Minimax Play at Wimbledon: Comment

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In a recent contribution, Mark Walker and John Wooders (2001) analyzed serve choices in Grand Slam tennis matches to provide an empirical test of the mixed strategy equilibrium. They argued convincingly that unlike subjects in laboratories, professional players have sufficient experience to play games well, and that they are also highly motivated to win these games.[1] Their results indicated that there were no statistical differences in win rates for male players across various strategies, which is consistent with the equilibrium prediction. They fairly noted, however, that even the top male players tended to switch from one strategy to another too often, resulting in serial dependence.

This paper reexamines the results of Walker and Wooders (2001) by collecting and analyzing a broader dataset, including men's, women's, and juniors' matches. We find that the support of the minimax hypothesis is stronger. The plays in our data pass all of the tests in Walker and Wooders (2001) and therefore are more consistent with the theory of equilibrium than those in Walker and Wooders (2001). In short, the two hypotheses implied by the equilibrium, i.e., the equal probability of winning serve directions and the serial independence of serves, are borne out in our data.

I. Data

Our original dataset is comprised of three major group matches (men, women, and juniors), which are either collected from videotapes or from first-hand observations. We have ten men's matches, nine women's matches, and eight juniors' matches. Since each match has four point games (depending on the server and the serving court), we have 40 men's, 36 women's, and 32 juniors' point games. All matches for men and women are from Grand Slam finals involving top-level players over the past two decades. It would therefore be fair to say that everyone within the sample is a highly motivated player. Since it is difficult to obtain data on junior players, the matches within the juniors' group include the finals, quarterfinals, and second-round matches in both tournaments and Grand Slams.[2] In accordance with the rules of tennis, men's Grand Slam matches can last for up to five sets, while women's and juniors' matches can last for up to three sets. (Tables 1–3 summarize the data.[3])

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[1] Pierre-Andre Chiappori, Steven Levitt, and Timothy Groseclose (2002) and Ignacio Palacios-Huerta (2003) examined datasets based on penalty kicks in professional soccer games. Since a typical tennis match can last for a considerable period of time, while penalty kicks in soccer rarely occur, the strategic situation tennis players encounter may differ from what soccer players face. Specifically, papers analyzing penalty kicks typically have to pool several matches to get enough data. For this reason, we compare our results to those in Walker and Wooders (2001).

[2] There are a few games missing at the beginning of three juniors' matches. These are Salerni versus Perediynis (Wimbledon 2000), Scherer versus Cvetkovic (Australian Open 2003) and Tsonga versus Feeney (Australian Open 2003). This does not, however, affect the continuity of our data.

[3] Three matches in Walker and Wooders (2001) are also included in our dataset. They are Borg versus McEnroe (Wimbledon 1980), McEnroe versus Borg (US Open 1980), and Sampras versus Agassi (US Open 1995). The number of serves and the number of choices of *R* and *L* that we record, however, are slightly different from those in Walker and Wooders (2001). We suspect that this occurs because we use a slightly different standard in defining *L* and *R*. Despite this slight difference, we record our data using the same criterion throughout all matches.

II. Replicating Walker and Wooders' (2001) Tests

We follow Walker and Wooders (2001) to model each point in a point game by a two-by-two payoff matrix.

$s\backslash r$	L	R
L	π_{LL}	π_{LR}
R	π_{RL}	π_{RR}

Let L stand for the direction of left of the receiver and R for right of the receiver. For each point, the server has to decide in which direction to serve. The receiver has to guess simultaneously whether the serve will be to his left or right. In each box π_{sr} denotes the server's probability of winning the point, where subscripts $s \in \{L, R\}$ and $r \in \{L, R\}$, respectively, denote the choices made by the server and the receiver for that point. Since the game is constant-sum, $1 - \pi_{sr}$ is the probability that the receiver wins. Following Walker and Wooders (2001), the same payoff matrix applies to each point in a given point game.

