Int J Adv Manuf Technol (2005) 26: 109–116
DOI 10.1007/s00170-003-1982-6

ORIGINAL ARTICLE

Su-Fen Yang

Dependent processes control for overadjusted process means

Received: 20 June 2003 / Accepted: 4 October 2003 / Published online: 29 March 2004

Abstract Overadjustment to processes may result in shifts in process mean, ultimately affecting the quality of products. An economic model is developed for the joint economic design of individual $X$and cause-selecting control charts to control both means of the dependent processes. The objective is to determine the design parameters of the proposed control charts that minimize the total quality control cost. A Markov chain approach is proposed to derive the economic- adjustment model. Application of the model is demonstrated through a numerical example.

Keywords Adjustment · Dependent processes control · Markov chain · Renewal reward processes · Special causes

1 Introduction

Control charts are an important tool of statistical quality control. These charts are used to monitor and maintain current control of a process. Deming [2] explains that a production worker can mistakenly overadjust or underadjust a process. He further explains that the control chart provides 'a rational and economic guide to minimize loss from both mistakes'. Economic design of control charts is first proposed by Duncan [3]. The pioneering work of Duncan is then extended by others. A review of the literature is available in Montgomery [6], Vance [9], or Ho and Case [4]. Economic design optimizes the model by considering the cost of underadjustment along with other costs; however, it assumes that the search for a special cause is perfect.

A common problem in statistical process control is process overadjustment. Information about the state of the process is available only through sampling. When a control chart indicates that the process is out of control, it requires adjustment. Sometimes the process may be adjusted unnecessarily, when a false alarm occurs. Saniga [8] describes that economic design of control charts does not consider statistical properties when selecting the design parameters for a control chart. Woodall [12] noted that the effect of overadjustment is an increase in variability. This increase in variability and resultant loss of quality can be quite significant. He describes the probability of Type I error in an economic design as being much higher than that in a statistical design. This results in greater false alarm frequency, which leads to overadjustment, and ultimately an increase in the variability of the quality characteristic. Collani et al [1] first solved this problem through an economic adjustment model for the $\overline{X}$ control chart with a single special cause that considers the effects of process overadjustment and underadjustment. Their model determines the design parameters of the $\overline{X}$ control chart that maximize the profitability of the process, or equivalently, minimize the cost of overadjustment and underadjustment. Yang and Rahim [13] propose a Makovian chain approach to derive the economic adjustment model for the $\overline{X}$ and S control charts that considers the effects of process mean and variance overadjustment and underadjustment. However, they only solve the problem for a single process.

Today, many industrial products are produced by several dependent Processes, not just one process. Consequently, it is not appropriate to monitor these processes with a control chart for each individual process, what is needed is an appropriate method for controlling the processes. Zhang [14] proposes the simple cause-selecting chart to monitor the second process of the two dependent processes. Wade and Woodall [11] review the basic principles of the cause-selecting chart for two dependent processes and suggest a modification to the use of a simple cause-selecting chart. They also examine the relationship between the simple cause-selecting chart and the multivariate T^2 control chart. In their opinion, the simple cause-selecting control chart has some advantages over the T^2 control chart. However, the process-control approach to effectively distinguish and monitor the dependent processes for an overadjusted process mean has not been addressed. This paper considers that the incoming quality of the first process and the outgoing quality of the second process can be affected by a special cause, resulting in

S.-F. Yang (✉)
Department of Statistics,
National Chengchi University Taipei,
116, Taiwan, R.O.C.
E-mail: yang@nccu.edu.tw
Fax: 886-2-29398024

shifts in the process mean due to overadjustment during operation. The individual X and cause-selecting control charts are used to signal the special cause, which result in a shift of the process mean. A Markovian chain approach is extended. The proposed approach allows easier derivation of the expected cycle time and the expected cycle cost than that of others. The paper is organized as follows. In the next section, the economic adjustment model is derived using a Markov chain approach. An optimization technique is used to determine the optimal design parameters of the individual X and cause-selecting control charts that minimize the cost of the production process. An example is provided to illustrate an application of the individual X and cause-selecting control charts. A brief summary concludes the paper.

2 Economic-adjustment model

2.1 Problem statement

In a production system, suppose that there are two dependent processes, which may have a failure mechanism. If the processes experience a failure mechanism, it goes out of control; otherwise, it is in control. The failure mechanism may occur only in the first process and shifts the mean of the quality variable (X) or the second process and shifts the mean of the quality variable (Y). The in-control process becomes out of control if it is overadjusted. The overadjustment means the operator adjusted the process when adjustment was unnecessary. The out-of-control process keeps out of control if it is underadjusted. The underadjustment means the operator did not adjust the process when adjustment was necessary. The quality variable Y is influenced by the quality variable X because the processes are dependent. How should we distinguish and detect the shifts of process mean on the two dependent processes? In this analysis, individual X and cause-selecting control charts are used to signal the need for adjustment of the first and the second processes, respectively. The problem here is: What is the economic dependent processes control policy? That is, what are the control charts and how does the overadjustment affect the performance of the processes control? Specifically, a sample of size n units of output is taken every hour (h), and the process is adjusted if its sample mean falls outside the control limits of its control chart. The objectives are to derive the economic adjustment model and to determine the parameters n, h, k_1 (control-limit coefficients of the individual X control chart) and k_2 (upper control-limit coefficient of cause-selecting control chart) so that the average long-term cost of the processes is minimized and the economic adjustment individual X and cause-selecting control charts are proposed.

