The Effects of Stochastic Volatility and Demand Pressure on the Monotonicity Property Violations

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In the literature, microstructure effects have been documented as determinants of the violations of the monotonicity property. In this article, we argue that in an order-driven market the violations are largely attributable to stochastic volatility and demand pressure. Using transaction prices for TAIEX options, we find that the monotonicity property is violated 34% (33%) of the time for call (put) options. We further find that either stochastic volatility or demand pressure alone can explain over 50% of the violations, while the portion of violations which are explained by neither stochastic volatility nor demand pressure is only 18%. Stochastic volatility can better explain violations of ATM option prices, while demand pressure can better explain violations of non-ATM option prices. Our empirical results affirm the inclusion of demand pressure by Gârleanu et al. [2009] into options pricing models.

Black and Scholes [1973] assume that the option price is a function of the underlying asset price and time. The underlying asset price is thus the only stochastic driving force of option prices. If the price of the underlying asset increases, then the call (put) option price will monotonically increase (decrease). This feature is referred to as the monotonicity property. It is shared by all option pricing models, which assume that the underlying asset follows a one-dimensional diffusion process. Examples of these models include Black and Scholes [1973]; Cox and Ross [1976]; and Rubinstein [1994]. Prior studies (e.g., Bakshi et al. [2000]; and Pérignon [2006]), however, have documented evidence of violations of the monotonicity property, implying that option prices are not generated by a univariate diffusion model. This highlights the inadequacy of one-dimensional diffusion models.

To compensate for the inadequacy, Merton [1976] first describes the underlying asset price dynamics as a jump stochastic process defined in continuous time. Based on Merton's work, Hull and White [1987] value a European call option on a stock that has a stochastic volatility.

Since then, more and more complicated option pricing models have been proposed, including the pure stochastic volatility (SV) model (Heston [1993]), a model that incorporates stochastic volatility with jumps in prices (SVJ) (Bates [2000]), and a model with contemporaneous jumps in volatility and prices (SVCJ) (Broadie et al. [2007]).

Using the average bid-ask quotes of S&P 500 options, Bakshi et al. [2000] examine the monotonicity property of a one-dimensional diffusion model. After controlling for time decay and market microstructure effects, they find that the violations of the property occur 7% to 16% (5% to 16%) of the time for call (put) options. Their finding indicates the possibility of a need to allow another state variable to evolve stochastically in the modeling process of the underlying asset price. This additional state variable may potentially explain why the property is violated. Using Heston's [1993] SV model for a simulation exercise, they show that option price changes exhibit patterns qualitatively similar to their documented violations.

Unlike Bakshi et al. [2000], Pérignon [2006] uses observed transaction prices for five (European, French, German, Swiss, and British) index option contracts and reports higher violation rates, 7% to 32% for calls and 6% to 35% for puts, than Bakshi, Cao, and Chen [2000]. Since the change in the option price when a violation occurs is less than the average bid–ask spread, Pérignon [2006] argues that market microstructure effects (e.g., the bid–ask bounce) and holiday/day-trade effects (arising from rational trading tactics) are the main causes of the violations.

Gârleanu et al. [2009] first incorporate demand pressure into their option pricing model. They regard the pressure of end-users' demand for derivatives as an exogenous variable in this model, maximize the dealer's value function, and compute equilibrium option prices. They argue that the price of an option increases with its demand pressure and that the incremental amount is proportional to the variance of the unhedgeable part of the option.

In this study, we test the validity of the monotonicity property using options on the Taiwan Stock Exchange Capitalization Weighted Stock Index (hereafter abbreviated as TAIEX). TAIEX options were the first European-style index options introduced by the Taiwan Futures Exchange (TAIFEX) in 2001. During the period from 2005 to 2013, the TAIEX options trading volume accounted for on average 71.2% of the total trading volume for all derivatives on the TAIFEX.[1]

Taiwan's futures market is an order-driven rather than quote-driven market. There is no real-time information about the bid and ask quotes of designated market makers. Thus, the measure for demand pressure proposed by Bollen and Whaley [2004] for a quote-driven market is not applicable in our study. We therefore use the measure developed by Shiu et al. [2010] specifically for an order-driven market. In addition, Taiwan's futures market is characterized by high individual participation. During the sample period from 2005 to 2013, individual participation, on average, accounted for 47% of the total derivative trading volume.[2] Our empirical results show that the violation rate of TAIEX options' monotonicity property is 34% (33%) of the time for call (put) options. We find that violations of the monotonicity property are mainly determined by stochastic volatility and demand pressure. About 54% (52%) of violations for call (put) options can be explained by stochastic volatility, and after incorporating net buying pressure as a factor in the model, the explanatory ratio increases to 82% (82%). Stochastic volatility can better explain the violations for ATM options, while demand pressure explains those for non-ATM options.

Our paper shares a key insight with Bollen and Whaley [2004] and Gârleanu et al. [2009]; namely, that an option's price is affected by its demand pressure. Bollen and Whaley [2004] find that the changes in option prices vary as a function of the changes in option demand. Gârleanu et al. [2009] further examine the relation between the level of option demand and the overall level of option prices. We extend their results and further argue that demand pressure can explain why the violations of the monotonicity property occur. Our empirical results affirm the inclusion of demand pressure by Gârleanu et al. [2009] into options pricing models.

