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# Nominal income targeting versus money growth targeting in an endogenously growing economy

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## Abstract

We find that local indeterminacy more easily emerges under a regime of nominal income targeting. Both targeting regimes are equally effective in influencing economic growth and inflation. The results thus favor money growth over nominal income as a nominal anchor.

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## 1. Introduction

The choice of an appropriate intermediate target for monetary policy has been one of the oldest debates in monetary economics. Over the last few decades, nominal income targeting has received considerable academic attention. To a large extent, the development of nominal income targeting has

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arisen as a result of the breakdown in the short-run relationships between monetary aggregates and nominal GDP due to large and unpredictable changes in payments industry technology and regulatory practices. Several economists thus consider nominal income to be a superior nominal anchor to monetary aggregates (Meade, 1978; Tobin, 1980; Taylor, 1985; McCallum, 1985).

There are two different criteria in the literature that are used to evaluate the relative stabilizing performance of alternative policy rules. The first criterion, proposed by Poole (1970), has to do with how closely different policy rules can on average keep crucial variables to their target values in a stochastic environment. The second criterion, proposed by Benhabib and Farmer (1994), is concerned with whether different policy rules lead to distinct patterns of local dynamics. Most of the existing studies on the relative stabilization between nominal income and monetary aggregates, including Bean (1983), Frankel and Chinn (1995), West (1986), Hall and Mankiw (1994), and McCallum and Nelson (1999), adopt the first criterion. This paper, however, departs from these studies and adopts the second criterion to evaluate the relative stabilization between targeting nominal income and targeting money supply. To be more specific, this paper sets up an endogenous growth model in which the behavioral relationship is derived based on a solid optimization, and uses it to study the relative desirability of nominal income targeting and money growth targeting by examining the macroeconomic stability properties as well as the growth and inflation rate effects under each targeting regime.<sup>1</sup> By means of such an optimizing model, our results indicate that, as long as the elasticity of the nominal interest rate with respect to the real balances/output ratio is sufficiently high, both targeting regimes are stabilizing in that they ensure the uniqueness of the equilibrium. However, local indeterminacy and endogenous growth fluctuations emerge more easily under a regime of nominal income targeting. When examining the long-run growth and inflation rate effects, we find that the two targeting regimes are equally effective. Our results obviously favor money growth over nominal income as the nominal anchor.

## 2. The model

The representative household's lifetime utility is given by:

$$U(c) = \int_0^{\infty} \frac{c^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt; \quad \rho > 0, \sigma > 1, \quad (1)$$

where  $c$  denotes consumption,  $\rho$  is the rate of time preference, and  $\sigma$  is the inverse of the intertemporal elasticity of substitution.<sup>2</sup>

In line with Rebelo (1991), output  $y$  is produced using a stock of broad-concept productive capital  $k$ ; that is,  $y = Ak$ , where  $A > 0$  stands for the total factor productivity. The household holds nominal money balances  $M$  to facilitate transactions of output. Let us denote  $m$  ( $\equiv M/P$ ) as real money balances with  $P$

<sup>1</sup> Recently, there has been a growing interest in the monetary dynamics literature in studying the link between money supply rules and macroeconomic stability (for example, Schmitt-Grohé and Uribe (2000), Benhabib et al. (2001a,b), and Meng (2002), etc.). Nevertheless, until now, no attention has been paid to the case of nominal income targeting.

<sup>2</sup> The assumption  $\sigma > 1$  is consistent with the empirical evidence presented in many recent studies, whose results suggest that the intertemporal elasticity of substitution is much less than one. See Agénor and Montiel (1999, p.468) for a summary.

representing the price level. Following Zhang (1996) and allowing for a balanced growth path, the transactions cost technology is summarized by a rate of loss in real output as follows:

$$\phi = \phi(m/y), \quad (2)$$

where  $\phi' < 0$ ,  $\phi'' \geq 0$ ,  $\lim_{m/y \rightarrow 0} \phi(m/y) = 1$ , and  $\lim_{m/y \rightarrow \infty} \phi(m/y) = \bar{\phi} \in (0, 1)$ .

The household also holds nominal government bonds  $B$  that pay the nominal interest rate  $R (> 0)$ . Let us denote the real financial wealth as  $a \equiv m + b$ , where  $b (\equiv B/P)$  represents real government bonds. The household's flow budget constraint is thus described by:

$$\dot{k} + \dot{a} = (1 - \phi)y - c + (R - \pi)a - Rm + \tau, \quad (3)$$

where an overdot denotes the time derivative,  $\pi \equiv \dot{P}/P$  is the inflation rate, and  $\tau$  represents real transfers from the government.

