The Role of Co-Kurtosis in the Pricing of Real Estate

Executive Summary. *Most prior studies in real estate ignore the existence of systematic kurtosis risk. Using a four-moment capital asset pricing model, this paper examines the impact of co-kurtosis on real estate pricing. It shows that, in the presence of kurtosis, the expected excess rate of return is related not only to the systematic variance and systematic skewness, but also to systematic kurtosis. Investors should request more compensation in terms of expected excess rate of return because they bearing higher co-kurtosis risk. The results point out that real estate systematic kurtosis displays significant risk-return characteristic, and that systematic variance and co-kurtosis are more important than co-skewness in pricing real estate securities. The findings offer additional insights into the measurement of real estate risk. The lack of consideration of systematic kurtosis may lead to an insufficient and irrational premium for the investment risk.*

by Chih-Yuan Yang*
Ming-Chi Chen**

The beginnings of a world boom in home prices in the late 1990s led to renewed interest in the residential real estate market. But with the recent collapse of the housing market along with the sub-prime crisis, real estate risk management has quickly attracted more research attention (Wong, Chau, and Yiu, 2007; Hinkelmann and Swidler, 2008; and Patel and Pereira, 2008).[1] Although the originating assets for these sub-prime mortgages only consist of residential real estate in the United States, once these sub-prime mortgages are packaged together (i.e., securitized) they are sold to investors all over the world. The advantage of this financial innovation is that risks can be distributed throughout the global financial system and held by those investors who wish to bear these risks, whether they are in New York or New Zealand. A harmful downside of the integrated global financial markets, however, is that shocks to the system are transmitted globally with much greater speed, leading to the re-pricing of risk, especially systematic kurtosis risk.

Since many kinds of risk[2] must be considered in real estate risk management, why does this paper focus on systematic kurtosis risk? Before answering, we must first clearly define systematic kurtosis risk. Systematic kurtosis, also called co-kurtosis (Christie-David and Chaudry, 2001), measures the likelihood that extreme returns will occur jointly between the given asset and market portfolio. From the perspective of asset-pricing, kurtosis of a given asset should be jointly analyzed with the kurtosis of the reference market because

*National Sun Yat-sen University, Taiwan or chikyang@gmail.com.

**National Sun Yat-sen University, Taiwan or mcchen@finance.nsysu.edu.tw.

an individual asset return is based on its contribution to the portfolio rather than its own distribution characteristics. That is, only the nondiversifiable portion of this kurtosis, known as systematic kurtosis, matters in equilibrium. Hwang and Satchell (1999) have suggested two possible explanations for the presence of coskewness and co-kurtosis in asset returns in emerging markets: non-stationarity resulting from growing degrees of market integration and the influences of non-economic factors, such as political and social factors.

In addition, it should be noted that extensive studies have demonstrated that the density function of real estate return distributions are often skewed and with relatively fat tails (Myer and Webb, 1993, 1994a; Young and Graff, 1995; Graff, Harrington, and Young, 1997; Lu and Mei, 1999; Liow and Sim, 2005; Young, Lee, and Devaney, 2006; and Young, 2008). Non-normal distributions of real estate returns[3] further revealed the importance of systematic kurtosis risk.[4] Chiao, Hung, and Srivastava (2003) emphasized that ignoring co-kurtosis may bias the estimates in tests for the risk-return trade-off. Accordingly, this could lead to an insufficient and irrational premium for the investment risk without considering the systematic kurtosis risk. The main motivation of this study is, therefore, to examine the role of co-kurtosis in the pricing of real estate. Furthermore, in order to help investors understand systematic kurtosis risk and reduce avoidable loss when making investment decisions in real estate markets, this paper considers the following questions:

1. With regard to public real estate, since its return volatility is generally higher than the market, could additional risk factors such as co-skewness and co-kurtosis be added to supplement the investment risk?
2. Whether the recognition of high-order moments can offer an alternative perspective as to the risk-adjusted returns on real estate? If so, to what extent can these systematic risk measures (co-skewness and co-kurtosis) explain the cross-sectional variation of real estate returns?
3. In the context of portfolio selection, does the co-kurtosis of property returns play a critical role when people are making investment decisions? Will investors request more compensation in terms of expected excess rate of return for bearing higher co-kurtosis risk?