One prior study that has a close connection to ours is Pérignon [2006]. In both papers, the validity of the monotonicity property is empirically tested using transaction prices of index options. However, several major differences exist. First, Pérignon uses data from developed Western countries, while we use data from an Asian emerging market. Second, we use nearest-the-money implied volatilities to investigate the stochastic volatility effect on the violation of monotonicity property, while Pérignon employs a matching procedure to discover the stochastic volatility effect. Third, Pérignon mainly attributes the causes of the violations to market microstructure effects and rational trading tactics. In contrast, we identify demand pressure as an additional cause after controlling for concurrent volatility changes and rational trading tactics. We find that violations of the property frequently occur when an imbalance between the demand for and the supply of options exists.

DEFINITION OF MONOTONICITY PROPERTY VIOLATION

We use intraday option transaction prices in this study and define four types of violations of the monotonicity property as follows:

Violation A: $\Delta C < 0$ *and* $\Delta S > 0$
Violation B: $\Delta C > 0$ *and* $\Delta S < 0$
Violation C: $\Delta P > 0$ *and* $\Delta S > 0$
Violation D: $\Delta P < 0$ *and* $\Delta S < 0$

where ΔC, ΔS, and ΔP represent the change in the call, underlying asset, and put prices, respectively.

The Taiwan Stock Exchange calculates TAIEX according to the latest transaction prices of all listed stocks, and publishes the index every minute during trading hours (9 a.m. to 1:30 p.m.).[3] Although TAIEX changes almost every minute, it never changes during a one-minute interval. TAIEX options could be traded at any time during trading hours (8:45 a.m. to 1:45 p.m.), while TAIEX cannot be traded directly. Furthermore, since the TAIEX options market is an order-driven market, there is no real-time information about the bid and ask quotes of designated market makers. Thus, we define violations of the monotonicity property based on the occurrence of options trades. The following rules are applied: 1) call price decreases (increases) between two consecutive transactions, and the concurrent change of TAIEX is positive (negative); and 2) put price increases (decreases) between two consecutive transactions, and the concurrent change of TAIEX is positive (negative).

Option prices are time-stamped to the second, and TAIEX to the minute. The simple mean of transaction prices during one minute is first calculated. For example, the simple average of all prices for a given option series during time interval [9:57:01 9:58:00] is calculated and time-stamped at 9:58. The change of the two consecutive mean prices is then compared with the concurrent change of TAIEX to determine whether a violation occurs.

EMPIRICAL DESIGN AND DATA

In the TAIEX market, there are market makers who do not usually post quotes, but instead wait for outside investors to put in a quote request through the TAIEX options trading system. The system then passes the request to all market makers, who must respond to the request in 20 seconds by giving the bid/ask quotes. The quote shown on the system actually is a firm limit order, not just a reference quote. It is placed on the order book and participates in matching. The system also displays the quote information and discloses for each option series the best bid and offer quotes on the market. Execution priority is given to orders with the better bid/offer prices (price priority). If the prices are the same, execution priority is given to orders that enter the system earlier (time priority). Under these two matching rules, investors who are eager for a buy (sell) order to be executed earlier may consider raising (lowering) their buying (selling) price. For instance, if investors determine that the market is temporarily down (up) and will soon go up (down), they would be willing to bid a price higher (lower) than the best offer (bid) quote on the market in order to get their order filled. This will cause the violation of the monotonicity property. However, this may not be the case in a quote-driven market. When the market is down (up) and market makers believe that the market will soon go up (down), their bid/offer prices probably will remain the same. Thus, the violation will not occur.

TAIEX options' expiration months include three near-term months followed by two additional months from the March quarterly cycle (March, June, September, and December). The expiration date for TAIEX options is the third Wednesday of the expiration month. The dataset employed in this study is intraday observations on 1) Taiwan Weighted Stock Index (TAIEX) per minute, and 2) the prices and volumes for the nearest-month TAIEX options. The TAIEX data are downloaded from the website of the Taiwan Stock Exchange (TWSE) (http://www.twse.com.tw), and the intraday data of TAIEX options from the database of the Taiwan Economic Journal. To alleviate expiration-related biases, we follow Ederington and Guan [2002, 2005]; and Shiu et al. [2010] to exclude the nearest-month options with fewer than eight days to expiration. We measure the risk-free interest rate using the simple average of the one-month time deposit interest rates of the five major banks in Taiwan, which are obtained from the Central Bank of the Republic of China.

Our sample period covers January 2005 to December 2013. Based on the ratio of the option's strike price to the TAIEX at the option's transaction time, options are classified into five moneyness categories. Using $K/S = 1$ as the central point, the range for each category is set as 0.04. Exhibit 1 lists the upper and lower bounds of the moneyness categories. Options with a strike price greater than 1.1 times or less than 0.9 times the TAIEX are excluded because their trading volume is typically small. Transactions occurring before 9 a.m.

EXHIBIT 1
Definitions for Moneyness Categories

Category	Label	Range
1	Deep in-the-money (DITM) call Deep out-of-the-money (DOTM) put	$0.90 \leqq K/S < 0.94$
2	In-the-money (ITM) call Out-of-the-money (OTM) put	$0.94 \leqq K/S < 0.98$
3	At-the-money (ATM) call At-the-money (ATM) put	$0.98 \leqq K/S < 1.02$
4	Out-of-the-money (OTM) call In-the-money (ITM) put	$1.02 < K/S < 1.06$
5	Deep out-of-the money (DOTM) call Deep in-the-money (DITM) put	$1.06 \leqq K/S \leqq 1.10$

Note: K is the strike price of the option and S is the spot price at option's transaction time. Options with K/S below 0.90 and above 1.10 are eliminated.