This game has a unique mixed-strategy Nash equilibrium when

$$\pi_{LL} < \pi_{LR},\ \pi_{LL} < \pi_{RL},\ \pi_{RR} < \pi_{LR}$$
$$\text{and } \pi_{RR} < \pi_{RL}.$$

This condition implies that when the receiver guesses correctly which direction the server will serve, the receiver is better prepared and, hence, his probability of winning is higher. Equilibrium theory from an analysis of the match implies two testable predictions. First, since the equilibrium is mixed, the probability of winning should be the same for each of the server's pure strategies. Second, since the server should maximize his probability of winning every point in a match, he should play out the equilibrium strategy at every point. Therefore his serve choices should be independently and identically distributed.[4]

A. *The Test for the Equal Probability of Winning*

Following Walker and Wooders (2001), we conduct both the Pearson chi-square goodness-of-fit test and the Kolmogorov-Smirnov test (henceforth, KS test) to see whether each of the server's pure strategies yields an equal probability of winning. Since these tests have been fully discussed in Walker and Wooders (2001), we directly report our results.

The first parts in Tables 1–3 summarize the results of the Pearson test. At the conventional 5- or 10-percent significance level, the hypothesis of the equal probability of winning hardly can be rejected for any of the groups. The number of rejections in the men's group (two at 5 percent, and six at 10 percent) is slightly higher than the number of rejections in the women's group (none at 5 percent and one at 10 percent) or the juniors' group (one at 5 percent and four at 10 percent).

In examining whether the data in each group are consistent with equilibrium theory, we perform both the Pearson joint test and the KS test.[5] Using the Pearson joint test, we find that the corresponding p-values are 0.067 for male players, 0.716 for female players, and 0.551 for juniors. The p-value for all 108 point games is 0.299. Therefore, for male players, the hypothesis of the equal probability of winning can be rejected at the 10-percent significance level. The null hypothesis predicting an equal probability of winning fares well for both female and junior players, however. It also fares well if we consider all 108 point games together. As for the KS test, the four left-hand panels in Figure 1 present a visual comparison, illustrating the cumulative distribution function (henceforth, CDF) of the p-values associated with Pearson's statistics for point games in a group and the CDF of a uniform distribution. The KS statistics are 0.778 for men, 0.577 for women, 0.646 for juniors, and 0.753 for all 108 point games, all of which are far from the critical value at either the 5-percent or 10-percent significance level.[6] In brief, except for the result of the

[4] For details, see Walker and Wooders (2000).

[5] The KS test is informative in deciding how the data are generated. See Jean Dickinson Gibbons and Subhabrata Chakraborti (2003, 148).

[6] The critical values of the KS statistic are as follows. For the sample size of 108 point games, the critical value is 1.36 at the 5-percent level and 1.22 at the 10-percent level. For the sample size of 40 point games, the critical value is 1.328 at the 5-percent level and 1.195 at the 10-percent level. For the sample size of 36 point games, the critical value is 1.326 at the 5-percent level and 1.194 at the 10-percent level. For the sample size of 32 point games, the critical value is 1.324 at the 5-percent level and 1.194 at the 10-percent level.