To answer these questions, this study presents the four-moment Capital Asset Pricing Model (CAPM), representing a pricing model for the systematic variance, co-skewness, and co-kurtosis of real estate returns. It shows that, in the presence of kurtosis, the expected excess rate of return is related not only to the systematic variance and systematic skewness, but also to systematic kurtosis. Using the monthly Real Estate Investment Trust (REIT) total return sample included in Center for Research in Security Prices (CRSP) from January 1965 to January 2007, we offer those systematic risk metrics and empirically test their abilities to explain the cross-sectional variations of property returns. The empirical results provide strong evidence that real estate systematic kurtosis displays significant risk-return characteristics, and systematic variance and co-kurtosis are more important than co-skewness in pricing real estate securities. This indicates that real estate investments have very different risk-return characteristics. The lack of consideration of higher moments in pricing real estate often leads to insufficient compensation for investment risk.

The main contribution of this paper is in providing a theoretical and empirical framework to explore the influence of co-kurtosis on real estate returns. This can assist investors in modeling their portfolio, making financial choices, pricing assets, trading investment risk, and dealing with the value at risk. Moreover, the empirical results show a robust relationship in which property returns would be influenced by their contribution to the reference portfolio. This study may be critically important in laying the groundwork for understanding how systematic kurtosis works in real estate returns and offers additional insight into the measurement of real estate risk.

The remainder of the paper is organized as follows. The next section provides a brief review of the relevant finance and real estate literature. Next, there is an interpretation of the fourth moment CAPM model, followed by a description of data sources and empirical framework. The empirical results are then reported and discussed. The paper closes with concluding remarks.

Literature Review

Since the seminal paper by Markowitz (1959), the equilibrium model of Sharpe (1964) and Lintner (1965), the CAPM has become an important tool in finance and real estate for the assessment of the cost of capital, portfolio performance and diversification, valuing investments, and choosing portfolio strategy. This particular theoretical framework restricts the risk-return trade-off to a simple mean-variance relationship, representing the expected rate of return of an individual asset with a measure of its systematic risk. Studies, however, provide only weak empirical evidence on this relationship, showing that the normality hypothesis has to be rejected (e.g., Fama and French, 1992; Davis, 1994; He and Ng, 1994; and Miles and Timmermann, 1996). In addition, Conover, Friday, and Howton (2000) identified an insignificant relationship between the U.S. REIT returns and the beta. Chiang, Lee, and Wisen (2005) also concluded that systematic variance has weak explanatory power in relation to variations in the U.S. REIT returns. Therefore, the hypothesis that CAPM is suitable for explaining real estate return variations is questionable.

Hence, some financial studies consider extensions of the traditional CAPM model that account for higher moment conditions, including systematic variance, systematic skewness, and systematic kurtosis. Rubinstein (1973), Ingersoll (1975), Kraus and Litzenberger (1976), and Sears and Wei (1985) extended the standard two-moment CAPM by incorporating a measure of systematic skewness risk and used the model to explain the risk-return trade-off. According to the three-moment model, investors are willing to pay a premium for assets with positive co-skewness with the market portfolio. The three-moment CAPM corrects for the apparent mispricing of high- and low-risk stocks encountered in the standard two-moment CAPM (Chiao, Hung, and Srivastava, 2003).