2.2 Description of the production process

When random samples of size one are taken from the second process at every sampling time interval h, we get pairs of observations (x, y). The model relating the two variables (X, Y) can take many forms. Because Y is influenced by X, we take one of the most useful models, the simple linear regression model. We let

$$Y_i\,|X_i = a_0 + a_1 X_i + \varepsilon_i, \quad i = 1, 2, 3, \ldots, n,$$

where a_0 and a_1 are constants, and ε_i is a random error, $\varepsilon_i \sim NID(0, \sigma_\varepsilon^2)$.

However, the model does not need to be linear; it can also be applied to a nonlinear model. To monitor the two dependent processes effectively, two control charts are constructed to control the first process and the second process, respectively. To monitor the first process, the individual X control chart is set up based on the in-control distribution of X. To monitor the second process, the specific quality of the second process can be specified by adjusting the effect of X on Y; that is, the specific quality is presented by the cause-selecting values, $e_i = Y_i|X_i - \hat{Y_i}\,|X_i$. The cause-selecting control chart is set up based on the in-control distribution of cause-selecting values.

Assume that, when the first process and the second process are all in control, $X \sim \mathrm{N}(\mu_X, \sigma_X^2)$ and $\sim \mathrm{N}(0, \sigma_e^2)$. When a special cause SC occurs, there may be a shift in the distribution of X to $X \sim \mathrm{N}(\mu_X + \delta_1\sigma_X, \sigma_X^2)$ with probability w or a shift in the distribution of $\sim \mathrm{N}(\delta_2\sigma_e, \sigma_e^2)$ with probability $1-w$, $\delta_1, \delta_2 \neq 0$. The time until occurrence of a special cause is assumed to be exponentially distributed with a mean of $1/\lambda$. It is also assumed that the processes are not self correcting and the time to sample and plot x is negligible.

An adjustment to the processes is performed if the sampled x value or/and cause-selecting value fall outside the control limits of the X or/and cause-selecting control charts, respectively, LCL_X, UCL_X, LCL_e and UCL_e, where

$$\begin{aligned} \mathrm{LCL}_X &= \mu_X - k_1\sigma_X \\ \mathrm{UCL}_X &= \mu_X + k_1\sigma_X \\ \mathrm{LCL}_e &= k_2\sigma_e \\ \mathrm{UCL}_e &= -k_2\sigma_e \end{aligned}$$

If the parameters μ_X, σ_X and σ_e are unknown, we take the in-control $\overline{x}$ (sample mean), $\overline{MR}_X$ (average moving range of X observations)$/d_2$, and $\overline{MR}_e$ (average moving range of cause-selecting values)$/d_2$ to be the estimates of μ_X, σ_X and σ_e, respectively, where d_2 is the factor for center line of the range chart.

The processes correct adjustment and overadjustment can take one of the forms following the alarm from the X chart or the cause-selecting chart:

- When the shift results in $X \sim \mathrm{N}(\mu_X + \delta_1\sigma_X, \sigma_X^2)$ and only X chart has an alarm, special cause is adjusted to let the mean of X be μ_X.
- When the shift results in $e \sim N(\delta_2\sigma_e, \sigma_e^2)$ and cause-selecting chart has an alarm, special cause is adjusted to let the mean of e be 0.
- When the processes are in control but only the X chart has an alarm, the first process is overadjusted to let $X \sim \mathrm{N}(\mu_X + \delta_1\sigma_X, \sigma_X^2)$.

- When the processes are in control but only the cause-selecting chart has an alarm, the second process is overadjusted to let $e \sim \mathrm{N}(\delta_2\sigma_e, \sigma_e^2)$.
- When the processes are in control but both X and cause-selecting charts have alarms, the first process is overadjusted to let $X \sim \mathrm{N}(\mu_X + \delta_1\sigma_X, \sigma_X^2)$ with probability w and the second process is overadjusted to let $e \sim \mathrm{N}(\delta_2\sigma_e, \sigma_e^2)$ with probability $1-w$ because a special cause can only influence one of the processes.
- When the first process is out of control but only the cause-selecting chart has an alarm, SC is adjusted, the second process is overadjusted to let $e \sim \mathrm{N}(\delta_2\sigma_e, \sigma_e^2)$ and the first process is correct adjusted to let $X \sim \mathrm{N}(\mu_X, \sigma_X^2)$.
- When the second process is out of control but only the X chart has an alarm, special cause is adjusted, the first process is overadjusted to let $X \sim \mathrm{N}(\mu_X + \delta_1\sigma_X, \sigma_X^2)$ and the second process is correct adjusted to let $e \sim \mathrm{N}(0, \sigma_e^2)$.
- When only the first process is out of control but X and The cause-selecting charts have alarms, the first process is correct adjusted to let $X \sim \mathrm{N}(\mu_X, \sigma_X^2)$ and the second process is overadjusted to let $e \sim \mathrm{N}(\delta_2\sigma_e, \sigma_e^2)$; similar to when only the second process is out of control but X and cause-selecting charts have alarms.

