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A new approach to estimating the metafrontier production function based on a stochastic frontier framework

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Abstract This paper proposes a new two-step stochastic frontier approach to estimate technical efficiency (TE) scores for firms in different groups adopting distinct technologies. Analogous to Battese et al. (J Prod Anal 21:91–103, 2004), the metafrontier production function allows for calculating comparable TE measures, which can be decomposed into group specific TE measures and technology gap ratios. The proposed approach differs from Battese et al. (J Prod Anal 21:91–103, 2004) and O'Donnell et al. (Empir Econ 34:231–255, 2008) mainly in the second step, where a stochastic frontier analysis model is formulated and applied to obtain the estimates of the metafrontier, instead of relying on programming techniques. The so-derived estimators have the desirable statistical properties and enable the statistical inferences to be drawn. While the within-group variation in firms' technical efficiencies is frequently assumed to be associated with firm-specific exogenous variables, the between-group variation in technology gaps can be specified as a function of some exogenous variables to take account of group-specific environmental differences. Two empirical applications are illustrated and the results appear to support the use of our model.

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1 Introduction

As first introduced by Hayami (1969) and Hayami and Ruttan (1970, 1971), the metaproduction function is based on the idea that all producers in the various production groups have potential access to an array of production technologies, but each may choose a particular technology, depending on specific circumstances, such as regulation, the environments, production resources, and relative input prices. These conditions inhibit firms in some groups from choosing the best technology from the array of the potential technology set. A production technology gap is the difference between the best technology and the chosen sub-technology, i.e., the group-specific frontier.

To estimate a metafrontier by simply pooling all the data of the various groups is not justifiable, as the so-derived metafrontier would not necessarily envelop the group-specific frontiers. It would also lack justification if one first estimated the individual group frontiers and then compared the technical efficiencies (TEs) among groups, because these TE scores are evaluated relative to different production frontiers, not relative to the metafrontier. A metafrontier production function model, developed by Battese et al. (2004) and O'Donnell et al. (2008), is able to disentangle the above difficulties. They proposed a mixed approach with a two-step procedure for estimating the metafrontier. They combined the stochastic frontier (SF) regression used in the first step to estimate the group-specific frontier with the mathematical programming techniques in the second-step to estimate the metafrontier. However, the potential difficulty with the two-step mixed approach comes from its second step estimation in that no statistical properties of the metafrontier estimators result, because it is a linear (or quadratic) programming algebraic calculation. Furthermore, no accounting for potentially different production environments facing firms can be incorporated into the estimation, not mentioning its incapability of isolating idiosyncratic shocks.

In this paper we propose a new two-step SF approach to estimate the group-specific frontiers and the metafrontier, respectively, and to decompose the efficiency scores of various groups into TE and technology gaps. It can easily be shown that the mixed approach is a special case of our proposed approach. The main difference between the two-step SF approach and that of Battese et al. (2004) and O'Donnell et al. (2008) is that the former's second-step estimation of the metafrontier is still based on the SF framework, rather than on a mathematical programming technique. Hence, our metafrontier estimation is a stochastic metafrontier (SMF) regression method, while the mixed approach is a deterministic metafrontier programming method. The new SMF method has the following merits. First, as we apply the conventional maximum likelihood method to estimating the parameters of the SMF regression, the usual statistical inferences can be performed without relying on simulations or bootstrapping, as opposed to mathematical programming techniques. Second, the SMF method can directly estimate the technology gaps by treating them as a conventional one-sided error term. This strategy allows us to separate the random shocks from the technology gaps, a well-known advantage of the SF technique over the programming technique. Conversely, the technology gaps obtained from the programming technique may be contaminated by random shocks. Lastly, since the second-step estimation of SMF is still based on a SF regression, the technology gaps represented by the one-sided term can be further specified as a function of environmental variables beyond the control of firms, such as the one considered by Huang and Liu (1994) and Battese and Coelli (1995). Such a specification is simultaneously built into the heteroscedastic inefficiency term, as pointed out by Kumbhakar and Lovell (2000). The foregoing merits are at the heart of the new approach in estimating the metafrontier function.

The rest of the paper is organized as follows. Section 2 formulates the proposed SMF method to estimate the metafrontier production function. Section 3 highlights the characteristics of the SMF method by conducting two empirical studies. In the first study, we use the same cross-country agricultural sector data used in O'Donnell et al. (2008) to make the empirical comparison on the estimates of the metafrontier based on the deterministic metafrontier programming method and the SMF method. In the second study, we present an empirical application of the SMF modeling using data from the hotel industry in Taiwan to measure the technical efficiency and the technology gaps of the chain- and the independently-operated hotels. In Sect. 4, we summarize and conclude the paper.

2 Formulation and estimation of the metafrontier production function

Suppose that, for the jth production group, for example, of a country or an industry, the SF of the ith decision making unit (DMU) or a firm in the tth period is modeled as

$$Y_{jit} = f_t^j(X_{jit})e^{V_{jit}-U_{jit}}, \quad j = 1,2,\ldots,J; \quad i = 1,2,\ldots,N_j; \quad t = 1,2,\ldots,T \tag{1}$$

where Y_{jit} and X_{jit} respectively denote the scalar output and input vector of the ith firm in the jth group at the tth period. We note that the function $f_t^j(.)$ of the production frontier is both subscripted by t and superscripted by j, that is, the individual group-specific production technology may vary across groups and across time. For example, $f_t^j(X_{jit}) = e^{X_{jit}\beta_t^j}$, where β_t^j denotes the parameters associated with the group-j frontier at the tth period. Following the standard SF modeling, the random errors V_{jit} represent statistical noise, and the non-negative random errors U_{jit} represent technical inefficiency. It is assumed that V_{jit}s are distributed independently and identically as $N(0, \sigma_v^{j2})$ and are independent of U_{jit}s, which follow the truncated-normal distribution as $N^+(\mu^j(Z_{jit}), \sigma_u^{j2}(Z_{jit}))$, i.e., truncated from below at zero and with the mode at $\mu^j(Z_{jit})$, where Z_{jit}s are some exogenous variables.[1] A firm's technical efficiency (TE) in production is then defined as

$$TE_{it}^j = \frac{Y_{jit}}{f_t^j(X_{jit})e^{V_{jit}}} = e^{-U_{jit}} \tag{2}$$

The technical efficiency can be associated with a set of within-group firm-specific exogenous variables Z_{jit}.

The common underlying metafrontier production function for all groups in the tth period is defined as $f_t^M(X_{jit})$, where the function is the same for all groups $j = 1, 2,\ldots, J$. The metafrontier $f_t^M(X_{jit})$ by definition envelops all individual groups' frontiers $f_t^j(X_{jit})$, which is expressed with the following relation,

$$f_t^j(X_{jit}) = f_t^M(X_{jit})e^{-U_{jit}^M}, \quad \forall j,\ i,\ t \tag{3}$$

where $U_{jit}^M \geq 0$. Hence, $f_t^M(.) \geq f_t^j(.)$ and the ratio of the jth group's production frontier to the metafrontier is defined as the technology gap ratio (TGR),

[1] See Huang and Liu (1994), Battese and Coelli (1995), Wang (2002), Huang (2005), and Lai and Huang (2010) for the specification, interpretation, and testing of various efficiency models related to the truncated-normal specification $N^+(\mu^j(Z_{jit}), \sigma_u^{j2}(Z_{jit}))$.