Compared with earlier studies on skewness preference, only a few studies have examined the impact of kurtosis on asset returns. Extending the three-moment CAPM, Fang and Lai (1997) argued that investors will not only sacrifice the expected excess returns in return for receiving the benefit of increasing the systematic skewness, but also require higher expected returns in return for bearing the systematic kurtosis risk. Using a GMM approach, Hwang and Satchell (1999) show that systematic kurtosis explains the emerging market returns better than systematic skewness. Christie-David and Chaudry (2001) employ the four-moment CAPM on the futures market and point out that systematic skewness and systematic kurtosis increase the explanatory power of the return generating process of the futures market. Ranaldo and Favre (2003) also consider the four-moment CAPM for pricing hedge funds and conclude that "the lack of consideration of higher moments may lead to an insufficient compensation for the investment risk."

In the real estate literature, Liu, Hartzell, and Grissom (1992) consider the presence of skewness for real estate assets using appraisal-based data. Employing the three-moment model of Kraus and Litzenberger (1976), they suggest that investors are willing to accept a lower expected return on real estate assets because of the lower negative coskewness. Using the Kraus and Litzenberger model, Vines, Hsieh, and Hatem (1994) examine the role of systematic covariance and co-skewness in the pricing of equity REITs. They find that systemic risk impacts return, as predicted, but there is no evidence indicating that co-skewness is a determinant of equity REIT return. This result is consistent with Cheng (2005), who finds that coskewness does not explain real estate returns well and thus is not a good systematic risk measure. In addition, Liow and Chan (2005) analyzed real estate securities returns in 19 countries and concluded that co-kurtosis is more important than coskewness in pricing global real estate securities. Although the central issue of Lee, Robinson, and Reed (2008) is downside risk, their findings demonstrate that high downside beta LPTs (listed property trusts) have higher co-kurtosis and lower coskewness and the inclusion of co-kurtosis and coskewness will lower the significance of downside beta in explaining LPT returns. But it should be noted that most real estate studies do not consider the existence of systematic kurtosis risk, except

Liow and Chan (2005) and Lee, Robinson, and Reed (2008).

According to the above review, the ability of higher-order co-moments (systematic variance, systematic skewness, and systematic kurtosis) to explain real estate returns appears to be the lack of a comprehensive theoretical and empirical framework. Hence, this study tries to fill this gap in the literature.

Modeling the Fourth Moment in the Capital Asset Pricing

Following Kraus and Litzenberger (1976) and Fang and Lai (1997), it is assumed that there is one risk-free asset and n risky assets in the market. Define R_f as the rate of return on the risk-free asset, R as a $n \times 1$ vector of the rates of returns of risky assets, $\overline{R}$ as a $n \times 1$ vector of the excepted rates of returns of risky assets, and V as a $n \times n$ variance-covariance matrix of the n risky assets. All assets are assumed to have limited liability and their returns are aggrandized only in the form of capital gains (i.e., no dividends). The market is perfect and competitive with no taxes and transaction costs and all investors hold homogeneous beliefs about the asset returns. Each investor seeks to maximize his or her expected utility, which can be represented by the mean, variance, skewness, and kurtosis of terminal wealth, subject to the budget constraint.

Beginning from the viewpoint of an individual investor, consider an investor who invests x_i of his original wealth in the ith risky asset, and $1 - \Sigma x_i$ in the risk-free asset. Assume the rate of return on the investor's portfolio of risky assets is nonsymmetrically distributed. Hence, the first four moments of the distribution of these portfolio excess returns are $X'(\overline{R} - R_f)$, $X'VX$, $E(X'(R - \overline{R})/\sqrt{X'VX})^3$, and $E(X'(R - \overline{R})/\sqrt{X'VX})^4$; where $X' = (x_1, x_2, \ldots, x_n)$ is a $n \times 1$ vector of the investor's holdings of risky assets. To simplify the model, the portfolio can be rescaled since the relative percentage invested in different assets is relevant. Use the standard deviation of the portfolio return to rescale the portfolio, and then let the variance of the portfolio return is unit (i.e., $X'VX = 1$). The investor's preference thus can be defined over the mean, variance, skewness, and kurtosis of the terminal wealth, subject to the unit variance. As in previous models, the increase of the mean and skewness of terminal wealth will increase the investor's utility. On the contrary, the increase of the kurtosis of terminal wealth represents the higher probability of the extreme outcome of terminal wealth, which will result in a benefit or cost to investors. Under the principle of diminishing marginal utility, consequently, the marginal utility of kurtosis is assumed to be negative in the following derivation.

