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A new nonparametric EWMA Sign Control Chart

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ABSTRACT

Many data in practice came from a population/process with a non-normal or often unknown distribution, hence the commonly-used Shewhart control chart, which requires normality of the monitoring statistics, is not suitable. In this paper, a new nonparametric EWMA Sign Control Chart is proposed for monitoring and detecting possible deviation from the process target. The sampling properties of the new monitoring statistics are examined and the average run lengths of the proposed chart are derived for evaluating its performance. An example is used to illustrate the proposed chart and compare with other existing charts, assuming normality. Furthermore, an arcsine transformed EWMA Sign Chart is examined and proposed. The average run lengths of the Arcsine EWMA Chart are more reasonable than those of the EWMA Sign Chart. The Arcsine EWMA Sign Chart is recommended if we were concerned with the proper values of the average run length.

1. Introduction

Control charts are commonly used tools to improve the quality of a process/service; like the *X*-bar, *R*, EWMA, and CUSUM charts for variables data; *p* and *c* charts for attributes data. However we need to know the sampling properties of the monitoring statistic in order to properly construct the chart and study the chart's behavior and assess its performance, including comparing with other existing charts. In most cases, a normal distribution or a distribution with a known form was assumed for variables data. Hence the question is: What if we had no knowledge of the underlying population distribution or the known distribution doesnot help us derive the necessary sampling properties? Using a nonparametric approach seems to be a reasonable alternative. Only a few researches had been done in this area; see, for example, Amin, Reynolds, and Baker (1995), Altukife (2003a, 2003b), Bakir (2004, 2006), Bakir and Reynolds (1979), Chakraborti and Eryilmaz (2007), Chakraborti and Graham (2007), Chakraborti, Van der Lann, and Van der Wiel (2001), Chakraborti and Van der Wiel (2008), Das and Bhattacharya (2008) and Ferrell (1953).

A major drawback of the Shewhart chart is its inability to detect small shifts, so a EWMA or CUSUM chart is used to rectify this deficiency. In practice, the in-control process mean may not be the process target. From Taguchi's philosophy (Gopalakrishnan, Jaraiedi, Iskander, & Ahmad, 2006), the target value is a vital process measurement. In this paper, we propose a new nonparametric version of the EWMA Sign Chart for variables data to monitor the deviation from the process target, without assuming a process distribution. The paper is organized as follows. Section 2 we discuss the construction of a newly proposed nonparametric EWMA Sign Chart and its performance. Section 3 we compare the performance of the EWMA Sign Chart with existing charts by numerical examples. Section 4 we propose an Arcsine EWMA Sign Chart for obtaining more reasonable average run lengths. Section 5 we summarize the findings and give a recommendation.

2. The EWMA Sign Chart

Assume that a critical quality characteristic, *X*, has a target value *T*. Let $Y = X - T$ and $p = P(Y > 0)$ = the 'Process Proportion'. If the process were in-control then $p = 0.5$, or the process were out-of-control, that is the deviation from the process target had changed, then $p = p_1 \neq 0.5$.

To monitor the deviation from the process target at any given time, a random sample of size n, $X_1, X_2, \ldots, X_n$, is taken from *X*. Similarly, we define

$$Y_j = X_j - T \text{ and } I_j = \begin{cases} 1, & \text{if } Y_j > 0, \\ 0, & \text{otherwise,} \end{cases} \quad \text{for } j = 1, 2, \ldots, n.$$

Let M be the total number of $Y_j > 0$, then $M = \sum_{j=1}^{n} I_j$ would follow a Binomial distribution with parameters $(n, 0.5)$ for an in-control process.

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Table 1
The k values with various combinations of (n, λ) under $ARL_0 \approx 370$.

