

Fuzzy- Evolvement Multi-Aim Majorization

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Abstract—The purpose of this paper is to address the multi-target majorization via combined fuzzy satisfied method and evolvement programming (E.P.) method. The concept of non-inferiority is employed to characterize a solution of the multi-target problem. Then, a fuzzy satisfied method based on evolvement programming is introduced to determine the optimal solution. As a result, the objective functions of the optimization problem are modeled with fuzzy sets to represent their imprecise nature. That also enables us to reduce the inaccuracies in decision-makers' judgments. A time-sharing computer program is implemented, and an application to a multi-target operation problem in feeder reconfiguration in electric power systems is demonstrated along with the computer outputs. In conclusion, the proposed solution algorithm allows for a more realistic problem formulation efficiently obtained the optimal solution for the tested system with a large search space.

Keywords: Multi-target programming, evolvement programming, fuzzy set, and feeder reconfiguration.

1. Introduction

The work presented here is to address the multi-target majorization via combined fuzzy

satisfied method and evolvement programming (E.P.) method. In recent years, customers have placed more stringent requirements on service utilities [1]. Decision-makers planning a system are typically confronted with multiple objective functions and these objective functions are generally of distinct types. Moreover, these objective functions are often non-commensurable. It is well known that much kind of methods to solve these problems have been proposed from different points of view [2-10]. One conceivable approach is to convert a multi-objective problem into a single objective problem by assigning distinct weights to each objective. However, this scheme is not totally satisfactory since distinct objectives cannot be evaluated under a common measure and there is no rational basis of determining adequate weights.

This paper presents the method that the objective functions of the optimization problems are modeled with fuzzy sets. Fuzzy sets were first introduced by Zadeh [11] as an effective means of solving non-probabilistic problems. Fuzzy sets are generally represented by a lower and upper boundary with a membership function. The higher the value of a membership function implies a greater satisfaction with the solution. The different objectives are easily integrated because all the membership function values of

these objectives are in the same range [0,1]. Moreover, this paper also develops a fuzzy satisfied method by using the evolutionary programming (EP) to solve the constrained multi-target problem since EP can readily achieve the global optimal solution [12].

The proposed method is implemented in a software package. Also, an application to an optimal operation of the feeder reconfiguration in electrical distribution power systems is presented. It is found that the effectiveness of the proposed solution algorithm is verified through numerical examples on the feeder reconfiguration of electrical distribution power systems. In conclusion, the proposed solution algorithm allows a more realistic problem formulation and obtains the optimal solution for the tested system with a large search space.

2. Multi-target majorization problems

The multi-objective optimization can be stated as $f_i(x), i=1,2,\dots,n_0$ (1)

Subject to

$$g_j(x) < 0, j=1,2,\dots,n_0 \quad (2)$$

Where, n_0 and n_c denote the number of objectives $f_i(x)$ and constraints $g_j(x)$, respectively. The individual objective function is simultaneously minimized subject to the given constraints. If some (generally all) of the objective functions are competing, no point x simultaneously minimizes all the objective functions. Restated, when objectives compete with each other, no "optimal solution" is available for the multi-target majorization problem. Instead of optimality, the concept of non-inferiority [6] is employed to characterize a

solution to the problem. The following two definitions are introduced to concisely define non-inferiority:

Definition: The feasible region, Ω , in the parameter space X , is the set of all designable parameters that satisfy the constraints, i.e.,

$$\Omega = \{ x \mid g(x) \leq 0 \} \quad (3)$$

Definition: The feasible region, Λ , in objective function space F , is the image by f of the feasible region

$$\Lambda = \{ f \mid f = f(x), x \in \Omega \} \quad (4)$$

Definition: A point $x^* \in \Omega$ is a local non-inferior point, if and only if, there does not exist a Δx for some neighborhood or x^* such that $(x^* + \Delta x) \in \Omega$ and

$$f_i(x^* + \Delta x) \leq f_i(x^*), i=1,2,\dots,n_0 \quad (5)$$

$$f_i(x^* + \Delta x) < f_k(x^*), \text{ for some } k \quad (6)$$

This image of a local non-inferior point is a local non-inferior solution.