To maximize the investor's expected utility for the end of period wealth, subject to the budget and unit variance constraints, the Lagrangian can be constructed as:

$$\text{Max } U\{X'(\overline{R} - R_f), E(X'(R - \overline{R}))^3, E(X'(R - \overline{R}))^4\} - \lambda(X'VX - 1), \qquad (1)$$

where λ is the Lagrangian multiplier of unit variance constants. Using the first-order conditions to solve for the investor's portfolio equilibrium conditions, yields:

$$\overline{R} - R_f = \frac{2\lambda}{U_1} VX - \frac{3U_2}{U_1} \text{Cov}[(X'(R - \overline{R}))^2, R] - \frac{4U_3}{U_1} \text{Cov}[(X'(R - \overline{R}))^3, R], \qquad (2)$$

where $\text{Cov}[(X'(R - \overline{R}))^i, R]$ is the $n \times 1$ covariance vector of asset return R and the portfolio return $(X'(R - \overline{R}))^i$ for $i = 2, 3$. U_i is the partial derivative with respect to the ith argument $i = 1, 2, 3$.

In order to move from the equilibrium conditions for individual investors to a model of market equilibrium, it is necessary to invoke the separation theorem, which assumes all investors hold the same probability beliefs and have identical wealth coefficients.[5] By the separation theorem, the portfolio held by investors must be the market portfolio in order to clear the market.

Let R_m be the market portfolio return with $R_m = X_m'(R - R_f)$ and thus $X_m'VX_m = 1$. The asset pricing model with skewness and kurtosis can then be derived from Equation (2) as:

$$\overline{R} - R_f = \alpha \operatorname{Cov}(R_m, R) + \beta \operatorname{Cov}(R_m^2, R) + \gamma \operatorname{Cov}(R_m^2, R), \quad (3)$$

where R_m^2 is the square of the standardized market portfolio return R_m, while R_m^3 is the cube of the standardized market portfolio return R_m. α, β, and γ can be regarded as the market prices of the systematic variance, systematic skewness, and systematic kurtosis, respectively.

Equation (3) is the four-moment CAPM[6] derived in Fang and Lai (1997). It shows that, in the presence of kurtosis, the expected excess rate of return is related not only to the systematic variance and systematic skewness, but also to systematic kurtosis. In addition, it is implied that only the systematic kurtosis rather than the total kurtosis of asset return is relevant in the asset valuation. Investors are compensated in terms of expected excess rate of return for bearing the systematic kurtosis risk. An intuitive explanation of the effect of kurtosis on asset pricing may rely on covariance as the appropriate risk measure. Ceteris paribus, the greater the covariance of the asset return and the cube of market portfolio return, the greater the extreme return, which cannot be diversified. This implies that there is higher systematic kurtosis risk contributed by the asset.

Data Description and Empirical Framework

Our sample consists of all REITs included in the CRSP monthly total return data from January 1965 to January 2007, with a minimum of 36 months (3 years) of time series returns. This criterion is intended to ensure that the time-series regressions are statistically valid. The final sample contains 382 REITs comprising 47,992 REIT-months. In addition, the return of the S&P 500 index is selected as the proxy of "market return." This choice of market return implies that the investigation is in the context of U.S. domestic portfolios diversification. Finally, yields on 3-month T-bills are also collected for the same period as a proxy for the risk-free rate.

