The Relationship between Relative Risk Aversion and Survivability

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Summary. As a follow-up to the work of [4] and [5], this paper continues to explore the relationship between wealth share dynamics and risk preferences in the context of an agent-based multi-asset artificial stock market. We simulate a multi-asset agent-based artificial stock market composed of heterogeneous agents with different degrees of relative risk aversion (RRA). A wide range of RRA coefficients has been found in the literature, and so far no unanimous conclusion has been reached. The agent-based computational approach as demonstrated in this paper proposes the possibility that in reality there may be such a wide *survival range* of the RRA coefficient. In addition, the time series plot of the wealth share dynamics indicates that the higher the risk aversion coefficient, the higher the wealth share. This result combined with our earlier result ([5]) well articulates the contribution of risk aversion to survivability.

Keyword: Risk Preferences, CRRA (Constant Relative Risk Aversion), Blume-Easley Theorem, Agent-Based Artificial Stock Markets, Genetic Algorithms

1 Motivation and Introduction

The contribution of *risk preference* to the survivability (wealth share) of investors has recently received a series of theoretic and simulation studies (e.g., [1], [19], [18], [2], [4], [5]). The results are mixed, depending on how we approach this issue. While the standard analytic approach proves the irrelevance of risk preference to survivability ([18], [2]), the agent-based computational approach indicates the opposite ([4], [5]). This kind of inconsistency, as quite often seen in the agent-based computational economics literature, simply reflects the sensitivity of the classical (analytical) results to the interacting heterogeneous boundedly-rational behavior.

[4] actually supports an earlier result obtained in [1], which is also known as the *Kelly criterion* in financial economics. This result basically points out the optimal type of risk preference, namely, the *CRRA (constant relative risk aversion coefficient) agent with an RRA coefficient of one.* Equivalently, it is the log utility function. [5] reestablishes this result, while in an agent-based computational setting. They examine the long-run wealth share dynamics of eleven different types of CRRA agents, with RRA coefficients ranging from 0 to 1 with increments of 0.1. They find that in finite time the wealth share is positively related to the CRRA coefficient, and in the long run, only the agents with high CRRA coefficients can survive. All others become extinct.

This paper is an extension of [5] in the sense that we wish to extend the earlier testing domain of the CRRA coefficient from [0, 1] to an even larger positive domain. In doing so, we are inquiring whether a higher degree of risk aversion can actually enhance the survivability of agents. Notice that the degree of risk aversion is not the original concern of either the Kelly criterion or the Blume-Easley theorem ([1]), both of which are only concerned with the dominance of the log-utility type agents. Risk aversion is involved because the log-utility agent is also known as a CRRA type of agent with an RRA coefficient of one. Now, is this the optimal degree of risk aversion? Will more risk-averse agents (i.e. those with RRA coefficients greater than one) be also driven out of the market when they are competing with the log-utility agents? Or, would higher risk aversion help them survive? These are the questions that we try to answer in this paper.

We consider these questions to be particularly relevant because the empirical literature actually suggests a large range of relative risk aversion coefficients. Some of them are exactly one or less than one, but many more are greater than one. Of course, it is doubtful whether one can directly compare our results with those empirical values, since they refer to quite different stories. However, given the prevailing empirical results on high risk aversion, it is definitely useful to know what makes them so, and our agent-based computational setting can serve as a good starting point.

The remainder of this paper is organized as follows. Section 2 gives a brief introduction to a simple multi-asset model, which is originally used in [1] and later extended and modified by [18]. Section 3 presents the artificial multi-asset market, which is an agent-based version of the analytical model presented in Section 2. Section 4 gives the experimental design. To justify the range of the relative risk aversion coefficient considered in this paper, it starts with a brief review of the literature on the empirical estimation of the RRA coefficient in Section 4.1, followed by the setting of other control parameters in Section 4.2. The simulation results are provided in Section 5, and are followed by the concluding remarks in Section 6.