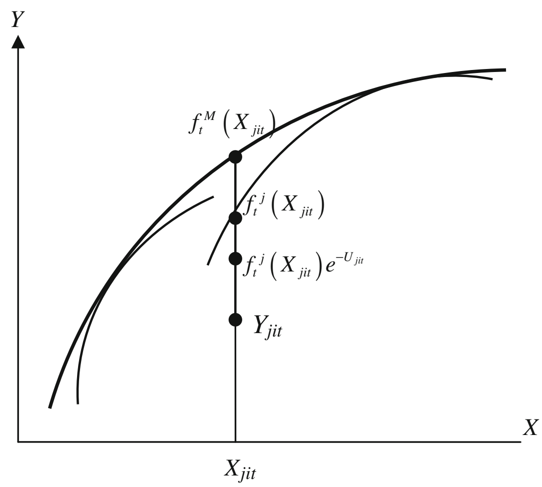

Fig. 1 Metafrontier production model

$$TGR_{it}^j = \frac{f_t^j(X_{jit})}{f_t^M(X_{jit})} = e^{-U_{jit}^M} \leq 1. \tag{4}$$

The existence of the technology gap is interpreted to be due to the choice of a particular technology that depends on the production environments, both economic and non-economic. The technology gap component U_{jit}^M in (4) is thus group, firm, and time specific. The TGR thus depends on the accessibility and extent of adoption of the available metafrontier production technology. Figure 1 illustrates the metafrontier production model. At a given input level X_{jit}, a firm's observed output Y_{jit} relative to the metafrontier $f_T{}^M(X_{jit})$ consists of three components: the TGR, $TGR_{it}^j = \frac{f_t^j(X_{jit})}{f_t^M(X_{jit})}$; the firm's technical efficiency, $TE_{it}^j = \frac{f_t^j(X_{jit})e^{-U_{jit}}}{f_t^j(X_{jit})} = e^{-U_{jit}}$; and the random noise component, $\frac{Y_{jit}}{f_t^j(X_{jit})e^{-U_{jit}}} = e^{V_{jit}}$, i.e.,

$$\frac{Y_{jit}}{f_t^M(X_{jit})} = TGR_{it}^j \times TE_{it}^j \times e^{V_{jit}}. \tag{5}$$

It should be emphasized that, although both the technology gap ratio $TGR_{it}^j \leq 1$ and the firm's technical efficiency $TE_{it}^j \leq 1$ are bounded, the metafrontier $f_t^M(X_{jit})$ does not necessary envelop all firms' observed outputs Y_{jit}. The unrestricted ratio in (5) distinguishes the metafrontier modeling using the stochastic frontier analysis (SFA) from the data envelopment analysis (DEA). By accounting for the random noise component, the decomposition in (5) can be expressed alternatively as

$$MTE_{jit} \equiv \frac{Y_{jit}}{f_t^M(X_{jit})e^{V_{jit}}} = TGR_{it}^j \times TE_{it}^j \tag{6}$$

where MTE_{jit} is defined as the firm's technical efficiency with respect to the metafrontier production technology $f_t^M(.)$ as opposed to the firm's technical efficiency TE_{it}^j with respect to the group-j production technology $f_t^j(.)$.

Empirical measurement of the above metafrontier model generally consists of two steps. Battese et al. (2004) and O'Donnell et al. (2008) propose a mixed approach of combining the maximum likelihood estimates of the group-specific SF regression in (1) with a mathematical optimization programming estimation of the metafrontier function in (3). More specifically, in the first step, the standard maximum likelihood (ML) estimation is applied to each group-specific frontier in (1) using the panel data of N_j firms in T periods, i.e.,

$$\ln Y_{jit} = \ln f_t^j(X_{jit}) + V_{jit} - U_{jit}, \quad i = 1, 2, \ldots, N_j; \\ t = 1, 2, \ldots, T \tag{7}$$

with the composite error $\varepsilon_{jit} = V_{jit} - U_{jit}$ and the distribution assumptions that $V_{jit} \sim N(0, \sigma_v^{j2})$ and $U_{jit} \sim N^+(\mu^j(Z_{jit}), \sigma_u^{j2}(Z_{jit}))$. Let $\hat{f}_t^j(X_{jit})$ be the maximum likelihood estimate of the group-j's specific frontier and the group-jth technical efficiency is estimated as the conditional expectation

$$\overset{\wedge}{TE}{}_{it}^j = \hat{E}\left(e^{-U_{jit}} | \hat{\varepsilon}_{jit}\right) \tag{8}$$

where $\hat{\varepsilon}_{jit} = \ln Y_{jit} - \ln \hat{f}_t^j(X_{jit})$ are the estimated composite residuals.

In the second step of the Battese et al. (2004) and O'Donnell et al. (2008) mixed approach, the metafrontier function $f_t^M(.)$ is obtained by solving the following linear programming problem using the estimated group-specific frontiers,

$$\min \sum_{j=1}^{J} \sum_{i=1}^{N_j} \sum_{t=1}^{T} \left| \ln f_t^M(X_{jit}) - \ln \hat{f}_t^j(X_{jit}) \right| \\ \text{subject to } \ln f_t^M(X_{jit}) \geq \ln \hat{f}_t^j(X_{jit}). \tag{9}$$

Alternatively, the metafrontier function can be obtained by minimizing the sum of squares of the deviations of the metafrontier function from the estimated group-specific frontiers,

$$\min \sum_{j=1}^{J} \sum_{i=1}^{N_j} \sum_{t=1}^{T} \left(\ln f_t^M(X_{jit}) - \ln \hat{f}_t^j(X_{jit}) \right)^2 \\ \text{subject to } \ln f_t^M(X_{jit}) \geq \ln \hat{f}_t^j(X_{jit}). \tag{10}$$

A major drawback of the above two-step mixed approach of Battese et al. (2004) and O'Donnell et al. (2008) is that, in the second-step, the metafrontier function $f_t^M(.)$is calculated using the mathematical programming techniques rather than estimated using regression techniques. It is difficult, unfortunately, to give a meaningful statistical interpretation to the computed metafrontier function, even though the group-specific frontiers are estimated by maximum likelihood. A more serious problem in the mixed approach is that, in the second-step, the estimated group-specific frontiers are used in the optimization programming to obtain the metafrontier

Table 1 Comparison of the agricultural metafrontier estimates

Variables	Stochastic metafrontier model (15)		LP metafrontier model (9)	
	Parameter estimates	Standard errors	Parameter estimates	Bootstrapped standard errors
Constant	12.2549***	2.3395	16.6959***	3.8117
$\ln x_1$	0.2603	0.2065	0.2464	0.3970
$\ln x_2$	0.0799	0.2278	0.9359***	0.3397
$\ln x_3$	2.0387***	0.1595	2.0474***	0.2619
$\ln x_4$	0.3547*	0.1686	0.1372	0.2154
$\ln x_5$	−1.5026***	0.4028	−2.3383***	0.7345
$\ln x_1 \times \ln x_1$	−0.0704***	0.0178	−0.0443*	0.0233
$\ln x_2 \times \ln x_2$	0.0298**	0.0105	0.0548***	0.0159
$\ln x_3 \times \ln x_3$	0.0989***	0.0120	0.0731***	0.0160
$\ln x_4 \times \ln x_4$	0.0686***	0.0066	0.0466***	0.0096
$\ln x_5 \times \ln x_5$	0.1518***	0.0441	0.2287***	0.0777
$\ln x_1 \times \ln x_2$	0.1315***	0.0194	0.1680***	0.0517
$\ln x_1 \times \ln x_3$	0.1087***	0.0167	0.0970***	0.0210
$\ln x_1 \times \ln x_4$	−0.1333***	0.0146	−0.1255***	0.0341
$\ln x_1 \times \ln x_5$	0.0219	0.0410	−0.0341	0.0747
$\ln x_2 \times \ln x_3$	−0.1157***	0.0171	−0.2060***	0.0483
$\ln x_2 \times \ln x_4$	−0.0395*	0.0196	−0.0189	0.0236
$\ln x_2 \times \ln x_5$	−0.0196	0.0362	−0.1364**	0.0632
$\ln x_3 \times \ln x_4$	−0.0128	0.0159	0.0195	0.0320
$\ln x_3 \times \ln x_5$	−0.2608***	0.0318	−0.1989***	0.0500
$\ln x_4 \times \ln x_5$	−0.0094	0.0226	0.0118	0.0266
dummy	−1.4173***	0.2532	−0.9397*	0.5102
$\sigma^{M2} = \sigma_v^{M2} + \sigma_u^{M2}$	0.1043***	0.0162		
$\bar{\gamma}^M = \sigma_v^{M2}/(\sigma_v^{M2}+\sigma_u^{M2})$	0.2904***	0.0967		
Log-Likelihood	13.2786			

The QML sandwich estimated standard errors are calculated for the stochastic metafrontier model. The standard errors of the LP method are obtained from bootstrapping

Following O'Donnell et al. (2008), we use a dummy variable to deal with the zero observations for the fertilizer input in the data set

