A JOINT TEST OF THE RATIONAL EXPECTATIONS-PERMANENT INCOME HYPOTHESIS UNDER SEASONAL COINTEGRATION

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This study re-evaluates the validity of the joint rational expectations-permanent income hypothesis under the framework of seasonal cointegration using seasonally unadjusted quarterly data from Austria, Canada and Taiwan. Evidence is found that the consumption change only depends on the innovations of the income and the unemployment rate changes, and that agents are rational in forming their expectations, i.e., the joint hypothesis is supported by the data used. However, with the same data set, a similar test based on non-seasonal cointegration tends to reject the joint hypothesis, since the test ignores completely the possible stochastic seasonalities that may contain important information, as has been pointed out by Wallis (1974), embodied in the data.

I. Introduction

The current study re-examines the permanent income hypothesis (PIH), which was developed by Friedman (1957) and has figured prominently in consumption theory over the past forty years, under the additional hypothesis of rational expectations. The rational expectations-permanent income hypothesis (RE-PIH) postulates that agents are forward looking and make consumption plans on the basis of their permanent income rather than their current income. This hypothesis implies that temporary changes in income should have less impact on consumption than changes in permanent income. Hall (1978) showed theoretically that, under rational expectations with fixed real interest rates, consumption changes are unpredictable since the desired consumption evolves as a random walk process.

According to Flavin (1985), the optimal consumption in period t, C_t, under the PIH is expressed in first differences as

$$\Delta C_t = \beta_p \Delta y_t^p + \beta_T(\Delta y_t - \Delta y_t^p), \tag{1}$$

where Δ denotes the first difference operator, y_t^p is the permanent income in period t, y_t is current income, and $y_t - y_t^p$ is transitory income. Equation (1) is quite a general form that nests special cases of both the PIH, when $\beta_T = 0$, and an extreme form of the Keynesian consumption function, when $\beta_p = \beta_T$. To avoid any potential misspecifications, equation (1) often extends to incorporating extra exogenous variables affecting consumption. Similar to the treatment of income, both the expected and unexpected components of the extra variables are included in equation (1). If the coefficients of the unexpected part of the extra variables are significantly different from zero, then the optimal consumption plan (1) is incorrect, which invalidates the PIH.

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The remainder of the paper is as follows. Section II briefly reviews the relevant studies. Section III tests the validity of RE-PIH under the framework of non-seasonal cointegration, while Section IV performs the same test in the context of SC. Section V concludes the study.

II. Literature Review

Following Hall (1978), numerous studies testing the PIH using different approaches have appeared and, in general, have rejected the hypothesis by formal statistical tests, with the exceptions of Hall (1978), Campbell (1987), and Tanner (1997). Among those studies containing negative evidence, Flavin (1981, 1985), Hall and Mishkin (1982), Zeldes (1989), and Lusardi (1996) found considerable predictive power for consumption and described it as an "excess sensitivity" of consumption to income. Campbell and Deaton (1989) showed that permanent income appears to be less smooth than measured income, and therefore the known smoothness of consumption cannot be explained by permanent income theory. Capital market imperfections or liquidity constraints, which prevent consumers from borrowing when income is temporarily low, may explain aspects of the data that are unexplained by the simple RE-PIH. Browning (1987), Campbell and Mankiw (1989, 1990, 1991), Jappelli and Pagano (1989), Wirjanto (1996), Deaton (1987), Hayashi (1982), Altonji and Siow (1987), and Villagomez (1997) all provide support for the use of liquidity constraints.

The durability of certain consumption goods is another possible reason often used to explain departures from the RE-PIH. Hayashi (1985), Mankiw (1982, 1985), Bernanke (1984, 1985), and Bar-Ilan and Blinder (1987) each examined the joint behaviour of income and durable purchases, while Hall (1989) produced an excellent survey. The theory of precautionary savings has also been used to explain the excess sensitivity of consumption (for example, see Normandin (1994), Caballero (1990), and Cantor (1985)).

Mankiw and Shapiro (1985) and Nelson (1987) argued that Flavin's (1981,1985) finding of excess sensitivity is most likely the result of her using detrended data, which were obtained by removing a fitted trend from each random walk process. A detrended random walk time series can become an autocorrelated stationary time series that is necessarily predictable (see Nelson and Kang (1981, 1984)). In addition, using original (raw or seasonally-unadjusted) data and an explicit model of seasonal effects, the simple PIH cannot be rejected, as has been shown by Miron (1986). Thus, the foregoing seems to suggest that failure to accept the RE-PIH may arise from inappropriately transforming the raw data, which distorts the structure of the data series.

Before estimating any macroeconomic model, the statistical properties of the model's individual variables have to be examined, especially with respect to whether these variables are stationary, otherwise, spurious regressions may result (see Granger and Newbold (1974)). In the literature on applied macroeconomics, seasonally-adjusted data, which are filtered using a seasonal adjustment process such as the X-11 method and are believed to contain no seasonal unit roots, have been used extensively.[1] Unfortunately, the typical X-11 procedure has been found to delete important information from the original data and, as a consequence, distorts the unit root tests and subsequent inference on cointegration (for example, see Jaeger and Kunst (1990) and Ghysels and Perron (1993)), when applying Johansen's (1988) multivariate cointegration methodology. Ghysels (1990), Ghysels *et al.* (1993), and Ericsson

[1] Another common procedure used to remove all the seasonal factors is the inclusion of seasonal dummies in a regression model. However, Abeysinghe (1991) showed that this procedure cannot eliminate the stochastic seasonality embedded in most of the macroeconomic series.

et al. (1994) have suggested to not employ this procedure to deseasonalise data. Elwood (1998) showed that seasonal filtering may be responsible for rejection of the RE-PIH in previous empirical studies, however, the alternative to conducting Johansen's cointegration test on original data may lead to a specification error, since possible stochastic seasonality is not formally addressed.

Recent developments in seasonal unit root tests (Hylleberg *et al*. (1990), Engle *et al*. (1993), and Ghysels *et al*. (1994)) and in seasonal cointegration (SC) (Lee (1992) and Johansen and Schaumburg (1999)) now allow empirical researchers to analyse seasonally-unadjusted data directly. Tests of SC are also carried out in a system context rather than variable by variable, and are similar to Johansen's approach. Furthermore, seasonal cointegration analysis is bolstered by empirical findings that vector autoregressive (VAR) models are able to fit macroeconomic data very well. Considerations such as these appear to imply that SC can provide a suitable framework for the test of RE-PIH. Elwood (1998) indeed tested the joint RE-PIH using a state-space/unobserved components model. However, he chose not to explicitly model stochastic seasonality in the context of SC.

Unlike previous studies (except for Sargent (1978)) which take rational expectations as given, this paper formally models the forecast rationality as a testable restriction in the context of the permanent-income theory of consumption. The purpose of this article is to then illustrate a procedure based on SC for testing the macroeconomic RE-PIH with seasonally-unadjusted data from Austria, Canada, and Taiwan, where such data are available. Moreover, a similar testing procedure using the same data set, but based on standard (non-seasonal or conventional) cointegration, is shown to be inappropriate due to the possible existence of seasonal unit roots and SC (Engle *et al*. (1989)). Specifically, Johansen's approach is used to exemplify how non-seasonal cointegration can distort statistical inference. This paper provides an alternative procedure, first developed by Ermini and Chang (1996) who tested the joint hypothesis of rationality and money neutrality under the framework of seasonal and non-seasonal cointegration, to the joint test of RE-PIH, subject to certain important modifications.

III. Test of RE-PIH under Standard Cointegration

Throughout this paper each variable used is assumed to be transformed by taking the natural logarithm. In addition to income, other variables regarded by previous studies as important in affecting agents' consumption decisions include the unemployment rate, the interest rate, the price level, wealth, the rate of inflation, the demand for money, government expenditures, and transfer payments. To keep the model tractable and comparable to related research to date, we consider a three-variable VAR model with endogenous variables consumption (*C*), measured either by total consumption (*TC*) or non-durable consumption and services (*NC*), disposable income (*DI*) (or gross domestic product *GDP*), and the unemployment rate (*UN*). The exact definitions of the above variables are shown in Appendix A.

A more appropriate measure of consumption would be consumption of non-durables and services plus the imputed service flow of the stock of durable consumption goods. To avoid potential distortion of useful information by the procedure of imputation, many empirical researchers in this area simply investigate *NC*. Both measures of *TC* and *NC* are adopted in this study, however, the latter is expected to outperform the former.

When the current income of a liquidity-constrained agent is temporarily low due to negative transitory income, then his actual consumption will be lower than the desired consumption plan (predicted by the RE-PIH to be a smooth time path over the agent's lifespan), because the agent is unable to borrow against expected future income. This implies that realized consumption and transitory income are positively correlated, although planned consumption may indeed be determined by the RE-PIH. During spell(s) of unemployment, an agent's current income is usually low, because of negative transitory income, which may make the agent's net worth incapable of smoothing his desired consumption path. Due to the unavailability of reliable quarterly data on asset values, the current study chooses the variable *UN* as a proxy for the proportion of the population subject to liquidity constraints (see Cuddington (1982), Flavin (1985), and Carroll and Summers (1987)). In other words, the variable *UN* accounts for one of the many possible sources of misspecification, i.e., the omission of liquidity constraints as an important factor of consumption decisions.