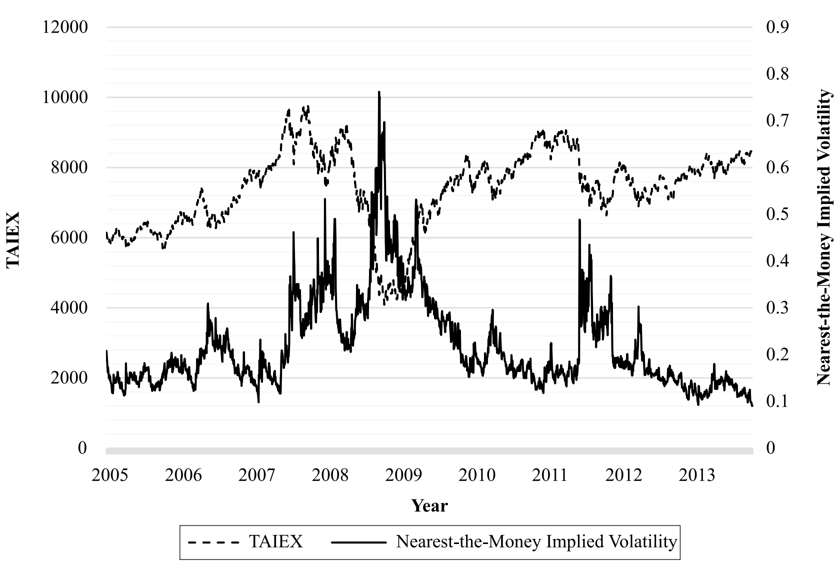
and after 1:30 p.m. are also deleted because the Taiwan Stock Exchange closes.

STOCHASTIC VOLATILITY

Bakshi et al. [2000] argue that stochastic volatility is the main reason why the monotonicity property is violated. Using the stochastic volatility model of Heston [1993] to conduct a simulation exercise, they find that the simulated results for violations ($\Delta S\Delta C < 0$ or $\Delta S\Delta P > 0$) are similar to the corresponding empirical results. Unlike Bakshi et al. [2000], we estimate the implied volatility of nearest-the-money options and use it as a proxy for the volatility of the TAIEX when exploring the relation between the monotonicity property violation and the stochastic volatility. Since TAIEX options are European options, the Black–Scholes model is employed to derive implied volatilities. In order to avoid the estimation error from dividend yields and the nonsynchronous transaction problem among options and stocks markets, implied TAIEX is created. Three steps are involved in creating the implied TAIEX, beginning with the calculation of the mean prices of all calls and puts during each one-minute interval. We then select the option pairs; that is, among all option pairs, those with the smallest difference in their mean prices. Finally, the put–call parity is applied to derive the implied TAIEX.[4]

Out-of-the-money options, whose strikes are nearest to the implied TAIEX, are chosen to calculate implied volatilities, denoted as V_t. If there are two implied volatilities, one for a call and another for a put, then the mean is calculated. Exhibit 2 depicts the TAIEX and the implied volatility at 13:30. As shown in this exhibit, the TAIEX was lowest during the 2008 financial crisis, but the implied volatility was highest. In general, they have the opposite trend. When the TAIEX is down, the implied volatility is up, and vice versa.

EXHIBIT 2
The Taiwan Stock Exchange Capitalization Stock Index (TAIEX) and Implied Volatility (V) during 2005 to 2013



DEMAND PRESSURE

Besides stochastic volatility, in this study we also argue that violations of the monotonicity property may be attributable to an imbalance between the option demand and supply forces. To measure the demand pressure for options, we adopt the approach suggested by Shiu et al. [2010]. Ever since July 16, 2008, all transactions have been split into at least two records, as a result of which the prices for the same transactions are not easily identifiable; consequently, we begin our analysis by calculating the mean prices and total trading volume for one-second intervals. Then, we use the changes of the two consecutive mean prices to define buyer-initiated and seller-initiated trade. If the mean transaction price is strictly higher (lower) than the preceding mean price, then the

Exhibit 3

The Descriptive Statistics for Time Interval, and Change of Implied Volatility and Demand Pressure During Two Consecutive Transactions

Type	Change of TAIEX	Mean	Median	St. Dev.	Min.	Max.	# of Obs.
Panel A: Time Interval							
Call	$\Delta S > 0$	1.73249	1	4.3906	1	250	2,205,352
Call	$\Delta S > 0$	1.72634	1	4.4470	1	259	2,261,386
Put	$\Delta S > 0$	1.67209	1	4.1761	1	256	2,167,239
Put	$\Delta S > 0$	1.68536	1	4.3089	1	255	2,217,329
Panel B: Change of Implied Volatility							
Call	$\Delta S > 0$	–0.00011	–0.00005	0.0029	–0.12096	0.10483	2,205,352
Call	$\Delta S > 0$	0.00009	0.00001	0.0030	–0.16841	0.23144	2,261,386
Put	$\Delta S > 0$	–0.00011	–0.00005	0.0025	–0.12905	0.13034	2,167,239
Put	$\Delta S > 0$	0.00009	0.00001	0.0026	–0.10467	0.10355	2,217,329
Panel C: Change of Demand Pressure							
Call	$\Delta S > 0$	0.01286	0	0.8112	–2	2	2,205,352
Call	$\Delta S > 0$	–0.01180	0	0.8103	–2	2	2,261,386
Put	$\Delta S > 0$	–0.01298	0	0.8088	–2	2	2,167,239
Put	$\Delta S > 0$	0.01290	0	0.8124	–2	2	2,217,329

trading volume for that one-second interval is classified as a buyer- (seller)-initiated trade. Finally, net demand is defined as buyer-initiated trades minus seller-initiated trades, and then is divided by the total trading volume during a one-minute period, denoted as DP_t.