The decision rule can result in an overadjustment following false alarm for either the first process or the second process or for both together. It is assumed that a transition in the process from in control to out of control during sampling is impossible. The following notation is used.

2.3 Defining the probabilities of overadjustment and underadjustment

α_X : Probability that the first process is overadjusted when the individual X control chart gives a false alarm, where

$$\alpha_X = 1 - P(\mathrm{LCL}_X \le X \le \mathrm{UCL}_X | X \sim \mathrm{N}(\mu_X, \sigma_X^2))$$
$$= 2\Phi(-k_1),$$

and $\Phi(.)$ is the cumulative probability of a normal distribution.

α_e : Probability that the second process is overadjusted when cause-selecting control chart gives a false alarm, where

$$\alpha_e = 1 - P(\mathrm{LCL}_e \le e \le \mathrm{UCL}_e | e \sim \mathrm{N}(0, \sigma_e^2))$$
$$= 2\Phi(-k_2).$$

α : Probability that either X or cause-selecting control chart indicates an alarm, when both processes are in control, where

$$\alpha = \alpha_X + \alpha_e - \alpha_X\alpha_e.$$

β_X : Probability that the first process is underadjusted because it is affected by a special cause, where

$$\beta_X = P(\mathrm{LCL}_X \le X \le \mathrm{UCL}_X | X \sim \mathrm{N}(\mu_X + \delta_1\sigma_X, \sigma_X^2))$$
$$= \Phi(k_1 - \delta_1) - \Phi(-k_1 - \delta_1).$$

β_e : Probability that the second process is underadjusted when it is affected by a special cause, where

$$\beta_e = P(\mathrm{LCL}_e \le X \le \mathrm{UCL}_e | e \sim N(\delta_2\sigma_e, \sigma_e^2))$$
$$= \Phi(k_2 - \delta_2) - \Phi(-k_2 - \delta_2).$$

2.4 Defining the terms associated with times and costs

T_f: expected time of overadjustment following a false alarm
T_{sc}: time before the special cause occurs in the process, $T_{sc} \sim \exp(\lambda_i)$
T_{sr}: expected time to search and repair the special cause
C_f: expected cost of overadjustment
C_0: production cost per unit time when the process is in control
C_1: production cost per unit time when the process is affected by a special cause
C_{sr}: expected cost to search and repair a special cause
a: fixed cost per sample and test
b: cost per unit sampled and tested
τ : expected arrival time of the special cause, given that it occurred in the first sampling interval, where $\tau = \frac{1-(1+\lambda h)\,\mathrm{e}^{-\lambda h}}{\lambda - \lambda\,\mathrm{e}^{-\lambda h}}$ (see Lorenzen and Vance [5])

2.5 Description of Markov chain

In order to use the Markov chain approach to derive the expected cycle time (ET) and the expected cycle cost (EC), all possible states at the end of each sampling and testing time must be examined. Depending on the state of the system, the transition probabilities and transition costs can be computed. There are 12 possible states at the end of every sampling and testing time, and these states are defined as as in Table 1.

These states can be classified into two types of states: transient states and absorbing states. States 6 and 11 are absorbing states, the others are transient states. Transition probability from state i to state j in time interval h is described in Appendix 1.

Table 1. Definition for each state

State	SC occurs and which process?	X chart signal?	Cause-selecting signal?	Process overadjustment and which process?	
1	No	No	No	No	—
2	No	Yes	No	Yes	I
3	No	No	Yes	Yes	II
4	No	Yes	Yes	Yes	I or II
5	Yes, X	No	No	No	—
6	Yes, X	Yes	No	No	—
7	Yes, X	No	Yes	Yes	II
8	Yes, X	Yes	Yes	Yes	II
9	Yes, CS	No	No	No	—
10	Yes, CS	Yes	No	Yes	I
11	Yes, CS	No	Yes	No	—
12	Yes, CS	Yes	Yes	Yes	I

The transition probability matrix is denoted as $\boldsymbol{P}_{11} = [\boldsymbol{P}_{i,j}]$, $i, j = 1, 2, 3, 4, 5, 7, 8, 9, 10, 12$; $\boldsymbol{P}_{12} = [P_{i,j}]$, $i = 1, 2, 3, 4, 5, 7, 8, 9, 10, 12$, j = 6, 11; zero matrix $\boldsymbol{0} = [P_{i,j}]$, $P_{i,j} = 0$ for $i = 6, 11$, $j = 1, 2, 3, 4, 5, 7, 8, 9, 10, 12$.

Identity matrix $\boldsymbol{I} = [P_{i,j}]$, $P_{i,j} = 1$ for $i, j = 6, 11$, and matrix $\boldsymbol{P}$ is the combination of submatrices $\boldsymbol{P}_{11}$, $\boldsymbol{P}_{12}$, $\boldsymbol{I}$, and $\boldsymbol{0}$. That is

$$\boldsymbol{P} = \begin{bmatrix} \boldsymbol{P}_{11} & \boldsymbol{P}_{12} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix}.$$

The cycle time is the time from the start of the process in control until an alarm is detected, repaired, and the process is restarted, or equivalently, it is the time from transient state 1 to reach state 6 or state 11. The state variable Y_t $(t = 0, h, 2h, \dots)$ is a Markov chain on the state $1, 2, \dots 16$ and so the Markov property can be effectively used to find the expected cycle time.