***, **, and * denote significant at the 1, 5, and 10 % levels, respectively

function. More specifically, consider the relation between the group-specific frontier and the metafrontier functions in (3),

$$\ln f_t^j(X_{jit}) = \ln f_t^M(X_{jit}) - U_{jit}^M. \tag{11}$$

Had the group-specific frontiers been known, the mathematical programming method would have been equivalent to the least-absolute deviations of U_{jit}^M,

$$\begin{aligned} &\min \sum_{j=1}^{J}\sum_{i=1}^{N_j}\sum_{t=1}^{T}\left|U_{jit}^M\right| \\ &= \sum_{j=1}^{J}\sum_{i=1}^{N_j}\sum_{t=1}^{T}\left|\ln f_t^M(X_{jit}) - \ln f_t^j(X_{jit})\right|, \end{aligned} \tag{12}$$

or the least-squares deviations of U_{jit}^M,

$$\begin{aligned} &\min \sum_{j=1}^{J}\sum_{i=1}^{N_j}\sum_{t=1}^{T}\left(U_{jit}^M\right)^2 \\ &= \sum_{j=1}^{J}\sum_{i=1}^{N_j}\sum_{t=1}^{T}\left(\ln f_t^M(X_{jit}) - \ln f_t^j(X_{jit})\right)^2 \end{aligned} \tag{13}$$

subject to the constraints, $U_{jit}^M \geq 0$.[2] However, since $f_t^j(X_{jit})$ are unknown a priori and the maximum likelihood estimates are not perfect, i.e., $\hat{f}_t^j(X_{jit}) \neq f_t^j(X_{jit})$, the degree of bias in applying (9) and (10) rather than (12) and (13) in the second step of the mixed approach is unknown.

The problems associated with the mixed approach of Battese et al. (2004) and O'Donnell et al. (2008) arise from using the mathematical programming technique and from omitting the error of $\hat{f}_t^j(X_{jit})$ in the estimation of $f_t^j(X_{jit})$. Consequently, the statistical properties of the metafrontier estimates in the second step are unknown. We propose an alternative method using the SF regression rather than the mathematical programming technique in the second-step estimation of the

[2] In the case that the group-specific frontiers are known, Schmidt (1976) has shown that the optimization programming problems in (12) and (13) correspond to the maximum likelihood estimates if the U_{jit}^M follows an exponential distribution and a half-normal distribution, respectively. Even in this case, however, the statistical properties of the maximum likelihood estimators cannot be obtained since the regularity condition for the maximum likelihood method is violated.

metafrontier that specifically takes into consideration the estimation error of $\hat{f}_t^j(X_{jit})$ in estimating $f_t^j(X_{jit})$.

Given the SFA estimates of the group-specific frontiers $\hat{f}_t^j(X_{jit})$ for all $j = 1,\ldots, J$ groups in (7) from the first step, the estimation error of the group-specific frontier is then,

$$\ln \hat{f}_t^j(X_{jit}) - \ln f_t^j(X_{jit}) = \varepsilon_{jit} - \hat{\varepsilon}_{jit} \tag{14}$$

Defining the estimation error as $V_{jit}^M = \varepsilon_{jit} - \hat{\varepsilon}_{jit}$, the metafrontier relation in (11) can be rewritten by replacing the unobserved group-specific frontiers $f_t^j(X_{jit})$ on the left-hand side with the estimates $\hat{f}_t^j(X_{jit})$, i.e.,

$$\ln \hat{f}_t^j(X_{jit}) = \ln f_t^M(X_{jit}) - U_{jit}^M + V_{jit}^M, \quad \forall\, i,\ t, j = 1, 2, .., J \tag{15}$$

Equation (15) resembles the conventional SF regression, and is therefore called the SMF regression.

The non-negative technology gap component $U_{jit}^M \geq 0$ is assumed to be distributed as truncated-normal,[3] i.e., $U_{jit}^M \sim N^+(\mu^M(Z_{jit}), \sigma_u^{M2}(Z_{jit}))$, and independent of V_{jit}^M. The mode $\mu^M(Z_{jit})$ of the truncated-normal is assumed to be a function of variables Z_{jit} that reflects the production environment of the ith firm encountered in the jth group at tth period, and the heteroscedastic variance $\sigma_u^{M2}(Z_{jit})$ reflects the production uncertainty.

The presence of V_{jit}^M is pivotal in formulating (15) as a stochastic, rather than a deterministic, setting. However, this stochastic setting may result in a potential problem in this second-stage estimation. Since $\ln \hat{f}_t^j(X_{jit})$ is the maximum likelihood estimator of the group-specific frontier in (7), it is reasonable to assume that the estimation error $V_{jit}^M = \varepsilon_{jit} - \hat{\varepsilon}_{jit}$ is to be asymptotically normally distributed with zero mean, but may not be independently, identically distributed (iid), since it contains the residuals from estimating the group frontiers, i.e., $\hat{\varepsilon}_{jit} = \ln Y_{jit} - \ln \hat{f}_t^j(X_{jit})$.[4] Thus, the usual SF likelihood function associated with (15), assuming iid in V_{jit}^M, is referred to as the quasi-likelihood function. Nevertheless, the derived quasi-maximum likelihood (QML) estimator is still consistent and asymptotically normal, but has invalid standard errors that have to be modified to account for the heteroscedasticity. Following White (1982), it requires computing the sandwich-form for the covariance matrix of the estimators in order to obtain the correct standard errors.[5]

The above proposed two-step SF approach allows for the estimated group-specific frontier to be larger than or equal to the metafrontier, i.e., $\hat{f}_t^j(X_{jit}) \geq f_t^M(X_{jit})$, due to the error of estimating $f_t^j(X_{jit})$. However, the metafrontier should be larger than or equal to the group-specific frontier, $f_t^j(X_{jit}) \leq f_t^M(X_{jit})$. The estimated TGR must always be less than or equal to unity,

$$\overset{\wedge}{TGR}{}_{it}^j = \hat{E}\left(e^{-U_{jit}^M} \middle| \hat{\varepsilon}_{jit}^M\right) \leq 1 \tag{16}$$

where $\hat{\varepsilon}_{jit}^M = \ln \hat{f}_t^j(X_{jit}) - \ln \widehat{f}_t^M(X_{jit})$ are the estimated composite residuals of (15). Furthermore, the estimated technology gap is a function of the production environments Z_{ij} via the mode $\mu^M(Z_{jit})$ and the heteroscedastic variance $\sigma_u^{M2}(Z_{jit})$.

In sum, the proposed new two-step approach of estimating the meta frontier consists of two SF regressions, (7) and (15),

$$\ln Y_{jit} = \ln f_t^j(X_{jit}) + V_{jit} - U_{jit}, \quad i = 1, 2, \ldots, N_j; \quad t = 1, 2, \ldots, T \tag{7}$$

$$\ln \hat{f}_t^j(X_{jit}) = \ln f_t^M(X_{jit}) + V_{jit}^M - U_{jit}^M, \quad \forall\, i,\ t, j = 1, 2, \ldots, J \tag{15}$$

where $\ln \hat{f}_t^j(X_{jit})$ is the estimates of the group-specific frontier from the first step in (7). Since the estimates $\ln \hat{f}_t^j(X_{jit})$ are group-specific, the regression (7) is estimated J times, one for each group ($j = 1,2,\ldots,J$). These estimates from all J groups are then pooled to estimate (15). The corresponding estimated meta technical efficiency (MTE) is equal to the product of the estimated TGR, (16), and the estimated individual firm's technical efficiency, (8), i.e.,

[3] The randomness of the technology gap component $U_{jit}^M \geq 0$ is justified on the assumption of the existence of a population distribution of an array of (possibly continuous) group frontiers $f_t^j(X_{jit})$. Given X_{jit}, the metafrontier is defined as the upper boundary of the support of the distribution of $f_t^j(X_{jit})$, i.e., $f_t^M(X_{jit}) = \sup\left(f_t^j(X_{jit}) \,\middle|\, G(f_t^j(X_{jit})) < 1\right)$ where $G(f_t^j(X_{jit}))$ is the distribution function of $f_t^j(X_{jit})$. Thus, in a random sample of J groups, the metafrontier $f_t^M(X_{jit})$ is the Jth order statistic. The non-negative technology gap component $U_{jit}^M = \ln f_t^M(X_{jit}) - \ln f_t^j(X_{jit}) \geq 0$ is random and is assumed to be distributed as a truncated-normal. This definition of the metafrontier differs from the standard metafrontier literature where the group frontiers $f_t^j(X_{jit})$, $j = 1,\ 2,\ldots,\ J$ are nonstochastic with $f_t^M(X_{jit}) = \max\left(f_t^j(X_{jit}) \,\middle|\, X_{jit},\quad j = 1, 2, \ldots, J\right)$.
We gratefully appreciate a referee for the constructive critiques on the stochastic nature of the randomness of the technology gap that has clarified and greatly improved the argument and exposition from the early version.

