Short Paper

Deadlock Control for Weighted Systems of Simple Sequential Processes with Resources Requirement (WS^3PR)

DANIEL YUH CHAO+
Department of Management and Information Science
National Cheng Chi University
Taipei, 116 Taiwan
E-mail: yaw@mis.nccu.edu.tw

Current deadlock control approaches for WS^3PR (*Weighted Systems of Simple Sequential Processes with Resources Requirement*) suffer from incorrect liveness characterization based on the concept of deadly marked siphons (*DMS*). We discover that nonlive transitions may exist even though there are no DMS. That is, the net model may be weakly live or in livelock states under no DMS. It is live under a new liveness condition: all siphons must be max*-controlled. We extend the liveness analysis for S^3PR (*systems of simple sequential processes with resources*) to WS^3PR (*Weighted Systems of Simple Sequential Processes with Resources Requirement*). We develop a new liveness condition called max*-controlled siphons to replace that of the absence of empty siphons. We propose further a deadlock control policy for WS^3PR by adding control nodes and arcs similar to that for S^3PR.

Keywords: Petri nets, siphons, traps, FMS, liveness, deadlock, control

1. INTRODUCTION

Flexible manufacturing systems (*FMS*) offer a very promising approach to the increase of productivity through state-of-the-art manufacturing technology. The modeling and control of *FMS* are part of the great challenge to the professionals in engineering, computer science, mathematics, and management. Petri net (*PN*) theory has been applied to specifications, validation, performance analysis, control code generation, and simulation for *FMS* [1].

Deadlock interrupts normal operation schedules significantly degrading the performance. Hence, it is important to design a *PN* model free of deadlocks. Generally, there are three types of approaches for handling deadlock problem. They are siphon-trap approach as mentioned in this paper; scheduling method [2, 3] and transitive matrix approach [4]. Scheduling method [2-4] assigns each task a time moment to execute during each iteration so that the system can run repetitively without deadlocks. Song and Lee [4] analyze the deadlock problem in Petri nets using the transitive matrix. The transitive matrix may explain all relations between the place and transitions in Petri nets. Since the

Received March 13, 2007; revised May 5, 2009; accepted May 21, 2009.
Communicated by Chin-Laung Lei.
+ The former name of the author is Yuh Yaw, which appeared in some of his earlier papers.

deadlock problem occurred by the relationship between more than two transitions based on the conflict places, they propose a find-deadlock-status algorithm after they define the deadlock-free condition in the transitive matrix.

The *FMS* model consists of a set of *working processes* (*WP*, Def. 4) competing for resources. A *WP* models a sequence of operations to manufacture a product. Circular wait for resources can bring the system into a deadlock where some *WP* can never finish.

Ezpeleta *et al.* proposed a class of nets called S^3PR (systems of simple sequential processes with resources) [5] where each *WP* is a state machine (*SM*) plus resource places. They showed that it is live as long as all siphons are controlled such that they never become empty of tokens (such nets denoted by N_f). Most recent deadlock control approaches [6-8] extend Ezpeleta's work.

They added a control place (and the associated control arcs) to every strict minimal siphon (*SMS*, see definitions in Appendix 1) such that liveness can be enforced. The method is simple and guarantees a success.

Because only one resource is used in each job stage and the processes are modeled using state machines (*SM*) in S^3PR, its modeling power is limited. It cannot model iteration statements (loop) in each *sequential process* (*SP*) as in [9] and the relationships of synchronization and communication in *SP*. At any state of a process, it cannot use multisets of resources. This paper proposes to progress one step forward by allowing multiple units of the same resource to be used at each job stage resulting in the S^3PGR^2 (systems of simple sequential processes with general resources requirement) model.

Park and Reveliotis [6] proposed S^3PGR^2 (*systems of simple sequential processes with general resources requirement*, see Fig. 1) based on the siphon construct of *deadly marked siphons* (*DMS*). They developed, based on the derived siphon-based liveness characterizations, a sufficiency test for the correctness of *CD-RAS* (conjunctive-disjunctive resource allocation system) *DAP* (*Deadlock Avoidance Policy*) that can be expressed as a set of invariants imposing control places, superimposed on the *PN* modeling the original *RAS* (Resource Allocation System) behavior. However, the absence of *DMS* is only necessary, but not sufficient, to the liveness of the model. An S^3PGR^2 net without *DMS* is deadlock-free but may be in livelock states.

Tricas and Martinez [10] proposed a similar system, called WS^3PSR (*weighted systems of simple sequential processes with several resources*). It differs from WS^3PR in two aspects: (1) A place can represent the use of various resources simultaneously. (2) There is no need of releasing resources used in the present state before advancing to the next state. However, the policy is very restrictive so that it sequentializes the flow in the siphon. The marking imposed to the control places limits the number of processes that can flow in the problematic areas to the minimal. They admitted that further work must be done to find better control places and better markings.

All these approaches are based on deadly marked siphons (*DMS*). A net with *DMS* is not live and conversely, a live net does not have *DMS*. However the absence of *DMS* does not guarantee the liveness of all transitions in the net; it may be weakly live or in livelock states. In order to provide optimum control, we need first to improve the condition for liveness.

Abdallah and Elmaraghy [11] proposed S^4PR − a generalization of S^3PR nets − to extend S^3PR and production Petri nets (*PPN*) nets to model systems that not only can use alternative resources, as in S^3PR nets, but also can utilize more than one resource simultaneously. They adopted a deadlock prevention policy by adding a control place for each siphon to remain max-controlled (Def. 7) for all reachable markings. However, it is only a sufficient condition implying that a live S^4PR may not be max-controlled.

We propose a relaxed liveness condition called max*-controlled siphons (Def. 9) in [12]. We propose further in this paper a deadlock control policy for WS^3PR by adding control nodes and arcs for each strict minimal siphon (*SMS*). Afterwards, we propose a deadlock control policy for WS^3PR by adding control nodes and arcs similar to that for S^3PR.

Section 2 presents the basis to understand the paper. Section 3 shows how general Petri nets (*GPN*) challenges the liveness analysis based on siphons and develops a new liveness condition for *GPN*. Section 4 presents a better liveness condition for WS^3PR. Section 5 presents the control policy for deadlock prevention in WS^3PR. Finally section 6 concludes the paper. However, in order to make the paper as self-contained as possible, Appendix 1 is included with the definitions of the main concepts related to models used in this paper.

2. PRELIMINARIES

In this paper, we consider only strongly connected nets.

Definition 1 A subnet $N_i = (P_i, T_i, F_i)$ of N is generated by $X = P_i \cup T_i$, if $F_i = F \cap (X \times X)$. It is an *I-subnet*, denoted by I, of N if $T_i = \bullet P_i$. $\boldsymbol{I_S}$ is the I-subnet (the subnet derived from $(S, \bullet S)$) of an *SMS* S. Note that $S = P(I_S)$; S is the set of places in I_S.

Property 1 [13] The linear combination of Y_1 and Y_2 is an S-invariant if both Y_1 and Y_2 are S-invariants. Furthermore, if Y is an S-invariant of N, then given an initial marking M_0, $\forall M \in R(N, M_0)$, $Y \bullet M = Y \bullet M_0$.

We follow [14-16] for the definitions of *handles*, *bridges*, *AB-handles*, and *AB-bridges* where A and B can be T or P. Roughly speaking, a "handle" is an alternate disjoint path between two nodes. A *PT*-handle starts with a place (as indicated by '*P*' in '*PT*-') and ends with a transition while a *TP*-handle starts with a transition and ends with a place.