2.6 Expected cycle time and cost

Let random variable T_i be the time until absorption from transient state i. Then, using the Markov property and conditioning on the first step,

$$\begin{aligned}
&P(T_i = h + T_{sr}) = P_{i,j} && \text{where } j = 6, 11,\ i \neq j \\
&P(T_i = h + T_f + T_j) = P_{i,j} && \text{where } i \neq 6, 11,\ j = 2, 3, 4 \qquad (1) \\
&P(T_i = h + T_f + T_{sr} + T_j) = P_{i,j} && \\
& && \text{where } i \neq 6, 11,\ j = 7, 8, 10, 12 \\
&P(T_i = h + T_j) = P_{i,j} && \text{where } i \neq 6, 11,\ j = 1, 5, 9
\end{aligned}$$

Equation 1 can be expressed in matrix form,

$$\boldsymbol{M} = h\boldsymbol{1} + \boldsymbol{P}_{11}\boldsymbol{M}_{sr1} + \boldsymbol{P}_{11}\boldsymbol{M} + \boldsymbol{P}_{12}\boldsymbol{M}_{sr2}$$

So

$$\boldsymbol{M} = h(\boldsymbol{I} - \boldsymbol{P}_{11})^{-1}\boldsymbol{1} + (\boldsymbol{I} - \boldsymbol{P}_{11})^{-1}\boldsymbol{P}_{11}\boldsymbol{M}_{sr1} + (\boldsymbol{I} - \boldsymbol{P}_{11})^{-1}\boldsymbol{P}_{12}\boldsymbol{M}_{sr2},$$

where $\boldsymbol{M}$ is a (10×1) vector, with the expected time up to absorption from transient state i, $i \neq 6, 11$.

$\boldsymbol{1}$ is a (10×1) vector, with elements 1,

$\boldsymbol{M}_{sr1}$i s a (10×1) vector, $\boldsymbol{M}_{sr1}^T = [0 \quad T_f \quad T_f \quad T_f \quad 0 \quad T_f + T_{sr} \quad T_f + T_{sr} \quad 0 \quad T_f + T_{sr} \quad Tf + T_{sr}]$,

$\boldsymbol{M}_{sr2}$ is a (2×1) vector, $\boldsymbol{M}_{sr2}^T = [T_{sr} \quad T_{sr}]$, $\boldsymbol{P}_{11}$ is defined as above.

The expected cycle time is the first element of vector $\boldsymbol{M}$, i.e. $\boldsymbol{M}_{\boldsymbol{1}}$ or $E(T_1)$.

Once the expected cycle time is obtained, the expected cycle cost must be calculated, and the economic adjustment model can be derived by taking the ratio of the expected cycle cost to the expected cycle time.

The derivation of the expected cycle cost uses the Markov property in a similar manner to that used for the expected cycle time. Let $C_{i,j}$ be the expected cumulative cost that is associated with transition from state i to j in time interval h; $i, j = 1, 2, \dots 12$. The calculation of $C_{i,j}$ is illustrated in Appendix 2.

The transition cost matrices are denoted as $\boldsymbol{C}_{11} = [C_{i,j}]$, $i, j = 1, 2, 3, 4, 5, 7, 8, 9, 10, 12$; $\boldsymbol{C}_{12} = [C_{i,j}]$, $i = 1, 2, 3, 4, 5, 7, 8, 9, 10, 12$, $j = 6, 11$; zero matrix $\boldsymbol{0} = [C_{i,j}]$, $C_{i,j} = 0$ for $i = 6, 11$, $j = 1, 2, 3, 4, 5, 7, 8, 9, 10, 12$; $\boldsymbol{C}_{22} = C_{sr}\boldsymbol{I}$, $\boldsymbol{I}$ is the identity matrix for $i, j = 6, 11$, and matrix $\boldsymbol{C}$ is the combination of submatrices $\boldsymbol{C}_{11}$, $\boldsymbol{C}_{12}$, $\boldsymbol{C}_{22}$, and $\boldsymbol{0}$. That is

$$\boldsymbol{C} = \begin{bmatrix} \boldsymbol{C}_{11} & \boldsymbol{C}_{12} \\ \boldsymbol{0} & \boldsymbol{C}_{22} \end{bmatrix}.$$

The cycle cost is the cumulative cost from the start of the process, in control, until an alarm is detected, the process is repaired and restarted, or equivalently, it is the cost from transient state 1 until it reaches an absorbing state.

Let random variable C_i be the cumulative cost up to absorption from transient state i, $i = 1, 2, \dots, 10$. Then using the Markov property and conditioning on the first step,

$$\begin{aligned}
&P(C_i = C_{i,j}) = P_{i,j} && \text{where } j = 6, 11,\ i \neq j \\
&P(C_i = C_{i,j} + C_j) = P_{i,j} && \text{where } i, j \neq 6, 11 \qquad (2)
\end{aligned}$$

Equation 2 can be expressed in matrix form as

$$\boldsymbol{U} = \boldsymbol{P}_{11} * \boldsymbol{C}_{11} + \boldsymbol{P}_{11}\boldsymbol{U} + \boldsymbol{P}_{12} * \boldsymbol{C}_{12},$$

where $*$ denotes the Hadamard product of the two matrices and $\boldsymbol{U}$ is a (10×1) vector with the expected cost up to absorption from transient state i, $i \neq 6, 11$.