[4] We thank an anonymous referee for making this observation.

[5] Let $\ln(\theta)$be the log-likelihood function of the parameter θ. The standard ML estimator has the inverse of the Fisher information matrix $I(\theta) = -E\left(\frac{\partial^2 \ln(\theta)}{\partial\theta\,\partial\theta^T}\right)$as the covariance matrix of the estimator $\hat{\theta}$. However, the QML's covariance matrix has the so-called sandwich form: $I^{-1}(\theta)[S(\theta)S^T(\theta)]I^{-1}(\theta)$where $S(\theta) = E\left(\frac{\partial \ln(\theta)}{\partial\theta}\right)$ is the score function. Johnston and DiNardo (1992), pages 428-430, has a brief discussion of the quasi-maximum likelihood estimation of misspecified models and the derivation of the covariance matrix.

$$\hat{MTE}_{it}^{j} = \hat{TGR}_{it}^{j} \times \hat{TE}_{it}^{j} \tag{17}$$

In conclusion, the key difference between our proposed new two-step SF approach and the two-step mixed approach is in the formulation of the SMF regression in (15) as opposed to the mathematical programming optimizations in (9) and (10). In the next section, we provide two empirical examples to illustrate and to compare the estimates of various technical efficiencies and the TGR obtained from the SMF method and the deterministic programming method.

A few remarks are worth mentioning on the comparison of the SMF model proposed in this paper with the SMF model of Battese and Rao (2002). In addition to the specification of the group-specific frontier in (7), the SMF model of Battese and Rao (2002) pools the data of all groups in formulating the metafrontier and the MTE, i.e.:

$$\ln Y_{jit} = \ln f_t^M(X_{jit}) + V_{jit}^* - U_{jit}^* \quad \forall j,\, i,\, t \tag{18}$$

where V_{jit}^* is the individual DMU's random noise error and the non-negative $U_{jit}^* \geq 0$ represents the DMU's meta technical inefficiency. These errors are distinguished from the errors in the group-specific frontiers. The model of Battese and Rao (2002) has often been criticized as being inconsistent in the data generating process (DGP), i.e., ln Y_{jit} are generated from both the distributions of $(V_{jit} - U_{jit})$ in (7) and $(V_{jit}^* - U_{jit}^*)$ in (18).[6] Furthermore, using (18), the MTE of the model of Battese and Rao (2002) is defined as $MTE_{jit} = e^{-U_{jit}^*} = \frac{Y_{jit}}{f_t^M(X_{jit})e^{V_{jit}^*}}$. Using (7), the decomposition MTE_{jit} becomes:

$$MTE_{jit} = \frac{f_t^j(X_{jit})}{f_t^M(X_{jit})} \times e^{-U_{jit}} \times \frac{e^{V_{jit}}}{e^{V_{jit}^*}} = TGR_{it}^j \times TE_{it}^j \times \frac{e^{V_{jit}}}{e^{V_{jit}^*}} \tag{19}$$

Thus, the MTE decomposition of the model in Battese and Rao (2002) is not exact, which is in contrast to the exact decomposition shown in (6) derived from the SMF model proposed in this paper.[7] We conclude, therefore, that to estimate a metafrontier by simply pooling all the data of the various groups is not justifiable due to the lack of a coherent data generating process and a lack of unique decomposition of the MTE so that the so-derived metafrontier would not necessarily envelop the group-specific frontiers.

3 Empirical examples

We conduct two empirical studies to exemplify the advantages of the new two-step SF approach over the two-step mixed approach of Battese et al. (2004) and O'Donnell et al. (2008). In the first example, we employ the country-level agricultural data, provided by the Food and Agriculture Organization (FAO) of the United Nations to make inter-country and inter-regional comparisons of agricultural efficiency. The data have been used by O'Donnell et al. (2008). In the second example, data from the hotel industry in Taiwan are employed to compare the operational efficiency between the chain- and independently-operated hotels. In both examples, we estimate and compare the TGR and the MTE with respect to the metafrontier technology derived from the two approaches.

3.1 Agricultural metafrontier production function

In the first example, using the FAO data, we re-estimate the model for a sample of 97 countries over the period of 1986–1990, and compare the results with those yielded by O'Donnell et al. (2008). These countries are divided into four groups (regions): African countries, American countries, Asian countries, and European countries. There is a single output (y_{jit}) defined as an aggregate of 185 agricultural commodities and five inputs, i.e., land ($x_{1\mathrm{jit}}$), machinery ($x_{2\mathrm{jit}}$), labor ($x_{3\mathrm{jit}}$), fertilizer ($x_{4\mathrm{jit}}$), and livestock ($x_{5\mathrm{jit}}$).[8] The SFA estimates of the group-specific frontier in (7), ln $f_t^j(X_{jit})$, and the metafrontier in (15), ln $f_t^M(X_{jit})$, are obtained by assuming a translog functional form with the above specified five inputs. To compare these results with those in O'Donnell et al. (2008), we follow the standard SFA normal-half normal specifications on the random variables: $V_{jit} \sim N(0, \sigma_v^{j2})$ and $U_{jit} \sim N^+(0, \sigma_u^{j2})$ for the

[6] For example, the conditional expectations of (7) is $E(\ln Y_{jit}|X_{jit}) = \ln f_t^j(X_{jit}) - E(U_{jit})$, while from (18) is $E(\ln Y_{jit}|X_{jit}) = \ln f_t^M(X_{jit}) - E(U_{jit}^*)$. These dual conditional expectations are inconsistent unless $\ln f_t^j(X_{jit}) = \ln f_t^M(X_{jit}) - E(U_{jit}) - E(U_{jit}^*)$, which is unlikely from the DGP of Battese and Rao (2002) specification. On the other hand, the current proposed SMF model has a unique DGP derived either from (7) or from $\ln Y_{jit} = \ln f_t^M(X_{jit}) + V_{jit} - U_{jit} - U_{jit}^M$ after substituting (11) into (7). This latter expression has the conditional expectation $E(\ln Y_{jit}|X_{jit}) = \ln f_t^M(X_{jit}) - E(U_{jit}) - E(U_{jit}^M)$, which is identical to the one obtained from (7) because of (11), $\ln f_t^j(X_{jit}) = \ln f_t^M(X_{jit}) - E(U_{jit}^M)$.

[7] To see the decomposition from the proposed two-step regressions in (7) and (15), we observe that, from (6), $MTE_{jit} \equiv \frac{Y_{jit}e^{-V_{jit}}}{f_t^M} = \frac{f_t^j}{f_t^M} \times e^{-U_{jit}}$. Since, from (14), $\hat{f}_t^j = f_t^j \times e^{V_{jit}^M}$, we have $MTE_{jit} = \frac{\hat{f}_t^j}{f_t^M e^{V_{jit}^M}} \times e^{-U_{jit}}$. Thus, from (15), we have the exact decomposition: $MTE_{jit} = e^{-U_{jit}^M} \times e^{-U_{jit}} = TGR_{jit} \times TE_{jit}$.

[8] We are grateful to Professor C.J. O'Donnell for providing the raw data. For a more detailed description of the input/output variables in our empirical application, please see O'Donnell et al. (2008).