Exhibit 3 provides the descriptive statistics for time interval, change of implied volatility, and demand pressure during two consecutive transactions. The mean of time interval is about 1.7 minutes, and the median is 1 minute. In reality, the mean and median of time interval are smaller than 1.7 minutes and 1 minute, respectively, because the minimum unit of time a minute in our research design. When the TAIEX increases (decreases), the mean and median change of implied volatility is negative (positive). The mean change of demand pressure for call options is positive (negative) as the TAIEX is up (down). When the TAIEX increases (decreases), the mean change of demand pressure for put options is negative (positive).

MONOTONICIY PROPERTY VIOLATIONS

Exhibit 4 presents the violation ratios of the monotonicity property for different years, option types, and moneyness categories. The violation ratio across years, option types, and moneyness categories ranged from 6.10% to 19.87%. ATM options have the highest violation ratios. OTM (DOTM) options have higher violation ratios than ITM (DITM). Also, OTM (ITM) options have higher violation ratios than DOTM (DITM). Since ATM options are traded most frequently, OTM are traded more frequently than ITM, and DITM are traded sparsely, our empirical results imply that TAIEX option trades are not only stimulated by the randomness of the TAIEX, but also by other factors. If they were only driven by the randomness of the TAIEX, more trades should not have more violations.

In addition, Violation D has higher ratios than Violation C. This indicates that the monotonicity violation of put option prices occurs more frequently as the TAIEX drops. Since the TAIEX put option could be regarded as an insurance product for market downside risk, demand for insurance, i.e., put options, might arise as the TAIEX drops. Comparing violation ratios across years, the monotonicity property was violated less during the period of the 2008 financial crisis, which occurred from about July 2008 to June 2009. Violation ratios in 2008 were smaller than those in 2007, while violation ratios in 2009 were smaller than those in 2010.

Does the monotonicity property violation bear any economic significance? If the violation reflects only market frictions (e.g., the transaction cost and the bid-ask spread), it seems that there is no need to expand the one-dimensional diffusion option pricing model to a two-state or more state variables model. We define violations depending on whether or not option transactions occur, but not on whether TAIEX changes. Thus, option market frictions are not the main

EXHIBIT 4
Violation Ratios by Year and by Moneyness

Violation	A ($\Delta C < 0, \Delta S > 0$)	B ($\Delta C > 0, \Delta S < 0$)	# of Obs.	C ($\Delta P > 0, \Delta S > 0$)	D ($\Delta P < 0, \Delta S < 0$)	# of Obs.
Category/Ratio (%)						
Year of 2005						
1	11.40	10.88	8,621	14.04	14.59	49,420
2	16.96	16.85	74,413	17.54	18.48	136,023
3	19.87	19.95	153,868	18.87	19.77	143,312
4	17.93	17.87	131,495	14.68	16.21	39,663
5	14.36	14.37	43,029	9.58	12.18	2,380
All	17.97	17.95	411,426	17.23	18.18	370,798
Year of 2006						
1	9.67	9.64	8,497	14.22	14.74	92,155
2	15.86	15.68	67,241	17.23	18.03	161,455
3	18.89	19.08	161,853	17.67	18.88	140,158
4	17.34	17.54	150,028	13.19	14.67	28,350
5	14.30	14.48	50,396	9.19	10.78	2,904
All	17.18	17.32	438,015	16.40	17.32	425,022
Year of 2007						
1	9.66	9.26	9,714	14.95	15.63	133,125
2	15.59	15.78	66,522	17.32	18.30	197,289
3	18.80	19.48	189,404	17.57	19.06	165,599
4	17.95	18.68	168,646	13.13	14.62	36,628
5	15.28	16.45	96,379	9.50	11.45	6,977
All	17.32	18.02	530,665	16.43	17.53	539,618
Year of 2008						
1	13.48	11.90	19,547	15.63	15.87	135,083
2	15.82	15.24	70,595	16.79	17.32	165,211
3	17.90	17.68	157,294	16.47	17.37	140,030
4	17.80	17.72	173,304	13.83	14.38	54,381
5	16.50	16.58	156,641	10.63	11.53	16,659
All	17.09	16.90	577,381	15.88	16.45	511,364
Year of 2009						
1	11.13	11.06	23,720	15.19	16.08	145,847
2	14.84	15.32	79,742	16.43	17.43	152,191
3	17.21	17.49	147,849	15.85	17.16	123,071
4	16.85	17.17	155,333	11.97	13.58	38,241
5	15.35	15.92	103,054	7.87	10.16	6,368
All	16.07	16.44	509,698	15.40	16.52	465,718
Year of 2010						
1	8.67	9.13	10,177	14.66	15.44	139,205
2	15.29	15.71	81,753	17.23	18.10	194,103
3	18.57	18.91	192,047	17.30	18.15	168,048
4	17.38	17.69	183,301	13.22	14.39	44,755
5	15.05	15.32	80,635	8.88	9.83	6,422
All	16.98	17.31	547,913	16.18	17.05	552,533

E X H I B I T 4 (*Continued*)