So $\boldsymbol{U} = (\boldsymbol{I} - \boldsymbol{P}_{11})^{-1}\boldsymbol{W}\boldsymbol{1}$, where $\boldsymbol{W} = [\boldsymbol{P}_{11} * \boldsymbol{C}_{11} \ \ \boldsymbol{P}_{12} * \boldsymbol{C}_{12}]$, and the first element of the vector, U_1, is the expected cycle cost.

2.7 Determination of optimal design parameters

Applying the property of renewal reward processes (Ross [7]), the objective function (L), the expected cost per unit time is derived by taking the ratio of the expected cycle cost (U_1) to the expected cycle time $(\boldsymbol{M}_{\boldsymbol{1}})$; $L = U_1/\boldsymbol{M}_{\boldsymbol{1}}$. The expected long-term loss is the function of design parameters h, k_1 and k_2; $\mathrm{L}(k_1, k_2, h)$. Hence, the optimal design parameters of the economic-adjustment design of the individual X and cause-selecting control charts can be determined by minimization of the objective function or cost model, that is $\mathrm{MinL}(k_1, k_2, h)$.

It may be noted that the proposed approach can also be used to derive the identical economic-adjustment model obtained by Collani et al. [1] if there is no second process, the expected time of overadjustment = 0, the expected time to search and repair a special cause = 0, the expected cost of incorrect adjustment = 0, the expected cost to search and repair a special cause = 0, and profit maximization is used, instead of cost minimization, in the single special-cause economic adjustment model.

3 A numerical example

In this section, we give an example to illustrate how the proposed method is used to solve a real process-control problem.

Assume that a cotton yarn factory produces cotton yarn in two dependent processes. The skein strength of the cotton yarn is denoted by the quality variable Y, which is produced in the current process. Yarn strength is the most important single index of spinning quality. Good yarn strength not only increases the range of usefulness of a given cotton but it indicates good spinning and weaving performance. The fiber length of the cotton yarn is denoted by the quality variable X, which is produced in the first process. The skein strength can be obtained from knowledge of fiber length, so their relationship can be found by analysis history data. When the process is in control, the average skein strength given fiber length is expressed as model $\hat{Y|X} = 11 + 1.1X$. The distributions of X and e are illustrated as follows, when the first and second processes are all in control:

$$X \sim \mathrm{N}(77.05, 5^2)$$
$$e \sim \mathrm{N}(0, 2.5^2)$$

In the production process, a machine could be out of control in either the first process or the second process. Because the machines do not tend to deteriorate with time, it is of prime concern in process control to be able to distinguish in which one of the processes the out-of-control situation occurs. An out-of-control situation occurring in the first process would cause only the mean of the X distribution to change or result in shifts in the process mean of X distribution due to overadjustment during operation. An out-of-control situation in the second process would cause only the mean of the e distribution to change or result in shifts in the process mean of the e distribution due to overadjustment during operation.

The individual X chart and cause-selecting chart that minimize cost of overadjustment and underadjustment are constructed to monitor the two processes effectively. To determine the optimal design parameters of the individual X chart and cause-selecting chart, the process and cost parameters are estimated as follows:

$\delta_1 = 2,\ \delta_2 = 2.5,\ \lambda_1 = 0.05,\ a = \$0.5,\ b = \$0.1,\ C_f = \$10,\ C_{sr} = \$35,\ T_f = 0.1(\text{hours}),\ T_{sr} = 0.4(\text{hours}),\ C_0 = \$5,\ C_1 = \$20,\ w = 0.5.$

The algorithm used to obtain the approximate optimum values $(h*, k_1*, k_2*)$ of the design values (h, k_1, k_2), with constraints $0 < k_1, k_2 < 6$, $0 < h \leq 8$, is a simple grid-search method yielding the following result: $h* = 0.9$, $k_1* = 1.2$, $k_2* = 2.6$.

That is, the upper and lower control limits of the economic X chart should be set at 88.05 and 71.05, respectively. The upper control limit of the cause-selecting chart should be set at 6.5; the lower control limit of the cause-selecting chart should be set at -6.5. To monitor the process states, every 0.9 hours, a sample of size 1 is taken and tested.

There are four possible results for the process. These outcomes with the associated actions are displayed in Table 2. Combination 1 means that the process is in control, so the process continues and the next sample is taken after 0.9 hours. Combination 2 means that the first process should be stopped and the special cause is adjusted. Combination 3 means that the second process should be stopped and the special cause is adjusted. Combination 4 means that either the first process or the second process should be stopped and the special cause is adjusted.