The linear empirical version of the four-moment CAPM of Equation (3) is rewritten as:

$$\overline{R}_i - R_f = b_1 Beta_i + b_2 CSK_i + b_3 CKU_i, \quad (4)$$

$$\text{where } Beta_i = \frac{\operatorname{Cov}[R_i - E[R_i], R_m - E[R_m]]}{\operatorname{Var}(R_m)}, \quad (5)$$

$$CSK_i = \frac{\operatorname{Cov}[R_i - E[R_i], (R_m - E[R_m])^2]}{E[(R_m - E[R_m])^3]}. \quad (6)$$

$$CKU_i = \frac{\operatorname{Cov}[R_i - E[R_i], (R_m - E[R_m])^3]}{[\operatorname{Var}(R_m)]^2}, \quad (7)$$

where $\overline{R}_i$ is the expected rate of return on the ith REIT (risky real estate asset). $Beta_i$, CSK_i, and CKU_i are the systematic variance, co-skewness, and co-kurtosis of REIT i, respectively. b_1, b_2, and b_3 are the market premiums for the respective risk.

The computation of Equations (5)–(7) assumes that asset returns are independently and identically distributed. However, several studies have suggested that real estate returns may not be independent. For instance, Firstenberg Ross, and Zisler (1988) and Liu, Hartzell, and Grissom (1992) suggest that "appraisal smoothing" may cause strong autocorrelation in real estate returns. Using NCREIF property indices, Myer and Webb (1994b) find non-normality and autocorrelation present in most of the nominal quarterly real estate series. They suggest that skewness should be examined based on conditional return distributions. However, Cheng (2005) claims that the empirical results of conditional skewness and co-skewness are almost identical to those of unconditional measures. He thinks this result is prospective, because linear transformation (such as auto-regression) of a random variable would not change the location of the distribution. Nevertheless, this paper still considers the effects of conditional high order co-moments to examine the robustness of empirical results. Extending the concept of Cheng (2005) and using the residuals of autoregressions of the asset return series, the linear empirical version of conditional high order comoments can be computed with the following equations:

$$\overline{R}_i - R_f = b_1 Beta_i^{cond} + b_2 CSK_i^{cond} + b_3 CKU_i^{cond}, \quad (8)$$

where Conditional Beta: $Beta_i^{cond} = \dfrac{E[\varepsilon_i \varepsilon_M]}{E[\varepsilon_M^2]}$ (9)

Conditional Co-Skewness: $CSK_i^{cond} = \dfrac{E[\varepsilon_i \varepsilon_M^2]}{E[\varepsilon_M^3]}$, (10)

Conditional Co-Kurtosis: $CKU_i^{cond} = \dfrac{E[\varepsilon_i \varepsilon_M^3]}{E[\varepsilon_M^4]}$ (11)

where ε_i is the autoregression residuals of real estate i's return, σ_ε is the standard error of the autoregression residuals of asset return, and ε_M is the autoregression residuals of market return.

Empirical Results

The empirical results have three major steps. The first is the examination of the normality of individual REITs. By using ordinary least squares regression analysis, the second estimates the cross-sectional correlations of systematic variance, co-skewness, co-kurtosis, and property excess returns. In addition, this study also checks for the robustness verification by adopting conditional, rather than unconditional, high order co-moments in the regression analysis.

Normality Tests

Five different normality tests were employed—Jarque-Bera, Lilliefors, Cramer-von Mises, Watson, and Anderson-Darling—with the results reported in Exhibit 1. It is apparent that the returns for REITs are not normally distributed due to the strong normality rejection results. The Jarque-Bera test revealed that more than half of REITs in the sample are not normally distributed. However, these results are not surprising since the individual REIT return distributions are, in general, highly peaked and positively skewed.