Definition: A point $x^* \in \Omega$ is a global non-inferior point, if and only, if there does not exist an $x \in \Omega$ such that

$$f_i(x) \leq f_i(x^*), i=1,2,\dots,n_0 \quad (7)$$

$$f_i(x) < f_k(x^*), \text{ for some } k \quad (8)$$

There are typically an infinite number of non-inferior points for a given multi-objective problem. A non-inferior point is the same as the intuitive notion of an optimum tradeoff solution since a design is non-inferior if improving an objective requires degradation in at least one of the other targets. Clearly, if a decision-maker attempts to generate non-inferior solutions to a multi-target problem when trying to reach a final design

3. Fuzzy Modeling

Considering the imprecise nature of each objective function of the optimization problems, these objective functions are formulated as fuzzy sets. A fuzzy set is typically represented by a membership function $\mu_{f_i}(\bar{X})$. The higher the value of the membership function implies a greater satisfaction with the solution. The membership function consists of a lower and upper boundary value together with a strictly monotonically decreasing and continuous function. The lower and upper bounds, $f_i^{\min}(\bar{X}), f_i^{\max}(\bar{X})$ of each objective function under given constraints are established to elicit a membership function $\mu_{f_i}(\bar{X})$, for each objective function $f_i(\bar{X})$. Then, a strictly monotonically decreasing and continuous function $h_i(f_i(\bar{X}))$, which can be linear or nonlinear, is determined. A membership function of a minimizing problem can be defined by:

$$\mu_{f_i}(\bar{X}) = \begin{cases} 1 & \text{or } \rightarrow 1, \text{ if, } f_i(\bar{X}) < f_i^{\min} \\ h_i(f_i(\bar{X})) & \text{if, } f_i^{\min} \leq f_i(\bar{X}) \leq f_i^{\max} \\ 0 & \text{or } \rightarrow 0, \text{ if, } f_i^{\max} < f_i(\bar{X}) \end{cases} \quad (9)$$

4. The fuzzy satisfied method

This section introduces a fuzzy satisfying algorithm based on EP to determine the non-inferior optimal solution for multi-target optimal problems. The decision-maker must

specify his expected value of the membership functions achievement to generate a candidate for the satisfied solution of the multi-objective problem. The expected value is a real number between $[0, 1]$ that represents the level of importance of each objective function. For the decision-maker's expected values $\bar{\mu}_{f_i}$, the following mini-max problem generates the optimal solution, which is closed to his requirements.

$$\text{Min}_{X \in \Omega} \{ \text{Max}_{i=1,2,\dots,n_0} [\bar{\mu}_{f_i} - \mu_{f_i}(\bar{X})] \} \quad (10)$$

The optimization technique can now be described as follows:

Step1: Input data and set the pointer, $\gamma = 0$.

Step2: Determine the upper and lower boundaries for every objective function, f_i min and f_i max, and elicit a strictly monotonically decreasing function to formulate the membership functions, $\mu_{f_i}(x)$.

Step3: Set the initial expected value of each objective function, $\bar{\mu}_{f_i} = 1, \text{ for } i=1,2,\dots,n_0$.

Step4: Apply EP (described in the next section) to solve the mini-max problem (equation 10).

Step5: Calculate the values of $\bar{X}, f_i(\bar{X})$, and $\mu_{f_i}(\bar{X})$ then go to next step if they are satisfactory. Otherwise, set the pointer, $\gamma = \gamma + 1$ and choose a new expected value $\bar{\mu}_{f_i}^{(\gamma)}$, $i=1,2,\dots,n_0$. Then go to step4.