The test procedure will basically follow a novel approach developed by Ermini and Chang (1996). Let

$$\Gamma(L)X_t = \mu + \varepsilon_t \tag{2}$$

be a VAR system with three endogenous variables, where $X_t = (DI_t\ UN_t\ C_t)'$, μ is a 3×1 vector of constants, ε_t is a 3×1 vector of innovations with zero means and constant variance covariance matrix, and $\Gamma(L) = I - \sum_{j=1}^{K}\Gamma_j L^j$ is a matrix polynomial of degree K in the lag operator L.[2,3] Equation (2) includes a multi-directional causality among the three variables and can be reformulated as a form composed of first-order differences and lagged levels (see Johansen (1988) and Johansen and Juselius (1990))

$$D(L)\Delta X_t = \Pi X_{t-K} + \mu + \varepsilon_t, \tag{3}$$

where $D(L) = I - \sum_{j=1}^{K-1} D_j L^j$ is a matrix polynomial of degrees $K-1$ with $D_j = -(I - \sum_{i=1}^{j}\Gamma_i)$, $j = 1, \ldots, K-1$, and $\Pi = -\Gamma(1)$.

Johansen (1988) and Johansen and Juselius (1990) developed maximum likelihood-based testing procedures of cointegration for a non-stationary VAR model which does not require pre-testing each single component of X_t for integration. Johansen's procedures are now extensively used in applied works, and are therefore not presented here in order to save space. Interested readers may refer to the aforementioned papers for details.

Consider a consumption equation

$$\begin{aligned} F(L)\Delta C_t = {} & G_1(L)[\Delta DI_t - P_{t-1}(\Delta DI_t)] \\ & + H_1(L)P_{t-1}(\Delta DI_t) + G_2(L)[\Delta UN_t - P_{t-1}(\Delta UN_t)] \\ & + H_2(L)P_{t-1}(\Delta UN_t) + v_t, \end{aligned} \tag{4}$$

where $F(L)$, $G_1(L)$, $G_2(L)$, $H_1(L)$, and $H_2(L)$ are all polynomial functions in L, and $P(.)$ denotes the forecast, not necessarily rational. Equation (4) states that consumption growth depends on both the predicted and unpredicted components of income and the unemployment rate growth.[4]

[2] The centred seasonal dummies are not included for the time being since their inclusion will not affect the following derivation of cross equation restrictions being used to test the RE-PIH. They are incorporated in the model when the estimation is actually performed.
[3] Notation I denotes the identity matrix having the same dimension as $\Gamma(L)$.
[4] This specification is a direct extension of Ermini and Chang (1996), in which they assumed that only the predicted and unpredicted component of money growth can explain output growth, and excluded the possibility that output growth may depend on the predicted and unpredicted components of other variables shown in the model.

Under the PIH, predicted income and unemployment rates affect consumption merely to the extent that their innovations contain new information, which helps to predict consumption. Consumption growth depends only on the unpredicted components of income and the unemployment rate, implying $H_1(L) = H_2(L) = 0$, $G_1(L) \neq 0$, and $G_2(L) \neq 0$. Moreover, forecast rationality can be jointly tested by imposing the additional restrictions that both $\Delta DI_t - P_{t-1}(\Delta DI_t)$ and $\Delta UN_t - P_{t-1}(\Delta UN_t)$ are pure white noise processes.

Suppose the income forecast is not rational. The term $P_{t-1}(\Delta DI_t)$ would then be a linear structure similar to the first equation of (3), while their parameters differ. If we rewrite the term $\sum_{j=1}^{K-1} D_j L^j$ of $D(L)$ in equation (3) as a 3×3 matrix, $[\sum_{m=1}^{K-1} d_{ij,m} L^m] \equiv [d_{ij}(L)]$, $i, j = 1, 2, 3$, and let X_j be the jth element of the vector X, then

$$P_{t-1}(\Delta DI_t) = \sum_{j=1}^{3} d_{1j}^*(L)\Delta X_{jt} + \mu_1^* + \alpha_1^* EC_{t-K}. \tag{5}$$

From above, the parameters with superscript * take different values from the corresponding ones in the rational forecast. Moreover, EC_{t-k} is the error correction term (ECT) if variables in vector X are found to be cointegrated using Johansen's procedure, and α_1^* is the adjustment coefficient (loading) toward the long-run stead state. Similarly, if the unemployment rate forecast is not rational as defined, then one would have

$$P_{t-1}(\Delta UN_t) = \sum_{j=1}^{3} d_{2j}^*(L)\Delta X_{jt} + \mu_2^* + \alpha_2^* EC_{t-K}. \tag{6}$$

Substituting equations (5) and (6) into the consumption equation (4), and replacing the third row of equation (3) with equation (4) so derived, the following three-variable error correction model can be obtained

$$\begin{bmatrix} \bar{D}(L) \\ N_1^*(L) N_2^*(L) N_3^*(L) \end{bmatrix} \begin{bmatrix} \Delta DI_t \\ \Delta UN_t \\ \Delta C_t \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha^*(L) \end{bmatrix} EC_{t-K} + \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3^* \end{bmatrix} + \varepsilon_t, \tag{7}$$

where $\bar{D}(L)$ is nothing, but the first two rows of $D(L)$ in system (3),

$$N_1^*(L) = d_{11}^*(L)[G_1(L) - H_1(L)] + d_{21}^*(L)[G_2(L) - H_2(L)] - G_1(L), \tag{8}$$

$$N_2^*(L) = d_{12}^*(L)[G_1(L) - H_1(L)] + d_{22}^*(L)[G_2(L) - H_2(L)] - G_2(L), \tag{9}$$

$$N_3^*(L) = F(L) + d_{13}^*(L)[G_1(L) - H_1(L)] + d_{23}^*(L)[G_2(L) - H_2(L)], \tag{10}$$

$$\alpha^*(L) = \alpha_1^*[G_1(L) - H_1(L)] + \alpha_2^*[G_2(L) - H_2(L)], \tag{11}$$

and μ_3^* is a linear combination of μ_1^* and μ_2^*.

In equations (7) to (11), once the error correction term EC_{t-K} and the degrees of the lag polynomials $N_i^*(L)$, $i = 1, 2, 3$, and $\alpha^*(L)$ are determined using macroeconomic data, the cross-equation restrictions corresponding to our test of the RE-PIH can then be derived.

Data used by this paper are collected from Austria, Canada, and Taiwan, where seasonally-unadjusted data are available. Appendix A summarises variable definitions, sample statistics, and seasonal unit root test results. As all series are found to be integrated at frequency zero, we proceed to the test of cointegration using Johansen's procedure. The results are shown in Appendix B and were computed using CATS in RATS computer software.

According to either the λ-max or the trace statistics, *NC* appears to be cointegrated with *DI* and *UN* in all the three economies, while *TC* is only marginally cointegrated with *DI* and *UN* for the Canadian data based on the trace statistics. Taiwan's *TC* is said to be cointegrated with the other two series if the λ-max statistic is used. It appears that variable *NC* tends to have a long-run equilibrium relationship with the other two variables in comparison with variable *TC* due, at least in part, to the fact that *TC* includes consumer durables other than the correct service flow of the stock of durables.

All the cointegrating coefficients have expected signs except for variable *DI* in the Canadian data. More specifically, an increase in income raises consumption, while an upswing in the unemployment rate is anticipated to lower the consumption level. Despite the fact that Canada's *DI* series has an unexpected negative impact on consumption (*TC* and *NC*), its coefficients are found to be statistically and insignificantly different from zero using the testing procedure proposed by Johansen and Juselius (1990). The test statistic is distributed as a Chi-square with one degree of freedom.[5] However, all loadings for the consumption functions (the first element in vector α), which represent the average speed of adjustment towards the long-run equilibrium, are negative, indicating that excess consumption in the current period over its long-run steady state will be partially corrected downward in the next period. The higher the loading is in absolute value, the faster the consumption's reversion to its steady state will be.

Given the optimal lag lengths *K* that are shown in Appendix B, the degrees of $F(L)$, $d^*_{1j}(L)$, and $d^*_{2j}(L)$ are then set to be $K-1$. In addition, $G_i(L)$ and $H_i(L)$, $i = 1, 2$, are found to be polynomials of degree one. These altogether imply that $N_j(L), j = 1, 2, 3$, are polynomials of degree *K*, and that $\alpha^*(L)$ is a polynomial of degree one, indicating that the ECT in (7) appears as EC_{t-K} and EC_{t-K-1}. To test the RE-PIH, the joint conditions of $H_1(L) = H_2(L) = 0$ (implied by the permanent income hypothesis), $d^*_{1j}(L) = d_{1j}(L)$, and $d^*_{2j}(L) = d_{2j}(L)$, $j = 1, 2$, must be imposed on equation (7). This leads to sets of non-linear cross-equation restrictions summarised in Appendix C, with notations similar to that found in Ermini and Chang (1996). The main difference between the restrictions of (A1) in Appendix C and those of Ermini and Chang (1996) is that (A1) contains an extra (the second) term, which arises from the assumption that consumption growth is associated with both the predicted and unpredicted components of ΔDI and ΔUN.