It should be noted that all normality tests confirmed that about half of the U.S. REITs in the sample are not normally distributed, where the normality assumptions can be rejected at least at the 5% level. These findings confirm that most

Exhibit 1
Normality Tests of REIT Returns

	Jarque-Bera	Lilliefors	Cramer-von Mises	Watson (U2)	Anderson-Darling
Percentage of Rejected Number REITs over the Sample with 10% Significance Level	73.04% (279)	58.64% (224)	70.94% (271)	69.63% (266)	71.99% (275)
Percentage of Rejected Number REITs over the Sample with 5% Significance Level	71.47% (273)	51.57% (197)	63.87% (224)	63.61% (243)	65.97% (252)
Percentage of Rejected Number REITs over the Sample with 1% Significance Level	63.09% (241)	39.53% (151)	52.62% (201)	53.14% (203)	53.66% (205)

Note: The hypothesis is normal distribution. The figures presented in this table represent the percentage of REITs in the sample rejected by normality tests, while the rejected numbers are reported in parentheses. Total sample is 382.

REITs in the U.S. are not normally distributed (Young, 2008). These results are consistent with previous studies where real estate return distributions, either in emerging or developed markets, are not necessarily normally distributed. From the standpoint of risk diversification, portfolios whose return distributions have positive or negative tails fatter than portfolios whose return distributions are Gaussian normal require more assets to produce the same risk reduction. More importantly, these also support the use of asymmetric risk measures over traditional risk measures.

Systematic Variance, Co-skewness, and Co-kurtosis Estimates

Exhibit 2 presents the coefficient estimates and the standard errors of the cross-sectional regressions of systematic risk in REITs returns. The results in Exhibit 2 show a strong correlation between excess returns and systematic variance. The coefficients of systematic variance in all regressions are negative and statistically significant at the 1% level related to real estate returns. According to the four-moment CAPM, there is a direct association between systematic variance and returns. This implies investors need to consider the covariance between real estate and the reference market because real estate systematic variance has a significant risk-return characteristic.

From Column (3) in Exhibit 2, we consider systematic kurtosis in the regression analysis, finding that the coefficient of co-kurtosis is negative and statistically significant at 1%. This result is consistent with Chiao, Hung, and Srivastava (2003) and emphasizes that ignoring the co-kurtosis may bias the estimates in tests for the risk-return trade-off. Additionally, it is not surprising that the inclusion of systematic variance and co-skewness will lower the significance of co-kurtosis, as in Columns 5 and 7 in Exhibit 2.

Consistent with previous studies, Cheng (2005) and Chen, Yang, and Kao (2008), the ability of systematic skewness in explaining real estate returns is questionable. Since co-skewness measures the relative skewness of individual assets to the market portfolio, these results indicate that asset returns may be only slightly affected by the individual asset's systematic skewness. It is likely that

Exhibit 2

Results of Cross-Sectional Regressions for Testing the Explanatory Ability of Systematic Risk Measures

$E(R_{it} - R_{ft})$	1	2	3	4	5	6	7
Constant	−0.0359 (0.0016)	−0.0453 (0.0010)	−0.0380 (0.0015)	−0.0359 (0.0016)	−0.0359 (0.0016)	−0.0379 (0.0015)	−0.0359 (0.0016)
Systematic variance	−0.0169*** (0.0023)			−0.0169*** (0.0023)	−0.0123*** (0.0043)		−0.0120*** (0.0043)
Co-skewness		2.62E-05 (3.72E-05)		2.82E-05 (3.49E-05)		3.97E-05 (3.53E-05)	3.26E-05 (3.50E-05)
Co-kurtosis			−0.0144*** (0.0022)		−0.0050* (0.0030)	−0.0145*** (0.0022)	−0.0054* (0.0029)
$\bar{R}^2$	0.1176	−0.0013	0.1026	0.116757	0.119078	0.1032	0.1189
F-Statistic	51.7589***	0.49607	44.5489***	26.1824***	26.7507***	22.9252***	18.1168***

Notes: This table presents the results of cross-sectional regressions for testing the explanatory ability of systematic risk measures in explaining the variation of real estate excess returns. The dependent variable is the historical average of REITs excess returns and the independent variables are systematic variance, co-skewness, and co-kurtosis. The standard errors are reported in parentheses. $N = 382$.