Table 2. Decision rules

Combinations	X chart signal?	Cause-selecting chart signal?	Which process stop?
1	No	No	No
2	Yes	No	First, adjust special cause
3	No	Yes	Second, adjust special cause
4	Yes	Yes	First, adjust special cause or Second, adjust special cause

4 Summary

A model of two dependent production processes is proposed, the quality of which can be affected by the occurrence of a special cause, which results in a shift in the mean of the first process or the second process. A shift in either may also result from overadjustment of the process when the process is in control. Deming [2] discusses this common situation for a single process in practice. The proposed model is an improvement to the economic design with a single process because it considers the effect of process overadjustment on two dependent processes. Using the proposed design, the processes may be distinguished and adjusted with minimum cost because the only information about process state available is from sampling.

A Markov chain approach is extended to derive the economic adjustment model of two dependent processes used to determine the design parameters of the X and cause-selecting control charts that together minimize the long-term cost resulting from processes overadjustment or underadjustment. It is demonstrated that the expression for the economic-adjustment model is easier to obtain through the proposed approach rather than by those of others. Several important extensions of the proposed model can be developed. It is straightforward to extend the proposed model to study other control charts, like attributes charts. One particularly interesting research area for the future involves the economic modeling of production processes subject to multiple special causes.

Acknowledgement Support for this research was provided in part by the National Science Council of the Republic of China, grant No. NSC-91-2118-M-004-001.

References

1. Collani EV, Saniga EM, Weigand (1994) Economic adjustment designs for $\overline{X}$ control charts. IIE Trans 26:37–43
2. Deming WE(1982) Quality, productivity and competition Position. MIT Press, Cambridge, MA
3. Duncan AJ (1956) The economic design of $\overline{X}$ charts used to maintain current control of a process. J Am Stat Assoc 51:228–242

4. Ho C, Case K (1994) An economic design of the zone control chart for jointly monitoring process centering and variation. Comput Indust Eng 26:213–221
5. Lorenzen T, Vance L (1986) The economic design of control charts: a unified approach. Technometrics 28(1):3–10
6. Montgomery DC (1980) The economic design of control charts: a review and literature survey. J Qual Technol 12:75–78
7. Ross SM (1993) Introduction to probability models. Academic, New York, NY
8. Saniga E (1979) Statistical control-chart designs with application to $\bar{X}$and R charts. Manage Sci 31:313–320
9. Vance LC (1983) A bibliography of statistical quality control chart techniques, 1970–1980. J Qual Technol 15:59-62
10. Wade R, Woodall W (1993) A review of analysis of cause-selecting control charts. J Qual Technol 25:161–169
11. Woodall D (1987), Conflicts between Deming's philosophy and the economic design of control charts. Frontiers Stat Qual Control 3:242–248
12. Woodall WH (1986) Weakness of the economic design of control charts. Technometrics 28:408–409
13. Yang S, Rahim A (2000) Economic statistical design for $\overline{X}$ and S^2 control charts: a Markov chain approach. Commun Stat Simul Comput 29:845-874
14. Zhang G (1984) A new type of control charts and a theory of diagnosis with control charts.World Quality Congress Transactions, American Society for Quality Control, pp. 75–85