2 A Simple Multi-Asset Model

The simulations presented in this paper are based on an agent-based version of the multi-asset market as [1] and [18] have studied. The market is complete in the sense that the number of states is equal to the number of assets, say M. At each date t, the outstanding volume of each asset is exogenously fixed at one unit. There are I investors in the market, each indexed by i. At time t asset m will pay dividends w_m if the corresponding state m occurs, and 0 otherwise. The behavior of states follows a finite-state stochastic process, which does not have to be stationary. The dividends w_m will be distributed among the I investors proportionately according to their owned share of the respective asset. The dividends can only be either re-invested or consumed. Hoarding is prohibited. If agent i chooses to consume c, her satisfaction is measured by her utility function $u(c)$. This simple multi-asset market clearly defines an optimization problem for each individual.

$$\max_{\{\{\delta^i_{t+r}\}_{r=0}^{\infty},\{\alpha^i_{t+r}\}_{r=0}^{\infty}\}} E\{\sum_{r=0}^{\infty}(\beta^i)^r u^i(c^i_{t+r}) \mid B^i_t\} \tag{1}$$

subject to

$$c^i_{t+r} + \sum_{m=1}^{M} \alpha^{i,*}_{m,t+r} \cdot \delta^{i,*}_{t+r} \cdot W^i_{t+r-1} \leq W^i_{t+r-1}, \quad \forall r \geq 0, \tag{2}$$

$$\sum_{m=1}^{M} \alpha^i_{m,t+r} = 1, \quad \alpha^i_{m,t+r} \geq 0, \quad \forall r \geq 0. \tag{3}$$

In Equation (1), u^i is agent i's temporal utility function, and β^i, also called the discount factor, reveals agent i's time preference. The expectation $E(\ )$ is taken with respect to the most recent belief B^i_t, which is a probabilistic model used to represent agent i's subjective belief regarding the stochastic nature of the state. The maximization problem asks for two sequences of decisions, one on saving, and one on portfolios, as denoted by

$$\{\{\delta^i_{t+r}\}_{r=0}^{\infty}, \{\alpha^i_{t+r}\}_{r=0}^{\infty}\},$$

where δ^i_t is the saving rate at time t, and

$$\alpha^i_t = (\alpha^i_{1,t}, \alpha^i_{2,t}, ..., \alpha^i_{M,t})$$

is the portfolio comprising the M assets.

Equations (2) and (3) are the budget constraints. W^i_t is the wealth of agent i at time t, which is earned from the dividends paid at time t. Notice that these budget constraints do not allow agents to consume or invest by borrowing.

The equilibrium price $\rho_{m,t}$ is determined by equating the demand for asset m with the supply of asset m, i.e.

$$\sum_{i=1}^{I} \frac{\alpha_{m,t}^{i,*} \cdot \delta_t^{i,*} \cdot W_{t-1}^i}{\rho_{m,t}} = 1, \quad m = 1, 2, ..., M. \tag{4}$$

Rearranging Equation (4), we obtain the market equilibrium price of asset m:

$$\rho_{m,t} = \sum_{i=1}^{I} \alpha_{m,t}^{i,*} \cdot \delta_t^{i,*} \cdot W_{t-1}^i. \tag{5}$$

3 The Agent-Based Multi-Asset Artificial Stock Market

An *agent-based computational version* of the Blume-Easley-Sandroni standard multi-asset model is developed in [4]. One of the mainstays of the agent-based computational economics is *autonomous agents.* ([20]) The idea of autonomous agents was initially presented in [12]. Briefly, these agents are able to learn and to adapt to the changing environment without too much external intervention, say, from the model designer. Their behavior is very much endogenously determined by the environment which they are interacting with. Accordingly, it can sometimes be very difficult to trace and to predict the resulting outcome, known as the *emergent behavior.*

In this paper, we follow what was initiated in [12], and equip our agents with the genetic algorithm to cope with the finite-horizon stochastic dynamic optimization problem, (1) – (3). The GA is applied here at two different levels, a high level (learning level) and a low level (optimization level). First, at the high level, it is applied as a *belief-updating scheme.* This is about the B_t^i appearing in (1). Agents start with some initial beliefs regarding state uncertainty which are technically characterized by parametric models, say, Markov processes. However, agents do not necessarily confine themselves with just stationary Markov processes. Actually, they can never be sure whether the underlying process will remain unchanged over time. So, they stay alert to that possibility, and keep on trying different Markov processes with different time frames (time horizons). Specifically, each belief can be described as "a kth order Markov process appearing over the last d days and may continue". These two parameters can be represented by a binary string, and a canonical GA is applied to evolve a population of these two parameters with a set of standard genetic operators.[3]

Once the belief is determined, the low-level GA is applied to solve the stochastic dynamic optimization problem defined in (1) – (3). Basically, we use Monte Carlo simulation to generate many possible ensembles consistent with the given belief and use them to evaluate a population of investment plans composed of a saving rate and a portfolio. GA is then applied to evolve this population of candidates.[4]

[3] Details can be found in [4], Appendix A.2.