An advantage of the Wald test in assessing the validity of non-linear restrictions is that only the unrestricted model need be estimated. When computing the Wald test statistic, potential conditional heteroskedasticity, which can bias the estimates of the variance-covariance matrix of the coefficient estimates in the model under consideration, needs to be taken into account. The Wald test statistics with and without correcting for conditional heteroskedasticity, denoted by W^* and W, respectively, and the corresponding *p*-values are shown in Table I. It can be seen that W^* is substantially different from W for all cases, indicating that heteroskedasticity may be a serious problem.

Given that W^* is more relevant than W, Table I fails to validate the cross-equation restrictions at the 10 per cent significance level for the VAR model involving total consumption, while conclusions for the non-durable consumption are mixed. Specifically, observations on Canadian non-durable consumption do not reject the RE-PIH, but Taiwan's *NC* does reject it, even at the 5 per cent level of significance. It seems that the inclusion of variable *NC* is inclined to support the RE-PIH, while the reverse is true when variable *TC* is incorporated.

[5] The *p*-values of the test statistics are 0.92 and 0.87, respectively, for the two cointegrating relationships of *NC* and *TC*.

Table I Wald testing results under EC model

	Total Consumption	*Non-Durable Consumption*
Austria (number of restrictions = 9)		
W^*	15.641	N.A.
p-value	0.075	
W	12.029	N.A.
p-value	0.212	
Canada (number of restrictions = 15)		
W^*	32.892	21.761
p-value	0.005	0.114
W	24.941	19.450
p-value	0.051	0.194
Taiwan (number of restrictions = 11)		
W^*	17.904	23.061
p-value	0.084	0.017
W	15.660	15.922
p-value	0.154	0.144

Notes: W^* Wald test statistics with correcting for conditional heteroskedasticity.
W Wald test statistics without correcting for conditional heteroskedasticity.

IV. Test of the RE-PIH under a Seasonal Cointegration Framework

This section attempts to test the validity of the RE-PIH using data from the same three countries, where the test is based on an SC and seasonal error correction model (SECM), which will be discussed in subsection a). Subsection b) derives the cross-equation restrictions implied by the RE-PIH in the context of SECM, and then investigates these restrictions with the data.

a) *Seasonal cointegration analysis*

Focusing on quarterly data, a relevant and simple specification for the data might be

$$\Delta_4 X_t = (1 - L^4)X_t = \varepsilon_t, \quad \varepsilon_t \text{ i.i.d.}, \tag{12}$$

which would capture the long-run peak and the seasonal peaks at zero as well as seasonal frequencies π, $\pi/2$, and $-\pi/2$ (or $3\pi/4$), respectively. Following Hylleberg *et al.* (1990) and Lee (1992), equation (12) can be expressed as

$$(1 - L^4)X_t = (1 - L)(1 + L + L^2 + L^3)X_t \equiv (1 - L)Y_{1t}, \tag{13}$$

$$= (1 + L)(1 - L + L^2 - L^3)X_t \equiv (1 + L)Y_{2t}, \tag{14}$$

$$= (1 + L^2)(1 - L^2)X_t \equiv (1 + L^2)Y_{3,t+1}. \tag{15}$$

The fourth difference operator contains four unit roots, i.e., ± 1 and $\pm i$. The unit root +1 produces an infinite peak at zero frequency which corresponds to a single period cycle; the root -1 generates an infinite peak at frequency π (also referred to as the biannual frequency) which corresponds to two cycles per year; the pair of complex roots $\pm i$ produces infinite peaks at $\pm\pi/2$ (also referred to as the annual frequency), corresponding to one cycle per year, or equivalently, to a quarter cycle per quarter. The unit root at frequency zero is the one analysed by Fuller (1976) and Dickey and Fuller (1979, 1981), and the remaining three roots are referred to as seasonal unit roots.

The term Y_{1t} in equation (13) is a non-stationary process with a unit root at frequency zero only, since the filter $S_1(L) = 1 + L + L^2 + L^3$ has removed all seasonal unit roots. Similarly, Y_{2t} and Y_{3t} are non-stationary processes with seasonal unit roots at frequencies π and $\pi/2$, respectively, due to the fact that filters $S_2(L) = 1 - L + L^2 - L^3$ and $S_3(L) = 1 + L^2$ have eliminated the unit roots at zero frequency, as well as at frequencies $\pi/2$ and π, respectively. Biannual and annual unit roots imply that the seasonal patterns respectively at six-month and twelve-month intervals will change permanently following a shock.

Assume that the quarterly data are generated by a general autoregression

$$\varphi(L)X_t = \varepsilon_t, \tag{16}$$

where ε_t's are i.i.d., and $\varphi(L)$ is a polynomnial function and finite-valued at the distinct, non-zero, possibly complex roots θ_k, $k = 1, \ldots, 4$. According to HEGY (1990), equation (16) can be written as

$$\varphi^*(L)\Delta_4 X_t = \pi_1 Y_{1t-1} + \pi_2 Y_{2t-1} + \pi_3 Y_{3t-2} + \pi_4 Y_{3t-1} + \varepsilon_t. \tag{17}$$

Testing the hypothesis that the roots of $\varphi(L)$ lie on the unit circle against the alternative that they lie outside the unit circle is equivalent to testing the hypothesis that $\varphi(\theta_k) = 0$, where θ_k is either ± 1, or $\pm i$. At the zero frequency this is a test for $\pi_1 = 0$ of equation (17), and at the biannual frequency it is $\pi_2 = 0$. The alternative hypotheses of the two tests are $\pi_i < 0, i = 1, 2$. For the complex roots, the one-sided test of $\pi_3 = 0$ against the alternative $\pi_3 < 0$ is used in this article, which implicitly assumes that $\pi_4 = 0$.

Because the roots $\pm i$ are complex conjugates, they cannot be distinguished from each other with quarterly data. It follows that in practice, researchers such as Lee (1992), Kunst (1993), Engle *et al.* (1993), Ermini and Chang (1996), Huang and Shen (1999), and Shen and Huang (1999) have concentrated their analyses of seasonal cointegration on the first three frequencies, i.e., at frequencies zero, π, and $\pi/2$. The current paper follows this convention. Although Lee's (1992) testing procedures for the presence of SC are simultaneously seasonal unit root tests, and thus do not require a pretest for each series under consideration for seasonal unit roots, the seasonal unit root tests are still conducted. By doing so, useful information on the variables' characteristics can be obtained, which will help the investigators judge the number of seasonal cointegration relationships, if any.

Graphs of a series and its transformed series using seasonal filters $S_i(L), i = 1, 2, 3$, may help to characterise and illuminate its stochastic structure and seasonality. Figures 1 to 3 contain plots of the three series *TC*, *DI* (or *GDP*), and *UN* with their seasonal transformations for the three countries, excluding the variable of non-durable consumption since the graphs are quite similar to those of *TC*. These figures appear to show that all series in the three economies exhibit some distinct stochastic seasonal patterns as the peaks of their seasonally-filtered series Y_{2t} and Y_{3t} have randomised values. This sharply contrasts with the case of deterministic seasonality that regularly reaches the same seasonal peak at different years. The formal statistical tests for seasonal unit roots are next conducted. Results of the tests using the Hylleberg *et al.* (1990) procedures are shown in Appendix A.

For Canadian data, all variables are found to be non-stationary at all seasonal frequencies no matter which combination of the deterministic components is included in the statistical