Step6: Print the most satisfied feasible

solution \bar{X} , $f_i(\bar{X})$ and $\mu_{f_i}(\bar{X})$ for $i = 1, 2, \dots, n_0$.

Notably, the decision-maker is only involved in step 5 as the sequence is generated automatically thereafter. The expected value (preferred degree) of an objective is achieved by the decision-maker's experiences or by simple trial and error. The decision-maker can determine the optimal compromised (or satisfied) solution for the mini-max problem from the fuzzy satisfying algorithm.

5. Evolvement programming

EP is applied to the multi-target optimization problems in this section. EP is different from conventional optimization method because it is a simulation approach based on a biological process. EP utilizes probability transition rules to select generations. Each individual competes with other individuals in a combined population of the old generation and the new generation mutated from the old generation. The winners in the combined population constitute the next generation. The state variable \bar{X} symbolizes a chromosome. The fitness function of \bar{X} is defined as follows:

$$C(\bar{X}) = \frac{1}{1 + obj(\bar{X})} \quad (11)$$

and

$$obj(\bar{X}) = \max_{i=1,2,\dots,N_0} [\mu_{f_i} - \mu_{f_i}(\bar{X})] \quad (12)$$

For a given μ_{f_i} , the solution reaches the

optimum as the fitness value increases. The detailed steps of EP are described as follows:

Step1: Input parameters.

Input the parameter of EP, such as the length of the individual and the population size, N_p .

Step2: Initialization

The initial population is determined by randomly selecting P_j from the set of the original solution space and their derivative according to the mutation rules. P_j is an individual, $j=1,2,\dots,N_p$, with N_s dimensions, where N_s denote the total number of gene.

Step3: Scoring

Calculate the fitness value of an individual by (11) and (12).

Step4: Mutation

Each P_j is mutated and assigned to P_{j+N_p} . The number of offspring n_j , for each individual P_j is decided by:

$$n_j = G[N_p \times \frac{C_j}{\sum_{j=1}^N C_j}] \quad (13)$$

Where, C_j denotes the fitness value of individual p_j , $G[x]$ rounds the elements of x to the integer. More offspring are generated from the individual as the fitness of an individual increases. A combined population is formed from the old generation and the new generation mutated from the old generation. Then, the corresponding fitness value is calculated by (11) and (12).

Step5: Competition

Each individual P_j in the combined population has to compete with some other individuals to have a chance to be transcribed to the next generation. All

individuals of the combined population are ranked in descending order of their corresponding fitness value. Then, the first N_p individuals are transcribed to the next generation.

Step6: Stop criterion.

Convergence is achieved when either the number of generations reaches the maximum generation number or the sampled mean fitness function values do not change noticeably at five consecutive generations. The process will go to stop if one of these conditions is met, but will otherwise return to the mutation step.

6. The Application Example

The proposed approach is implemented in a software package and its effectiveness is verified through numerical examples on the feeder reconfiguration of electrical distribution power systems. There are two types of switches in distribution feeders, i.e. sectionalizing switches (normally closed) and tie switches (normally open). The network reconfiguration chooses the status of these switches for a new topological structure in the normal state to enhance service reliability and reduce power losses. The network reconfiguration problem is a complex non-linear combinatorial problem since the status of switches is non-differentiable and the normally open tie switches must be determined to satisfy the system requirements. An optimal radial configuration s^* for all possible feeder configurations S_i for a radial distribution system of n_b buses, is developed by changing the open and closed switches

of feeders to : (i)reduce power loss, (ii) increase of feeders to: (iii)improve power quality such that the operating constraints are satisfied.

(A)Objective function

(1) Power loss reduction

Minimizing the real power loss of feeders is selected as the first objective function for the feeder reconfiguration problem since reducing the real power loss of distribution feeders is the main purpose of feeder reconfiguration. The objective function can be calculated as follows:

$$f_1 = \sum_{i=1}^{n_b} r_i \frac{P_i^2 + Q_i^2}{|V_i|^2} \quad (14)$$

Where, n_b denotes the total number of branches in the considered system, r_i , V_i , P_i , and Q_i denote the resistance, voltage, real power and reactive power of branch i , respectively. The lower value of f_1 implies that the system has a lower power loss.