Definition 2 The handle $H = [n_s n_1 n_2 \ldots n_k n_e]$ *to a subnet* N' (similar to the handle of a tea pot) is an elementary directed path from n_s in N' to another node n_e in N'; any other node in H is not in N'. H is said to be a handle in $N' \cup H$. If $n_s \in P$, $n_e \in P$, $n_s = n_e$ ($n_s \neq n_e$), H is called a *PP-circuit* (*PP-handle*). H^{TP} (H^{PT}, H^{PP}, H^{TT}) denotes a *TP*-handle (*PT*-handle, *PP*-handle, *TT*-handle) to I_S. H_1 is a *XY*-handle where X and Y can be T or P. X is T (P) if $n_s \in T$ ($n_s \in P$). Y is T (P) if $n_e \in T$ ($n_e \in P$). H_1 is a *resource handle* if all places in H_1 are resource places.

Definition 3 A *simple sequential process* (S^2P) is a net $N = (P \cup \{p^0\}, T, F)$ where: (1) $P \neq \emptyset$, $p^0 \notin P$ (p^0 is called the process idle or initial or final state); (2) N is strongly connected state machine; and (3) every circuit of N contains the place p^0.

Definition 4 A *simple sequential process with weighted resources requirement* (WS^2PR), also called a *working process* (*WP*), is a net $N = (P \cup \{p^0\} \cup P_R, T, F)$ so that (1) The subnet generated by $X = P \cup \{p^0\} \cup T$ is an S^2P; (2) $P_R \neq \varnothing$ and $(P \cup \{p^0\}) \cap P_R = \varnothing$; (3)$\forall p \in P$, $\forall t \in \bullet p$, $\forall t' \in p\bullet$, $\exists r_p \in P_R$, $\bullet t \cap P_R = t'\bullet \cap P_R = \{r_p\}$; (4) The two following statements are verified: (a) $\forall r \in P_R$, $\bullet\bullet r \cap P = r\bullet\bullet \cap P \neq \varnothing$; (b) $\forall r \in P_R$, $\bullet r \cap r\bullet = \varnothing$; (5) $\bullet\bullet\,(p^0) \cap P_R = (p^0)\bullet\bullet \cap P_R = \varnothing$; (6) $\forall p \in H(r)$, $F(t_1, r) = F(r, t_2)$, where $H(r) = \bullet\bullet r \cap P$ denotes the set of holders of r (operation places that use r), $t_1 \in p\bullet \cap \bullet r$, $t_2 \in \bullet\bullet t_1 \cap r\bullet$. $\forall p \in P$, p is called an operation place. $\forall r \in P_R$, r is called a resource place.

Definition 5 Let $N = (P \cup \{p^0\} \cup P_R, T, F)$ be a WS^2PR. An initial marking is called an *acceptable initial marking* for N *iff*: (1) $M_0(p_0) \geq 1$; (2) $M_0(p) = 0$, $\forall p \in P$; and (3) $M_0(r) \geq \max_{t \in r\bullet} F(r, t)$, $\forall r \in P_R$. The couple (N, M_0) is called an *acceptably marked* WS^2PR.

Definition 6 A system of WS^2PR (WS^3PR) is defined recursively as follows: (1) An WS^2PR is a WS^3PR; (2) Let $N_i = (P_i \cup P_i^0 \cup P_{Ri}, T_i, F_i)$, $i \in \{1, 2\}$ be two WS^3PR so that $(P_1 \cup P_1^0) \cap (P_2 \cup P_2^0) = \varnothing$, $P_{R1} \cap P_{R2} = P_C\ (\neq \varnothing)$ and $T_1 \cap T_2 = \varnothing$. The net $N = (P \cup P^0 \cup P_R, T, F)$ resulting from the composition of N_1 and N_2 via P_C (denoted $N_1\, o\, N_2$) which is defined as follows: (1) $P = P_1 \cup P_2$ is the set of operation places; (2) $P^0 = P_1^0 \cup P_2^0$; (3) $P_R = P_{R1} \cup P_{R2}$ is the set of resource places; (4) $T = T_1 \cup T_2$ and (5) $F = F_1 \cup F_2$ is also a WS^3PR. A directed path (circuit, subnet) Γ in N is called a resource path (circuit, subnet) if $\forall p \in \Gamma$, $p \in P_R$.

Lemma 1 [21, 22] (1) A subnet N' is the I of a minimal siphon *iff* each handle in N' is a *PP*- or *TP*- or *virtual PT*-handle (virtual means containing only two nodes) and there are none of *PP*-, *TP*-, and virtual *PT*-handles to N'; (2) $P(N')$ is an *SMS iff* there is a non-virtual *PT*- handle to N'', which is a subnet of N' without any *TP*-handles.

Example: In Fig. 1, first find a circuit $c_b = [p_{22}\ t_{10}\ p_{26}\ t_{16}\ p_{22}]$. Second add *TP*-handles $[t_{16}\ p_{18}\ t_{17}\ p_{26}]$ and $[t_{10}\ p_{10}\ t_6\ p_{22}]$ plus *PP*-handle $[p_{22}\ t_3\ p_{10}]$ to get I_{S_1} and $S_1 = P(I_{S_1}) = \{p_{10}, p_{18}, p_{22}, p_{26}\}$ with a nonvirtual *PT*-handle $[p_{26}\ t_9\ p_{13}\ t_{10}]$ (more than two nodes) to c_b.

The rest of the *SMS* are shown in Table 1.

3. THE GPN CHALLENGE

An S^3PR is live if no siphons ever become empty [5], not necessary true for WS^3PR since it is a general Petri net (*GPN*). Hence, we tackle the problem first for an arbitrary GPN. One can no longer ensure liveness by making no siphons emptiable as illustrated in Fig. 2 where $t_3(t_1)$ is (not) live with only one siphon $D = P$. It is deadlock-free but not live; we call it weakly live (called N_w). This implies the change of the condition "no siphons emptiable" to the new "max-controlled" [18]. To understand this, we explore the condition under which a *GPN* behaves like an *OPN* (ordinary *PN*). It occurs if $\forall M \in R(N, M_0)$, $\exists p \in P$, $M(p) \geq F(p, t)$ (the arc weight from p to t), $\forall t \in p\bullet$. That is for every reachable marking, there is a max-marked p (see Def. 7); *i.e.*, the amount of its tokens is greater than the weight of any outgoing arc. A counter example is shown in Fig. 2 with no max-marked places. Even though p_2 has a token, it can never fire t_1 due to the weight of 2 between p_2 and t_1; while that in Fig. 3 is due to the blocking by the presence of unmarked p_1.

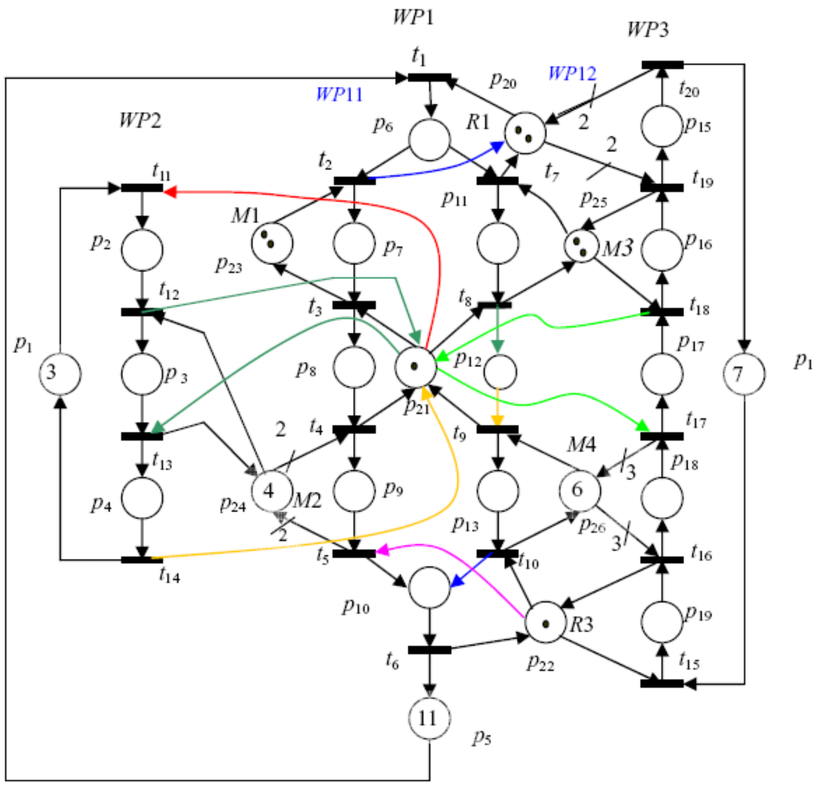

Fig. 1. An example of *WS^{3}PR* (weighted systems of simple sequential processes with resources requirement).