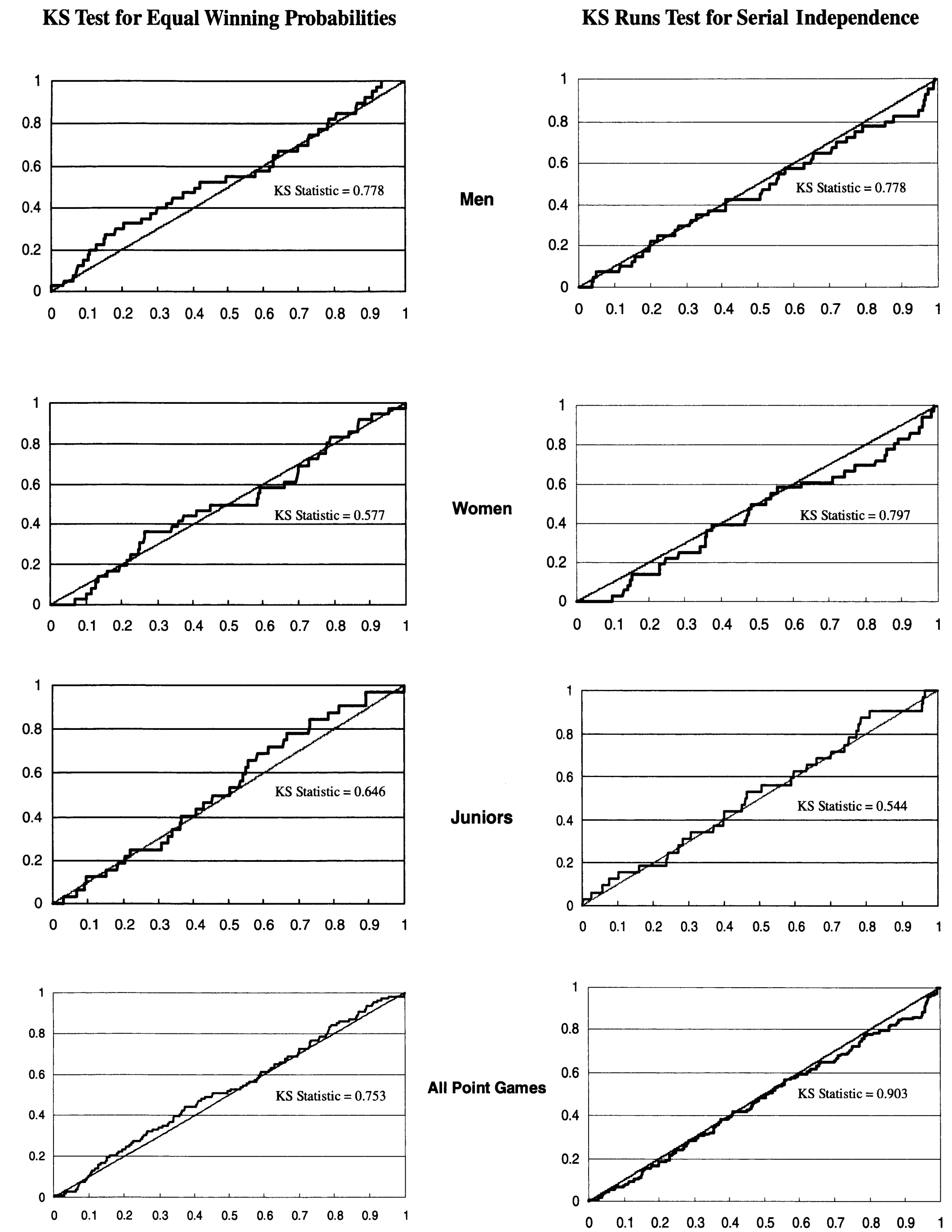

FIGURE 1. KOLMOGOROV-SMIRNOV TEST

Pearson joint test for men, we cannot reject the null hypothesis that, jointly, players in each group are behaving in accordance with equilibrium. These findings are consistent with Walker and Wooders (2001).

B. *The Test for Serial Independence*

Walker and Wooders (2001) used the runs test to examine serial independence. Recall that a run is the maximal string of consecutive iden-