Table 2 Summary statistics for various agricultural efficiency measures

	Two-step mixed approach				Two-step stochastic frontier approach			
	Mean	SD	Min	Max	Mean	SD	Min	Max
Group-specific technical efficiency (TE)								
Argentina	0.959	0	0.959	0.959	0.959	0	0.959	0.959
Australia	0.950	0	0.950	0.950	0.950	0	0.950	0.950
Brazil	0.895	0	0.895	0.895	0.895	0	0.895	0.895
China	0.936	0	0.936	0.936	0.936	0	0.936	0.936
India	0.944	0	0.944	0.944	0.944	0	0.944	0.944
Indonesia	0.563	0	0.563	0.563	0.563	0	0.563	0.563
Netherlands	0.972	0	0.972	0.972	0.972	0	0.972	0.972
South Africa	0.935	0	0.935	0.935	0.935	0	0.935	0.935
UK	0.968	0	0.968	0.968	0.968	0	0.968	0.968
USA	0.917	0	0.917	0.917	0.917	0	0.917	0.917
(1) Africa	0.505	0.249	0.190	0.972	0.505	0.249	0.190	0.972
(2) The Americas	0.824	0.137	0.519	0.981	0.824	0.137	0.519	0.981
(3) Asia	0.719	0.195	0.362	0.981	0.719	0.195	0.362	0.981
(4) Europe	0.823	0.151	0.514	0.982	0.823	0.151	0.514	0.982
All countries	0.707	0.233	0.190	0.982	0.707	0.233	0.190	0.982
Technology gap ratio (TGR)								
Argentina	0.982	0.011	0.969	1.000	0.889	0.002	0.886	0.893
Australia	0.969	0.034	0.924	1.000	0.873	0.010	0.860	0.882
Brazil	0.799	0.020	0.772	0.823	0.828	0.010	0.813	0.839
China	0.997	0.004	0.991	1.000	0.827	0.010	0.812	0.838
India	0.740	0.041	0.696	0.788	0.814	0.007	0.806	0.822
Indonesia	0.830	0.039	0.777	0.869	0.896	0.003	0.893	0.899
Netherlands	0.959	0.030	0.918	1.000	0.927	0.005	0.921	0.934
South Africa	0.607	0.006	0.601	0.615	0.694	0.005	0.689	0.702
UK	0.559	0.007	0.552	0.568	0.713	0.008	0.705	0.722
USA	0.987	0.007	0.982	1.000	0.905	0.001	0.904	0.907
(1) Africa	0.752	0.206	0.308	1.000	0.824	0.099	0.565	0.934
(2) The Americas	0.751	0.161	0.435	1.000	0.797	0.086	0.588	0.919
(3) Asia	0.738	0.197	0.328	1.000	0.829	0.060	0.709	0.949
(4) Europe	0.664	0.210	0.250	1.000	0.802	0.100	0.537	0.934
All countries	0.727	0.198	0.250	1.000	0.814	0.088	0.537	0.949
Metafrontier technical efficiency (MTE)								
Argentina	0.942	0.011	0.929	0.959	0.853	0.002	0.850	0.857
Australia	0.921	0.032	0.878	0.950	0.831	0.010	0.818	0.839
Brazil	0.715	0.018	0.691	0.736	0.740	0.009	0.727	0.750
China	0.933	0.004	0.928	0.936	0.776	0.009	0.763	0.787
India	0.698	0.038	0.657	0.744	0.765	0.007	0.758	0.773
Indonesia	0.468	0.022	0.438	0.490	0.514	0.002	0.512	0.515
Netherlands	0.932	0.029	0.892	0.972	0.901	0.005	0.895	0.908
South Africa	0.568	0.005	0.562	0.575	0.649	0.005	0.644	0.656
UK	0.541	0.007	0.535	0.550	0.690	0.008	0.683	0.699
USA	0.906	0.006	0.901	0.917	0.830	0.001	0.829	0.832
(1) Africa	0.362	0.185	0.122	0.972	0.407	0.190	0.140	0.870
(2) The Americas	0.615	0.157	0.381	0.959	0.654	0.122	0.429	0.857
(3) Asia	0.537	0.212	0.119	0.950	0.593	0.168	0.292	0.872
(4) Europe	0.541	0.194	0.213	0.975	0.656	0.136	0.457	0.908
All countries	0.506	0.211	0.119	0.975	0.570	0.189	0.140	0.908

Since the two-step mixed approach and the two-step stochastic frontier approaches specify an identical group-specific frontier, the top panel shows identical group-specific technical efficiency (TE) estimates group-specific frontier in (7), and $V^M_{jit} \sim N(0, \sigma^{M2}_v)$ and $U^M_{jit} \sim N^+(0, \sigma^{M2}_u)$ for the metafrontier in (15).

Since the group-specific frontier is specified exactly the same as in O'Donnell et al. (2008), the parameter estimates are omitted here. The second-step parameter estimates of the metafrontier are presented in Table 1.[9] For comparison with the estimates obtained from the SMF regression in (15), we also provide the bootstrapped standard errors for the LP estimates obtained from (9). The results do not show a significant difference in either the magnitude or the sign of the metafrontier estimates between the two methods. However, with the SMF estimate of the variance ratio, $\bar{\gamma}^M = \frac{\sigma^{M2}_v}{\sigma^{M2}_v + \sigma^{M2}_u} = 0.2794$, the empirical evidence confirms the bias in the LP estimates caused by replacing the group-specific frontiers $\ln f^j_t(X_{jit})$with the estimate $\ln \hat{f}^j_t(X_{jit})$ in (12) without taking into account the error of estimation. If the group-specific frontier estimates were perfect, the sampling error in (14) would be zero, i.e., $V^M_{jit} = 0$, or, equivalently, the SMF specification in (15) would have a zero variance ratio,[10] i.e., $\bar{\gamma}^M = 0$. The statistically significant estimate of $\bar{\gamma}^M$shown in Table 1 confirms the bias in the LP estimates of the metafrontier.

Table 2 reports the sample estimates of various efficiency scores, including TGR, TE, and MTE for selected countries and all groups. For the purpose of comparison, estimates obtained from the mixed approach of O'Donnell, et al. (2008) are listed side-by-side with the estimates obtained from the SF approach. Since both approaches adopt the SF regression in the first step to estimate the group-specific technical efficiency, the top panel in Table 2 shows the identical technical efficiency estimates from both approaches. However, it is the estimates of TGR and MTE obtained from the second step that deserve a close comparison.

Based on the SMF regression estimates, the average TGRs range narrowly between 0.797 for the American group and 0.829 for the Asian group, with a higher overall mean value of 0.814 than the LP metafrontier programming estimate of 0.727. This outcome reveals that the agricultural production technology adopted in the Asian group countries is in general slightly closer to the best available production technology than is the American group counterpart. Although variant, the technologies taken by the sample countries in different regions seem close to one another.

The mean overall technical efficiency scores against the metafrontier (MTE) vary from 0.407 for Africa to 0.656 for Europe. The ranking seems to be dominated by the component of the TE score due to its larger variation among regions. For instance, the average TGR for the African countries is as high as 0.824 (the second highest), but its average TE score is merely 0.505 (the lowest), a result that pulls the mean MTE of Africa down to the bottom (0.407).

The results from the two-step mixed approach of O'Donnell et al. (2008) are reproduced on the left half of Table 2. It is clear that the mean values of TGR in the four groups are smaller than those achieved by the SMF method, together with larger variations (standard deviations), leading to lower average values of MTE. These results may be attributed to the fact that the TGRs obtained from the programming technique are likely to be contaminated by random shocks because the technique is deterministic and unable to isolate the influence of the shocks. Conversely, the SMF method allows for the separation of random shocks from the inefficiency, a well-known advantage of the SFA over the programming technique. In view of the empirical evidence, the SMF method that uses the SF regression framework in the second step seems to be preferable.

3.2 The hotel industry metafrontier production function

In the second example, we collect unbalanced panel data of the hotel industry in Taiwan over the period 1998–2008. The data are taken from the annual report of Taiwan's Tourism Bureau, the Ministry of Transportation and Communications, Taiwan. A hotel with four or five stars is classified as an international tourist hotel. This type of hotel is further categorized into either a chain- or an independently-operated hotel, based on the patterns of its operational structure. A hotel is said to be chain-operated if it participates in a domestic parent consortium by passing a strict qualification assessment and by joining a cooperative management contract that clearly specifies their respective rights and responsibilities. This type of hotel usually belongs to a company that owns more than one subsidiary hotel, located in different regions around Taiwan. A hotel is said to be independently operated otherwise. These are not related to any domestic chain organization and are responsible for their own management decisions. The

[9] Note that the QML sandwich estimated standard errors are calculated and presented in Tables 1 (for FAO data) and 5 (for hotel data). The unadjusted estimated standard errors without taking into account heteroscedasticity are not shown to save space. For FAO data, most of the unadjusted standard errors are slightly greater than those of QML sandwich estimates and the number of significant parameter estimates, at least at the 10 % level, are the same. However, for hotel data, the reverse is true, i.e., most of the unadjusted standard errors are somewhat less than those of the QML sandwich estimates. Consequently, there are fewer coefficients reaching statistical significance in Table 5.