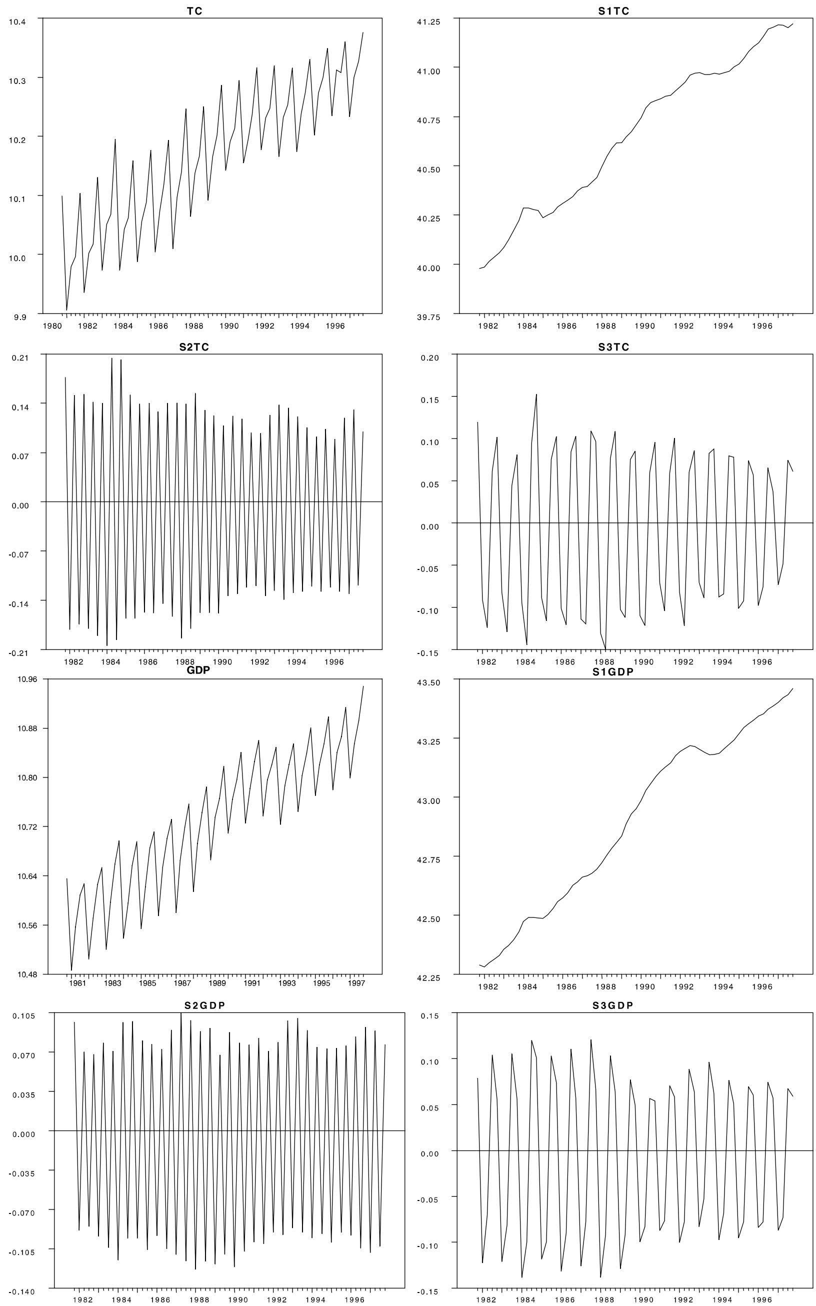

Figure 1. Austria

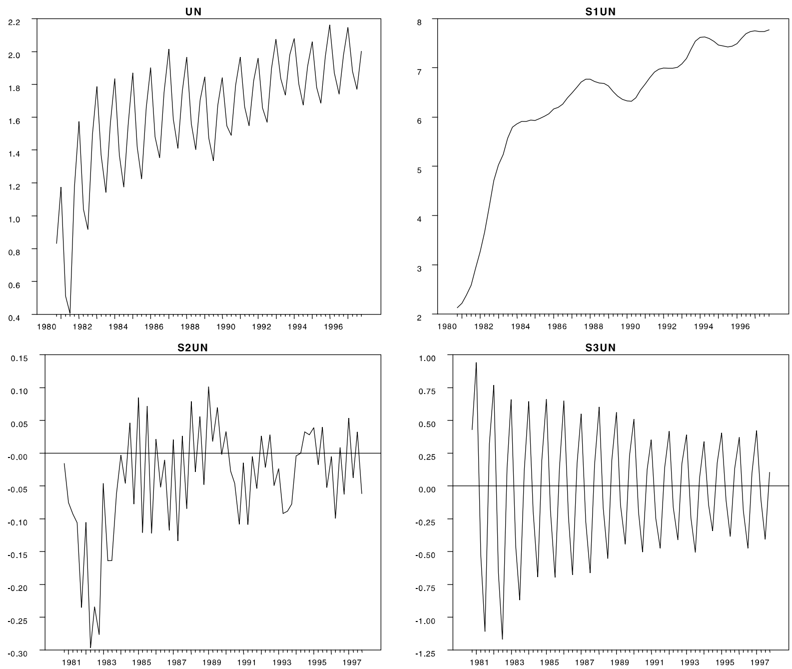

Figure 1. (Continued)

model. The tests on Austrian data reflect that *TC* and *GDP* are seasonally integrated at both seasonal frequencies. Moreover, *UN* exhibits a seasonal unit root at the annual frequency, while it is stationary at the biannual frequency when seasonal dummies are absent. All the series for Taiwan contain seasonal unit roots at both the π and $\pi/2$ frequencies.

Equation (2) can be reformulated as the isomorphic seasonal error-correction model (SECM)

$$Q(L)\Delta_4 X_t = \sum_{j=1}^{4} \Pi_j Y_{j,t-1} + \mu + \varepsilon_t, \tag{18}$$

where $Y_{j,t}$, for $j = 1, 2, 3$, are already defined in equations (13) to (15), $Y_{4t} = Y_{3,t+1}$, $Q(L) = I - \sum_{j=1}^{K-4} Q_j L^j$ is a matrix polynomial of degrees $K - 4$, and $Q(L)$ and Π_j's, $j = 1, \ldots, 4$, are suitably redefined from $\Gamma(L)$ of equation (2) (see Lee (1992)). Matrices Π_j's, $j = 1, \ldots, 4$, contain useful information on long-run behaviours of the series in the seasonal system (18). Lee (1992) extended Johansen's maximum likelihood inference on non-seasonal cointegration to seasonal cointegration, where similar rank conditions on Π_j's are applied to the system at each of the three frequencies. More specifically, the likelihood ratio test statistics for the null hypotheses that