Table 1. All *SMS* for the net in Fig. 1.

SMS	places
S_1	$p_{10}, p_{18}, p_{22}, p_{26}$
S_2	$p_4, p_{10}, p_{15}, p_{20}, p_{21}, p_{22}, p_{23}, p_{24}, p_{25}, p_{26}$
S_3	$p_4, p_{10}, p_{16}, p_{21}, p_{22}, p_{24}, p_{25}, p_{26}$
S_4	$p_4, p_{10}, p_{17}, p_{21}, p_{22}, p_{24}, p_{26}$
S_5	$p_4, p_9, p_{13}, p_{15}, p_{20}, p_{21}, p_{23}, p_{24}, p_{25}, p_{26}$
S_6	$p_4, p_9, p_{13}, p_{16}, p_{21}, p_{24}, p_{25}, p_{26}$
S_7	$p_4, p_9, p_{13}, p_{17}, p_{21}, p_{24}, p_{26}$
S_8	$p_4, p_9, p_{12}, p_{15}, p_{20}, p_{21}, p_{23}, p_{24}, p_{25}$
S_9	$p_4, p_9, p_{12}, p_{16}, p_{21}, p_{24}, p_{25}$
S_{10}	$p_4, p_9, p_{12}, p_{17}, p_{21}, p_{24}$
S_{11}	$p_2, p_4, p_8, p_{10}, p_{15}, p_{20}, p_{21}, p_{22}, p_{23}, p_{25}, p_{26}$
S_{12}	$p_2, p_4, p_8, p_{13}, p_{15}, p_{20}, p_{21}, p_{23}, p_{25}, p_{26}$
S_{13}	$p_2, p_4, p_8, p_{10}, p_{16}, p_{21}, p_{22}, p_{25}, p_{26}$
S_{14}	$p_2, p_4, p_8, p_{13}, p_{16}, p_{21}, p_{25}, p_{26}$
S_{15}	$p_2, p_4, p_8, p_{10}, p_{17}, p_{21}, p_{22}, p_{26}$
S_{16}	$p_2, p_4, p_8, p_{13}, p_{17}, p_{21}, p_{26}$
S_{17}	$p_2, p_4, p_8, p_{12}, p_{15}, p_{20}, p_{21}, p_{23}, p_{25}$
S_{18}	$p_2, p_4, p_8, p_{12}, p_{16}, p_{21}, p_{25}$

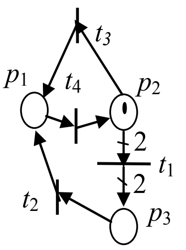

Fig. 2. An example of weakly live *GPN* where the only siphon $\{p_1, p_2, p_3\}$ never gets empty of tokens.

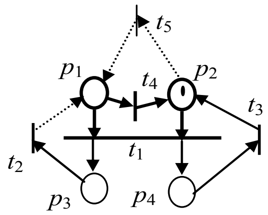

Fig. 3. An example of weakly live net where the only siphon $\{p_1, p_2, p_3, p_4\}$ never gets empty of tokens.

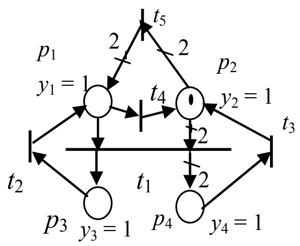

Fig. 4. The general *PN* version of that in Fig. 3.

However, if $\exists M \in R(N, M_0)$, $M(p_2) > 2$, then p_2 in Fig. 2 is max-marked and it is no longer weakly live since it behaves likes an *OPN*. Further, let's compare two *PN* with identical nodes, arcs and initial marking except one is weighted and dead (a *GPN* in Fig. 4) and the other is an *OPN* and N_w (Fig. 3). However, if $M_0(p_2) = 2$ in Fig. 4, then it is max-marked and behaves likes an *OPN* and is an N_w also.

In the above examples, there is only one siphon D and we say that D is max-controlled (Def. 8) since for every reachable marking, there exists a max-marked p in D. In general, when all siphons are max-controlled, then it behaves like an *OPN*. Thus, for a WS^3PR, it is live as long as all siphons are max-controlled. We will show that this condition can be relaxed in the next section. For the moment, we present some basic theories of max-controlled siphons.

Definition 7 An output arc (p_i, t) of p_i is called enabled (disabled) if $M(p_i) < F(p_i, t)$ ($M(p_i) \geq F(p_i, t)$) where $F(p_i, t)$ is the arc weight from p_i to t. p_i is called max-marked under M, if $\forall t \in p_i\bullet, M(p_i) \geq F(p_i, t)$; *i.e.*, all output arcs of p_i are enabled. Let $D = \{p_1, p_2, \ldots, p_K\}$. D is called max-marked under M, if $\exists p \in D$, p is max-marked. If D is a siphon, it is said to be max-controlled *iff* D is max-marked under any reachable marking. Let Y be an S-invariant with components y_i and $M_D = [a_x(p_1) - 1\ a_x(p_2) - 1\ \ldots\ a_x(p_K) - 1\ 0\ 0\ \ldots\ 0]^T$ a marking where $a_x(p) = \max_{t \in p\bullet} F(p, t)$ is the maximal weight of all outgoing arcs from p. That is, $\forall p \in D$, $M(p) = a_x(p) - 1$; $\forall p \in P\backslash D$, $M(p) = 0$, where $P\backslash D = \{x \mid x \in P, x \notin D\}$. The weighted sum of tokens under M_D is

$$W(M_D) = W_D = M_D^T \bullet Y = \textstyle\sum_k (a_x(p_k) - 1) \bullet y_k.$$

Definition 8 Let Y be an S-invariant with components y_i, $\forall p_i \in P$, and $D \subseteq P$ a siphon of N. The siphon is called max-controlled by Y under M_0 *iff* the weighted sum of tokens $W(M_0) = Y^T \bullet M_0 > W_D$, and Y satisfies the ***negative-property*:** $\forall p_k \in D$, $y_k > 0$ and $\forall p_i \in P\backslash D$: $y_i \leq 0$.

Lemma 2 [18] Let (N, M_0) be a net-system and $D \subseteq P$ a siphon of N. If D is max-controlled by an S-invariant Y under M_0, then $\forall M \in R(N, M_0)$: D is max-marked under M.

By this lemma, if it is initially max-marked, it remains so for all reachable markings.

Lemma 3 [18] For a dead Petri net (N, M_0), there exists a non-max-controlled siphon at M_0.

Lemma 4 [18] N is deadlock-free under M_0 if every siphon D is max-controlled by an S-invariant under M_0.