λ	$ARL_0 \approx 370$											
n	0.05	0.1	0.15	0.2	0.25	0.3	0.4	0.5	0.6	0.7	0.8	0.9
9	2.48	2.70	2.77	2.83	2.85	2.85	2.88	2.87	2.86	2.83	2.85	2.94
10	2.49	2.69	2.77	2.84	2.86	2.86	2.89	2.88	2.86	2.85	2.81	2.70
11	2.49	2.70	2.78	2.84	2.88	2.88	2.89	2.89	2.88	2.86	2.85	2.79
12	2.49	2.68	2.79	2.84	2.86	2.89	2.90	2.90	2.90	2.89	2.89	2.91
13	2.50	2.69	2.78	2.84	2.87	2.89	2.90	2.91	2.90	2.90	2.88	2.95
14	2.49	2.69	2.80	2.85	2.86	2.89	2.91	2.92	2.91	2.89	2.88	2.81
15	2.49	2.69	2.78	2.84	2.87	2.90	2.91	2.92	2.91	2.91	2.91	2.89
16	2.49	2.68	2.79	2.84	2.86	2.91	2.92	2.92	2.91	2.91	2.91	2.96
17	2.48	2.70	2.79	2.85	2.87	2.89	2.93	2.92	2.93	2.92	2.91	2.83
18	2.50	2.70	2.79	2.84	2.86	2.90	2.92	2.95	2.92	2.92	2.92	2.89
19	2.48	2.70	2.79	2.86	2.87	2.89	2.92	2.93	2.93	2.93	2.92	2.96
20	2.47	2.71	2.77	2.84	2.89	2.89	2.92	2.93	2.93	2.92	2.92	2.89
21	2.50	2.67	2.79	2.85	2.88	2.90	2.93	2.92	2.93	2.93	2.93	2.90
22	2.49	2.70	2.79	2.84	2.88	2.90	2.93	2.93	2.94	2.93	2.94	2.96
23	2.49	2.70	2.79	2.84	2.88	2.90	2.93	2.94	2.94	2.93	2.93	2.90
24	2.49	2.70	2.78	2.85	2.87	2.91	2.93	2.94	2.94	2.94	2.94	2.92
25	2.49	2.69	2.80	2.85	2.88	2.90	2.93	2.95	2.94	2.94	2.94	2.97

2.1. The proposed nonparametric EWMA Sign Chart

Monitoring deviation from the process target is equivalent to monitoring changes in Process Proportion, $p = p_1 \neq 0.5$. We propose a 'EWMA Sign Chart' as follows:

First, we define EWMA monitoring statistics

$$EWMA_{M_i} = \lambda M_i + (1 - \lambda) EWMA_{M_{i-1}} \quad 0 < \lambda \leqslant 1, \tag{1}$$

where M_i represents the ith sequentially recorded number of $Y_i(>0)$ from the process. Adopting the starting value, $EWMA_{M_0}$, as the mean of M; that is $EWMA_{M_0} = n/2$. The mean and variance of $E(EWMA_{M_i}) = n/2$ and $Var(EWMA_{M_i}) = \frac{\lambda[1-(1-\lambda)^{2i}]}{2-\lambda}(1/4n)$. If time is infinite then $Var(EWMA_{M_i}) = \frac{\lambda}{2-\lambda}(1/4n)$.

The control limits for the EWMA Sign Chart are usually based on the asymptotic standard deviation of the control statistic. Hence we could construct the EWMA Sign Chart as follows:

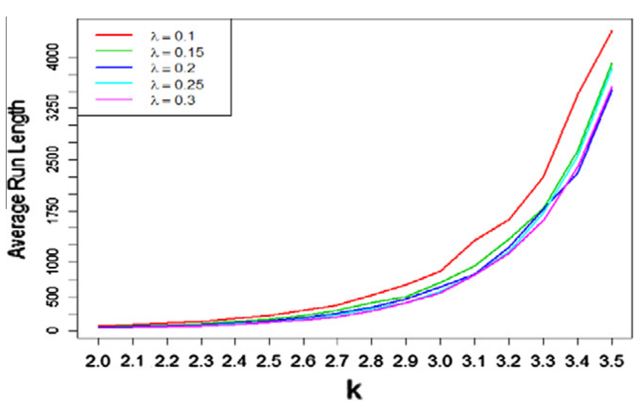

Fig. 1. ARL_0 of the EWMA Sign Chart for various (k, λ) under $n = 10$.