$$H_{0j} : \Pi_j = \alpha_j \beta_j', \quad j = 1, \ldots, 4,$$

(i.e., that there are at most r_j cointegrating vectors at the jth frequency) is

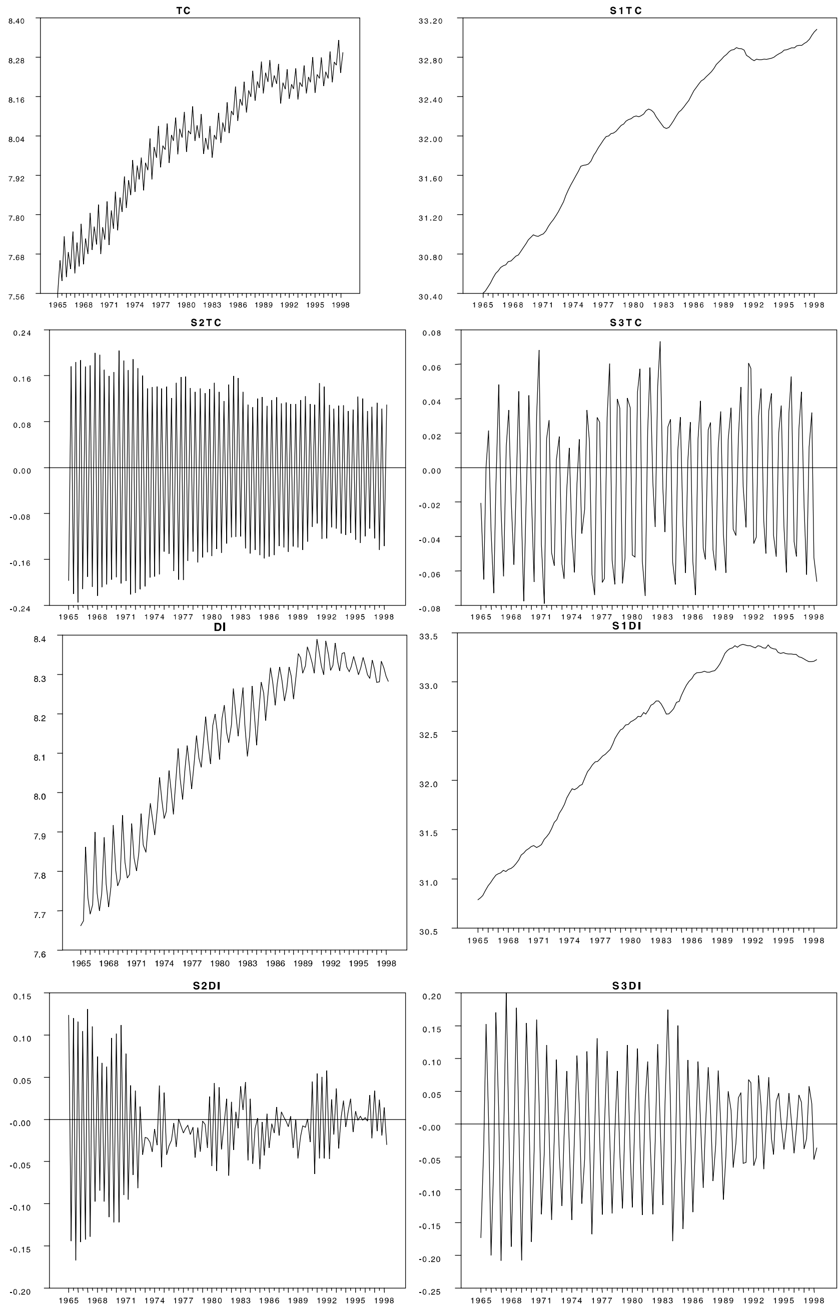

Figure 2. Canada

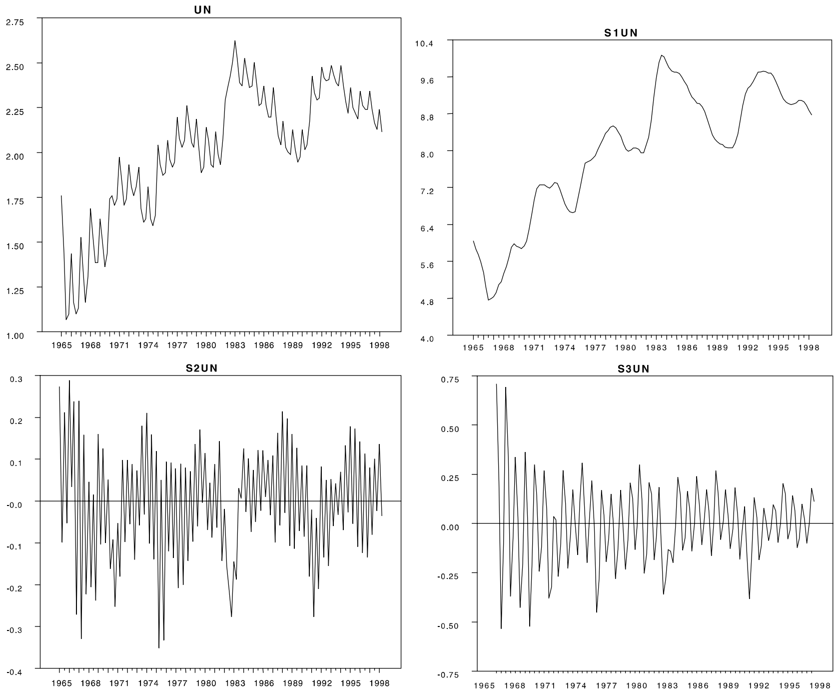

Figure 2. (Continued)

$$h_{0j} = -T \sum_{i=r_j+1}^{3} \ln(1 - \hat{\lambda}_{ji}),$$

where α_j's and β_j's are $3 \times r_j$ matrices.

The parameters α_j's and β_j's are not estimable, because they form an over-parameterisation of the model. Instead, the space spanned by β_j, denoted by sp(β_j), $j = 1, \ldots, 4$, can be estimated. Values $\hat{\lambda}_{j1} > \hat{\lambda}_{j2} > \hat{\lambda}_{j3}$ are the eigenvalues computed from the estimation of sp(β_j) and are sorted in descending order. The asymptotic distributions of the test statistics h_{0j}'s, $j = 1, \ldots, 4$, will depend on which deterministic components are included. Readers are suggested to refer to Johansen (1988), Johansen and Juselius (1990), Lee (1992), Lee and Siklos (1995), Johansen and Schaumburg (1999), and Franses and Kunst (1999).

Lee (1992) provided various quantile distributions of the likelihood ratio test statistics for cointegration and seasonal cointegration, but failed to include any deterministic components. Thus, these distributions are not applicable when the statistical models contain intercept, time trend, or seasonal dummy terms. Although Johansen and Schaumburg (1999) considered the role of these deterministic components, they nevertheless focused their analysis mainly on complex roots. The finite sample quantile distributions, including the model's deterministic parts, have to be constructed prior to the tests of seasonal cointegration being actually performed. The empirical finite sample quantile distributions are obtained by Monte Carlo experiments through 30,000 replications by utilising the GAUSS programming language.

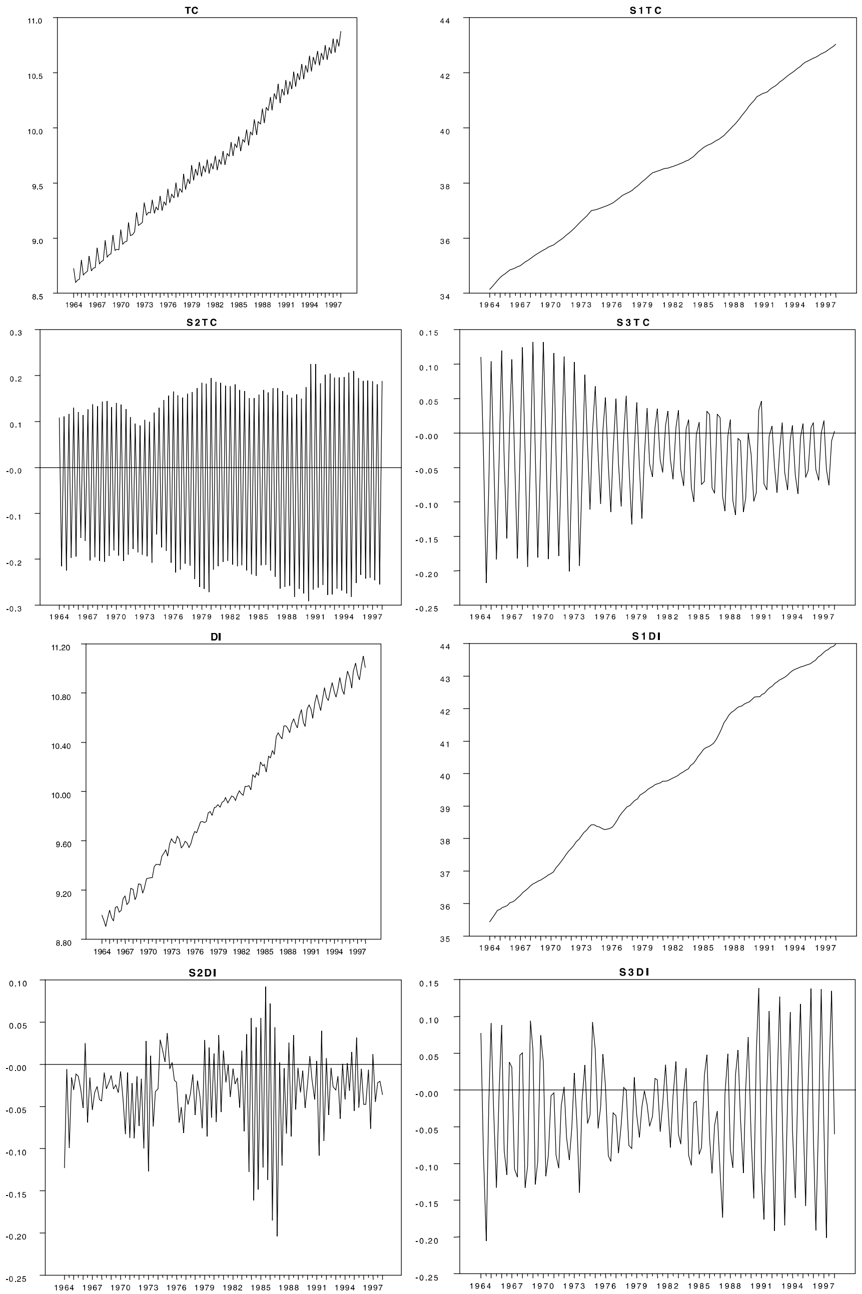

Figure 3. Taiwan

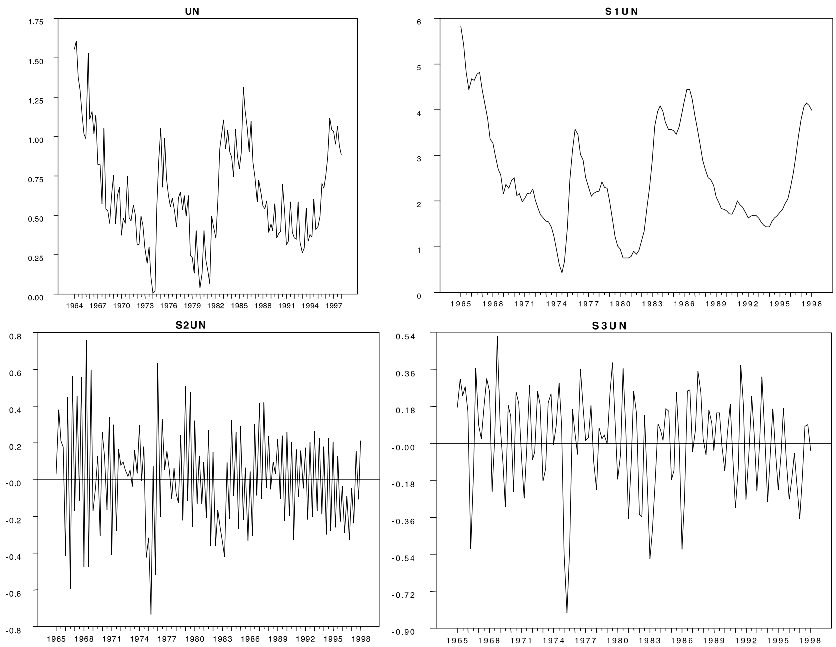

Figure 3. (Continued)

Applying the maximum likelihood inference, the likelihood ratio test results for seasonal cointegration at each seasonal frequency are shown in Table II.