The support of an S-invariant is also a set of places where the weighted total number of tokens is conserved. A minimal siphon is in one such support. Hence all the unloaded tokens remain in the support of the invariant. If they are also in that of another invariant (ν), the minimal siphon is said to be *invariant-controlled* [17]. By controlling the number of tokens in ν, we may prevent the minimal siphon from being non-max-marked or completely unloaded in the *OPN* case.

For WS^3PR, the condition of max-controlled siphons may be overly constrained as shown in the next section.

4. A BETTER LIVENESS CONDITION FOR WS^3PR

The relaxation of the condition is shown in Fig. 5 (a) where $M_0(p_1) = 1$, $M_0(p_1') = 1$, $M_0(r_1) = 6$, $M_0(r_2) = 3$. There is only one *SMS* $S = \{r_1, r_2, p_3, p_2'\}$. We push as many tokens out from S as possible to make some transitions dead. t_1 and t_2 can never fire and the rest are live; hence it is deadlock free. Note that t_2 is neither live nor potentially firable even under M_0; hence it is not quasi-live. After adding a token to r_2, it becomes live and also quasi-live since all transitions are potentially firable (*i.e.*, quasi-live, defined in Appendix 1) under M_0. However, neither r_1 nor r_2 is max-marked; hence S is not max-controlled. Rather, it is max*-controlled motivating us to relax the liveness condition as follows.

Definition 9 Let $N = (P \cup P^0 \cup P_R, T, F)$ be a WS^3PR. A siphon D in N is said to be max*-controlled, *iff* $\forall M \in R(N, M_0)$, $\exists r \in D_R$ $(= D \cap P_R)$ is max-marked in the resource subnet υ of the I_D or $\exists p \in I_D \cap P$, p is max-marked. Let $a_y(r) = \max_{t \in r\bullet \cap \upsilon} F(r, t)$, $r, t \in \upsilon$ (for all output t which are in υ). $r(p)$ in D_R $(D_p = D \cap P)$ is called max*-marked under M, if $a_y(r) \le M(r)$ $(M(p) > 0)$. Let Y be an S-invariant with components y_i and $M_D' = [a_y(r_1) - 1\ a_y(r_2) - 1\ \ldots\ a_y(r_K) - 1\ 0\ 0\ \ldots\ 0]^T$ a marking such that $\forall r_i \in D_R$, $M(r_i) = a_y(r_i) - 1$, $i = 1, 2, \ldots, K$ and $\forall p \in P \cup \{p^0\} \cup P_R \backslash D_R$, $M(p) = 0$. The weighted sum of tokens under M_D is

$$W'(M_D) = W_D' = M_D'^T \bullet Y = \textstyle\sum_k (a_y(r_k) - 1) \bullet y_k.$$

Note that $I_D \backslash \upsilon$ is a set of *PP*-, *TP*- and virtual *PT*-handles to I_D by Lemma 1. Only *PP*-handles contain output arcs from $r \in D_R$ and may not be disabled as those in υ. We do not consider H^{PP} above. An example of H^{PP} $([r_2 t_2''\ p_2''\ t_1''\ r_2])$ is shown in Fig. 5 (b) where all transitions in WP_3 are live, while all transitions in WP_1 and WP_2 are dead. The net is said to be in a livelock state.

We did not define $a_y(p)$ for $p \in D_p$, since otherwise $a_y(p) = 0$ in M_D' – the same as those not in D. Because $F(p, t) = 1$ $\forall p \in D_p$, p is max-marked (or max*-marked in Def. 9) under M, if $M(p) > 0$.

Further, it is not live, yet siphon S is not deadly marked. Unlike S^3PR, the absence of deadly marked siphons (*DMS*) does not imply the liveness of a WS^3PR. Simple extension of *DMS* is insufficient for the liveness analysis of WS^3PR.

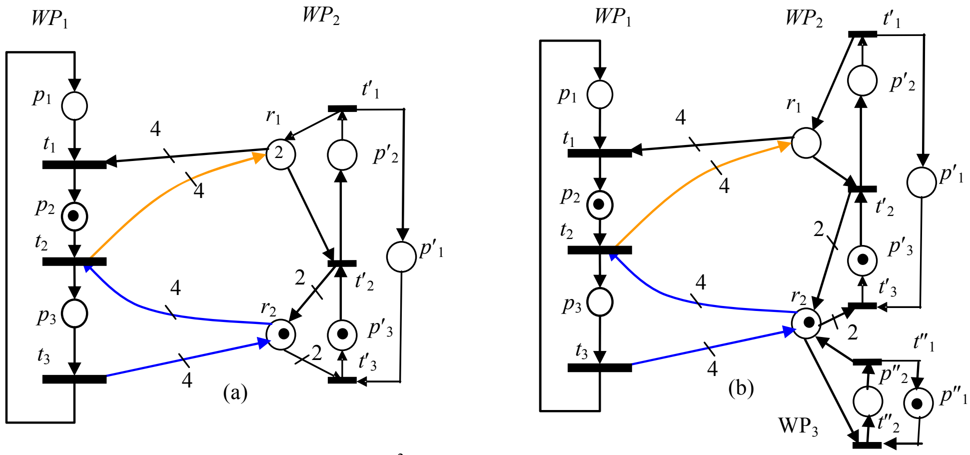

Fig. 5. (a) An example of weakly live WS^3PR ($M_0(p_1) = 1$, $M_0(p_1') = 1$, $M_0(r_1) = 6$, $M_0(r_2) = 3$). t_1, t_2 and t_3 can never fire. The rest are live. After adding a token to r_2, it becomes live. It is live if all transitions are potentially firable from M_0 or WP_1 and WP_2 are quasi-live; (b) An example of $H^{PP}{}_c = [r_2\, t_2''\, p_2''\, t_1''\, r_2]$. The siphon is not deadly marked and it is not live.

Definition 10 Let Y be an S-invariant with components y_i, $\forall p_i \in P$, and $D \subseteq P$ a siphon of N. The siphon is called max*-controlled by Y under M_0 *iff* the weighted sum of tokens $W(M_0) = Y^T \bullet M_0 > W_D'$ and Y satisfies the *negative-property*: $\forall p_k \in D$, $y_k > 0$ and $\forall p_i \in P \backslash D$: $y_i \leq 0$, where $P \backslash D = \{x \,|\, x \in P, x \notin D\}$.

When we design a system, we first select a Y satisfying the negative-property and then assign an initial marking M_0 such that $W(M_0) > W_D'$ to ensure that the system remains max*-controlled under all reachable markings as shown below.

Lemma 5 [12] Let (N, M_0) be a net-system and let $D \subseteq P$ a siphon of N. If D is max*-controlled by the S-invariant under M_0, then $\forall M \in R(N, M_0)$: D is max*-marked under M.

By this lemma, if (N, M_0) is initially max*-marked, it remains so for all reachable markings. This facilitates the verification of max*-controlled siphons. Note that the condition $W(M_0) = Y^T \bullet M_0 > W_D'$ in Def. 10 is only sufficient (not necessary) for D to be max*-marked under M.

Proposition 1 [5] Let $N = (P \cup P^0 \cup P_R, T, F)$ be a WS^3PR and D ($\neq \Phi$) a siphon so that it does not contain the support of any S-invariant. Then we have $|D \cap P_R| > 1$.

Thus, any SMS in a WS^3PR contains at least two resource places.

Theorem 1 [12] Let (N, M_0) be a marked WS^3PR, $M \in R(N, M_0)$ and $t' \in T$ a dead transition under M. Then $\exists M' \in R(N, M)$, $\exists D$ a siphon so that D is nonempty (*i.e.*, not a null set) and non-max*-marked.

In Fig. 5 (b), t_2 is dead and S is non-max*-marked but not deadly marked since t_2'' is live.