Violation	A ($\Delta C < 0, \Delta S > 0$)	B ($\Delta C > 0, \Delta S < 0$)	# of Obs.	C ($\Delta P > 0, \Delta S > 0$)	D ($\Delta P < 0, \Delta S < 0$)	# of Obs.
Year of 2011						
1	7.50	7.30	9,741	14.24	14.74	155,630
2	14.25	14.11	72,707	16.50	17.04	197,432
3	17.76	17.92	190,156	16.43	17.26	172,165
4	16.94	17.10	187,366	12.19	13.69	53,094
5	15.03	15.62	103,712	8.82	9.67	9,685
All	16.36	16.55	563,682	15.37	16.07	588,006
Year of 2012						
1	7.99	8.32	9,943	14.13	14.38	137,672
2	14.08	14.23	78,209	16.19	16.76	183,589
3	17.23	17.26	180,470	16.50	17.12	165,776
4	16.54	16.48	177,479	12.61	13.72	53,735
5	14.17	14.68	88,687	7.35	7.80	5,142
All	15.86	15.96	534,788	15.33	15.88	545,914
Year of 2013						
1	6.81	8.28	4,612	14.31	14.38	82,824
2	15.70	15.51	57,140	17.11	17.84	180,452
3	18.75	18.53	188,085	17.83	18.62	164,969
4	17.06	16.89	148,689	13.39	14.67	26,560
5	13.09	13.31	25,208	6.10	6.95	820
All	17.28	17.12	423,734	16.62	17.29	455,625

reason for violations. As will be shown later, stochastic volatility and demand pressure may have impacts on option prices.

UNIVARIATE ANALYSIS

Exhibits 5 and 6 show that the explanatory power of volatility change and demand pressure change for different years and moneyness categories for call and put options, respectively. Those that can be explained by neither stochastic volatility nor demand pressure are denoted as "abnormal." About 55% and 62% of violations for call options can be explained by stochastic volatility and demand pressure, respectively. Only 18% of violations for call options can be explained by neither stochastic volatility nor demand pressure. About 51% of violations for put options can be explained by stochastic volatility, and after incorporating demand pressure as a factor, the explanatory ratio increases to 82%. In addition, stochastic volatility better explains the violations for ATM options, while demand pressure better explains the violations for non-ATM options. Furthermore, stochastic volatility is the better explanation for the violations as options prices decrease, while demand pressure better explains an increase in options prices. Abnormal ratios for call options are lowest during 2009, but lowest for put options during 2013.

REGRESSION ANALYSIS

To assess the effects of stochastic volatility and demand pressure on the violation of monotonicity property, we estimate regression models as follows:

$$\begin{aligned} Violation(\cdot) = \alpha + \beta_1 \Delta V + \beta_2 \Delta DP + \beta_3 Crisis \\ + \beta_4 Moneyness + \beta_5 \Delta t \\ + \beta_6 Daytrade + \varepsilon \end{aligned} \tag{1}$$

where $Violation(\cdot)$ denotes Violation A, B, C, or D. For Violation A (B), $Violation(\cdot)$ takes the value 1 if a Violation A (B) occurs, and 0 if it does not. $\Delta V = 1$ if implied

EXHIBIT 5

Explanatory Power of Volatility Change and Demand Pressure Change for Call Prices

	A ($\Delta C < 0$, $\Delta S > 0$)				B ($\Delta C > 0$, $\Delta S < 0$)			
Violation	**# of Obs.**	**$\Delta V < 0$**	**$\Delta DP < 0$**	**Abnormal**	**# of Obs.**	**$\Delta V > 0$**	**$\Delta DP > 0$**	**Abnormal**
Category/Ratio (%)								
Year of 2005								
1	983	50.25	70.60	14.34	938	47.23	73.45	14.50
2	12,617	50.23	64.93	17.21	12,539	49.15	65.17	17.65
3	30,580	56.78	56.32	19.15	30,700	55.78	56.03	19.52
4	23,582	50.89	61.45	18.95	23,504	49.83	61.07	19.46
5	6,177	51.76	70.71	13.83	6,182	49.48	73.08	13.75
All	73,939	53.27	60.82	18.24	73,863	52.13	60.83	18.64
Year of 2006								
1	822	53.41	78.71	10.10	819	48.84	74.36	13.06
2	10,663	52.77	69.10	14.64	10,545	51.09	69.08	14.98
3	30,566	56.77	57.11	18.76	30,876	56.16	57.11	18.83
4	26,012	52.88	60.78	18.65	26,321	52.19	61.00	18.41
5	7,205	53.75	70.10	13.93	7,295	53.21	70.06	13.57
All	75,268	54.53	61.56	17.58	75,856	53.71	61.55	17.58
Year of 2007								
1	938	48.72	81.02	10.13	900	55.00	78.78	9.56
2	10,370	53.15	69.88	14.45	10,500	52.63	70.32	14.17
3	35,610	56.96	58.40	18.15	36,899	57.13	58.37	18.30
4	30,266	53.52	58.83	19.01	31,499	53.92	59.13	18.85
5	14,730	54.41	66.33	15.11	15,853	54.70	66.29	15.16
All	91,914	54.90	61.34	17.44	95,651	55.15	61.44	17.43
Year of 2008								
1	2,634	54.33	75.44	11.24	2,326	51.63	75.80	12.04
2	11,169	55.99	70.51	12.90	10,760	54.78	70.20	13.90
3	28,157	58.43	60.76	16.59	27,808	57.51	60.72	17.29
4	30,850	54.97	58.16	18.87	30,714	53.88	57.81	19.27
5	25,838	54.61	63.10	16.76	25,964	53.77	63.40	16.77
All	98,648	55.96	62.05	16.79	97,572	54.93	61.92	17.28
Year of 2009								
1	2,640	54.92	77.01	10.98	2,624	51.60	76.30	12.12
2	11,832	55.05	69.74	13.87	12,215	53.72	70.24	13.91
3	25,448	59.12	60.88	16.58	25,860	58.74	60.29	16.92
4	26,176	55.15	60.82	17.59	26,672	55.02	59.74	18.09
5	15,815	54.43	65.13	16.19	16,409	54.01	64.65	16.02
All	81,911	56.22	63.48	16.26	83,780	55.67	62.92	16.52
Year of 2010								
1	882	52.61	76.64	11.68	929	50.38	77.40	11.63
2	12,504	55.02	68.23	14.76	12,841	53.95	69.42	14.98
3	35,665	57.99	58.03	18.16	36,318	56.75	58.24	18.48
4	31,850	55.28	61.00	17.75	32,428	53.93	61.10	17.99
5	12,135	56.66	69.57	13.41	12,350	54.36	69.76	14.03
All	93,036	56.44	62.10	16.88	94,866	55.03	62.42	17.19