TABLE 1—TEST OF EQUAL WINNING PROBABILITIES AND RUNS TEST IN MEN'S TENNIS

			Serve direction			Points won		Win rate				Runs		
Index match	Server	Court	L	R	Total	L	R	L	R	Pearson statistic	p-value	r_i	$F(r_i - 1)$	$F(r_i)$
1 80 WIMBLEDON	**Borg**	Ad	15	74	89	10	51	0.667	0.689	0.029	0.864	27	0.529	0.741
2 80 WIMBLEDON	**Borg**	Deuce	32	57	89	26	36	0.813	0.632	3.174	0.075**	44	0.638	0.715
3 80 WIMBLEDON	McEnroe	Ad	45	30	75	29	19	0.644	0.633	0.010	0.922	42	0.863	0.908
4 80 WIMBLEDON	McEnroe	Deuce	47	38	85	31	29	0.660	0.763	1.086	0.297	51	0.951	0.970**
5 80 U.S. OPEN	**McEnroe**	Ad	42	36	78	23	27	0.548	0.750	3.450	0.063**	42	0.395	0.476
6 80 U.S. OPEN	**McEnroe**	Deuce	60	28	88	38	16	0.633	0.571	0.309	0.579	32	0.031	0.050
7 80 U.S. OPEN	Borg	Ad	23	53	76	15	33	0.652	0.623	0.060	0.806	34	0.548	0.639
8 80 U.S. OPEN	Borg	Deuce	27	53	80	18	27	0.667	0.509	1.797	0.180	33	0.139	0.207
9 85 ROLAND GARROS	**Wilander**	Ad	36	12	48	23	8	0.639	0.667	0.030	0.862	18	0.285	0.396
10 85 ROLAND GARROS	**Wilander**	Deuce	18	36	54	10	18	0.556	0.500	0.148	0.700	26	0.563	0.669
11 85 ROLAND GARROS	Lendl	Ad	39	9	48	20	6	0.513	0.667	0.697	0.404	19	0.903	1.000
12 85 ROLAND GARROS	Lendl	Deuce	30	19	49	13	9	0.433	0.474	0.077	0.782	27	0.748	0.839
13 89 WIMBLEDON	**Becker**	Ad	32	31	63	29	16	0.906	0.516	11.743	0.001*	32	0.400	0.502
14 89 WIMBLEDON	**Becker**	Deuce	41	25	66	29	15	0.707	0.600	0.805	0.370	28	0.116	0.173
15 89 WIMBLEDON	Lendl	Ad	50	35	85	27	23	0.540	0.657	1.166	0.280	45	0.698	0.773
16 89 WIMBLEDON	Lendl	Deuce	60	31	91	39	19	0.650	0.613	0.122	0.727	44	0.650	0.725
17 89 U.S. OPEN	**Becker**	Ad	48	10	58	31	4	0.646	0.400	2.090	0.148	15	0.076	0.184
18 89 U.S. OPEN	**Becker**	Deuce	44	27	71	27	15	0.614	0.556	0.234	0.629	37	0.693	0.781
19 89 U.S. OPEN	Lendl	Ad	34	22	56	19	16	0.559	0.727	1.617	0.203	29	0.585	0.693
20 89 U.S. OPEN	Lendl	Deuce	33	26	59	18	21	0.545	0.808	4.463	0.035*	28	0.245	0.337
21 92 AUSTRALIAN OPEN	**Courier**	Ad	34	18	52	22	12	0.647	0.667	0.020	0.888	32	0.988	0.994*
22 92 AUSTRALIAN OPEN	**Courier**	Deuce	30	20	50	20	12	0.667	0.600	0.231	0.630	29	0.850	0.912
23 92 AUSTRALIAN OPEN	Edberg	Ad	40	6	46	24	3	0.600	0.500	0.215	0.643	11	0.213	0.529
24 92 AUSTRALIAN OPEN	Edberg	Deuce	26	16	42	17	8	0.654	0.500	0.973	0.324	22	0.135	0.253
25 95 ROLAND GARROS	**Muster**	Ad	27	8	35	21	4	0.778	0.500	2.333	0.127	15	0.672	0.878
26 95 ROLAND GARROS	**Muster**	Deuce	30	8	38	23	5	0.767	0.625	0.654	0.419	14	0.479	0.615
27 95 ROLAND GARROS	Chang	Ad	38	2	40	22	0	0.579	0.000	2.573	0.109	5	0.146	1.000
28 95 ROLAND GARROS	Chang	Deuce	21	24	45	16	12	0.762	0.500	3.268	0.071**	29	0.940	0.968
29 95 U.S. OPEN	**Sampras**	Ad	19	38	57	11	29	0.579	0.763	2.054	0.152	27	0.510	0.639
30 95 U.S. OPEN	**Sampras**	Deuce	30	28	58	20	24	0.667	0.857	2.870	0.090**	28	0.256	0.350
31 95 U.S. OPEN	Agassi	Ad	41	13	54	31	11	0.756	0.846	0.463	0.496	27	0.989	1.000*
32 95 U.S. OPEN	Agassi	Deuce	35	25	60	21	14	0.600	0.560	0.096	0.757	26	0.106	0.163
33 00 AUSTRALIAN OPEN	**Agassi**	Ad	30	25	55	19	12	0.633	0.480	1.304	0.254	22	0.031	0.056
34 00 AUSTRALIAN OPEN	**Agassi**	Deuce	32	28	60	23	21	0.719	0.750	0.075	0.785	38	0.959	0.978**
35 00 AUSTRALIAN OPEN	Kafelnikov	Ad	28	24	52	15	14	0.536	0.583	0.119	0.730	24	0.172	0.255
36 00 AUSTRALIAN OPEN	Kafelnikov	Deuce	31	27	58	21	15	0.677	0.556	0.910	0.340	30	0.461	0.568
37 01 WIMBLEDON	**Ivanisevic**	Ad	48	26	74	30	21	0.625	0.808	2.628	0.105	33	0.279	0.377
38 01 WIMBLEDON	**Ivanisevic**	Deuce	60	23	83	41	17	0.683	0.739	0.246	0.620	28	0.035	0.057
39 01 WIMBLEDON	Rafter	Ad	27	32	59	20	24	0.741	0.750	0.007	0.935	33	0.721	0.802
40 01 WIMBLEDON	Rafter	Deuce	31	33	64	22	23	0.710	0.697	0.012	0.911	29	0.130	0.190
Joint test			1414	1076	2490	914	689	0.646	0.640	54.157	0.067			