5. CONTROL POLICY FOR DEADLOCK PREVENTION IN WS^3PR

This section extends the deadlock prevention technique for S^3PR by Ezpeleta *et al.* to WS^3PR. For each *SMS S*, we add a control place V_S and control arcs exactly the same as in [5]. The only difference is the initial marking at V_S and some arcs are weighted. The following definitions help subsequent discussions.

Definition 11 Let $N = O^k_{i=1} N_i = (P \cup P^0 \cup P_R, T, F)$ be a WS^3PR and $\prod$ be the set of SMS in N. Given $S \in \prod$, where $S = S_P \cup S_R$, $S_R = S \cap P_R$, $S_P = S \backslash P_R$, $[S] = (\cup_{r \in S_R} H(r)) \backslash S$ denotes the set of holders, corresponding to resources in S, which do not belong to S. $[S^i] = [S] \cap P_i$, $i \in I_N = \{1, 2, \ldots, k\}$. $[S]$ is called S's complementary set. $\overline{N_i}$ denotes the S^2P corresponding to N_i.

Definition 12 [5, 13] Let $N = (P, T, F)$ be an S^2P. (1) Let C be a circuit of N, $||C||$ the set of nodes in it, $|\,||C||\,|$ the length of C and x, y two nodes of C, We say that x is *previous* to y in C *iff* there exists a path in C from x to y, the length of which is greater than 1 and which does not pass p^0. This fact is denoted by $x \rightarrow_C y$. (2) Let x and y be two nodes of N. We say that x is *previous* to y in N *iff* there exists a circuit C such that $x \rightarrow_C y$. This fact is denoted as $x \rightarrow_N y$. (3) Let x and $\theta \subseteq (P \cup T)$ be a node and a set of nodes of N, respectively. Then $x \rightarrow_N \theta$ *iff* there exists a node $y \in \theta$ such that $x \rightarrow_N y$. $\theta \rightarrow_N x$ *iff* there exists a node $y \in \theta$ such that $y \rightarrow_N x$.

Definition 13 Let $N = O^k_{i=1} N_i = (P \cup P^0 \cup P_R, T, F)$ be a WS^3PR and $\prod$ be the set of *SMS* in N. $\prod^+$: $T \rightarrow \wp(\prod)$ ($\wp(\prod)$ is the power set of $\prod$) is a mapping where $\prod^+(t) = \{S \in \prod \mid t \rightarrow_{\overline{N_i}} [S^i]\}$. $\prod^-$: $T \rightarrow \wp(\prod)$ is a mapping where $\prod^-(t) = \{S \in \prod \mid [S^i] \rightarrow_{\overline{N_i}} t\}$. $\forall i \in \{1, 2, \ldots, k\}$, $\forall S \in \prod$, $P_S = \cup_{i=1}{}^k P_S^i$, $P_S^i = [S^i] \cup \{p \in P_i \mid p \rightarrow_{\overline{N_i}} [S^i]\}$.

Definition 14 Let (N, M_0) be a marked $WS^3PR = (P \cup P^0 \cup R, T, F)$. The net $(N_A, M_{0A}) = (P \cup P^0 \cup R \cup P_A, T, F \cup F_A, M_{0A})$ is the *controlled system* of (N, M_0) *iff* (1) $P_A = \{V_S \mid S \in \prod\}$ is the set of additional control places such that there exists a bijective mapping from $\prod$ into it; (2) $F_A = F_A^1 \cup F_A^2 \cup F_A^3$ where $F_A^1 = \{(V_S, t) \mid t \in P^0\bullet, S \in \prod^+(t)\}$, $F^2_A = \{(t, V_S) \mid t \in [S]\bullet, S \notin \prod^+(t)\}$, $F_A^3 = \cup^k_{i=1}\{(t, V_S) \mid t \in T_i \backslash P^0\bullet, S \notin \prod^-(t), \bullet t \cap P_i \subseteq P_S^i, t \nrightarrow_{\overline{N_i}} [S^i]\}$, and (3) M_{0A} is defined as follows:

(a) $\forall p \in P \cup P^0 \cup P_R$, $M_{0A}(p) = M_0(p)$,
(b) $\forall V_S \in P_A$, $M_{0A}(V_S) = \lfloor 1/b \bullet (M_0(S) - W_S' - 1) \rfloor$ if $1/b \bullet (M_0(S) - W_S' - 1) \geq 1$, else $M_{0A}(V_S) = 1$. b is a constant to be determined below.

Determination of (*b*) $M_{0A}(V_S)$ is assigned in Def. 14.(b) to make the controlled net max*-controlled as explained below. First, we need to find the S-invariant Y in Def. 10. Y must be such that $\forall p_k \in S, y_k > 0$ and $\forall p_i \in P \backslash S$: $y_i \leq 0$ (see Def. 10). Set $Y = Y_S + b \bullet Y_V$ (also an S-invariant by Property 1 where Y_S and Y_V are the S-invariants associated with S and V_S respectively and defined as follows.

Y_S: y_j, $y_j = 1$, $\forall p_j \in S_R$, or $y_j = a_r(p_j) = F(p_j\bullet \cap \bullet r, r)$, $\forall p_j \in H(r)$, $r \in S_R$, and $y_j = 0$ for all other p_j.

Y_V: y_j, $y_j = -1$, $\forall p_j \in P_S$, or $p_j = V_S$, and $y_j = 0$ for all other p_j.

Select $b = \max a_r(p_j) = \max Y_S(p_j)$, where $p_j \in P_S$ so that $\forall p_k \in S$, $y_k > 0$ and $\forall p_i \in P \backslash S$: $y_i \leq 0$. To make $W(M_0) = Y^T \bullet M_0 > W_S'$, set $M_{0A}(V_S) = \lfloor 1/b \bullet (M_0(S) - W_S' - 1) \rfloor$ if $1/b \bullet (M_0(S) - W_S' - 1) \geq 1$, else $M_{0A}(V_S) = 1$.

Lemma 7 $\forall S \in \prod$, $Y = Y_S + b \bullet Y_V$ satisfies the negative property.

Proof: $\forall p_j \in S_R$, $Y(p_j) = Y_S(p_j) + b \bullet Y_V(p_j) = 1 + b \bullet 0 = 1 > 0$ ($P_S \cap S_R = \Phi$). $\forall p_j \in P_S \cap [S]$, $Y(p_j) = Y_S(p_j) + b \bullet Y_V(p_j) = Y_S(p_j) + (\max Y_S(p_j)) \bullet -1 \leq 0$. $\forall p_j \in (P_S \backslash [S]) \cup \{V_S\}$, $Y(p_j) = Y_S(p_j) + b \bullet Y_V(p_j) = 0 - b < 0$. For all other p_j, $Y(p_j) = 0$. Thus, Y satisfies the negative property. ❑

Table 2 lists the values of Y, Y_S, and Y_V of all *SMS* of the net in Fig. 1 where $y_{x\text{-}y}$ indicates the y value for places p_x to p_y. For instance, 3_{18} $(-1_{11\text{-}12})$ implies that $y = 3$ (-1) for place p_{18} (p_{11} to p_{12}). This is to avoid the long vector form containing 25 components.

Proposition 2 Let (N_A, M_{0A}) be the controlled system (as defined in Def. 14) of a marked WS^3PR, (N, M_0). Then S is max*-controlled.