The representative household treats  $\pi$  and  $\tau$  as given and maximizes (1) subject to (2) and (3) by choosing a sequence  $\{c, m, a, k\}_{t=0}^{\infty}$ . By letting  $\lambda$  be the shadow value of wealth, the optimum conditions necessary for the representative household are:

$$c^{-\sigma} = \lambda, \quad (4)$$

$$R = -\phi', \quad (5)$$

$$\dot{\lambda}/\lambda = \rho - (R - \pi), \quad (6)$$

$$\dot{\lambda}/\lambda = \rho - (1 - \phi)A - \phi'(m/k), \quad (7)$$

together with Eqs. (2) and (3), and the transversality conditions of  $a$  and  $k$ :

$$\lim_{t \rightarrow \infty} \lambda a e^{-\rho t} = 0, \quad (8a)$$

$$\lim_{t \rightarrow \infty} \lambda k e^{-\rho t} = 0. \quad (8b)$$

Eq. (4) indicates that the household equates the marginal utility of consumption to the marginal utility of wealth. Eq. (5) indicates that the nominal interest rate equals the marginal benefit of holding real money balances. By putting (5)–(7) together, we can infer that the inflation rate  $\pi$  is:

$$\pi = -(1 - \phi)A - \phi'[1 + (m/k)]. \quad (9)$$

Differentiating (4) with respect to time and plugging the resulting equation into (6), we have the standard Keynes–Ramsey rule:

$$\frac{\dot{c}}{c} = \frac{(R - \pi) - \rho}{\sigma}. \quad (10)$$

Eq. (10) indicates that consumption rises (falls) as the real interest rate  $R - \pi$  exceeds (falls short of) the rate of time preference  $\rho$ .

By definition, the real money balances evolve through time according to:

$$\dot{m} = (\mu - \pi)m, \quad (11)$$

where  $\mu$  is the growth rate of nominal money balances. Under a regime of nominal income targeting, the central bank adjusts the money supply to whatever level is needed for the nominal income target to prevail. When targeting money growth, on the other hand,  $\mu$  is kept constant.

The government issues money and bonds to finance its expenditures on interest payments and lump-sum transfers. The flow budget constraint of the government is thus given by:

$$\dot{M}/P + \dot{b} = (R - \pi)b + \tau. \quad (12)$$

The economy's consolidated budget constraint can now be obtained by combining (3), (11) and (12):

$$\dot{k} = A(1 - \phi)k - c, \quad (13)$$

which reveals that a fraction  $\phi$  of real output flows from capital accumulation to transactions services.

### 3. Nominal income targeting vs. money growth targeting

#### 3.1. Targeting nominal income

Under a regime of nominal income targeting, the following relationship must hold:

$$\pi + \gamma_y = \bar{n}, \quad (14)$$

where  $\gamma_y (\equiv \dot{y}/y)$  denotes the growth rate of real output and  $\bar{n}$  is the government's target for the nominal income growth rate. Given  $Ak$  production technology, (13) and (14) imply

$$(1 - \phi)A - x = \bar{n} - \pi, \quad (15)$$

where  $x \equiv c/k$  is the consumption/capital ratio. Eqs. (9) and (15) simultaneously solve the real balances/output ratio  $z \equiv m/y$  and inflation rate  $\pi$  as  $z = z(x; \bar{n})$  and  $\pi = \pi(x; \bar{n})$ , respectively, with  $\partial z / \partial x = \partial z / \partial \bar{n} = \{(\phi' / z) \cdot [\zeta(1 + Az) - Az]\}^{-1}$ ,  $\partial \pi / \partial x = \partial \pi / \partial \bar{n} = \zeta(1 + Az)[\zeta(1 + Az) - Az]^{-1}$ , and  $\zeta \equiv -d \ln R / d \ln z > 0$  denoting the elasticity of the nominal interest rate with respect to  $z$ .

The differential equation of the transformed variable  $x$  is derived from (10) and (13):

$$\dot{x} = \left\{ \frac{[1 - \phi(z)]A + \phi'(z)Az - \rho}{\sigma} - [1 - \phi(z)]A + x \right\} x, \quad (16)$$

where  $z = z(x; \bar{n})$ . Linearizing (16) around the steady state gives

$$\dot{x} = \Delta(x - \tilde{x}), \quad (16a)$$

where  $\tilde{x}$  is the stationary value of  $x$  under nominal income targeting and

$$\Delta = \frac{\zeta[\sigma + A\tilde{z}(\sigma - 1)]\tilde{x}}{\sigma[\zeta(1 + A\tilde{z}) - A\tilde{z}]} \geq 0, \text{ as } \zeta \geq \zeta^* \equiv A\tilde{z}/(1 + A\tilde{z}) \in (0, 1). \quad (17)$$