[4] Details can be found in [4], Appendix A.1.

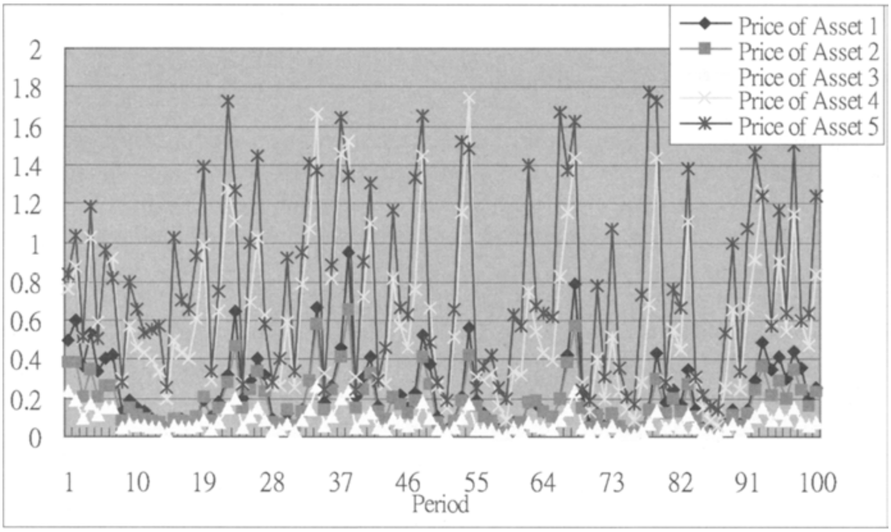

Fig. 1. Time Series Plot of the Prices of Assets; M =5.

In sum, the high-level GA finds an appropriate belief, and under that belief the low-level GA searches for the best decisions regarding savings and portfolios. This style of adaptive design combines *learning how to forecast* with *learning how to optimize*, a distinction made in [3]. These two levels of GA, however, do not repeat themselves with the same frequency. As a matter of fact, the belief-updating scheme is somewhat slow, whereas the numerical optimization scheme is more frequent. Intuitively, changing our belief in the meta-level of the world tends to be slower and less frequent than just fine-tuning or updating some parameters associated with a given structure. In this sense, the idea of *incremental learning* is also applied to our design of autonomous agents.

To simulate this agent-based multi-asset artificial stock market, a software called *AIE-ASM Version 5.0* is written using *Delphi, Version 7.0.* In each single run, we generate a series of artificial data. At the micro level, it includes the dynamics of agents' beliefs, investment behavior, and the associated wealth

$$\{B_t^{i,*}, \delta_t^i, \alpha_t^i, W_t^i\}_{t=1}^{100}, \quad i = 1, ..., I.$$

At the aggregate level, we observe the asset price dynamics

$$\{\rho_{m,t}\}_{t=1}^{100}, \quad m = 1, ..., M.$$

Figure 1 displays the time series plot of prices in a five-asset market. In this specific simulation, the state follows an *i.i.d.* process.

4 Experimental Design

4.1 Empirical RRA Coefficients

This paper and the coming experimental design are both very much motivated by the existing intensive empirical studies on the RRA coefficient. Therefore, we would like to give a brief survey here to present a general flavor on the wide dispersion of various estimates. For convenience we shall denote the RRA coefficient by ϱ.