(2) Increased system security

Network managers heavily emphasize relieving network over loads. A distribution feeder can relieve over loads by increasing the loading margin of the feeder and its corresponding transformer. An effective strategy to increase the loading margin of heavily loaded feeders is to transfer part of their loads to lightly loaded feeders (load balancing).Hence, load balancing is a major objective of feeder reconfiguration, and can be expressed as follows:

$$f_2 = \max \text{ of } \left[\left(\frac{\sum_{i=1}^{n_h} I_i}{n_h} \right), i = 1, 2, \dots, n_h \right] \quad (15)$$

Where, I_i denotes the current on branch I (transformer i). The lower value of f_2 indicates that the loading margin of heavily loaded feeders (transformers) in a system is increased and the system is more secure.

(3) Improved power quality

Voltage is an important security and service quality index. Therefore, the deviation of bus voltage must be considered as an objective and not merely a constraint. It is calculated as follows:

$$f_3 = \max \text{ of } \left[|V_i - 1.0|, i = 1, 2, \dots, n_b \right] \quad (16)$$

Where n_b denotes the total number of buses, V_i is the voltage on bus I in pu unit. The lower the value of f_3 implies that the bus voltage approaches 1, i.e., the voltage profile is better.

(B) Constraints

Two constraints are considered in the feeder reconfiguration problem for the formulation:

- (1) The radial structure of network must be maintained.
- (2) All loads must be served.

7. Simulation Results

A time-sharing computer program is implemented in C++ with man-machine procedures based on the proposed algorithm. One of the Tai-Power Company's distribution systems is tested by the proposed method. This system includes two transformers, 10 feeders, 102 branches, 13 tie lines, 102 buses and 204 switches. Figure 1 illustrates the network

structure of the system, while Table 1 depicts the critical parameters of the objective functions. Three loading levels (heavy, medium and light) are adopted to illustrate the effectiveness of the practical operation. Table 2 summarizes the results of the test case, which includes the power loss, voltage profile, load balancing, and location of the tie (open) switch. The power loss has been reduced by approximately 50%, and the power quality and system security have also been enhanced after reconfiguration for each load level as compared with the data before reconfiguration. The test case verifies that the proposed method can be implemented in a practical system and that the run time is fast for application in an on-line system. Indeed, a reconfiguration plan was obtained in under 36 seconds on a Pentium-CELERON 300A PC for all cases.

TABLE 1

PARAMETERS OF OBJECTIVE FUNCTIONS

Objective function	Parameter
Power loss feeders	$f_1^{\min} = 0.4 f_1(x_0), f_1^{\max} = 2 f_1(x_0)$
Margin loading of feeders (transformers)	$f_2^{\min} = 0.3 \text{ pu}, f_2^{\max} = 0.6 \text{ pu}$
Deviation of bus voltage	$f_3^{\min} = 0.05 \text{ pu}, f_3^{\max} = 0.1 \text{ pu}$

Remark: (i) $f_1(x_0)$ represents the original power loss for the system before reconfiguration. (ii) The lower and upper bounds f_1^{\min} and f_1^{\max} depend on the constraints of the considered problem. For example, let $f_2^{\max} = 0.1 \text{ pu}$ if the bus voltage is limited in the range (0.9-1.1)pu.