[10] In standard stochastic frontier analysis, it is the variance ratio $\gamma^M = \sigma^{M2}_u / \left(\sigma^{M2}_v + \sigma^{M2}_u\right)$ that tests the hypothesis of $U^M_{jit} = 0$, i.e., the average verse frontier model. On the other hand, the complement of the variance ratio, $\bar{\gamma}^M = 1 - \gamma^M = \sigma^{M2}_v / \left(\sigma^{M2}_v + \sigma^{M2}_u\right)$, allows us to test the hypothesis of $V^M_{jit} = 0$, i.e., which is the deterministic verse the stochastic frontier model.

Table 3 Summary statistics of the hotel data

Variables	Chain-operated (number of observations = 237)				Independently operated (number of observations = 385)			
	Mean	SD	Min	Max	Mean	SD	Min	Max
Output variable								
Total revenue (y)	65.96	56.57	4.50	271.77	55.05	59.32	1.60	316.54
Input variable								
Number of guest rooms (x_1)	300.98	154.35	50	606	312.72	159.28	96	873
Floor space of the catering division (x_2)	1683.05	2103.94	80	17477	1354.89	2964.50	48	5296
Number of full-time employees (x_3)	360.79	232.42	53	1196	313.62	231.50	25	1230
Other operating expenses (x_4)	25.87	19.00	2.35	101.24	23.88	24.61	0.65	135.73
First-step environment variable								
F-age	18.18	12.67	1.00	56.00	20.13	11.88	1.00	53.00
F-DI	0.11	0.31	0.00	1.00	0.20	0.40	0.00	1.00
F-DC (%)	51.52	31.33	2.62	99.44	40.63	29.44	3.70	100.00
Second-step environment variable								
I-HHI	237.80	11.43	223.05	259.72	237.80	11.43	223.05	259.72
I-Center	0.10	0.30	0.00	1.00	0.13	0.33	0.00	1.00
I-South	0.30	0.45	0.00	1.00	0.18	0.39	0.00	1.00
I-East	0.14	0.34	0.00	1.00	0.12	0.32	0.00	1.00

Total number of observations of the hotel data is 622

All the dollar-valued variables are measured in millions of New Taiwan dollars (NT$) and are deflated by the GDP deflator of Taiwan with the base year 2006

chain-operated hotels may enjoy some advantages, such as a branding effect, better access to knowledge and new innovations, resources and information sharing, and economies of scale and scope. However, the independently operated hotels may also have some advantages, such as high managerial autonomy and operational flexibility.

The data set comprises observations on one output variable and four inputs. The output variable (y_{jit}) is defined as the overall operational revenue of a hotel, including guest room revenues, food and beverage revenues, and other operating revenues. The four inputs include the total number of full-time employees (x_{1jit}), the total number of guest rooms (x_{2jit}), the total floor area of the catering division (x_{3jit}), and other operating expenses (x_{4jit}), including utilities, materials, maintenance fees, and other operating costs. The items of revenues and expenses are all measured in millions of New Taiwan dollars (NT$) and are deflated by the GDP deflator of Taiwan with the base year 2006.

Several environmental variables (Z_{jit}) are considered in the formulation of group-specific and industry-specific frontiers. The firm-specific environmental variables in the first-step estimation of the group-specific frontier in (7) are identified as the possible impact on firm-specific technical efficiency, while the industry-specific environmental variables in the second-step estimation of the metafrontier in (15) are identified as the possible impact on the group-specific technology gap ratio. The firm-specific environmental variables are:

1. *F-age and F-age*^{2}: the age of a hotel and square of the hotel's age.
2. *F-DI*: the degree of internationalization of a hotel. It is a dummy variable with a value of unity if the hotel cooperates with an international hotel organization, and zero otherwise. An international hotel may enjoy the advantage of the international management systems to improve its managerial efficiency by joining, e.g., franchise-chains, outsourcing, and membership in an international hotel association (Hwang and Chang, 2003). We thus expect that variable *F-DI* is negatively associated with inefficiency.
3. *F-DC*: the proportion of domestic customers to total customers. Past empirical evidence has shown that foreign customers have higher brand loyalty and require higher quality of services. It is anticipated that there is a positive association between *F-DC* and inefficiency.

Four other industry-specific environmental variables are also identified to be included in the second-step estimation of the metafrontier:

Table 4 The hotel group-specific stochastic frontier estimates

Variables	Chain-operated		Independently operated	
	Parameter estimates	Standard errors	Parameter estimates	Standard errors
Constant	−24.2576***	3.1214	29.4490***	2.5897
$\ln x_1$	−9.0851***	1.0219	−0.8630	0.9872
$\ln x_2$	0.5986	0.8607	−0.5883	0.4579
$\ln x_3$	6.5017***	0.7172	−2.3445***	0.4910
$\ln x_4$	−1.7428	1.2077	3.8944***	1.0219
$\ln x_1 \times \ln x_1$	−0.9807***	0.1982	0.4584***	0.1206
$\ln x_2 \times \ln x_2$	−0.0258	0.0251	0.0527	0.0355
$\ln x_3 \times \ln x_3$	−0.5255***	0.0888	0.1518**	0.0610
$\ln x_4 \times \ln x_4$	−0.3396	0.2099	0.3618**	0.1611
$\ln x_1 \times \ln x_2$	0.2029**	0.1003	0.1621*	0.0905
$\ln x_1 \times \ln x_3$	1.4813***	0.1743	−0.0151	0.1595
$\ln x_1 \times \ln x_4$	−0.1619	0.2578	−0.8618***	0.2531
$\ln x_2 \times \ln x_3$	−0.1385	0.1366	0.0788	0.0816
$\ln x_2 \times \ln x_4$	0.1519	0.1512	−0.3904***	0.1230
$\ln x_3 \times \ln x_4$	0.3594*	0.2176	−0.1091	0.1932
t	0.3784**	0.1699	0.1249	0.1164
t^2	−0.0012	0.0023	−0.0002	0.0027
$t \times \ln x_1$	−0.0033	0.0146	−0.0020	0.0127
$t \times \ln x_2$	−0.0013	0.0049	−0.0096	0.0064
$t \times \ln x_3$	−0.0231*	0.0133	−0.0086	0.0104
$t \times \ln x_4$	0.0193	0.0174	0.0268*	0.0159
Group-specific environmental variables				
Constant	−3.8011***	0.4100	−0.9155***	0.2611
F-age	0.0357**	0.0168	0.0416***	0.0102
F-age^2	−0.0013*	0.0006	−0.0011***	0.0003
F-DI	−0.1929**	0.0978	−0.9314***	0.1056
F-DC	2.7232***	0.2237	0.8039***	0.1864
$\sigma^{j2} = \sigma_u^{j2} + \sigma_v^{j2}$	0.1751***	0.0290	0.0566***	0.0077
$\gamma^j = \sigma_u^{j2}/(\sigma_u^{j2} + \sigma_v^{j2})$	0.9292***	0.0199	0.6714***	0.0241
Log-Likelihood	132.4695		122.7812	

***, **, and * denote significant at the 1, 5, and 10 % levels, respectively

1. *I-HHI*: the Herfindahl–Hirschman index. This is a commonly used measure of market concentration in an industry, indicating the degree of competitive pressure. It is calculated by taking a square of the market share of total annual tourists per hotel competing in the market, and then summing over all of the hotels under consideration.
2. *I-Center*: a regional dummy variable, with a value of unity if the hotel is located in central Taiwan, and zero otherwise.
3. *I-South*: a regional dummy variable, with a value of unity if the hotel is located in southern Taiwan, and zero otherwise.
4. *I-East*: a regional dummy variable, with a value of unity if the hotel is located in eastern Taiwan, and zero otherwise. Hotels operating in northern Taiwan are arbitrarily chosen as the normalization.