Let us look at this from the left extreme, i.e. $\varrho \leq 1$. While [5] already questions the survivability of the CRRA agents with ϱ being less than 1, empirical studies supporting small ϱ still exist. The lowest value of ϱ is found in [16], which is only 0.3. A little above this is [11]'s estimate, that ranges between 0.3502 and 0.9903. The one that seems to best fit our early simulation result of [5] is [8], who find values of RRA of around unity. Instead of using real economic data, [10] derived their estimate from a laboratory experiment with human subjects, which lay between 0.6 and 1.4. At the other extreme, there are cases of $\varrho \geq 10$, such as 18 in [17] and 30 in [14]. These estimates are so large and are puzzling to many economists, having become part of the very famous *equity premium puzzle*. Next to those big ones, [13] also find a double-digit RRA in their estimates, which is close to 12. Despite these two extremes, most estimates suggest a moderate range for ϱ, and include the [1,3] interval ([9]), the [2,3] interval ([15]), [21]), the [2,5] interval ([6]), etc.

4.2 Control Parameters

According to the brief review above, we have set ϱ to lie in the [0.5, 5] interval. It starts from 0.5 with increments of 0.5 and continues up to 5. In this setting, we have a total of 10 types of CRRA agents, and assign 5 market participants to each type of agent. Hence, there are fifty agents in the market in total. There are also 5 assets available in the market ($M = 5$), corresponding to 5 states. Asset m pays dividends $6 - m$ ($m = 1, 2, ...5$). Two stochastic processes are considered in the experiments, namely, *iid* and the *first-order Markov*. Each is employed for one half of the total number of runs. The parameters of these two stochastic processes are also randomly generated in such a way that the axioms of the probability function are satisfied. Parameter values for the low-level and high-level GA can be found in [4], 4.1. We have 100 independent runs of the same experiment, and each run lasts for 100 market periods (T=100).

5 Simulation Results

Figure 2 shows the time series of the wealth share among the 10 types of agents. Notice that each line is based on an average of 100 simulations. The diagram seems to clearly indicate that the higher the RRA coefficient, the

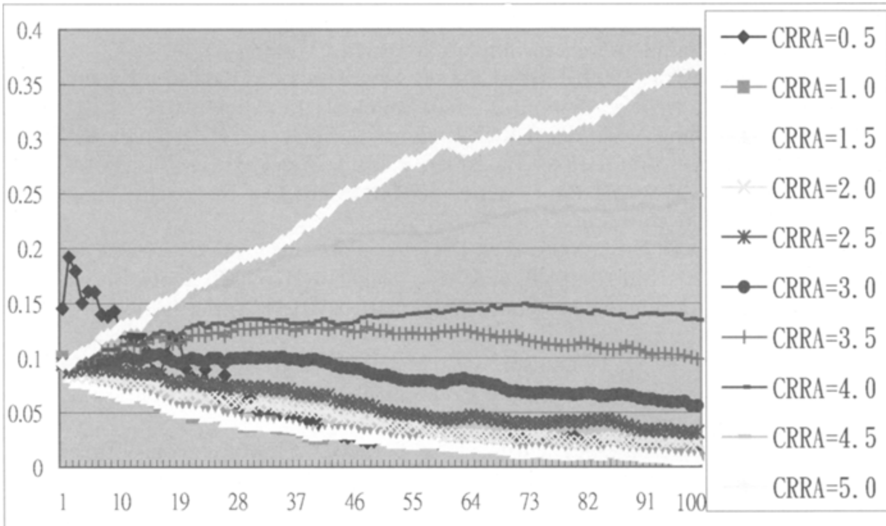

Fig. 2. Wealth Share Dynamics

higher the wealth share. For example, the wealth share of the agents with the highest risk aversion coefficient ($\varrho = 5$) shows strong growth from 10% in the initial period to 35% in the final period. Unlike what the Blume-Easley theorem ([1]) predicts, the log-utility agents ($\varrho = 1$) do not survive well in this case. Instead, their wealth share keeps on declining toward zero. Agents with lower values of ϱ share the same destiny. They are all dominated by agents with high risk aversion coefficient.

6 Concluding Remarks

This study can be possibly related to the extensive literature on the empirical estimation of the RRA coefficients. A wide range of RRA coefficients has been found in the literature, and so far no unanimous conclusion has been reached. The agent-based computational approach as demonstrated in this paper proposes the possibility that in reality there may be such a wide *survival range* of the relative risk aversion coefficient. Furthermore, the agents' wealth share is positively related to their RRA coefficients. This result combined with our earlier result (Figure 3, [5]) well articulates the contribution of risk aversion to survivability.

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