Since the asymptotic distributions of the cointegration test statistics at the zero frequency can be shown to be the same as those of Johansen's test statistics (as summarised in Appendix B), it is not necessary to repeat the same likelihood ratio statistics for cointegration tests in Table II.[6] Table II shows that data from Austria and Taiwan are found to be seasonally cointegrated at frequency $\pi/2$, while a seasonal cointegration relationship exists at frequency π for the Canadian data. Though these seasonal cointegration relationships can be interpreted as equilibrium conditions, their estimated coefficients are difficult to interpret in an economically meaningful way, as economic theories are generally only applicable to the zero (long-run) frequency.

b) *The cross-equation restrictions*

As with equation (4), the consumption function is now written in fourth difference form as

$$\begin{aligned} F(L)\Delta_4 C_t = {} & G_1(L)[\Delta_4 DI_t - P_{t-1}(\Delta_4 DI_t)] + H_1(L)P_{t-1}(\Delta_4 DI_t) \\ & + G_2(L)[\Delta_4 UN_t - P_{t-1}(\Delta_4 UN_t)] + H_2(L)P_{t-1}(\Delta_4 UN_t) + v_t. \qquad (19) \end{aligned}$$

[6] The testing results on the presence of cointegration at zero frequency are exactly the same as Johansen's test. It follows that they would have the same cointegration coefficients if cointegrating relationships do exist. However, if the cointegration vector contains deterministic terms, such as an intercept and a linear time trend, then these terms' parameter estimates of the seasonal model differ from those of Johansen's procedure. Moreover, elements in the loading matrix also differ, but have the same signs.

Table II Likelihood ratio statistics for seasonal cointegration tests

Austria	(lag = 5, N = 64)					
Variables	(*TC GDP UN*)			(*NC GDP UN*)		
Rank	Freq. π	Freq. $\pi/2$	Freq. π (90%)	Freq. $\pi/2$ (90%)	Freq. π	Freq. $\pi/2$
0	28.697	37.814	32.810	37.855		
1	11.312	20.530	16.951	21.323		
2	1.523	7.936	6.762	9.562		
Vectors of Cointegration Coefficients						
Freq. π		None				
Freq. $\pi/2$		(1 −0.2222 0.2091)				
Vectors of Loading Coefficients						
Freq. π		None				
Freq. $\pi/2$		(0.1350 0.0623 0.3534)				
Canada	(lag = 8, N = 126)					
Variables	(*TC DI UN*)			(*NC DI UN*)		
Rank	Freq. π	Freq. $\pi/2$	Freq. π (90%)	Freq. $\pi/2$ (90%)	Freq. π	Freq. $\pi/2$
0	35.889	31.805	29.679	35.129	37.318	31.264
1	15.436	13.009	16.122	20.422	16.059	14.261
2	5.960	1.325	6.585	9.333	6.346	0.540
Vectors of Cointegration Coefficients						
Freq. π		(1 0.7060 1.3921)			(1 0.8041 1.7300)	
Freq. $\pi/2$		None			None	
Vectors of Loading Coefficients						
Freq. π		(0.0164 0.0378 0.2401)			(0.0091 0.0306 0.2244)	
Freq. $\pi/2$		None			None	
Taiwan	(lag = 6, N = 131)					
Variables	(*TC DI UN*)			(*NC DI UN*)		
Rank	Freq. π	Freq. $\pi/2$	Freq. π (90%)	Freq. $\pi/2$ (90%)	Freq. π	Freq. $\pi/2$
0	24.168	63.122	29.607	35.101	26.127	60.596
1	10.556	16.030	16.117	20.444	12.501	14.264
2	2.409	3.861	6.609	9.415	3.841	4.204
Vectors of Cointegration Coefficients						
Freq. π		(1 −0.4895 3.7688)			(1 −1.2814 7.9358)	
Freq. $\pi/2$		None			None	
Vectors of Loading Coefficients						
Freq. π		None			None	
Freq. $\pi/2$		(−0.0013 0.0028 0.1749)			(−0.0009 0.0015 0.0822)	

Notes: All test statistics presented are the trace values. (90%) under the frequency name means that numbers in that column represent the critical values for that frequency at the 10% level. Three centred seasonal dummies are always included in the statistical models.

Except for the notation Δ_4, the other notations in equation (19) are similarly defined as those in equation (4).

Under the PIH, consumption growth is only affected by the innovations of income and the unemployment rate, which imposes $H_1(L) = H_2(L) = 0$, $G_1(L) \neq 0$, and $G_2(L) \neq 0$. If the income forecast is irrational, then the forecast of income, $P_{t-1}(\Delta_4 DI)$, exhibits a structure similar to the first row of (18), but with distinct parameters. Furthermore, if the prediction of unemployment rate $P_{t-1}(\Delta_4 UN)$ is not rational as well, then an analogous structure results. Re-writing the left-hand side term of equation (18) as $\sum_{m=1}^{k-4} q_{ij,m}L^m \equiv q_{ij}(L)$, $i, j = 1, 2, 3$, and letting X_j be the j th element of vector X, then

$$P_{t-1}(\Delta_4 DI_t) = \mu_1^* + \sum_{j=1}^{3} q_{1j}^*(L)\Delta_4 X_{jt} + \alpha_{11}^* SEC_{1,t-1} + \alpha_{12}^* SEC_{2,t-1}, \tag{20}$$

and

$$P_{t-1}(\Delta_4 UN_t) = \mu_2^* + \sum_{j=1}^{3} q_{2j}^*(L)\Delta_4 X_{jt} + \alpha_{21}^* SEC_{1,t-1} + \alpha_{22}^* SEC_{2,t-1}, \tag{21}$$

where the parameters with superscript * differ from their rational forecast counterparts. Moreover, $SEC_{1,t-1}$ is the ECT at frequency zero, and $SEC_{2,t-1}$ represents the seasonal ECT at either frequency π or $\pi/2$, if variables in vector X constitute seasonal cointegration relationships.

After substituting equations (20) and (21) into the consumption equation (19), the derived consumption equation is further used to replace the third row of system (18). This turns out to be an "unrestricted" SECM as follows.

$$\begin{bmatrix} \bar{Q}(L) \\ N_1^*(L)N_2^*(L)N_3^*(L) \end{bmatrix} \begin{bmatrix} \Delta_4 DI_t \\ \Delta_4 UN_t \\ \Delta_4 C_t \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3^* \end{bmatrix} + \begin{bmatrix} \alpha_{11} \\ \alpha_{21} \\ \alpha_1^*(L) \end{bmatrix} SEC_{1,t-1} + \begin{bmatrix} \alpha_{12} \\ \alpha_{22} \\ \alpha_2^*(L) \end{bmatrix} SEC_{2,t-1} + \varepsilon_t, \tag{22}$$

where $\bar{Q}(L)$ corresponds to the first two rows of $Q(L)$ in equation (18), μ_3^* is a linear combination of μ_1^* and μ_2^*, and

$$N_1^*(L) = q_{11}^*(L)[G_1(L) - H_1(L)] + q_{21}^*(L)[G_2(L) - H_2(L)] - G_1(L), \tag{23}$$

$$N_2^*(L) = q_{12}^*(L)[G_1(L) - H_1(L)] + q_{22}^*(L)[G_2(L) - H_2(L)] - G_2(L), \tag{24}$$

$$N_3^*(L) = F(L) + q_{13}^*(L)[G_1(L) - H_1(L)] + q_{23}^*(L)[G_2(L) - H_2(L)], \tag{25}$$

$$\alpha_1^*(L) = -\alpha_{11}^*[G_1(L) - H_1(L)] - \alpha_{21}^*[G_2(L) - H_2(L)], \tag{26}$$

$$\alpha_2^*(L) = -\alpha_{12}^*[G_1(L) - H_1(L)] - \alpha_{22}^*[G_2(L) - H_2(L)]. \tag{27}$$

Given that the optimal lag lengths have been determined as 5, 8, and 6 for Austria, Canada, and Taiwan in Section III under standard cointegration, respectively, and that the lag polynomials of $F(L)$, $q_{1j}^*(L)$, and $q_{2j}^*(L)$ have equal lag lengths, the lag polynomials of the three economies are in turn set to have orders 1, 4, and 2. Through sequential likelihood ratio tests, all the orders of $G_i(L)$ and $H_i(L)$, $i = 1, 2$, for these countries are chosen as 1. Accordingly, the orders of $N_j^*(L)$, $j = 1, 2, 3$, are 2, 5, and 3, respectively. Finally, the two seasonal error-correction terms appear as $SEC_{m,t-1}$ and $SEC_{m,t-2}$, for $m = 1, 2$. It is also noted that seasonal dummies are always included in equation (22) during estimation.