Table 3 summarizes the sample statistics of the hotel data, including the output, inputs, and environmental variables for each group. The table shows that in general the chain-operated hotels are bigger than the independently operated hotels as measured in revenue, floor space, employees, and operating expenses, except in the number of guest rooms. Independently operated hotels are older with higher degree of internationalization and less dependent on domestic travelers. Finally, independently operated hotels tend to be clustered in northern and central Taiwan, while chain-operated hotels are more likely to be located in the east and south.

Table 5 The estimates of the hotel industry's metafrontier

Variables	SMF approach		LP approach		QP approach	
	Parameter estimates	Standard errors	Parameter estimates	Bootstrapped standard errors	Parameter estimates	Bootstrapped standard errors
Constant	21.5271***	0.8105	44.9219***	7.6791	34.5604***	6.5627
$\ln x_1$	−0.2575	0.7409	−2.6209***	0.8546	−2.1401**	0.8597
$\ln x_2$	−0.3298	0.3228	−0.1366	0.4432	0.0631	0.3723
$\ln x_3$	−1.3979***	0.2425	−4.5161***	1.1713	−3.1862***	1.0280
$\ln x_4$	2.3157***	0.3950	6.3931***	1.1867	4.8265***	1.0900
$\ln x_1 \times \ln x_1$	0.1913***	0.0554	0.1505	0.1141	0.1830**	0.0872
$\ln x_2 \times \ln x_2$	0.0185	0.0174	0.0085	0.0174	0.0176	0.0164
$\ln x_3 \times \ln x_3$	0.1390***	0.0333	0.3500***	0.0948	0.2696***	0.0832
$\ln x_4 \times \ln x_4$	0.2690***	0.0696	0.6753***	0.1103	0.5244***	0.1090
$\ln x_1 \times \ln x_2$	0.2781***	0.0401	0.2681***	0.0678	0.2566***	0.0562
$\ln x_1 \times \ln x_3$	−0.1099	0.1177	0.3143***	0.1210	0.2219*	0.1284
$\ln x_1 \times \ln x_4$	−0.3867***	0.1377	−0.8539***	0.1918	−0.7523***	0.1613
$\ln x_2 \times \ln x_3$	0.0088	0.0604	−0.0462	0.0654	−0.0811	0.0603
$\ln x_2 \times \ln x_4$	−0.2486***	0.0701	−0.0950	0.0845	−0.0670	0.0683
$\ln x_3 \times \ln x_4$	−0.1183	0.0879	−0.7381**	0.1958	−0.5260***	0.1784
t	0.1729***	0.0615	0.0809	0.0679	0.0855	0.0679
t^2	−0.0015	0.0011	−0.0014*	0.0008	−0.0015*	0.0009
$t \times \ln x_1$	−0.0140***	0.0054	−0.0211***	0.0056	−0.0191***	0.0048
$t \times \ln x_2$	−0.0045	0.0031	−0.0016	0.0029	−0.0028	0.0030
$t \times \ln x_3$	−0.0114**	0.0051	−0.0009	0.0055	−0.0014	0.0056
$t \times \ln x_4$	0.0327***	0.0073	0.0156**	0.0064	0.0166**	0.0065
Second-step environment variables						
Constant	−1.7309***	0.4717				
I-HHI	−4.1817	3.4712				
I-Center	0.9904***	0.3754				
I-South	0.1485	0.1957				
I-East	0.0149	0.5224				
$\sigma^{M2} = \sigma_v^{M2} + \sigma_u^{M2}$	0.0539***	0.0057				
$\bar{\gamma}^M = \sigma_v^{M2}/(\sigma_v^{M2}+\sigma_u^{M2})$	0.0705***	0.0189				
Log-likelihood	801.0357					

The QML sandwich estimated standard errors are calculated for the SMF model. The standard errors of LP and QP models are obtained using the bootstrapping method

***, **, and * denote significant at the 1, 5, and 10 % levels, respectively

The group-specific production frontier (7) is also specified as a standard translog form, incorporating the time trend as an independent variable to capture the technological change. The technical inefficient term U_{jit} is assumed to be a function of a set of environmental variables as proposed by Huang and Liu (1994), and Battese and Coelli (1995). More specifically, it is assumed that $U_{jit} \sim N^+(\mu^j(Z_{jit}), \sigma_u^{j2})$ where $\mu^j(Z_{jit})$ is a linear function of the firm-specific variables (Z_{jit}) defined above. Table 4 reports the estimates of the hotel group-specific SFs for both chain-operated and independently operated hotels. In general, the overall group-specific translog production frontiers fit both types reasonably well. Most coefficient estimates are statistically significant. The estimates of the environmental variables at the bottom of the panel present an interesting interpretation. In particular, there exists a U-shaped relationship between a hotel's age and technical efficiency for each group, as the coefficient estimates of the square of a hotel's age are negative.[11] Evidence appears to support the presence of learning-by-doing that prompts

[11] An environmental variable that has a positive (negative) coefficient implies that the variable exerts a negative (positive) impact on technical efficiency.

Table 6 Summary statistics of various hotel industry efficiency measures

	SMF estimates				QP estimates			
	Mean	SD	Min	Max	Mean	SD	Min	Max
Chain-operated hotels								
TGR	0.9730	0.0248	0.7970	0.9904	0.9259	0.0616	0.5719	1.0000
TE	0.9218	0.0562	0.6525	0.9790	0.9218	0.0562	0.6525	0.9790
MTE	0.8970	0.0593	0.6310	0.9623	0.8535	0.0758	0.3965	0.9685
Independently operated hotels								
TGR	0.9698	0.0085	0.9277	0.9882	0.8630	0.0657	0.6137	1.0000
TE	0.8699	0.1046	0.4584	0.9811	0.8699	0.1045	0.4584	0.9811
MTE	0.8436	0.1013	0.4483	0.9599	0.7506	0.1059	0.3700	0.9659
Overall								
TGR	0.9710	0.0168	0.7970	0.9904	0.8870	0.0710	0.5719	1.0000
TE	0.8897	0.0927	0.4584	0.9811	0.8897	0.0927	0.4584	0.9811
MTE	0.8639	0.0914	0.4483	0.9623	0.7898	0.1078	0.3700	0.9685

hotels to be more productively efficient due to the accumulation of knowledge and working experience. Moreover, the degree of internationalization (*F-DI*) stimulates technical efficiency for each group, showing that cooperation with an international organization is conducive for the sample hotels to improve their managerial capacities and quality of service. One is led to conclude that a hotel with higher proportion of domestic customers tends to have a lower technical efficiency, since the coefficient estimate of *F-DC* is significantly positive. This result is consistent with Hwang and Chang (2003), who find that the managerial efficiency of a hotel that caters mostly to domestic customers is lower than for hotels catering mainly to foreign tourists.

It is crucial to verify whether chain-operated hotels and independently operated hotels share the same technology. If the hotel data are truly generated from a single production frontier, implying that they adopt the same underlying technology, then it is not necessary to estimate the metafrontier production function. We apply the likelihood ratio test for the null hypothesis that the production frontiers are the same for the two groups of hotels in Taiwan. The value of the restricted log-likelihood function under the null hypothesis is 176.9694, while the unrestricted value is 255.2507, which is the sum of the two log-likelihood function values shown in Table 4. This leads to the likelihood ratio statistic of 156.5626, which is the twice of the difference between the unrestricted and the restricted log-likelihood function values. Thus the null hypothesis is decisively rejected. Evidence is found to support the fact that the sample hotels are operating under heterogeneous technologies. The existence of potential production technology gap further justices the estimation of the metafrontier production function in hotel operation.