Notes: Each row corresponds to a point game. We index each point game from 1 to 40. Each row contains the following information in order: (1) the index of the point game, (2) the match and year of the point game, (3) the server, (4) the serving court, (5) the number of times the server chooses L, (6) the number of times the server chooses R, (7) the total number of serves, (8) the number of times the server chooses L and wins, (9) the number of times the server chooses R and wins, (10) the proportionate number of times the server wins if he chooses L, (11) the proportionate number of times that the server wins if he chooses R, (12) the Pearson statistic and its p-value, (13) the number of runs, (14) the probability of having the number of runs one less than that in (13) or even fewer, and (14) the probability of having the number of runs equal to that in (13) or fewer. Note that the winner of each match is indicated in bold type.

* Denotes rejection at 5 percent.

** Denotes rejection at 10 percent.

TABLE 2—TEST OF EQUAL WINNING PROBABILITIES AND RUNS TEST IN WOMENS' TENNIS

			Serve direction			Points won		Win rate				Runs		
Index match	Server	Court	L	R	Total	L	R	L	R	Pearson statistic	p-value	r_i	$F(r_i - 1)$	$F(r_i)$
1 85 AUSTRALIAN OPEN	**Navratilova**	Ad	21	20	41	17	13	0.810	0.650	1.328	0.249	25	0.831	0.898
2 85 AUSTRALIAN OPEN	**Navratilova**	Deuce	22	14	36	12	4	0.545	0.286	2.338	0.126	20	0.689	0.800
3 85 AUSTRALIAN OPEN	Evert	Ad	18	16	34	9	8	0.500	0.500	0.000	1.000	23	0.946	0.975
4 85 AUSTRALIAN OPEN	Evert	Deuce	5	32	37	2	17	0.400	0.531	0.298	0.585	10	0.466	0.610
5 87 WIMBLEDON	**Navratilova**	Ad	21	7	28	17	6	0.810	0.857	0.081	0.776	11	0.281	0.502
6 87 WIMBLEDON	**Navratilova**	Deuce	29	6	35	17	3	0.586	0.500	0.151	0.698	11	0.331	0.647
7 87 WIMBLEDON	Graf	Ad	13	16	29	7	11	0.538	0.688	0.677	0.411	17	0.671	0.793
8 87 WIMBLEDON	Graf	Deuce	11	20	31	8	13	0.727	0.650	0.194	0.660	19	0.905	0.963
9 87 U.S. OPEN	**Navratilova**	Ad	25	12	37	14	9	0.560	0.750	1.244	0.265	15	0.148	0.259
10 87 U.S. OPEN	**Navratilova**	Deuce	24	10	34	16	9	0.667	0.900	1.975	0.160	13	0.133	0.252
11 87 U.S. OPEN	Graf	Ad	13	11	24	8	6	0.615	0.545	0.120	0.729	12	0.273	0.433