TABLE 2

RESULTS OF THE TEST CASE

Loading level	Low		Medium		Heavy	
	Before reconfig	After reconfig	Before reconfig	After reconfig	Before reconfig	After reconfig
Power loss (KW)	205.27	104.66	325.66	168.17	476.29	246.05

Reduce loss rate (%)	—	49.0%	—	48.4%	—	48.4%
Max of deviation of bus voltage (pu)	0.039	0.0320	0.049	0.025	0.060	0.030 (pu)
Load Balancing Index (%)	38.3%	22.2%	48.2%	28.0%	58.3%	33.6%
Min of the margin loading	56.0%	72.2%	44.6%	65.1%	33.0%	58.0%
CPU time (second)	—	35	—	36	—	36
Locations of the tie switch (# of bus)	T (103) T (104) T (105) T (106) T (107) T (108) T (109) T (110) T (111) T (112) T (113) T (114) T (115)	S (22,12) S (16,15) S (19,1) S (106) S (107) S (46,45) S (43,50) S (52,46) S (47,54) S (91,95) S (67,68) S (63,64) S (92,91)	T (103) T (104) T (105) T (106) T (107) T (108) T (109) T (110) T (111) T (112) T (113) T (114) T (115)	S (22,12) S (16,15) S (19,1) S (92,91) S (38,42) S (46,52) S (109) S (45,44) S (111) S (106) S (67,68) S (63,64) S (91,95)	T (103) T (104) T (105) T (106) T (107) T (108) T (109) T (110) T (111) T (112) T (113) T (114) T (115)	S (22,12) S (16,15) S (19,1) T (106) T (108) T (107) S (43,50) S (46,45) S (47,45) S (92,91) S (67,68) S (63,64) T (115)

8. Conclusion

This work has presented a multi-objective algorithm based on the fuzzy satisfying method and EP for the multi-objective optimization problems. Fuzzy sets are utilized for modeling the objective functions to represent their imprecise nature. EP was applied to our solution algorithm to derive the optimal solution because EP utilizes a multi-path search to solve the problem with nonlinear and non-differentiable objective functions. Finally, the proposed method has been implemented and tested on a feeder reconfiguration problem which is a complex non-linear combinatorial problem. The test results lead to the following conclusions.

1. The proposed solution algorithm allows for a more realistic problem formulation since this technique does not require the objective function to be differentiable or continuous.
2. The proposed method efficiently obtained the optimal solution for the tested system

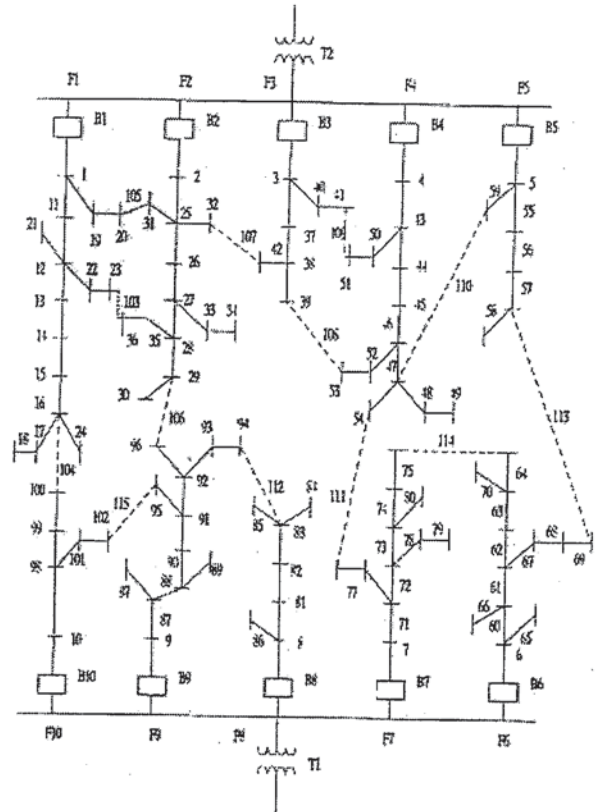
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(a)system configuration



(b) switch location



(c) tie switch location

- F_n : No. of feeder
- B_n : No. of breaker
- : Breaker
- : Feeder line
- : Tie line
- n : No. of tie switch

Fig.1.Network structure of the testing system.