Table III Wald test statistics under SECM

	Total Consumption	*Non-Durable Consumption*
Austria (number of restrictions = 4)		
W^*	4.191	N.A.
p-value	0.381	
W	3.392	N.A.
p-value	0.495	
Canada (number of restrictions = 10)		
W^*	14.741	9.230
p-value	0.142	0.510
W	14.941	10.162
p-value	0.134	0.426
Taiwan (number of restrictions = 6)		
W^*	8.002	8.605
p-value	0.238	0.197
W	7.796	8.158
p-value	0.253	0.227

Notes: W^* Wald test statistics with correcting for conditional heteroskedasticity.
W Wald test statistics without correcting for conditional heteroskedasticity.

To test the RE-PIH under the framework of SECM, the joint conditions of PIH, $H_1(L) = H_2(L) = 0$, and of rationality, $q^*_{1j}(L) = q_{1j}(L)$ and $q^*_{2j}(L) = q_{2j}(L)$, for $j = 1, 2$, must be imposed on equation (22). This leads to five sets of non-linear cross-equation restrictions for the three countries shown in Appendix D with each set applicable to either total consumption or non-durable consumption (excluding Austria). The Wald test statistics are presented in Table III.

Since all the p-values of the test statistics in Table III exceed 0.1, it is thus evident that applying an SC approach to analyse seasonally-unadjusted data cannot reject the joint hypothesis of RE-PIH at the 10 per cent level of significance. These results are sharply different from those of the conventional cointegration analysis summarised in Table I, due at least partially to the existence of the seasonal error correction term associated with either frequency π or $\pi/2$, which is ignored by conventional cointegration analysis. This omission is innocuous at frequency π for both Austria and Taiwan and $\pi/2$ for Canada, as no cointegration relationships are detected at the corresponding frequencies. At frequency $\pi/2$ for both Austria and Taiwan and π for Canada, this omission is not legitimate in that there exists a significant SC relationship at the respective frequencies. Overlooking these SC relationships can result in misspecification and produce serious consequences. Therefore, the emergence of the seasonal ECT, $SEC_{2,t-1}$, in equation (22) corrects for this specification error.

If a set of X-11 adjusted data is heuristically used, then similar problems result. The X-11 filter is applied to each series of a VAR system with N variables, which removes seasonal unit roots (as well as possible deterministic seasonality) at each seasonal frequency [see Ghysels (1998)]. If an SC relationship exists, say at frequency π, then only $N - 1$ seasonal unit roots remain in the system at the frequency. Consequently, the employment of the filter tends to undesirably overdifferentiate a system component at frequency π, leading to mis specification again.

The application of SECM to seasonally-unadjusted data has proved to be successful in avoiding the potential distortion of sample information and overdifference of the model. There are two purposes for incorporating the additional variable *UN*. One, it is chosen in an attempt to reflect the severity of liquidity constraints. As stated in Section II, the prevalence of liquidity constraints restricts agents from achieving their planned consumption, which results in an excess sensitivity of consumption to current (transitory) income, even though agents are rational and forward looking. Two, it is used as a specification test. Agents are said to be 'myopic' if their consumption is responsible to transitory income other than *UN*, implying that equation (1) is an appropriate specification of the consumption function. This helps explain the observed excess sensitivity of consumption to current income.

V. Conclusions

This article extends a new econometric modelling, first developed by Ermini and Chang (1996), who tested the macroeconomic rational expectations hypothesis of rationality and money neutrality, to the joint test of the validity of RE-PIH for three countries, where seasonally-unadjusted data are available. The testing procedure is novel, because the SC framework is applied for the first time to the area of agents' behaviours of consumption decisions. Under SC, a researcher conducting empirical studies need not deseasonalise the data series by series, such as using the X-11 procedure, so that valuable information embedded in the data will not be deleted by the specific adjusting procedure and the potential problem of overdifferencing is avoided. In addition, the non-linear cross-equation restrictions required to carry out the joint test can still be derived with a slight extension, where both the error-correction terms at zero and seasonal frequencies must be considered.

The presence of a seasonal ECT in equation (22) is crucial, in that it eliminates specification error due to the utilisation of either raw or seasonally-adjusted data adopted by the conventional error correction framework. Based on Wald tests under SC, one cannot reject the RE-PIH at the 10 per cent significance level for all three economies, while most of the Wald test statistics under conventional ECM reject the hypothesis at the same confidence level. This indicates that the specification error indeed distorts the empirical inference. One may further infer that the application of deseasonalised procedures series by series can lead to the same consequence as well. Therefore, the SECM tends to be superior to the conventional ECM.

Appendix A: Data Source and Seasonal Unit Root Tests

Quarterly raw data are employed in the current paper, which are taken from MEI (Main Economic Indicator) and INTLINE data banks for Austria and Canada, and from AREMOS data bank for Taiwan. The consumption series is measured by both real total consumption and real non-durable consumption, except for Austria, since the latter is not available. Variable income is represented by either real gross domestic products (Austria) or real disposable income (Canada and Taiwan). Both real consumption and real income are further divided by population. Finally, *per capita* real consumption and income together with the unemployment rate (expressed as percentage points) are taken by natural logarithms, and these logarithmic variables are actually used in the empirical study. Sample statistics are available upon request.

The seasonal unit root testing results at frequency zero, π, and $\pi/2$ using the procedures developed by Hylleberg *et al*. (1990) and Engle *et al*. (1993) are presented in the following table. Here, *TC* denotes (log) real *per capita* total consumption, *NC* denotes (log) real *per*

	$t: \pi_1$	$t: \pi_2$	$t: \pi_3$
Austria			
TC (lag = 5, N = 64)			
c, DUM	−1.241	−1.593	−2.214
c, T, DUM	−2.194	−1.440	−2.234
GDP (lag = 5, N = 64)			
c, DUM	−0.877	−2.903	−1.883
c, T, DUM	−1.929	−2.766	−1.918
UN (lag = 7, N = 61)			
c, DUM	−1.634	−3.488b	−1.854
c, T, DUM	−3.097	−3.319b	−2.342
Canada			
TC (lag = 6, N = 139)			
c, DUM	−1.941	−1.035	−3.081
c, T, DUM	−1.481	−1.041	−3.031
NC (lag = 6, N = 139)			
c, DUM	−2.147	−0.926	−3.065
c, T, DUM	−1.486	−0.929	−3.028
DI (lag = 5, N = 140)			
c, DUM	−2.713	−1.984	−2.025
c, T, DUM	0.838	−2.015	−1.928
UN (lag = 7, N = 138)			
c, DUM	−1.628	−2.410	−0.957
c, T, DUM	−2.603	−2.394	−0.933
Taiwan			
TC (lag = 6, N = 139)			
c, DUM	0.208	−1.835	−1.085
c, T, DUM	−1.644	−1.832	−1.071
NC (lag = 6, N = 139)			
c, DUM	0.530	−2.122	−1.266
c, T, DUM	−1.281	−2.115	−1.253
DI (lag = 5, N = 140)			
c, DUM	−0.728	−2.496	−1.185
c, T, DUM	−2.425	−2.468	−1.214
UN (lag = 6, N = 127)			
c, DUM	−2.941b	−2.517	−2.372
c, T, DUM	−2.804	−2.507	−2.397

N: sample size

Critical Values (taken from Hylleberg *et al.* (1990))

5% (N = 136)	t: π_1	t: π_2	t: π_3
c, DUM	−2.94	−2.90	−3.44
c, T, DUM	−3.52	−2.93	−3.44
1% (N = 136)	t: π_1	t: π_2	t: π_3
c, DUM	−3.56	−3.49	−4.06
c, T, DUM	−4.15	−3.57	−4.05

capita non-durable consumption, *DI* (*GDP*) represents (log) real *per capita* disposable income (gross domestic product), and *UN* represents (log) unemployment rate.

The optimal lag lengths are determined by using Akaike Information Criterion. Two combinations of the deterministic components, which include intercept (c), linear trend (T), and seasonal dummies (DUM), are considered. For Austria and Canadian data, all series are integrated at zero frequency under 5 per cent and 1 per cent levels of significance, irrespective of the deterministic parts. Taiwan's data have similar properties, except that *UN* is stationary when (c, DUM) are accommodated in the regression model.

Appendix B: Results of Conventional Cointegration Test

The optimal lag lengths are determined by the sequential likelihood ratio tests. Three centred seasonal dummies are always included in the models. In the current paper, matrix Π in equation (3) is a 3×3 matrix. To test the null hypothesis of $\text{rank}(\Pi) \leq r < 3$ against the alternative that $\text{rank}(\Pi) > r$, one can equivalently express the null hypothesis as $\Pi = \alpha\beta'$s, where α and β are both $3 \times r$ matrices. Term β is composed of r column vectors of cointegrating coefficients, and α is the corresponding loading matrix. One can merely estimate the space spanned by α and β, denoted by sp(α) and sp(β), as they form an over-parameterisation of the model (see Johansen, 1988, and Johansen and Juselius, 1990), and the estimates of sp(α) and sp(β) can be used to test the above hypothesis.