Proof: There are two cases: (1) $1/b \bullet (M_0(S) - W_S' - 1) \geq 1$. $\forall M_A \in R(N_A, M_{0A})$, $W = Y \bullet M = Y \bullet M_0 = M_0(S) - b \bullet M_{0A}(V_S) = M_0(S) - b \bullet \lfloor 1/b \bullet (M_0(S) - W_S' - 1) \rfloor > W_S'$ (two cases: (a) $M_0(S) - W_S' - 1 = b \bullet k$. $W = W_S' + 1$. (b) $M_0(S) - W_S' - 1 = b \bullet k + k'$, $b > k' > 0$. $W = W_S' + k + 1$). Thus, S is max*-controlled by Def. 10. (2) Otherwise, $M_{0A}(V_S) = 1$. Now $M_A(V_S) = 1$ (S is max*-marked.) or 0 (The token at V_S fires the output transition to remove some tokens from an $r \in S$. Since by Proposition 1, there exist other $r' \in S$ and they remain at $M_{0A}(r')$; hence r' remains to be max-marked and S is max*-controlled even though $W = Y \bullet M = Y \bullet M_0 = M_0(S) - b \leq W_S'$ since $1/b \bullet (M_0(S) - W_S' - 1) < 1$.) ❑

The following lemma is obvious since every firing sequence of the controlled system remains to be such for the uncontrolled one.

Lemma 8 Let (N_A, M_{0A}) be the controlled system of a marked WS^3PR (N, M_0), and σ a firing sequence of (N_A, M_{0A}). Then σ is a firing sequence of (N, M_0).

This lemma will be used to prove Lemma 9.

Lemma 9 Let (N_A, M_{0A}) be the be controlled system of a marked WS^3PR (N, M_0), $t \in T$, and $M_A \in R(N_A, M_{0A})$ be a reachable marking. Then t is not dead under M_A in (N_A, M_{0A}).

Proof: Assume $t \in T_i$ and prove by induction over the number of tokens, denoted by K^{M_A}, in the system not in idle states. Assume $K^{M_A} > 0$. Since all siphons are max*-marked under M_A by Proposition 2, there exists $t' \in T \backslash P^0 \bullet$ so that $M_{A|N}[t'\rangle$ ($M_{A|N}$ denotes the projection of M_A onto N) by Theorem 1 (t' is firable in the uncontrolled system), and since $\bullet t' \cap (\bigcup_{S \in \prod} V_S) = \varnothing$ (t' is not an output transition of any V_S), we have $M_A[t'\rangle$ (t' is also friable in (N_A, M_A)). Iterating this reasoning, we can fire a sequence σ' of transitions by Lemma 8

Table 2. The values of *Y*, Y_S, and Y_V of all *SMS* of the net in Fig. 1.

SMS	Y_S	Y_V	$Y_S + bY_V$
S_1	$1_{10} + 1_{13} + 3_{18} + 1_{19} + 1_{22} + 1_{26}$	$-1_6 - 1_{11\text{-}13} - 1_{19} - 1_{VS1}$	$-1_6 + 1_{10} - 1_{11\text{-}12} + 3_{18} + 1_{22} + 1_{26} - 1_{VS1}$
S_2	$1_{2\text{-}4} + 1_{6\text{-}8} + 2_9 + 1_{10\text{-}13} + 2_{15} + 1_{16\text{-}17}$ $+ 3_{18} + 1_{19} + 1_{20\text{-}26}$	$-1_{2\text{-}3} - 1_{6\text{-}9} - 1_{11\text{-}13} - 1_{16\text{-}19} - 1_{VS2}$	$1_4 - 2_{2\text{-}3} - 2_{6\text{-}8} - 1_9 + 1_{10} - 2_{11\text{-}13} + 2_{15}$ $- 2_{16\text{-}17} - 2_{19} + 1_{20\text{-}26} - 1_{VS2}$
S_3	$1_{2\text{-}4} + 1_8 + 2_9 + 1_{10\text{-}13} + 1_{16\text{-}17} + 3_{18}$ $+ 1_{19} + 1_{21\text{-}22} + 1_{24\text{-}26}$	$-1_{2\text{-}3} - 1_{6\text{-}9} - 1_{11\text{-}13} - 1_{17\text{-}19} - 1_{VS3}$	$1_4 - 2_{2\text{-}3} - 2_{6\text{-}8} - 1_9 + 1_{10} - 2_{11\text{-}13} + 1_{16}$ $- 2_{17} - 2_{19} + 1_{21\text{-}22} + 1_{24\text{-}26} - 1_{VS3}$
S_4	$1_{2\text{-}4} + 1_8 + 2_9 + 1_{10} + 1_{12\text{-}13} + 1_{17}$ $+ 3_{18} + 1_{19} + 1_{21\text{-}22} + 1_{24} + 11_{26}$	$-1_{2\text{-}3} - 1_{6\text{-}9} - 1_{11\text{-}13} - 1_{18\text{-}19} - 1_{VS4}$	$1_4 - 2_{2\text{-}3} - 3_{6\text{-}7} - 2_8 - 1_9 + 1_{10} - 3_{11}$ $- 2_{11\text{-}13} + 1_{17} - 2_{19} + 1_{21\text{-}22} + 1_{24} + 11_{26}$ $- 1_{VS4}$
S_5	$1_{2\text{-}4} + 1_{6\text{-}8} + 2_9 + 1_{11\text{-}13} + 2_{15} + 1_{16\text{-}17}$ $+ 3_{18} + 1_{20\text{-}21} + 1_{23\text{-}26}$	$-1_{2\text{-}3} - 1_{6\text{-}8} - 1_{11\text{-}12} - 1_{16\text{-}19} - 1_{VS5}$	$1_4 - 2_{2\text{-}3} - 2_{6\text{-}8} + 2_9 - 2_{11\text{-}13} + 1_{13} + 2_{15}$ $- 2_{16\text{-}17} - 3_{19} - 1_{20\text{-}21} + 1_{23\text{-}26} - 1_{VS5}$
