utility agents. The rest are all driven out, including the CARA agents and the CAPM believers. In the second experiment, we further test for the significance of the RRA coefficient by assuming that all agents are CRRA types, and it is found that the agents' wealth share is affected by how close their RRA coefficients are to 1.

[9] To see this, the risk-return plot is drawn in Figure 4. The continuous frontier line is constructed by smoothly connecting the eight points on the frontier. The eight points on the frontier correspond to agents with RRA values of 1, 0.9, 0.8, 0.7, 0.6, 0.5, 0.4, 0.3. From this point of view, their portfolio performance offer them equal survivability. Furthermore, while the other three types of agents do not lie exactly on the frontier, they are not far away from it.

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