$$\bar{R}_i - R_f = b_1 Beta_i + b_2 CSK_i + b_3 CKU_i.$$

*Significant at the 1% level.
**Significant at the 5% level.
***Significant at the 10% level.

investors exclude real estate assets from their short-term portfolio due to the illiquidity of the property market. Furthermore, the main purpose of holding an owner-occupied home is to live in it, rather than for investment. These two reasons can partially explain why co-skewness does not effectively explain the variation of real estate returns. Our findings are, nevertheless, not completely consistent with the previous real estate literature (Liu, Hartzell, and Grissom, 1992; Myer and Webb, 1994; Liow and Chan, 2005; and Lee, Robinson, and Reed, 2008). But because these studies provide different viewpoints and complement each other, the overall compatibility of the results is all the more impressive.

In summary, the empirical results point out that real estate systematic kurtosis has a significant risk-return characteristic, and systematic variance and co-kurtosis are more important than co-skewness in pricing real estate securities. The findings of this study offer additional insight into the measurement of real estate risk. The lack of consideration of systematic kurtosis may lead to an insufficient and irrational premium for the investment risk.

Robustness: Conditional Systematic Risk

This study also checks for the robustness of conditional high order co-moments in the regression analysis due to the autocorrelation in real estate returns suggested in the literature (Firstenberg Ross, and Zisler, 1988; Liu, Hartzell, and Grissom, 1992; and Myer and Webb, 1994). Recalling Equations 9 to 11, we start by using the auto-regression residuals of asset returns to compute conditional high order co-moments, and then put them into empirical analysis instead of the variables in the regressions reported in Exhibit 2. Exhibit 3 shows that the coefficients of conditional co-kurtosis, but not conditional co-skewness, are negative and statistically significant. Clearly, the outcome of conditional high order co-moments is very similar to the result for unconditional variables in Exhibit 2. Our results are compatible with Cheng (2005), who stated that "this is expected because linear transformation (such as auto regression) of a random variable only changes the location, not the shape, of the distribution." Therefore, we can confirm the influence of high order co-moments on real estate returns when the effect of autocorrelations in the return series is removed.

Exhibit 3
Results of Cross-Sectional Regressions for Testing the Explanatory Ability of Conditional Systematic Risk

$E(R_{it} - R_{ft})$	8	9	10	11	12	13	14
Constant	−0.0384	−0.0452	−0.0377	−0.0384	−0.0377	−0.0377	−0.0377
	(0.0014)	(0.0010)	(0.0015)	(0.0014)	(0.0015)	(0.0015)	(0.0015)
Conditional systematic variance	−0.0153***			−0.0152***	−0.0112***		−0.0110**
	(0.0021)			(0.0022)	(0.0043)		(0.0043)
Conditional co-skewness		1.19E-06		2.24E-07		5.29E-07	2.68E-07
		(8.84E-07)		(8.44E-07)		(8.44E-07)	(8.44E-07)
Conditional co-kurtosis			−0.0147***		−0.0048*	−0.0145***	−0.0049*
			(0.0022)		(0.0025)	(0.0022)	(0.0027)
$\overline{R}^2$	0.1176	0.0021	0.1045	0.1154	0.1181	0.1031	0.1160
F-Statistic	51.7636***	1.8131	45.4804***	25.8538***	26.5102***	22.9002***	17.6651***

Notes: This table presents the results of cross-sectional regressions for testing the explanatory ability of conditional systematic risk measures in explaining the variation of real estate excess returns. The dependent variable is the historical average of REITs excess returns and the independent variables are conditional systematic variance, conditional co-skewness, and conditional co-kurtosis respectively. The standard errors are reported in parentheses. $N = 382$.

$$\overline{R}_i - R_f = b_1 Beta_i^{cond} + b_2 CSK_i^{cond} + b_3 CKU_i^{cond}.$$

*Significant at the 1% level.
**Significant at the 5% level.
***Significant at the 10% level.