S_6	$1_{2\text{-}4} + 1_8 + 2_9 + 1_{11\text{-}13} + 1_{16\text{-}17} + 3_{18}$ $+ 1_{21} + 1_{24\text{-}26}$	$-1_{2\text{-}3} - 1_{6\text{-}8} - 1_{11\text{-}12} - 1_{17\text{-}19} - 1_{VS6}$	$1_4 - 2_{2\text{-}3} - 3_{6\text{-}7} - 2_8 + 2_9 - 2_{11\text{-}12} + 1_{13}$ $+ 1_{16} - 2_{17} - 3_{19} + 1_{21} + 1_{24\text{-}26} - 1_{VS6}$
S_7	$1_{2\text{-}4} + 1_8 + 2_9 + 1_{12\text{-}13} + 1_{17} + 3_{18}$ $+ 1_{21} + 1_{24} + 1_{26}$	$-1_{2\text{-}3} - 1_{6\text{-}8} - 1_{11\text{-}12} - 1_{18\text{-}19} - 1_{VS7}$	$1_4 - 2_{2\text{-}3} - 3_{6\text{-}7} - 2_8 + 2_9 - 3_{11} - 2_{12} + 1_{13}$ $+ 1_{17} - 3_{19} + 1_{21} + 1_{24} + 1_{26} - 1_{VS7}$
S_8	$1_{2\text{-}4} + 1_{6\text{-}8} + 2_9 + 1_{11\text{-}12} + 2_{15} + 1_{16\text{-}17}$ $+ 1_{20\text{-}21} + 1_{23\text{-}25}$	$-1_{2\text{-}3} - 1_{6\text{-}8} - 1_{11} - 1_{16\text{-}19} - 1_{VS8}$	$1_4 + 2_9 + 1_{12} + 2_{15} - 1_{18\text{-}19} + 1_{20\text{-}21} + 1_{23\text{-}25}$ $- 1_{VS8}$
S_9	$1_{2\text{-}4} + 1_8 + 2_9 + 1_{11\text{-}12} + 1_{16\text{-}17} + 1_{21}$ $+ 1_{24} + 1_{25}$	$-1_{2\text{-}3} - 1_{6\text{-}8} - 1_{11} - 1_{17\text{-}19} - 1_{VS9}$	$1_4 - 1_{6\text{-}7} + 2_9 + 1_{12} + 1_{16} - 1_{18\text{-}19} + 1_{21}$ $+ 1_{24} + 1_{25} - 1_{VS9}$
S_{10}	$1_{2\text{-}4} + 1_8 + 2_9 + 1_{12} + 1_{17} + 1_{21} + 1_{24}$	$-1_{2\text{-}3} - 1_{6\text{-}8} - 1_{VS10}$	$1_4 - 1_{6\text{-}7} + 2_9 + 1_{12} + 1_{17} + 1_{21} + 1_{24}$ $- 1_{VS10}$
S_{11}	$1_2 + 1_4 + 1_{6\text{-}8} + 1_{10\text{-}13} + 2_{15} + 1_{16\text{-}17}$ $+ 3_{18} + 1_{19} + 1_{20\text{-}23} + 1_{25\text{-}26}$	$-1_{6\text{-}8} - 1_{11\text{-}13} - 1_{16\text{-}19} - 1_{VS11}$	$1_2 + 1_4 - 2_{6\text{-}8} + 1_{10} - 2_{11\text{-}13} + 2_{15} - 2_{16\text{-}17}$ $- 21_{19} + 1_{20\text{-}23} + 1_{25\text{-}26} - 1_{VS11}$
S_{12}	$1_2 + 1_4 + 1_{6\text{-}8} + 1_{11\text{-}13} + 2_{15} + 1_{16\text{-}17}$ $+ 3_{18} + 1_{20\text{-}21} + 1_{23} + 1_{25\text{-}26}$	$-1_{6\text{-}7} - 1_{11\text{-}12} - 1_{16\text{-}19} - 1_{VS12}$	$1_2 + 1_4 - 2_{6\text{-}7} + 1_8 - 2_{11\text{-}12} + 1_{13} + 2_{15} -$ $2_{16\text{-}17} - 31_{19} + 1_{20\text{-}21} + 1_{23} + 1_{25\text{-}26} - 1_{VS12}$
S_{13}	$1_2 + 1_4 + 1_8 + 1_{10\text{-}13} + 1_{16\text{-}17} + 3_{18}$ $+ 1_{19} + 1_{21\text{-}22} + 1_{25\text{-}26}$	$-1_6 - 1_{11\text{-}13} - 1_{17\text{-}19} - 1_{VS13}$	$1_2 + 1_4 - 3_6 + 1_8 + 1_{10} - 2_{11\text{-}13} + 1_{16}$ $- 2_{17} - 2_{19} + 1_{21\text{-}22} + 1_{25\text{-}26} - 1_{VS13}$
S_{14}	$1_2 + 1_4 + 1_8 + 1_{11\text{-}13} + 1_{16\text{-}17} + 3_{18}$ $+ 1_{21} + 1_{25\text{-}26}$	$-1_6 - 1_{11\text{-}12} - 1_{17\text{-}19} - 1_{VS14}$	$1_2 + 1_4 - 3_6 + 1_8 - 2_{11\text{-}12} + 1_{13} + 1_{16} - 2_{17}$ $- 31_{19} + 1_{21} + 1_{25\text{-}26} - 1_{VS14}$
S_{15}	$1_2 + 1_4 + 1_8 + 1_{10} + 1_{12\text{-}13} + 1_{17}$ $+ 3_{18} + 1_{19} + 1_{21\text{-}22} + 1_{26}$	$-1_6 - 1_{11\text{-}13} - 1_{18\text{-}19} - 1_{VS15}$	$1_2 + 1_4 - 3_6 + 1_8 + 1_{10} - 3_{11} - 2_{12\text{-}13} + 1_{17}$ $- 2_{19} + 1_{21\text{-}22} + 1_{26} - 1_{VS15}$
S_{16}	$1_2 + 1_4 + 1_8 + 1_{12\text{-}13} + 1_{17} + 3_{18}$ $+ 1_{21} + 1_{26}$	$-1_6 - 1_{11\text{-}12} - 1_{18\text{-}19} - 1_{VS16}$	$1_2 + 1_4 - 3_6 + 1_8 - 3_{11} - 2_{12} + 1_{13} + 1_{17}$ $- 31_{19} + 1_{21} + 1_{26} - 1_{VS16}$
S_{17}	$1_2 + 1_4 + 1_{6\text{-}8} + 1_{11\text{-}12} + 2_{15} + 1_{16\text{-}17}$ $+ 1_{20} + 1_{21} + 1_{23} + 1_{25}$	$-1_{6\text{-}7} - 1_{11} - 1_{16\text{-}19} - 1_{VS17}$	$1_2 + 1_4 + 1_8 + 1_{12} + 2_{15} - 1_{18\text{-}19} + 1_{20}$ $+ 1_{21} + 1_{23} + 1_{25} - 1_{VS17}$
S_{18}	$1_2 + 1_4 + 1_8 + 1_{11} + 1_{12} + 1_{16} + 1_{17}$ $+ 1_{21} + 1_{25}$	$-1_6 - 1_{11} - 1_{17\text{-}19} - 1_{VS18}$	$-1_6 + 1_2 + 1_4 + 1_8 + 1_{12} + 1_{16} - 1_{18\text{-}19} + 1_{21}$ $+ 1_{25} - 1_{VS18}$