$$UCL_{EWMA_M} = n/2 + k\sqrt{\frac{\lambda}{2-\lambda}(1/4n)},$$
$$CL_{EWMA_M} = n/2,$$
$$LCL_{EWMA_M} = n/2 - k\sqrt{\frac{\lambda}{2-\lambda}(1/4n)}$$

and plot $EWMA_M$ on the chart. If any $EWMA_M \geqslant UCL_{EWMA_M}$ or $EWMA_M \leqslant LCL_{EWMA_M}$, the process is deemed to be out-of-control.

The two chart parameters, k and λ, are chosen that they would satisfy certain average run length (ARL) requirements.

Note that the in-control ARL, denoted ARL_0, of the EWMA Sign Chart depends on the values of n, k and λ, i.e., $ARL_0 = f(n,k,\lambda)$.

2.2. Designing a chart (n, k, λ) with $ARL_0 \approx 370$

The average run length evaluates the performance of a new chart. Following Lucas and Saccucci (1990), the ARL's of the EWMA Sign Chart are evaluated by Markov chain approach.

Since the statistic M in the EWMA statistic follows a binomial distribution, $ARL_0 = 370$ is not always achievable exactly for all combinations of (n, k, λ). Hence, we will find a control chart with k such that its ARL_0 is close to 370 (i.e. $ARL_0 \approx 370$) for each combination of $n = 9(1)25$ and $\lambda = 0.05(0.05)0.3(0.1)0.9$ (see Table 1). We found that the k values are very close for $n = 9(1)25$ under the specified λ.

Fig. 1 shows us the ARL_0's for various combinations of (k, λ) under $n = 10$. We found that ARL_0 increases when k increases under a specified λ; when λ decreases, the difference among ARL_0's is smaller for smaller k but the difference among ARL_0's becomes larger for larger k. Using the same approach, the ARL_0's for various combinations of (k, λ) under $n = 11(1)25$ could be calculated.

The control limits of the EWMA Sign Chart are function of n, k, and λ. We adopt $k = 2.49$ and $\lambda = 0.05$ under $ARL_0 \approx 370$, the control limits of the EWMA Sign Chart for $n = 9(1)25$ are listed in Table 2.

Table 2
EWMA Sign Control Chart limits for $k = 2.49$, $\lambda = 0.05$, $ARL_0 \approx 370$.

n	CL	LCL	UCL	n	CL	LCL	UCL	n	CL	LCL	UCL
9	4.5	3.90	5.10	15	7.5	6.73	8.27	21	10.5	9.59	11.41
10	5	4.37	5.63	16	8	7.20	8.80	22	11	10.06	11.94
11	5.5	4.84	6.16	17	8.5	7.68	9.32	23	11.5	10.54	12.46
12	6	5.31	6.69	18	9	8.15	9.85	24	12	11.02	12.98
13	6.5	5.78	7.22	19	9.5	8.63	10.37	25	12.5	11.50	13.50
14	7	6.25	7.75	20	10	9.11	10.89				

Table 3
The ARL_1 values under $\lambda = 0.05$ and $k = 2.49$.