The dynamic feature reported in (16a) hinges on the sign of  $\Delta$ . In particular, given that  $x$  is a jump variable, the monetary equilibrium is locally determinate if  $\Delta > 0$ . If  $\Delta < 0$ , on the other hand, then there is

a continuum of equilibrium trajectories that converges to the steady state, and hence local indeterminacy emerges. Therefore, we have the following proposition:

**Proposition 1.** *Under a regime of nominal income targeting, the monetary equilibrium is locally determinate if the elasticity of the nominal interest rate with respect to the real balances/output ratio is sufficiently high ( $\zeta > \zeta^*$ ). When the elasticity of the nominal interest rate with respect to the real balances/output ratio ( $\zeta < \zeta^*$ ) is sufficiently low, local indeterminacy will arise.*

The steady-state solution is obtained by setting  $\dot{x}=0$ , which gives  $\tilde{x}=\tilde{x}(\bar{n})$ . By substituting  $\tilde{x}=\tilde{x}(\bar{n})$  into  $z=z(x;\bar{n})$  and,  $\pi=\pi(x;\bar{n})$  the steady-state solutions to the real balances-output ratio and inflation rate are  $\tilde{z}=\tilde{z}(\bar{n})$  and  $\tilde{\pi}=\tilde{\pi}(\bar{n})$ . Sequentially, we can infer from (14) that the stationary economic growth rate is  $\tilde{y}=\tilde{y}(\bar{n})$ . The comparative-static results with respect to the stationary rates of economic growth and inflation in response to the nominal income target are reported as follows:

$$\frac{\partial \tilde{\pi}}{\partial \bar{n}} = \frac{\sigma(1 + A\tilde{z})}{\sigma + (\sigma - 1)A\tilde{z}} > 1, \tag{18a}$$

$$\frac{\partial \tilde{y}}{\partial \bar{n}} = \frac{-A\tilde{z}}{\sigma + (\sigma - 1)A\tilde{z}} < 0. \tag{18b}$$

Eq. (18a) (18b) indicates that a rise in the nominal income target raises the inflation rate and lowers the economic growth rate in the long run. Intuitively, the central bank expands money growth in order for the nominal income growth to rise. The resulting higher inflation rate discourages the households from holding real money balances, and thus causes the real balances/output ratio to fall. Therefore, a larger fraction of real output is devoted to transactions services, which lowers the marginal product of capital. The lower marginal product of capital in turn discourages investment and thus reduces the rate of economic growth.

### 3.2. Targeting money growth

Under a regime of money growth targeting, the dynamic system in terms of the transformed variables  $x$  and  $z$  is as follows:

$$\dot{x} = \left\{ \frac{[1 - \phi(z)]A + \phi'(z)Az - \rho}{\sigma} - [1 - \phi(z)]A + x \right\} x, \tag{16}$$

$$\dot{z} = \{ \mu + (1 - \phi)A + \phi'(1 + Az) - [1 - \phi(z)]A + x \} z, \tag{19}$$

where (19) is derived from (9) (11) (13). Let us denote the stationary values of  $x$  and  $z$  under money growth targeting as  $\hat{x}$  and  $\hat{z}$ , respectively. The dynamic system  $(x, z)$  linearized around the steady state is given by

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = J \begin{bmatrix} x - \hat{x} \\ z - \hat{z} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (\mu - \mu_0), \tag{20}$$

where

$$J = \begin{bmatrix} 1 & \phi' \hat{z} (\zeta - \sigma) / \sigma \zeta \\ 1 & (\phi'' / \zeta) [\zeta (1 + A\hat{z}) - A\hat{z}] \end{bmatrix}.$$

It follows from (20) that the trace and the determinant of  $J$ , respectively, are:

$$\text{Tr}(J) = \left( \hat{\phi}'' / \varsigma \right) \left[ \varsigma(1 + A\hat{z}) - A\hat{z} \right] + 1 \geq 0 ; \quad \text{as } \varsigma \geq \varsigma^{**} \equiv A\hat{z} / \left[ (1 + A\hat{z}) + 1 / \hat{\phi}'' \right],$$

$$\text{Det}(J) = \left( \hat{\phi}'' / \sigma \right) [\sigma + A\hat{z}(\sigma - 1)] > 0.$$

Since  $x$  and  $z$  are jump variables, the monetary equilibrium is locally determinate if  $\text{Tr}(J) > 0$ , which indicates that the Jacobian matrix  $J$  has two roots with positive real parts. If  $\text{Tr}(J) < 0$ , then the two roots both have negative real parts, so that local indeterminacy emerges. Thus, we have the following proposition:

**Proposition 2.** *Under a regime of money growth targeting, the monetary equilibrium is locally determinate if the elasticity of the nominal interest rate with respect to the real balances-output ratio is sufficiently high ( $\varsigma > \varsigma^{**}$ ). If the elasticity of the nominal interest rate with respect to the real balances/output ratio ( $\varsigma < \varsigma^{**}$ ) is sufficiently low, local indeterminacy will arise.*

Propositions 1 and 2 point out that, under both targeting regimes, a sufficiently high elasticity of the nominal interest rate with respect to the real balances-output ratio  $\varsigma$  can ensure the uniqueness of the equilibrium transitional dynamic path around the BGP. However, comparing the critical values of both targeting regimes,  $\varsigma^*$  and  $\varsigma^{**}$ , reveals that  $\varsigma^{**} < \varsigma^*$ . This result indicates that local indeterminacy more easily emerges under a regime of nominal income targeting and contributes to the following proposition:

**Proposition 3.** *Local indeterminacy emerges more easily under a regime of nominal income targeting than under a regime of money growth targeting.*

From (20) with  $\dot{x} = \dot{z} = 0$ , we can derive the following steady-state relationships:  $\hat{x} = \hat{x}(\bar{n})$  and  $\hat{z} = \hat{z}(\bar{n})$ . By substituting  $\hat{z} = \hat{z}(\bar{n})$  into (9), we then have  $\hat{\pi} = \hat{\pi}(\bar{n})$ . Finally,  $\hat{\gamma} = \hat{\gamma}(\bar{n})$  is obtained given that  $\hat{\gamma} = [1 - \phi(\hat{z})]A - \hat{x}$ . The comparative-static results with respect to the stationary rates of economic growth and inflation in response to the money growth target are reported as follows:

$$\frac{\partial \hat{\pi}}{\partial \mu} = \frac{\sigma(1 + A\hat{z})}{\sigma + (\sigma - 1)A\hat{z}} > 1, \quad (21a)$$

$$\frac{\partial \hat{\gamma}}{\partial \mu} = \frac{-A\hat{z}}{\sigma + (\sigma - 1)A\hat{z}} < 0. \quad (21b)$$

Therefore, a rise in the money growth target also raises the inflation rate and lowers the economic growth rate in the long run. The intuition behind this is that an increase in the money growth rate raises the inflation rate and consequently discourages the households from holding real money balances. This tends to reduce the real balances/output ratio and to increase the transactions cost. As a result, the marginal product of capital declines, and this in turn discourages investment and economic growth.

By comparing (18a) (18b) with (21a) (21b), it is interesting to discover that both monetary rules have the same comparative-static results. This leads us to our final proposition:

**Proposition 4.** *Nominal income targeting and money growth targeting are equally effective in influencing the stationary rates of economic growth and inflation.*

The reasoning behind Proposition 4 is quite clear when we rewrite (11) as  $\pi + \gamma_m = \mu$ , where  $\gamma_m \equiv \dot{m}/m$  denotes the growth rate of real money balances. Given that, along the BGP real money balances and real

output grow at the same rate,  $\pi + \gamma_m = \mu$  and  $\pi + \gamma_y = \bar{n}$  in (14) imply that  $\mu = \bar{n}$  in the long run. Therefore, it is not surprising that targeting nominal income and targeting money growth will have the same balanced growth effect.

#### 4. Conclusions

Before ending this paper, three points should be mentioned. First, although our assumption that the intertemporal elasticity of substitution falls short of unity is consistent with much of the recent empirical evidence, it is well known that there is still no consensus on the estimate of it. However, it is easy to demonstrate that relaxing this assumption will not markedly change the results derived in the present paper; money growth targeting remains superior to nominal income targeting in terms of stabilizing the economy. Secondly, although this paper introduces money into the economy through a transactions-cost technology, we can further show that Propositions 3 and 4 hold if we instead adopt the cash-in-advance or the money-in-the-utility approach.<sup>3</sup> Therefore, our conclusion that in an endogenously growing economy money growth targeting dominates nominal income targeting is indeed robust. Third, nominal income growth targeting has been recommended by scholars who are concerned with economies that are buffeted by stochastic shocks and are characterized by some type of nominal price stickiness or wage stickiness (see, for example, Bean (1983), Aizenman and Frenkel (1986), and McCallum and Nelson (1999), among others). However, neither of these features is included in our model. A promising subject for future research would be to incorporate either nominal price stickiness or wage stickiness into the model, and then to use it to examine the robustness of our conclusions.

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<sup>3</sup> The detailed mathematical results are available upon request.

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