Rank	*Eighen-value*	*λ-max*	*Trace*	*λ-max 90%*	*Trace 90%*	*Eighen-value*	*λ-max*	*Trace*
Austria						(lag = 5, N = 64) (*TC GDP UN* Intercept)		
r = 0				16.13	39.08	0.3999	32.68	52.36
r = 1				12.39	22.95	0.2064	14.79	19.68
r = 2				10.56	10.56	0.0735	4.88	4.88
					$\beta' = (1\ -0.732\ 0.197\ -0.003)$ $\alpha = (-0.187\ -0.074\ -1.205)$			
Canada	(lag = 8, N = 126) (*NC DI UN* Intercept)					(lag = 8, N = 126) (*TC DI UN* Intercept)		
r = 0	0.1541	21.09	34.02	14.09	31.88	0.1348	18.24	31.62
r = 1	0.0874	11.52	12.93	10.29	17.79	0.0911	12.04	13.39
r = 2	0.0111	1.41	1.41	7.50	7.50	0.0106	1.34	1.34
	$\beta' = (10.556\ 1.292\ -16.165)$ $\alpha = (-0.002\ -0.002\ -0.033)$					$\beta' = (1\ 1.176\ 0.683\ -19.995)$ $\alpha = (-0.001\ -0.001\ -0.032)$		
Taiwan	(lag = 6, N = 131) (*NC DI UN*)					(lag = 6, N = 131) (*TC DI UN*)		
r = 0	0.1190	16.60	27.29	13.39	26.70	0.1077	14.93	23.03
r = 1	0.0697	9.46	10.69	10.60	13.31	0.0486	6.52	8.10
r = 2	0.0094	1.23	1.23	2.71	2.71	0.0120	1.58	1.58
	$\beta' = (1\ -1.080\ 0.148)$ $\alpha = (-0.031\ 0.059\ 0.462)$					$\beta' = (1\ -1.210\ 0.807)$ $\alpha = (-0.004\ 0.017\ -0.122)$		

APPENDIX C: CROSS-EQUATION RESTRICTIONS UNDER NON-SEASONAL ERROR CORRECTION MODEL

Let ϕ_{ij}^{k} be the (i, j)-th element of the 3×3 coefficient matrix at lag k of equation (7), and φ_{j}^{s} is the jth element of the vector of loading coefficients corresponding to the terms of EC_{t-k}, for $s = k$, and EC_{t-k-1}, for $s = k + 1$. Using these notations, the set of non-linear cross-equation restrictions imposed by the RE-PIH for Canadian data can be derived as

$$\begin{aligned}
\phi_{11}^{1}\phi_{31}^{1} + \phi_{21}^{1}\phi_{32}^{1} + \phi_{31}^{2} &= 0, \qquad \text{(A1)}\\
\phi_{12}^{1}\phi_{31}^{1} + \phi_{22}^{1}\phi_{32}^{1} + \phi_{32}^{2} &= 0,\\
\phi_{11}^{2}\phi_{31}^{1} + \phi_{21}^{2}\phi_{32}^{1} + \phi_{31}^{3} &= 0,\\
\phi_{12}^{2}\phi_{31}^{1} + \phi_{22}^{2}\phi_{32}^{1} + \phi_{32}^{3} &= 0\\
\phi_{11}^{3}\phi_{31}^{1} + \phi_{21}^{3}\phi_{32}^{1} + \phi_{31}^{4} &= 0,\\
\phi_{12}^{3}\phi_{31}^{1} + \phi_{22}^{3}\phi_{32}^{1} + \phi_{32}^{4} &= 0,\\
\phi_{11}^{4}\phi_{31}^{1} + \phi_{21}^{4}\phi_{32}^{1} + \phi_{31}^{5} &= 0,\\
\phi_{12}^{4}\phi_{31}^{1} + \phi_{22}^{4}\phi_{32}^{1} + \phi_{32}^{5} &= 0,\\
\phi_{11}^{5}\phi_{31}^{1} + \phi_{21}^{5}\phi_{32}^{1} + \phi_{31}^{6} &= 0,\\
\phi_{12}^{5}\phi_{31}^{1} + \phi_{22}^{5}\phi_{32}^{1} + \phi_{32}^{6} &= 0,\\
\phi_{11}^{6}\phi_{31}^{1} + \phi_{21}^{6}\phi_{32}^{1} + \phi_{31}^{7} &= 0,\\
\phi_{12}^{6}\phi_{31}^{1} + \phi_{22}^{6}\phi_{32}^{1} + \phi_{32}^{7} &= 0,\\
\phi_{11}^{7}\phi_{31}^{1} + \phi_{21}^{7}\phi_{32}^{1} + \phi_{31}^{8} &= 0,\\
\phi_{12}^{7}\phi_{31}^{1} + \phi_{22}^{7}\phi_{32}^{1} + \phi_{32}^{8} &= 0,\\
\varphi_{1}^{8}\phi_{31}^{1} + \varphi_{2}^{8}\phi_{32}^{1} + \varphi_{3}^{9} &= 0.
\end{aligned}$$

For Austrian data, the number of cross-equation restrictions is nine, which are composed of the first eight restrictions of (A1) and $\varphi_{1}^{5}\phi_{31}^{1} + \varphi_{2}^{5}\phi_{32}^{1} + \varphi_{3}^{6} = 0$. The number of cross-equation restrictions for Taiwan's data is eleven, consisting of the first ten restrictions of (A1) and $\varphi_{1}^{6}\phi_{31}^{1} + \varphi_{2}^{6}\phi_{32}^{1} + \varphi_{3}^{7} = 0$.

APPENDIX D: CROSS-EQUATION RESTRICTIONS UNDER SECM

Let ϕ_{ij}^{k} be the (i, j)-th element of the 3×3 coefficient matrix at the kth lag of equation (10), and φ_{ij}^{s} is the jth element of the column vector of loading coefficients associated with the seasonal error-correction terms $SEC_{s,t-i}$ for $s = 1$ (frequency zero), 2 (frequency π), or 3 (frequency $\pi/2$) and $i = 1, 2$. These notations are then used to derive sets of non-linear cross-equation restrictions implied by the RE-PIH hypothesis for the three countries as follows.

(i) Austria (total consumption only)

$$\phi_{11}^1\phi_{31}^1 + \phi_{21}^1\phi_{32}^1 + \phi_{31}^2 = 0,$$

$$\phi_{12}^1\phi_{31}^1 + \phi_{22}^1\phi_{32}^1 + \phi_{32}^2 = 0,$$

$$\varphi_{11}^1\phi_{31}^1 + \varphi_{12}^1\phi_{32}^1 + \varphi_{23}^1 = 0,$$

$$\varphi_{11}^3\phi_{31}^1 + \varphi_{12}^3\phi_{32}^1 + \varphi_{23}^3 = 0.$$

(ii) Canada (both total and non-durable consumption)

$$\phi_{11}^1\phi_{31}^1 + \phi_{21}^1\phi_{32}^1 + \phi_{31}^2 = 0,$$

$$\phi_{12}^1\phi_{31}^1 + \phi_{22}^1\phi_{32}^1 + \phi_{32}^2 = 0,$$

$$\phi_{11}^2\phi_{31}^1 + \phi_{21}^2\phi_{32}^1 + \phi_{31}^3 = 0,$$

$$\phi_{12}^2\phi_{31}^1 + \phi_{22}^2\phi_{32}^1 + \phi_{32}^3 = 0$$

$$\phi_{11}^3\phi_{31}^1 + \phi_{21}^3\phi_{32}^1 + \phi_{31}^4 = 0,$$

$$\phi_{12}^3\phi_{31}^1 + \phi_{22}^3\phi_{32}^1 + \phi_{32}^4 = 0,$$

$$\phi_{11}^4\phi_{31}^1 + \phi_{21}^4\phi_{32}^1 + \phi_{31}^5 = 0,$$

$$\phi_{12}^4\phi_{31}^1 + \phi_{22}^4\phi_{32}^1 + \phi_{32}^5 = 0,$$

$$\varphi_{11}^1\phi_{31}^1 + \varphi_{12}^1\phi_{32}^1 + \varphi_{23}^1 = 0,$$

$$\varphi_{11}^2\phi_{31}^1 + \varphi_{12}^2\phi_{32}^1 + \varphi_{23}^3 = 0.$$

(iii) Taiwan (both total and non-durable consumption)

$$\phi_{11}^1\phi_{31}^1 + \phi_{21}^1\phi_{32}^1 + \phi_{31}^2 = 0,$$

$$\phi_{12}^1\phi_{31}^1 + \phi_{22}^1\phi_{32}^1 + \phi_{32}^2 = 0,$$

$$\phi_{11}^2\phi_{31}^1 + \phi_{21}^2\phi_{32}^1 + \phi_{31}^3 = 0,$$

$$\phi_{12}^2\phi_{31}^1 + \phi_{22}^2\phi_{32}^1 + \phi_{32}^3 = 0,$$

$$\varphi_{11}^1\phi_{31}^1 + \varphi_{12}^1\phi_{32}^1 + \varphi_{23}^1 = 0,$$

$$\varphi_{11}^3\phi_{31}^1 + \varphi_{12}^3\phi_{32}^1 + \varphi_{23}^3 = 0.$$

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