Conclusion

Real estate is a multi-dimensional asset that can be regarded both as a durable consumer good offering a flow of services such as shelter, and as a commodity for investment by which rental income or capital gains are earned. Along with the trend towards asset-backed securitization and portfolio globalization, the investment risks of real estate have played increasingly important roles in the global financial markets. That the domestic subprime crisis in the U.S. quickly developed into a worldwide crisis affecting global stocks and foreign markets is a dramatic example. Therefore, investors cannot ignore the systematic kurtosis risk, which measures the jointly occurring possibility of the extreme returns of those assets in their portfolios. Very little literature, however, has directly addressed the importance of co-kurtosis risk. In order to identify this risk, we present the four-moment CAPM, representing a pricing model for the systematic variance, co-skewness, and co-kurtosis of the real estate returns. Drawing from the theoretical model, the expected excess rate of return is related not only to the systematic variance and systematic skewness, but also to systematic kurtosis. Investors will require more compensation in terms of expected excess rate of return because they bear the co-kurtosis risk.

In addition, this study proposes these systematic risk metrics and empirically tests their abilities in explaining the cross-sectional variations of property returns using the monthly REIT total return sample included in CRSP from January 1965 to January 2007. The empirical results provide strong evidence that real estate systematic kurtosis has a significant risk-return characteristic, and that systematic variance and co-kurtosis are more important than co-skewness in pricing real estate securities. This shows that the lack of consideration of higher moments in pricing real estate in many cases leads to an insufficient compensation for the investment risk.

The main contribution of this paper is in providing a theoretical and empirical framework to explore the influence of co-kurtosis on real estate returns. The findings demonstrate the importance of co-kurtosis risk and provide additional insight into the risk-return characteristics of the real estate market. Further research could investigate other types of real estate returns, like real estate indices and stocks. It would be rewarding to consider how different trading types affect systematic kurtosis risks in the pricing of real estate returns.

Endnotes

1. Shiller (2008) described the recent "subprime crisis" as the result of failure to mange risks.
2. For example, liquid risk, interest risk (He, Myer, and Webb, 1996; and Koutmos and Pericli, 1999), asymmetric risk (Cheng, 2005), and inflation risk (Chen and Sing, 2006; and Hoesli, Lizieri, and MacGregor, 2008).
3. Some real estate studies show that the normality of asset returns appears to be sensitive to the data frequency employed. Using conventional statistical approaches on monthly data, Lee (2002) reported positive skewness in the majority of markets while Booth, Matysiak, and Ormerod (2002) found evidence of kurtosis, and in particular fat tails. In contrast, Maurer, Reiner, and Sebastian (2004) found no evidence of either skewness or kurtosis using quarterly data from the United Kingdom. This result is consistent with Lizieri and Ward (2000), who found that quarterly data generally fits a normal distribution, whereas monthly returns were found to be non-normal. But Young, Lee, and Devaney (2006) used the IPD U.K. annual data from1981 to 2003 and emphatically rejected the normality of distributions of individual property returns.
4. Chen, Yang, and Kao (2008) indicate that kurtosis can depict some features of asset return distributions that skewness cannot capture. For example, a return distribution with zero-skewness but high kurtosis still brings higher investment risk than normal.
5. A necessary and sufficient condition for the composition of each investor's optimal risk asset to be the same is that all investors have identical wealth linear coefficients and are under the same probability beliefs. The same assumption has been used by Cass and Stiglitz (1970), Kraus and Litzenberger (1976), Cox, Ingersoll, and Ross (1985), Fang and Lai (1997), and Chiao, Hung, and Srivastava (2003).
6. In the mean-variance framework, the systematic skewness and kurtosis would not be priced and Equation 3 collapses to the CAPM. In the three-moment framework, systematic kurtosis is not priced and Equation 3 is reduced to the Kraus and Litzenberger (1976) version of three-moment CAPM.

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