Appendix 1

Row 1			
$P(1,1) = \exp(-\lambda h)(1-\alpha_X)(1-\alpha_e)$	$P(1,2) = \exp(-\lambda h)\alpha_X(1-\alpha_e)$	$P(1,3) = \exp(-\lambda h)(1-\alpha_X)\alpha_e$	$P(1,4) = \exp(-\lambda h)\alpha_X\alpha_e$
$P(1,5) = (1-\exp(-\lambda h))\beta_X(1-\alpha_e)$	$P(1,6) = (1-\exp(-\lambda h))(1-\beta_X)(1-\alpha_e)$	$P(1,7) = (1-\exp(-\lambda h))\beta_X\alpha_e$	$P(1,8) = (1-\exp(-\lambda h))(1-\beta_X)\alpha_e$
$P(1,9) = (1-\exp(-\lambda h))(1-\alpha_X)\beta_e$	$P(1,10) = (1-\exp(-\lambda h))\alpha_X\beta_e$	$P(1,11) = (1-\exp(-\lambda h))(1-\alpha_X)(1-\beta_e)$	$P(1,12) = (1-\exp(-\lambda h))\alpha_X(1-\beta_e)$
Row 2			
$P(2,1) = 0$	$P(2,2) = 0$	$P(2,3) = 0$	$P(2,4) = 0$
$P(2,5) = \beta_X(1-\alpha_e)$	$P(2,6) = (1-\beta_X)(1-\alpha_e)$	$P(2,7) = \beta_X\alpha_e$	$P(2,8) = (1-\beta_X)\alpha_e$
$P(2,9) = 0$	$P(2,10) = 0$	$P(2,11) = 0$	$P(2,12) = 0$
Row 3			
$P(3,1) = 0$	$P(3,2) = 0$	$P(3,3) = 0$	$P(3,4) = 0$
$P(3,5) = 0$	$P(3,6) = 0$	$P(3,7) = 0$	$P(3,8) = 0$
$P(3,9) = (1-\alpha_X)\beta_e$	$P(3,10) = \alpha_X\beta_e$	$P(3,11) = (1-\alpha_X)(1-\beta_e)$	$P(3,12) = \alpha_X(1-\beta_e)$
Row 4			
$P(4,1) = 0$	$P(4,2) = 0$	$P(4,3) = 0$	$P(4,4) = 0$
$P(4,5) = 0$	$P(4,6) = 0$	$P(4,7) = 0$	$P(4,8) = 0$
$P(4,9) = 0$	$P(4,10) = 0$	$P(4,11) = 0$	$P(4,12) = 0$
Row 5			
$P(5,1) = 0$	$P(5,2) = 0$	$P(5,3) = 0$	$P(5,4) = 0$
$P(5,5) = \beta_X(1-alpha_e)$	$P(5,6) = (1-\beta_X)(1-\alpha_e)$	$P(5,7) = \beta_X\alpha_e$	$P(5,8) = (1-\beta_X)\alpha_e$
$P(5,9) = 0$	$P(5,10) = 0$	$P(5,11) = 0$	$P(5,12) = 0$
Row 6			
$P(6,1) = 0$	$P(6,2) = 0$	$P(6,3) = 0$	$P(6,4) = 0$
$P(6,5) = 0$	$P(6,6) = 1$	$P(6,7) = 0$	$P(6,8) = 0$
$P(6,9) = 0$	$P(6,10) = 0$	$P(6,11) = 0$	$P(6,12) = 0$
Row 7			
$P(7,1) = 0$	$P(7,2) = 0$	$P(7,3) = 0$	$P(7,4) = 0$
$P(7,5) = 0$	$P(7,6) = 0$	$P(7,7) = 0$	$P(7,8) = 0$
$P(7,9) = 0$	$P(7,10) = 0$	$P(7,11) = 0$	$P(7,12) = 0$
Row 8			
$P(8,1) = 0$	$P(8,2) = 0$	$P(8,3) = 0$	$P(8,4) = 0$
$P(8,5) = 0$	$P(8,6) = 0$	$P(8,7) = 0$	$P(8,8) = 0$
$P(8,9) = \exp(-\lambda_1 h)\cdot\beta_{\overline{X_2}}$	$P(8,10) = \exp(-\lambda_1 h)(1-\beta_{\overline{X_2}})\cdot Psc_1$	$P(8,11) = \exp(-\lambda_1 h)(1-\beta_{\overline{X_2}}) Psc_2$	$P(8,12) = \exp(-\lambda_1 h)(1-\beta_{\overline{X_2}}) Psc_{12}$
Row 9			
$P(9,1) = 0$	$P(9,2) = 0$	$P(9,3) = 0$	$P(9,4) = 0$
$P(9,5) = 0$	$P(9,6) = 0$	$P(9,7) = 0$	$P(9,8) = 0$
$P(9,9) = \exp(-\lambda_1 h)\cdot\beta_{\overline{X_2}}$	$P(9,10) = \exp(-\lambda_1 h)(1-\beta_{\overline{X_2}})\cdot Psc_1$	$P(9,11) = \exp(-\lambda_1 h)(1-\beta_{\overline{X_2}}) Psc_2$	$P(9,12) = \exp(-\lambda_1 h)(1-\beta_{\overline{X_2}}) Psc_{12}$

Row 10			
P(10,1) $= 0$	P(10,2) $= 0$	P(10,3) $= 0$	P(10,4) $= 0$
P(10,5) $= 0$	P(10,6) $= 0$	P(10,7) $= 0$	P(10,8) $= 0$
P(10,9) $= 0$	P(10,10) $= 0$	P(10,11) $= 0$	P(10,12) $= 0$
Row 11			
P(11,1) $= 0$	P(11,2) $= 0$	P(11,3) $= 0$	P(11,4) $= 0$
P(11,5) $= 0$	P(11,6) $= 0$	P(11,7) $= 0$	P(11,8) $= 0$
P(11,9) $= 0$	P(11,10) $= 0$	P(11,11) $= 1$	P(11,12) $= 0$
Row 12			
P(12,1) $= 0$	P(12,2) $= 0$	P(12,3) $= 0$	P(12,4) $= 0$
P(12,5) $= 0$	P(12,6) $= 0$	P(12,7) $= 0$	P(12,8) $= 0$
P(12,9) $= \exp(-\lambda_1 h) \cdot \beta_{\overline{X_1}}$	P(12,10) $= \exp(-\lambda_1 h)(1-\beta_{\overline{X_1}}) \cdot Psc_1$	P(12,11) $= \exp(-\lambda_1 h)(1-\beta_{\overline{X_1}})\ Psc_2$	P(12,12) $= \exp(-\lambda_1 h)(1-\beta_{\overline{X_1}})\ Psc_{12}$