12 87 U.S. OPEN	Graf	Deuce	12	14	26	6	10	0.500	0.714	1.254	0.263	14	0.430	0.594
13 92 ROLAND GARROS	**Seles**	Ad	34	15	49	23	6	0.676	0.400	3.293	0.070**	25	0.810	0.903
14 92 ROLAND GARROS	**Seles**	Deuce	29	22	51	16	13	0.552	0.591	0.078	0.780	22	0.096	0.155
15 92 ROLAND GARROS	Graf	Ad	33	27	60	14	16	0.424	0.593	1.684	0.194	29	0.282	0.376
16 92 ROLAND GARROS	Graf	Deuce	36	27	63	17	17	0.472	0.630	1.539	0.215	30	0.270	0.362
17 92 U.S. OPEN	**Seles**	Ad	13	13	26	7	6	0.538	0.462	0.154	0.695	18	0.919	0.966
18 92 U.S. OPEN	**Seles**	Deuce	18	9	27	13	7	0.722	0.778	0.096	0.756	17	0.939	0.984
19 92 U.S. OPEN	Sanchez	Ad	9	25	34	2	10	0.222	0.400	0.916	0.339	17	0.828	0.947
20 92 U.S. OPEN	Sanchez	Deuce	21	12	33	12	8	0.571	0.667	0.290	0.590	14	0.145	0.246
21 97 WIMBLEDON	**Hingis**	Ad	26	14	40	14	8	0.538	0.571	0.040	0.842	21	0.668	0.794
22 97 WIMBLEDON	**Hingis**	Deuce	15	29	44	8	16	0.533	0.552	0.013	0.908	21	0.454	0.598
23 97 WIMBLEDON	Novotna	Ad	14	20	34	8	12	0.571	0.600	0.028	0.868	23	0.966	0.987*
24 97 WIMBLEDON	Novotna	Deuce	29	14	43	13	10	0.448	0.714	2.686	0.101	23	0.814	0.905
25 99 ROLAND GARROS	**Graf**	Ad	22	21	43	13	10	0.591	0.476	0.568	0.451	30	0.985	0.994*
26 99 ROLAND GARROS	**Graf**	Deuce	23	20	43	14	12	0.609	0.600	0.003	0.954	26	0.831	0.899
27 99 ROLAND GARROS	Hingis	Ad	36	9	45	14	6	0.389	0.667	2.250	0.134	16	0.515	0.636
28 99 ROLAND GARROS	Hingis	Deuce	32	18	50	17	10	0.531	0.556	0.027	0.869	22	0.217	0.312
29 00 U.S. OPEN	**V. Williams**	Ad	11	21	32	5	14	0.455	0.667	1.347	0.246	16	0.510	0.654
30 00 U.S. OPEN	**V. Williams**	Deuce	17	20	37	10	13	0.588	0.650	0.149	0.699	16	0.096	0.168
31 00 U.S. OPEN	Davenport	Ad	14	14	28	8	11	0.571	0.786	1.474	0.225	14	0.280	0.427
32 00 U.S. OPEN	Davenport	Deuce	10	21	31	4	12	0.400	0.571	0.797	0.372	14	0.331	0.478
33 02 AUSTRALIAN OPEN	**Capriati**	Ad	13	29	42	6	16	0.462	0.552	0.293	0.588	16	0.107	0.180
34 02 AUSTRALIAN OPEN	**Capriati**	Deuce	20	22	42	11	13	0.550	0.591	0.072	0.789	22	0.442	0.569
35 02 AUSTRALIAN OPEN	Hingis	Ad	33	16	49	17	6	0.515	0.375	0.850	0.357	21	0.245	0.366
36 02 AUSTRALIAN OPEN	Hingis	Deuce	26	23	49	16	9	0.615	0.391	2.452	0.117	21	0.078	0.128
Joint test			748	639	1387	415	370	0.555	0.579	30.758	0.716			