EXHIBIT 5 (*Continued*)

Violation	A ($\Delta C < 0$, $\Delta S > 0$) # of Obs.	$\Delta V < 0$	$\Delta DP < 0$	Abnormal	B ($\Delta C > 0$, $\Delta S < 0$) # of Obs.	$\Delta V > 0$	$\Delta DP > 0$	Abnormal
Year of 2011								
1	731	50.62	75.92	12.31	711	47.26	76.51	12.10
2	10,363	52.23	68.17	16.23	10,262	52.58	69.64	15.48
3	33,774	56.13	57.78	19.40	34,083	56.37	58.29	18.75
4	31,740	54.11	59.60	18.90	32,037	54.07	59.46	18.85
5	15,590	55.56	66.03	15.47	16,196	54.42	66.28	15.47
All	92,198	54.86	61.11	18.15	93,289	54.76	61.46	17.80
Year of 2012								
1	794	52.14	75.69	10.96	827	46.55	77.03	11.73
2	11,011	50.54	68.48	15.66	11,132	50.41	68.79	15.28
3	31,096	54.29	57.80	19.33	31,150	54.82	57.86	19.13
4	29,350	51.43	59.55	19.95	29,251	51.01	59.46	19.55
5	12,569	53.67	69.99	14.07	13,018	52.10	69.67	14.42
All	84,820	52.70	61.77	18.21	85,378	52.45	61.82	17.98
Year of 2013								
1	314	52.55	71.97	13.38	382	44.50	71.20	14.92
2	8,970	51.17	65.16	17.46	8,863	48.03	66.32	17.75
3	35,270	54.56	58.17	19.36	34,845	53.72	58.34	19.61
4	25,361	51.80	63.48	17.75	25,110	50.47	63.74	18.08
5	3,300	55.94	75.27	10.85	3,356	51.97	75.66	11.98
All	73,215	53.24	61.70	18.16	72,556	51.77	62.05	18.47

Note: Those can be explained by neither volatility change nor demand pressure change are denoted as "Abnormal."

EXHIBIT 6
Explanatory Power of Volatility Change and Demand Pressure Change for Put Prices

Violation	C ($\Delta P > 0$, $\Delta S > 0$) # of Obs.	$\Delta V > 0$	$\Delta DP > 0$	Abnormal	D ($\Delta P < 0$, $\Delta S < 0$) # of Obs.	$\Delta V < 0$	$\Delta DP < 0$	Abnormal
Category/Ratio (%)								
Year of 2005								
1	6,939	49.69	73.96	13.03	7,211	51.78	72.60	13.77
2	23,865	50.50	61.81	19.01	25,139	52.16	61.22	18.60
3	27,050	57.27	58.34	18.26	28,332	58.41	58.13	18.05
4	5,821	50.03	68.65	15.99	6,430	52.69	67.99	15.16
5	228	46.49	68.42	16.67	290	54.48	70.69	11.38
All	63,903	53.22	62.31	17.76	67,402	54.81	61.82	17.49
Year of 2006								
1	13,107	49.29	70.98	14.72	13,585	50.95	69.32	15.32
2	27,822	49.52	60.69	20.16	29,116	50.42	59.91	20.43
3	24,772	53.77	59.99	19.19	26,456	54.45	59.65	19.05
4	3,740	48.26	72.27	14.57	4,160	50.72	70.05	15.31
5	267	47.94	75.66	11.24	313	51.12	76.04	10.22
All	69,708	50.92	63.05	18.46	73,630	51.98	62.19	18.66

EXHIBIT 6 (*Continued*)