n	p																		
	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95
9	4	4	5	5	7	9	12	21	57	384	57	21	12	9	7	5	5	4	4
10	3	4	4	5	6	8	11	19	52	371	52	19	11	8	6	5	4	4	3
11	3	4	4	5	6	8	11	18	48	370	48	18	11	8	6	5	4	4	3
12	3	4	4	5	6	7	10	17	46	380	45	17	10	7	6	5	4	4	3
13	3	3	4	5	6	7	10	16	43	377	43	16	10	7	6	5	4	3	3
14	3	3	4	4	5	7	9	15	41	378	40	15	9	7	5	4	4	3	3
15	3	3	4	4	5	7	9	15	39	386	39	15	9	7	5	4	4	3	3
16	3	3	3	4	5	6	9	14	37	371	36	14	9	6	5	4	3	3	3
17	3	3	3	4	5	6	8	14	35	384	35	14	8	6	5	4	3	3	3
18	3	3	3	4	5	6	8	13	34	375	34	13	8	6	5	4	3	3	3
19	3	3	3	4	5	6	8	13	33	388	33	13	8	6	5	4	3	3	3
20	3	3	3	4	4	6	8	12	32	389	32	12	8	6	4	4	3	3	3
21	2	3	3	4	4	5	7	12	30	379	30	12	7	5	4	4	3	3	2
22	2	3	3	4	4	5	7	12	29	383	29	12	7	5	4	4	3	3	2
23	2	3	3	4	4	5	7	11	28	383	28	11	7	5	4	3	3	3	2
24	2	3	3	4	4	5	7	11	28	381	27	11	7	5	4	3	3	3	2
25	2	3	3	4	4	5	7	11	27	377	27	11	7	5	4	3	3	3	2

2.3. The ARL_1 of the EWMA Sign Chart

The out-of-control average run length, ARL_1, of the EWMA Sign Chart is also a function of n, k, and λ. Considering in-control proportion $p = 0.5$, $\lambda = 0.05$, $k = 2.49$, $ARL_0 \approx 370$, the expected out-of-control proportion $p = p_1 = 0.05(0.05)0.45$, $0.55(0.05)0.95$, $n = 9(1)25$, the ARL_1's are calculated and given in Table 3.

Table 3 showed that the ARL_1's are inversely related to n and $|p_1 - 0.5|$. When $p = 0.5$, the numbers are the ARL_0's, which are close to the desired value of 370. The values in this table seem reasonable.

3. Example

We will use the following example from Montgomery (2009) to illustrate the proposed chart.

The fill volume of soft-drink beverage bottles is an important quality characteristic. The volume is measured (approximately) by placing a gauge over the crown and comparing the height of the liquid in the neck of the bottle against a coded scale. On this scale, a reading of zero corresponds to the correct fill height. Fifteen samples of size $n = 10$ have been analyzed, and the fill heights are shown below:

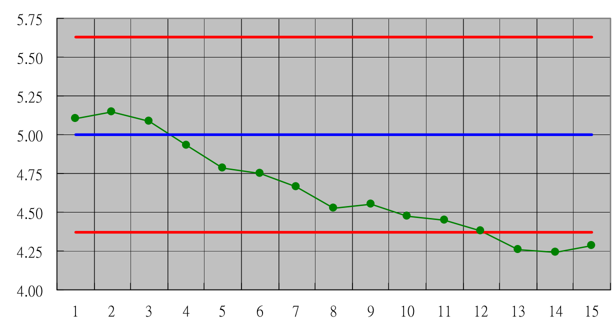

Fig. 2. EWMA Sign Chart.

Here, sample size = 10, number of samples = 15, target value = 0. Choose $ARL_0 \approx 370$, $\lambda = 0.05$ and $k = 2.49$. From Table 2, $CL = 5$, $LCL = 4.36$ and $UCL = 5.63$.

Hence M = Sum of positive differences $(X_i - 0)$, $i = 1, 2, \ldots, 15$.

The EWMA Sign Chart is thus constructed and the monitoring statistics $EWMA_M$'s are plotted in Fig. 2.

The plot shows that the data appear to be out-of-control from sample 13.