Note: Each column contains the same information as noted in Table 1, except the index runs from 1 to 36.

tical serve directions. The idea is to find the number of runs in the list of the direction of serves according to the order observed. If there are too many runs, then players switch directions too often to be random. On the other hand, if there are too few runs, then they switch direction too infrequently to be random. We omit the details of the test. Walker and Wooders (2001) found that there were too many runs in male players' choices, which led to their conclusion that even the best tennis players tend to switch from one direction to another too often, resulting in serial dependence. Our results are quite different, with the null hypothesis being rejected in only a few point games in our dataset. The results are reported in the second parts

TABLE 3—TEST OF EQUAL WINNING PROBABILITIES AND RUNS TEST IN JUNIORS' TENNIS

Index	match	Server	Court	Serve direction L	Serve direction R	Total	Points won L	Points won R	Win rate L	Win rate R	Pearson statistic	p-value	Runs r_i	Runs $F(r_i - 1)$	Runs $F(r_i)$
1	96 AVVENIRE TOURNAMENT	**Middelton**	Ad	17	10	27	11	5	0.647	0.500	0.564	0.453	12	0.189	0.320
2	96 AVVENIRE TOURNAMENT	**Middelton**	Deuce	22	11	33	17	7	0.773	0.636	0.688	0.407	18	0.771	0.866
3	96 AVVENIRE TOURNAMENT	Kalvaria	Ad	21	8	29	12	4	0.571	0.500	0.120	0.730	10	0.077	0.156
4	96 AVVENIRE TOURNAMENT	Kalvaria	Deuce	11	15	26	8	7	0.727	0.467	1.766	0.184	9	0.016	0.042**
5	00 WIMBLEDON	**Salerni**	Ad	8	8	16	3	4	0.375	0.500	0.254	0.614	10	0.595	0.786
6	00 WIMBLEDON	**Salerni**	Deuce	8	9	17	3	7	0.375	0.778	2.837	0.092**	10	0.500	0.702
7	00 WIMBLEDON	Perediynis	Ad	6	8	14	2	4	0.333	0.500	0.389	0.533	5	0.028	0.086
8	00 WIMBLEDON	Perediynis	Deuce	10	6	16	3	1	0.300	0.167	0.356	0.551	9	0.497	0.706
9	02 AVVENIRE TOURNAMENT	**Gonzalez**	Ad	13	11	24	5	7	0.385	0.636	1.510	0.219	12	0.273	0.433
10	02 AVVENIRE TOURNAMENT	**Gonzalez**	Deuce	9	13	22	6	4	0.667	0.308	2.764	0.096**	11	0.305	0.472
11	02 AVVENIRE TOURNAMENT	Sanchez	Ad	15	6	21	7	3	0.467	0.500	0.019	0.890	11	0.668	0.871
12	02 AVVENIRE TOURNAMENT	Sanchez	Deuce	13	9	22	4	2	0.308	0.222	0.196	0.658	11	0.305	0.472
13	03 AUSTRALIAN OPEN (Qrt)	**Baqhdatis**	Ad	9	7	16	9	4	1.000	0.571	4.747	0.029*	7	0.108	0.231
14	03 AUSTRALIAN OPEN (Qrt)	**Baqhdatis**	Deuce	10	12	22	9	9	0.900	0.750	0.825	0.364	12	0.425	0.605
15	03 AUSTRALIAN OPEN (Qrt)	Evans	Ad	12	8	20	5	5	0.417	0.625	0.833	0.361	14	0.920	0.971
16	03 AUSTRALIAN OPEN (Qrt)	Evans	Deuce	18	6	24	12	2	0.667	0.333	2.057	0.151	10	0.392	0.569
17	03 AUSTRALIAN OPEN (2nd)	**Bauer**	Ad	19	12	31	11	6	0.579	0.500	0.185	0.667	15	0.319	0.466
18	03 AUSTRALIAN OPEN (2nd)	**Bauer**	Deuce	6	27	33	5	17	0.833	0.630	0.917	0.338	12	0.673	0.792
19	03 AUSTRALIAN OPEN (2nd)	Kerber	Ad	28	12	40	13	3	0.464	0.250	1.607	0.205	20	0.747	0.840
20	03 AUSTRALIAN OPEN (2nd)	Kerber	Deuce	21	20	41	12	11	0.571	0.550	0.019	0.890	19	0.173	0.264
21	03 AUSTRALIAN OPEN (2nd)	**Dellacqua**	Ad	18	7	25	15	5	0.833	0.714	0.446	0.504	13	0.741	0.908
22	03 AUSTRALIAN OPEN (2nd)	**Dellacqua**	Deuce	21	6	27	15	3	0.714	0.500	0.964	0.326	8	0.062	0.139
23	03 AUSTRALIAN OPEN (2nd)	Kim	Ad	6	28	34	4	17	0.667	0.607	0.074	0.785	10	0.216	0.347
24	03 AUSTRALIAN OPEN (2nd)	Kim	Deuce	13	21	34	8	10	0.615	0.476	0.624	0.429	16	0.282	0.415
25	03 AUSTRALIAN OPEN (2nd)	**Scherer**	Ad	11	7	18	7	5	0.636	0.714	0.117	0.732	9	0.296	0.484
26	03 AUSTRALIAN OPEN (2nd)	**Scherer**	Deuce	11	9	20	6	6	0.545	0.667	0.303	0.582	15	0.955	0.985**
27	03 AUSTRALIAN OPEN (2nd)	Cvetkovic	Ad	6	6	12	4	3	0.667	0.500	0.343	0.558	7	0.392	0.608
28	03 AUSTRALIAN OPEN (2nd)	Cvetkovic	Deuce	6	7	13	5	4	0.833	0.571	1.040	0.308	9	0.733	0.879
29	03 AUSTRALIAN OPEN (2nd)	**Tsonga**	Ad	11	5	16	10	4	0.909	0.800	0.374	0.541	9	0.626	0.846
30	03 AUSTRALIAN OPEN (2nd)	**Tsonga**	Deuce	8	10	18	6	7	0.750	0.700	0.055	0.814	4	0.000	0.003*
31	03 AUSTRALIAN OPEN (2nd)	Feeney	Ad	14	6	20	7	3	0.500	0.500	0.000	1.000	13	0.956	1.000**
32	03 AUSTRALIAN OPEN (2nd)	Feeney	Deuce	12	8	20	4	6	0.333	0.750	3.333	0.068**	11	0.480	0.663
Joint test				413	338	751	248	185	0.600	0.547	30.327	0.551			