	C ($\Delta P > 0$, $\Delta S > 0$)				D ($\Delta P < 0$, $\Delta S < 0$)			
Violation	**# of Obs.**	**$\Delta V > 0$**	**$\Delta DP > 0$**	**Abnormal**	**# of Obs.**	**$\Delta V < 0$**	**$\Delta DP < 0$**	**Abnormal**
Year of 2007								
1	19,897	49.14	67.84	16.36	20,802	49.39	66.86	16.58
2	34,173	49.26	60.36	20.24	36,099	49.32	59.59	20.40
3	29,103	52.88	60.17	19.33	31,561	52.98	59.97	19.08
4	4,809	48.83	71.01	14.18	5,354	48.66	70.99	14.16
5	663	46.30	75.57	11.76	799	46.68	75.34	11.51
All	88,645	50.38	62.67	18.68	94,615	50.50	62.09	18.69
Year of 2008								
1	21,110	48.96	62.35	18.88	21,443	49.68	62.57	18.61
2	27,732	49.75	58.85	20.82	28,615	50.76	59.03	20.10
3	23,062	51.70	62.66	17.99	24,326	52.38	62.72	17.82
4	7,519	47.87	72.87	13.81	7,819	48.79	72.66	13.90
5	1,771	47.49	74.93	12.70	1,920	48.02	77.81	11.88
All	81,194	49.88	62.49	18.68	84,123	50.71	62.70	18.30
Year of 2009								
1	22,147	48.39	63.58	18.40	23,446	49.21	62.37	18.91
2	25,003	49.39	59.52	20.61	26,521	50.02	58.50	21.04
3	19,507	52.04	63.86	17.84	21,113	52.82	63.76	17.40
4	4,576	49.02	74.41	12.19	5,192	48.42	72.52	13.79
5	501	49.10	76.05	9.18	647	51.78	72.02	11.59
All	71,734	49.78	63.02	18.56	76,919	50.45	62.18	18.82
Year of 2010								
1	20,402	48.62	67.32	16.30	21,492	49.86	67.19	16.21
2	33,451	48.49	58.46	21.32	35,139	49.69	58.87	20.46
3	29,068	51.26	60.02	19.62	30,502	52.49	60.08	19.05
4	5,917	45.45	71.67	14.65	6,440	46.88	70.95	14.02
5	570	42.11	78.25	11.58	631	45.01	78.76	10.46
All	89,408	49.18	61.99	19.12	94,204	50.41	62.12	18.52
Year of 2011								
1	22,158	50.89	64.94	16.57	22,935	50.89	64.65	16.89
2	32,577	50.51	57.66	20.55	33,633	50.80	57.95	20.07
3	28,287	53.13	59.37	18.84	29,710	52.93	59.77	18.52
4	6,472	49.15	71.51	13.41	7,266	49.90	70.19	13.78
5	854	47.66	73.54	14.17	937	48.67	77.59	10.14
All	90,348	51.30	61.12	18.47	94,481	51.40	61.28	18.23
Year of 2012								
1	19,451	51.46	67.29	15.77	19,799	50.97	67.77	15.75
2	29,732	52.81	58.80	19.50	30,765	52.05	58.72	20.12
3	27,351	56.46	59.91	17.58	28,374	55.98	59.42	18.29
4	6,776	52.04	71.22	13.15	7,373	52.41	70.24	13.71
5	378	50.53	70.90	13.23	401	54.86	75.81	11.22
All	83,688	53.62	62.20	17.46	86,712	53.13	62.07	17.94
Year of 2013								
1	11,849	50.49	71.80	13.72	11,906	52.66	71.50	13.62
2	30,867	52.47	61.07	18.39	32,193	53.53	60.99	18.16
3	29,422	56.63	60.17	17.55	30,722	57.68	59.49	17.34
4	3,556	50.03	68.42	16.00	3,896	55.11	66.45	15.20
5	50	42.00	70.00	20.00	57	63.16	73.68	8.77
All	75,744	53.66	62.75	17.22	78,774	55.10	62.27	17.00

Note: Those can be explained by neither volatility change nor demand pressure change are denoted as "Abnormal."

volatility decreases (increases), and 0 otherwise. $\Delta DP = 1$ if demand pressure decreases (increases), and 0 if it does not. For Violation C (D), *Violation*$(\cdot)$ takes the value 1 if a Violation C (D) occurs, and 0 if it does not. $\Delta V = 1$ if volatility increases (decreases), and 0 otherwise. $\Delta DP = 1$ if demand pressure increases (decreases), and 0 if it does not. *Crisis* = 1 if a violation occurs between July 2008 and June 2009, and 0 otherwise. *Moneyness* = 1 if the option is an ATM option, and 0 if it is not. Δt denotes the time length during two consecutive transactions. Following Pérignon [2006], we include the variable Daytrade to control for the day-trade effect. Daytrade takes the value 1 if it is time-stamped after 12 noon, and 0 otherwise. Some investors may be unwilling to hold positions overnight. As the time approaches 1:45 p.m., they thus tend to sacrifice part of their profit in order to close their positions. They will sell call options at a lower (higher) price, or buy put options at a higher (lower) price when the stock index goes up (down) so as to increase their chance for matching.[5]

Since the dependent variable in Equation (1) can take only two values, 1 and 0, the Probit and Logistic models are employed to estimate the regression coefficients. The empirical results are showed in Exhibit 7. As shown in this exhibit, the results for these two models are similar. No matter which type of violations, the regression coefficients for ΔV and ΔDP are all positive and significant at the 1% significance level, and have the highest student *t* values, except Violation A for ΔV. This means that ΔV and ΔDP can mainly explain why the monotonicity violation happens. The coefficients for *Crisis* are significantly negative, implying that with monotonicity, fewer violations occur during the 2008 financial crisis. The coefficients for *Moneyness* are positive significantly, which affirms that ATM options violate more often than non-ATM options. Since ATM options are traded most frequently, it motivates us to include the variable Δt, the time interval between two consecutive transactions, into the regression models. The coefficients for Δt are negative significantly. The longer the time length, the lower the violation ratio.