Sample	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	M	$EWMA_M$
1	2.5	0.5	2	−1	1	−1	0.5	1.5	0.5	−1.5	7	5.10
2	0	0	0.5	1	1.5	1	−1	1	1.5	−1	6	5.15
3	1.5	1	1	−1	0	−1.5	−1	−1	1	−1	4	5.09
4	0	0.5	−2	0	−1	1.5	−1.5	0	−2	−1.5	2	4.93
5	0	0	0	−0.5	0.5	1	−0.5	−0.5	0	0	2	4.79
6	1	−0.5	0	0	0	0.5	−1	1	−2	1	4	4.75
7	1	−1	−1	−1	0	1.5	0	1	0	0	3	4.66
8	0	−1.5	−0.5	1.5	0	0	0	−1	0.5	−0.5	2	4.53
9	−2	−1.5	1.5	1.5	0	0	0.5	1	0	1	5	4.55
10	−0.5	3.5	0	−1	−1.5	−1.5	−1	−1	1	0.5	3	4.47
11	0	1.5	0	0	2	−1.5	0.5	−0.5	2	−1	4	4.45
12	0	−2	−0.5	0	−0.5	2	1.5	0	0.5	−1	3	4.38
13	−1	−0.5	−0.5	−1	0	0.5	0.5	−1.5	−1	−1	2	4.26
14	0.5	1	−1	−0.5	−2	−1	−1.5	0	1.5	1.5	4	4.25
15	1	0	1.5	1.5	1	−1	0	1	−2	−1.5	5	4.24

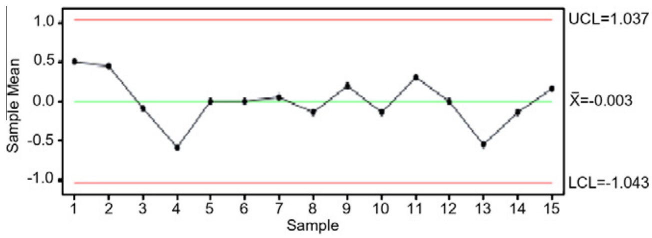

Fig. 3. *X*-bar chart.

The corresponding *X*-bar chart (Montgomery, 2009), which requires normality, in Fig. 3 gives not the same conclusion of an out-of-control process.

4. The average run length of the Arcsine EWMA Sign Chart

Table 3 shows that the ARL_0's of the EWMA Sign Chart are not the commonly known value of 370 when $p = 0.5$. These results do not seem to be reasonable. The cause for this is that the binomial distribution is asymmetric for small or moderate sample size n. To rectify this problem, we would apply the arcsine transformation (Mosteller & Youtz, 1961). That is, let $Y = \sin^{-1}\left(\sqrt{\frac{M}{n}}\right)$ and then the distribution of Y would be approximately normal with mean $\sin^{-1}(\sqrt{p})$ and variance $1/(4n)$.

A revised $EWMA_Y$ chart, Arcsine EWMA Sign Chart is then constructed as follows:

$$EWMA_{Y_i} = \lambda Y_i + (1-\lambda)EWMA_{Y_{i-1}} \quad 0 < \lambda \leqslant 1. \tag{2}$$

Using the mean of Y as the starting value, $EWMA_{Y_0}$; that is $EWMA_{Y_0} = \sin^{-1}\sqrt{0.5}$. The mean and variance of $EWMA_{Y_i}$ are $E(EWMA_{Y_i}) = \sin^{-1}\sqrt{0.5}$ and $Var(EWMA_{Y_i}) = \frac{\lambda[1-(1-\lambda)^{2i}]}{(2-\lambda)}(1/4n)$. If time gets sufficiently large then $Var(EWMA_{Y_i}) = \frac{\lambda}{2-\lambda}(1/4n)$.

For an in-control process, i.e. $p = 0.5$, the control limits and the center line of the Arcsine $EWMA_Y$ Sign Chart are:

$$\begin{aligned} UCL &= \sin^{-1}\left(\sqrt{0.5}\right) + k\sqrt{\frac{\lambda}{(2-\lambda)}(1/4n)}, \\ CL &= \sin^{-1}\left(\sqrt{0.5}\right), \\ LCL &= \sin^{-1}\left(\sqrt{0.5}\right) - k\sqrt{\frac{\lambda}{(2-\lambda)}(1/4n)} \end{aligned} \tag{3}$$

and plot $EWMA_Y$ on the chart. If any $EWMA_Y \geqslant UCL_{EWMA_Y}$ or $EWMA_Y \leqslant LCL_{EWMA_Y}$, the process is deemed to be out-of-control.