Table 5 summarizes the estimates of the SMF Eq. (15), together with the estimates obtained from the linear and quadratic programming (LP and QP). The SMF inefficient term U_{jit}^{M} is assumed to be a function of a set of environmental variables, i.e., $U_{jit}^{M} \sim N^{+}(\mu^{M}(Z_{jit}), \sigma_{u}^{M2})$ where $\mu^{M}(Z_{jit})$ is a linear function of the industry-specific variables Z_{jit} : the Herfindahl–Hirschman index and the regional dummies defined above. The two sets of parameter estimates from the LP and QP models are quite close to each other with the same signs, while they deviate significantly from those of the SMF estimates. Furthermore, the SMF estimates show some significant industry-wide environmental impact on the metafrontier production function. The negative coefficient estimate of *I-HHI* implies that the higher concentration of the hotel industry, the closer its production frontier is to the metafrontier. However, this coefficient estimate is not significant. Although the coefficient estimates of the three regional dummies are all positive, only that of *I-Center* is significant, implying that hotels located in the central region are operating under inferior technology. Hotels in northern Taiwan appear to adopt the best technology in providing services to domestic and foreign tourists. We further note that the SMF estimate of the variance ratio,$\bar{\gamma}^{M} = \frac{\sigma_{v}^{M2}}{\sigma_{v}^{M2}+\sigma_{u}^{M2}} = 0.0705$, is statistically significant. Thus, as in the previous example on the agricultural metafrontier production function, the empirical evidence confirms the bias in the LP and QP estimates of the metafrontier.

Table 6 reports the sample statistics of various efficiency scores for the two groups of hotels.[12] The average

[12] Because the results from the LP model are almost identical to those from the QP model, we report only the results from the QP model to save space.

Table 7 Summary statistics of various hotel industry efficiency measures without industry-specific environmental variables

	Half-Normal with time-invariant				Half-Normal with time-varying			
	Mean	SD	Min	Max	Mean	SD	Min	Max
Chain-operated								
TGR	0.9448	0.0469	0.7917	0.9934	0.9423	0.0459	0.7758	0.9928
TE	0.9218	0.0562	0.6525	0.9790	0.9218	0.0562	0.6525	0.9790
MTE	0.8709	0.0675	0.5799	0.9686	0.8685	0.0656	0.6054	0.9673
Independently operated								
TGR	0.8862	0.0478	0.7285	0.9921	0.8813	0.0442	0.7282	0.9920
TE	0.8699	0.1046	0.4584	0.9811	0.8699	0.1046	0.4584	0.9811
MTE	0.7709	0.1017	0.3989	0.9623	0.7667	0.1002	0.3976	0.9634
Overall								
TGR	0.9085	0.0553	0.7285	0.9934	0.9046	0.0537	0.7282	0.9928
TE	0.8897	0.0927	0.4584	0.9811	0.8897	0.0927	0.4584	0.9811
MTE	0.8090	0.1024	0.3989	0.9686	0.8055	0.1014	0.3976	0.9673

group-specific TE scores show a TE = 0.9218 for the chain-operated hotels and a TE = 0.8699 for the independently operated hotels. The chain-operated hotels seem to be more technically efficient with respect to their own peer group than the independently operated hotels. Furthermore, the SMF estimates show that the chain-operated hotels seem to be slightly more efficient in adopting the best available hotel-operating technology as measured in the technology gap ratio (TGR). The chain-operated has TGR of 0.9730 versus TGR of 0.9698 for the independently operated hotels.[13] In all, the chain-operated hotels are more technically efficient in operation with respect to the hotel industry in Taiwan as measured by the metafrontier technical efficiency (MTE). While the chain-operated hotels are found to outperform independently operated hotels in terms of both TGR and TE measures, the TE scores play a more important role in the determination of the ranking in MTE, suggesting that the primary source of inefficiency comes from managerial inefficiency, rather than the technology undertaken.

These results obtained from the SMF method are in contrast to the estimates based on the quadratic programming method of estimating the metafrontier. While the QP estimates show, as in the SMF, consistent evidence of better operational performance of the chain-operated hotels in terms of the group-specific TE, TGR, and MTE, they show a much wider variation than the SMF estimates as is evident in the standard deviation of TGR and MTE. As we recall, the LP and QP models are essentially deterministic and are likely to be confounded by random shocks. Table 6 confirms that the TGR and MTE measures obtained under QP are inclined to be less than those of SMP.[14]

The SMF results shown in Table 5 and the technology gap ratio given in Table 6 are obtained with the assumption that industry competitiveness pressure (*I-HHI*) and location (regional dummies) have a significant impact on the technology gap term U_{jit}^{M} of the metafrontier regression in (15). To highlight the importance of incorporating these industry-specific environment variables in the estimation of the technology gap ratio (TGR), we re-estimate the SMF without the inclusion of these variables and present the results in Table 7. Table 7 reports the results from two specifications: a time-invariant and a time-varying technology gap term U_{jit}^{M}. As in the work of Battese and Coelli (1992), the time-invariant U_{jit}^{M} has the half-normal distribution as $U_{jit}^{M} \sim N^{+}(0, \sigma_u^{M2})$, while the time-varying model has the specification that $U_{jit}^{M} = -\eta^{M}(t - T)U_{ji}^{M}$, i.e., a linear time-trend with a half-normal distribution, $U_{ji}^{M} \sim N^{+}(0, \sigma_u^{M2})$, on the time-invariant part.[15] Regardless of which model on U_{jit}^{M} is specified, the statistically significant estimates of the variance ratio $\bar{\gamma}^{M}$ consistently indicate the preference of the SMF method over the mathematical programming method in estimating the metafrontier production function.[16] Table 7 shows descriptive statistics of

[13] We perform a *t* test for the hypothesis that the average value of the TGR of the chain-operated hotels is the same as that of the independently operated hotels. The *p* value of the test statistic is equal to 0.0564, which is significant at the 10 % level. Empirically this might indicate that the average TGR of the former type of hotels is greater than that of the latter type of hotels.

[14] By construction, the LP and QP estimates always result in a few TGR scores to be 1 as shown in the Table 6. However, as argued, the LP and QP estimates are likely on average to be smaller than the MTE estimates.

[15] We also estimate models with the TGR set as a truncated-normal random variable with and without the time-varying structure of Battese and Coelli (1992). Since the results are similar to those of Table 7, we do not show them here.

[16] We do not show the estimation results from the models without considering industry-wide environment variables to save space. The results are available upon request from the authors.

the TGRs obtained from the model without considering the industry-wide environment variables. The average TGRs lie between 0.88 (for the independently operated hotels) and 0.94 (for the chain-operated hotels). These results are small than those reported in Table 6, in which industry-wide environmental differences are included. The exclusion of the industry-wide environment variables from the metafrontier function leads on average to something of an underestimation of the TGR and MTR. Environmental heterogeneity indeed plays some role for a hotel in the determination of its technology. One is urged to consider those variables in order to correctly evaluate the TGR and obtain comparable technical efficiencies for firms running under different technologies.

4 Concluding remarks

This paper proposes a new approach in estimating metafrontier production function. It is based on a two-step SFA procedure that estimates the group-specific frontiers and the firm's technical efficiency in the first step followed by the metafrontier and group-specific technology gap ratio in the second step, both under the framework of the SFA. The novelty of the new approach is in the treatment of metafrontier estimation under the framework of standard stochastic frontier analysis, which is contrary to the popular mathematical programming approach of Battese et al. (2004) and O'Donnell et al. (2008). The proposed SMF method in the second step has several advantages over the deterministic metafrontier programming method: It allows for making relevant statistical inferences on the metafrontier estimates; it purges the so-derived TGR measures from the influence of random shocks and errors of group-specific frontier estimation; and it allows for identifying the sources of variation in group-specific TGR with environmental variables beyond the control of firms.

We provide two empirical examples to illustrate the new modeling and compare the results from our two-step SF approach with those of the two-step mixed approach. Both empirical results show that the use of the deterministic metafrontier programming method tends to underestimate the TGR and MTE, possibly due to the fact that the measure of TGR from the programming method is confounded with random shocks and its inability to linking with environmental differences. Conversely, the proposed SMF method is able to purge the random shocks from inefficiency, as well as to take into account of the environmental impacts.

As for the hotel data in Taiwan, we verify that the omission of environmental variables from the second-step estimation causes an underestimation of the TGRs and results in an erroneous conclusion about what is the primarily source of the overall technical inefficiency. Furthermore, evidence is found that the industry-wide environmental variables are the crucial determinants of the TGRs for the two groups of hotels. It is essential then to encompass them particularly in a metafrontier model to obtain reliable measures of TGRs. In turn, the comparisons of TGR and MTE among the firms in different groups make sense and offer meaningful implications. An extension to productivity changes in the context of the proposed SMF approach for firms belonging to dissimilar groups is certainly worthy of further examination.

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