Note: Each column contains the same information as noted in Table 1, except the index runs from 1 to 32.

of Tables 1–3. At the 5-percent significance level, there are two rejections for male players and two rejections for female players, both as a result of too many runs, and one rejection for junior players as a result of too few runs. At the 10-percent significance level, there are four rejections for male players and two rejections for female players, both as a result of too many runs, and four rejections for juniors (two for too many runs and two for too few runs). It is interesting to note that of all these rejections, only junior players violate the null hypothesis as a result of too few runs (switching direction too infrequently).

As for the joint test, the KS statistics of the joint null hypothesis, that the serves are serially independent within a group, are 0.778 for men, 0.797 for women, and 0.544 for juniors. If we consider all 108 point games together, the KS statistic is 0.903. The *p*-values of these KS statistics are all far from the rejection region under the conventional significance level.[7] The four right-hand panels in Figure 1 offer a visual comparison of the uniform CDF and the CDF of the *p*-values associated with the runs test for point games in a group. In sum, we cannot reject the null hypothesis that, jointly, serves within each group are serially independent. This is more consistent with the theory of equilibrium than the result of the runs test in Walker and Wooders (2001).[8]

[7] Refer to footnote 6 for the critical values of the KS statistics.

[8] Alternatively, the other statistic of interest is the length of the longest run. The idea is that a run of extreme length can be used as a criterion for rejection of randomness. In our data, the rejection number based on this is three for men, two for women and two for juniors at the 10-percent significance level. In brief, the result that players' behaviors are serially independent still holds.

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