Let us use the same example in Section 3, construct the Arcsine $EWMA_Y$ Sign Chart and the monitoring statistics $EWMA_Y$'s are plotted in Fig. 4.

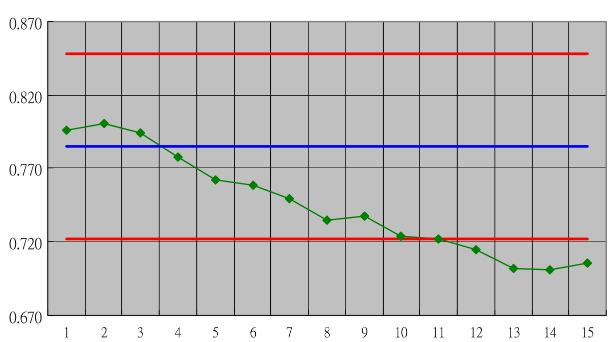

Fig. 4. The Arcsine $EWMA_Y$ Sign Chart.

The plot shows that the data appear to be out-of-control from sample 12 but not sample 13. It shows that the Arcsine EWMA Sign Chart performs a little bit better than the EWMA Sign Chart. In order to evaluate the performance of this new chart, we would calculate the chart's *ARLs*. Table 4 gives these values.

Now the $ARL_0 = 370$ and ARL_1's are smaller than those of EWMA Sign Chart. The results seem more reasonable and the detection ability of the Arcsine $EWMA_Y$ Sign Chart is a little bit better than the EWMA Sign Chart.

5. Conclusion

A new nonparametric chart, the EWMA Sign Chart, to monitor the deviation from process target for variables data is proposed. It provides an alternative when the underlying distribution is unknown or non-normal. It shows it performs quite well. However, we would recommend a modified version, the Arcsine EWMA Sign Chart if we are concerned with attaining the proper ARL values.

Acknowledgements

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Table 4
The *ARL* values of Arcsine $EWMA_Y$ Sign Chart ($\lambda = 0.05$, $k = 2.49$).

n	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95
9	3	4	4	5	6	8	12	20	56	370	56	20	12	8	6	5	4	4	3
10	3	3	4	5	6	8	11	19	52	370	52	19	11	8	6	5	4	3	3
11	3	3	4	5	6	8	11	18	48	370	48	18	11	8	6	5	4	3	3
12	3	3	4	4	6	7	10	17	45	370	45	17	10	7	6	4	4	3	3
13	3	3	4	4	5	7	10	16	42	370	42	16	10	7	5	4	4	3	3
14	2	3	3	4	5	7	9	15	40	370	40	15	9	7	5	4	3	3	2
15	2	3	3	4	5	6	9	15	38	370	38	15	9	6	5	4	3	3	2
16	2	3	3	4	5	6	9	14	36	370	36	14	9	6	5	4	3	3	2
17	2	3	3	4	5	6	8	13	35	370	35	13	8	6	5	4	3	3	2
18	2	3	3	4	4	6	8	13	33	370	33	13	8	6	4	4	3	3	2
19	2	3	3	4	4	6	8	13	32	370	32	13	8	6	4	4	3	3	2
20	2	2	3	3	4	5	8	12	31	370	31	12	8	5	4	3	3	2	2
21	2	2	3	3	4	5	7	12	30	370	30	12	7	5	4	3	3	2	2
22	2	2	3	3	4	5	7	12	29	370	29	12	7	5	4	3	3	2	2
23	2	2	3	3	4	5	7	11	28	370	28	11	7	5	4	3	3	2	2
24	2	2	3	3	4	5	7	11	27	370	27	11	7	5	4	3	3	2	2
25	2	2	3	3	4	5	7	11	26	370	26	11	7	5	4	3	3	2	2

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