EXHIBIT 7
Regression Analysis of Violations

Violation	ΔV	ΔDP	Crisis	Moneyness	Δt	Daytrade	Log Likelihood	# of Obs.
Panel A: Probit Model								
A	0.0715**	0.6428**	–0.0263**	0.0973**	–0.0138**	0.012**	–1351452	2,205,352
($\Delta C < 0, \Delta S > 0$)	(1616.54)	(130976)	(85.56)	(2713.46)	(2491.05)	(17.48)		
B	0.1288**	0.6403**	–0.029**	0.1013**	–0.0109**	0.0323**	–1377856	2,261,386
($\Delta C > 0, \Delta S < 0$)	(5372.31)	(132688)	(103.82)	(3001.62)	(1797.06)	(130.67)		
C	0.1522**	0.6632**	–0.0423**	0.0786**	–0.0155**	0.0241**	–1298067	2,167,239
($\Delta P > 0, \Delta S > 0$)	(7082.13)	(134486)	(190.37)	(1630.77)	(2252.68)	(68.74)		
D	0.0952**	0.6635**	–0.0398**	0.083**	–0.0103**	0.0314**	–1344086	2,217,329
($\Delta P < 0, \Delta S < 0$)	(2867.18)	(139066)	(170.69)	(1883.42)	(1479.6)	(121.14)		
Panel B: Logistic Model								
A	0.1188**	1.0481**	–0.0476**	0.1504**	–0.0262**	0.019**	–1351613	2,205,352
($\Delta C < 0, \Delta S > 0$)	(1639.94)	(127594)	(102.28)	(2384.8)	(2083.85)	(16.09)		
B	0.2137**	1.0464**	–0.0521**	0.158**	–0.02**	0.0522**	–1378044	2,261,386
($\Delta C > 0, \Delta S < 0$)	(5418.18)	(129399)	(122)	(2680.71)	(1473.09)	(125.49)		
C	0.2527**	1.0884**	–0.0742**	0.1246**	–0.0289**	0.0394**	–1298175	2,167,239
($\Delta P > 0, \Delta S > 0$)	(7112.88)	(130969)	(212.11)	(1498.91)	(1851.62)	(66.64)		
D	0.1575**	1.0861**	–0.07**	0.1319**	–0.0189**	0.0511**	–1344188	2,217,329
($\Delta P < 0, \Delta S < 0$)	(2871.81)	(135706)	(191.52)	(1748.17)	(1207.13)	(117.51)		

Notes: Violation $= \alpha + \beta_1 \Delta V + \beta_2 \Delta DP + \beta_3 Crisis + \beta_4 Moneyness + \beta_5 \Delta t + \beta_6 Daytrade + \varepsilon.$

*For Violation A (B), Violation $(\cdot)$ takes the value 1 if a Violation A (B) occurs, and 0 if it does not. $\Delta V = 1$ if volatility decreases (increases), and 0 otherwise. $\Delta DP = 1$ if demand pressure decreases (increases), and 0 if it does not. For Violation C (D), Violation $(\cdot)$ takes the value 1 if a Violation C (D) occurs, and 0 if it does not. $\Delta V = 1$ if volatility increases (decreases), and 0 otherwise. $\Delta DP = 1$ if demand pressure increases (decreases), and 0 if it does not. Crisis = 1 if a Violation occurs during July, 2008 to June, 2009, and 0 otherwise. Moneyness = 1 if the option is an ATM option, and 0 if it is not. Δt denotes the time length between two consecutive transactions. Daytrade takes the value 1 if it is stamped at time after 12:00 noon, and 0, otherwise. The number in parentheses is the Chi-Square value of each regression parameter. ** indicates statistical significance at the 0.01 level.*

Finally, this empirical result also confirms that as the time approaches 1:45 p.m., investors tend to sacrifice part of their profit in order to close their positions, and thus the monotonicity violation happens.

CONCLUSION

The monotonicity property is a common feature of one-dimensional diffusion models. Literature has documented empirical evidence of violations of this property. In this article, we examine whether violations occur in an order-driven market characterized by high individual participation and, if they do, what causes them. Using transaction prices for TAIEX options from 2005 to 2013, we find that violations do occur. We argue that these violations are not largely attributable to microstructure effects and find that stochastic volatility and demand pressure are the main factors in explaining the likelihood of violations.

Unlike Bakshi et al. [2000], we use the nearest-the-money implied volatility as a proxy for the volatility of the stock index. We document evidence that 53% of the violations can be explained by the implied volatility, highlighting the importance of expanding the one-dimensional diffusion model to a two-factor model by including volatility as another factor. Taking into consideration the characteristics of TAIEX options and using the net buying pressure measure developed by Shiu, Pan, Lin, and Wu [2010], we find that demand pressure alone can explain 62% of violations, and in combination with stochastic volatility, more than 80% of these violations can be explained. Our results affirm the inclusion of demand pressure by Gârleanu et al. [2009] into options pricing models.

ENDNOTES

[1]Source: http://www.taifex.com.tw/chinese/index.asp.

[2]Source: http://www.taifex.com.tw/chinese/index.asp.

[3]Taiwan Stock Exchange calculates TAIEX and publishes the index every 15 seconds since January 17, 2011.

[4]The correlation coefficients between the TAIEX and the implied TAIEX for each year are about 0.99, with the implied TAIEX being smaller than the TAIEX. Since cash dividends are not considered in deriving the implied TAIEX, this measure is actually much the same as the ex-dividend TAIEX.

[5]Some trading strategies may have a natural impact on the market that tends to produce non-monotonic results. For instance, a covered call involves buying the underlying (tending to push its price upward) and simultaneously selling a call option (thus pushing its price down). In our study, however, a covered call often involves selling a call option and buying the index futures (rather than the spot index itself) because the index is not tradable. Therefore, it is difficult to examine to what extent such strategies may affect our results.

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