Appendix 2

Row 1			
C(1,1) $= (C_0 h) + (a+bn)$	C(1,2) $= (C_0 h) + (a+bn) + C_f$	C(1,3) $= (C_0 h) + (a+bn) + C_f$	C(1,4) $= (C_0 h) + (a+bn) + C_f$
C(1,5) $= C_0\tau_1 + C_1(h-\tau_1) + (a+bn)$	C(1,6) $= C_0\tau_1 + C_1(h-\tau_1)$ $+(a+bn) + C_{sr}$	C(1,7) $= C_0\tau_1 + C1(h-\tau_1)$ $+(a+bn) + C_f$	C(1,8) $= C_0\tau_1 + C_0(h-\tau_1)$ $+(a+bn) + C_{sr} + C_f$
C(1,9) $= C_0\tau_2 + C_0(h\tau_2 -) + (a+bn)$	C(1,10) $= C_0\tau_2 + C_0(h-\tau_2)$ $+(a+bn) + C_f$	C(1,11) $= C_0\tau_2 + C_0(h-\tau_2)$ $+(a+bn) + C_{sr}$	C(1,12) $= C_0\tau_2 + C_0(h-\tau_2)$ $+(a+bn) + C_{sr} + C_f$
Row 2			
C(2,1) $= 0$	C(2,2) $= 0$	C(2,3) $= 0$	C(2,4) $= 0$
C(2,5) $= C_1 h + (a+bn)$	C(2,6) $= C_1 h + (a+bn) + C_{sr}$	C(2,7) $= C_1 h + (a+bn) + C_{f_2}$	C(2,8) $= C_1 h + (a+bn) + C_{sr} + C_f$
C(2,9) $= 0$	C(2,10) $= 0$	C(2,11) $= 0$	C(2,12) $= 0$
Row 3			
C(3,1) $= 0$	C(3,2) $= 0$	C(3,3) $= 0$	C(3,4) $= 0$
C(3,5) $= 0$	C(3,6) $= 0$	C(3,7) $= 0$	C(3,8) $= 0$
C(3,9) $= C_2 h + (a+bn)$	C(3,10) $= C_2 h + (a+bn) + C_{f_1}$	C(3,11) $= C_2 h + (a+bn) + C_{sr_2}$	C(3,12) $= C_2 h + (a+bn) + C_{sr_2} + C_{f_1}$
Row 4			
C(4,1) $= 0$	C(4,2) $= 0$	C(4,3) $= 0$	C(4,4) $= 0$
C(4,5) $= 0$	C(4,6) $= 0$	C(4,7) $= 0$	C(4,8) $= 0$
C(4,9) $= 0$	C(4,10) $= 0$	C(4,11) $= 0$	C(4,12) $= 0$
Row 5			
C(5,1) $= 0$	C(5,2) $= 0$	C(5,3) $= 0$	C(5,4) $= 0$
C(5,5) $= C_1 h + (a+bn)$	C(5,6) $= C_1 h + (a+bn) + C_{sr}$	C(5,7) $= C_1 h + (a+bn) + C_f$	C(5,8) $= C_1 h + (a+bn) + C_{sr} + C_f$
C(5,9) $= 0$	C(5,10) $= 0$	C(5,11) $= 0$	C(5,12) $= 0$
Row 6			
C(6,1) $= 0$	C(6,2) $= 0$	C(6,3) $= 0$	C(6,4) $= 0$
C(6,5) $= 0$	C(6,6) $= C_{sr_1}$	C(6,7) $= 0$	C(6,8) $= 0$
C(6,9) $= 0$	C(6,10) $= 0$	C(6,11) $= 0$	C(6,12) $= 0$
Row 7			
C(7,1) $= 0$	C(7,2) $= 0$	C(7,3) $= 0$	C(7,4) $= 0$
C(7,5) $= 0$	C(7,6) $= 0$	C(7,7) $= 0$	C(7,8) $= 0$
C(7,9) $= 0$	C(7,10) $= 0$	C(7,11) $= 0$	C(7,12) $= 0$

Row 8			
C(8,1) = 0	C(8,2) = 0	C(8,3) = 0	C(8,4) = 0
C(8,5) = 0	C(8,6) = 0	C(8,7) = 0	C(8,8) = 0
C(8,9) = $C_2h + (a + bn)$	C(8,10) = $C_2h + (a + bn) + C_f$	C(8,11) = $C_2h + (a + bn) + C_{sr}$	C(8,12) = $C_2h + (a + bn) + C_{sr} + C_f$
Row 9			
C(9,1) = 0	C(9,2) = 0	C(9,3) = 0	C(9,4) = 0
C(9,5) = 0	C(9,6) = 0	C(9,7) = 0	C(9,8) = 0
C(9,9) = $C_2h + (a + bn)$	C(9,10) = $C_2h + (a + bn) + C_f$	C(9,11) = $C_2h + (a + bn) + C_{sr}$	C(9,12) = $C_2h + (a + bn) + C_{sr} + C_f$
Row 10			
C(10,1) = 0	C(10,2) = 0	C(10,3) = 0	C(10,4) = 0
C(10,5) = 0	C(10,6) = 0	C(10,7) = 0	C(10,8) = 0
C(10,9) = 0	C(10,10) = 0	C(10,11) = 0	C(10,12) = 0
Row 11			
C(11,1) = 0	C(11,2) = 0	C(11,3) = 0	C(11,4) = 0
C(11,5) = 0	C(11,6) = 0	C(11,7) = 0	C(11,8) = 0
C(11,9) = 0	C(11,10) = 0	C(11,11) = C_{sr}	C(11,12) = 0
Row 12			
C(12,1) = 0	C(12,2) = 0	C(12,3) = 0	C(12,4) = 0
C(12,5) = $C_1h + (a + bn)$	C(12,6) = $C_1h + (a + bn) + C_{sr}$	C(12,7) = $C_1h + (a + bn) + C_f$	C(12,8) = $C_1h + (a + bn) + C_{sr} + C_f$
C(12,9) = 0	C(12,10) = 0	C(12,11) = 0	C(12,12) = 0