until the token reaches its inevitable (as said in [5]) idle state; *i.e.*, $M_A[\sigma'\rangle M_A'$. Eventually we will reach M_{0A} where $K^{M_A} = 0$. We can prove in the same way as for the uncontrolled system (N, M_0) that t is live under M_{0A} since $M_0(V_S) > 1$ and $F(V_S, t) = 1$ $\forall t \in V_S\bullet$, $\forall S \in \prod$, thus we conclude t is live for any K^{M_A} and we are done. ❑

We now prove the liveness as follows.

Theorem 2 Let (N_A, M_{0A}) be the controlled system of a marked WS^3PR (N, M_0). Then (N_A, M_{0A}) is live.

***Proof*:** By Lemma 9, $\forall M_A \in R(N_A, M_{0A})$, no $t \in T$ is dead. Thus, the controlled model is live. ❑

Table 3 shows the new places and arcs added as well as $M_0(V_{Si})$ using the control policy for the WS^3PR in Fig. 1.

Table 3. The complementary sets [*S*], the control *PN* elements added by, and the auxiliary variables (*b* & W_S') associated with, the control policy.

	[S]	$V_S\bullet$	$\bullet V_S$	b	$M_0(S)$	W_S'	$M_0(V_S)$
1	$\{p_{13}, p_{19}\}$	$\{t_1, t_{15}\}$	$\{t_{16}, t_{10}, t_2\}$	1	7	2	4
2	$\{p_2, p_3, p_6, p_7, p_8, p_9, p_{11}, p_{12}, p_{13}, p_{16}, p_{17}, p_{18}, p_{19}\}$	$\{t_1, t_{11}, t_{15}\}$	$\{t_5, t_{10}, t_{13}, t_{19}\}$	3	18	4	4
3	$\{p_2, p_3, p_8, p_9, p_{11}, p_{12}, p_{13}, p_{17}, p_{18}, p_{19}\}$	$\{t_1, t_{11}, t_{15}\}$	$\{t_5, t_{10}, t_{13}, t_{18}\}$	3	14	3	3
4	$\{p_2, p_3, p_8, p_9, p_{12}, p_{13}, p_{18}, p_{19}\}$	$\{t_1, t_{11}, t_{15}\}$	$\{t_5, t_{10}, t_{13}, t_{17}\}$	3	12	3	2
5	$\{p_2, p_3, p_6, p_7, p_8, p_{11}, p_{12}, p_{16}, p_{17}, p_{18}\}$	$\{t_1, t_{11}, t_{15}\}$	$\{t_4, t_9, t_{13}, t_{19}\}$	3	17	2	4
6	$\{p_2, p_3, p_8, p_{11}, p_{12}, p_{17}, p_{18}\}$	$\{t_1, t_{11}, t_{15}\}$	$\{t_4, t_9, t_{13}, t_{18}\}$	3	13	1	3
7	$\{p_2, p_3, p_8, p_{12}, p_{18}\}$	$\{t_1, t_{11}, t_{15}\}$	$\{t_4, t_9, t_{13}, t_{17}\}$	3	11	1	3
8	$\{p_2, p_3, p_8, p_6, p_7, p_{11}, p_{16}, p_{17}\}$	$\{t_1, t_{11}, t_{15}\}$	$\{t_4, t_8, t_{13}, t_{19}\}$	1	11	1	9
9	$\{p_2, p_3, p_8, p_{11}, p_{17}\}$	$\{t_1, t_{11}, t_{15}\}$	$\{t_4, t_8, t_{13}, t_{18}\}$	1	7	1	5
10	$\{p_2, p_3, p_8\}$	$\{t_1, t_{11}\}$	$\{t_4, t_7, t_{13}\}$	1	5	1	3
11	$\{p_6, p_7, p_{11}, p_{12}, p_{13}, p_{16}, p_{17}, p_{18}, p_{19}\}$	$\{t_1, t_{15}\}$	$\{t_3, t_{10}, t_{19}\}$	3	14	3	3
12	$\{p_6, p_7, p_{11}, p_{12}, p_{16}, p_{17}, p_{18}\}$	$\{t_1, t_{11}, t_{15}\}$	$\{t_3, t_9, t_{19}\}$	3	13	1	3
13	$\{p_{11}, p_{17}, p_{12}, p_{18}, p_{13}, p_{19}\}$	$\{t_1, t_{15}\}$	$\{t_2, t_{10}, t_{18}\}$	3	10	2	2
14	$\{p_{11}, p_{12}, p_{17}, p_{18}\}$	$\{t_1, t_{15}\}$	$\{t_2, t_9, t_{18}\}$	3	9	0	2
15	$\{p_{12}, p_{13}, p_{18}, p_{19}\}$	$\{t_1, t_{15}\}$	$\{t_2, t_{10}, t_{17}\}$	3	8	2	1
16	$\{p_{12}, p_{18}\}$	$\{t_1, t_{15}\}$	$\{t_2, t_9, t_{17}\}$	3	7	0	2
17	$\{p_6, p_7, p_{11}, p_{16}, p_{17}\}$	$\{t_1, t_{15}\}$	$\{t_3, t_8, t_{19}\}$	1	7	1	5
18	$\{p_{11}, p_{17}\}$	$\{t_1, t_{15}\}$	$\{t_2, t_8, t_{18}\}$	1	3	0	2

6. CONCLUSION

We have pioneered the concept of max*-controlled siphons and proved that any WS^3PR is live if all strict minimal siphons are max*-controlled. We conjecture that the condition of max*-controlled siphons can be extended to more complicated models such as *SAPGR* (systems of arbitrary processes with general resources), where multiple types and units of resources may be used at a job stage.

We also pioneered the first correct method to control WS^3PR to avoid deadlocks. As for S^3PR, the method suffers from expensive computation of all *SMS* since the number of which grows exponentially with the number of places. However, the solution of siphons can be performed offline.

Li *et al.* [13] proposed simpler Petri net controllers by adding control places to elementary siphons only (generally a much smaller set than that of *SMS* in large Petri nets). In the mean time, it controls all other *SMS* too. Thus, the number of control places is much smaller and, therefore, is suitable for large-scale Petri nets. *SMS* can be divided into two groups: elementary and dependent; characteristic *T*-vectors of the latter are linear combinations of that of the former. However, it applies to ordinary Petri net models only and hence it is interesting to apply the concept of max*-controlled siphons to extend the elementary-siphon-approach to the control of WS^3PR.

Future work may extend the results to cases where resources may be unreliable and the corresponding deadlock handling method must be fault tolerant [19, 20].

REFERENCES

1. D. Y. Chao, "A computer aided design technique for flexible manufacturing systems synthesis utilizing Petri nets," *Computer-Aided Design*, *Engineering*, *and Manufacturing: Techniques and Applications*, *Operational Methods in Computer Aided Design*, Vol. 3, 2001, pp. 8.1-8.64.
2. X. Gang and Z. Wu, "A kind of deadlock-free scheduling method based on Petri net," in *Proceedings of the 7th IEEE International Symposium on High Assurance Systems Engineering*, 2002, pp. 195-201.
3. H. Huang, J. Li, and W. Zhou, "Deadlock-free design of JSP with multi-resource sharing," in *Proceedings of International Conference on Computer and Electrical Engineering*, 2008, pp. 412-416.
4. Y. Song and J. Lee, "Deadlock analysis of Petri nets using the transitive matrix," in *Proceedings of the 41st Society of Instrument and Control Engineers Annual Conference*, Vol. 2, 2002, pp. 689-694.
5. J. Ezpeleta, J. M. Colom, and J. Martinez, "A Petri net based deadlock prevention policy for flexible manufacturing systems," *IEEE Transactions on Robotic and Automation*, Vol. 11, 1995, pp. 173-184.
6. J. Park and S. A. Reveliotis, "Deadlock avoidance in sequential resource allocation systems with multiple resource acquisitions and flexible routings," *IEEE Transactions on Automatic Control*, Vol. 46, 2001, pp. 1572-1583.
7. Y. Huang, "Modeling, analysis, deadlock prevention and cell controller implementation for flexible manufacturing systems," Department of Electrical Engineering, National Taiwan University of Science and Technology, Taipei, 2001.
8. K. Barkaoui and I. B. Adallah, "Deadlock avoidance in FMS on structural theory of Petri nets," in *Proceedings of INRIA/IEEE Symposium on Emerging Techonologies and Factory Automation*, Vol. 2, 1995, pp. 499-510.
9. M. Jeng, X. Xie, and S. L. Chung, "ERCN* merged nets for modeling degraded behavior and parallel processes in semiconductor manufacturing systems," *IEEE Transactions on Systems*, *Man*, *Cybernetics – Part A: Systems and Humans*, Vol. 34, 2004, pp. 102-112.
10. F. Tricas and J. Martinez, "An extension of the liveness of concurrent sequential processes competing for shared resources," in *Proceedings of IEEE International Conference System*, *Man*, *Cybernetics*, 1995, pp. 4119- 4124.
11. I. B. Abdallah and A. Elmaraghy, "Deadlock prevention and avoidance in FMS: A Petri net based approach," *The International Journal of Advanced Manufacturing*, Vol. 14, 1998, pp. 704-715.

12. D. Y. Chao, "Max*-Controlled siphons for liveness of *WS^3PR*," *IEEE Proceedings Control Theory and Applications*, Vol. 1, 2007, pp. 933-936.
13. Z. Li and M. Zhou, "Elementary siphons of Petri nets and their application to deadlock prevention in flexible manufacturing systems," *IEEE Transactions on Systems, Man, Cybernetics – Part A*, Vol. 34, 2004, pp. 38-51.
14. D. Y. Chao and J. A. Nicdao, "Liveness for synchronized choice Petri nets," *Computer Journal*, Vol. 44, 2001, pp. 124-136.
15. D. Y. Chao, "Extended synchronized choice nets," *Computer Journal*, Vol. 46, 2003, pp. 505-523.
16. D. Y. Chao and D. T. Wang, "Two theoretical and practical aspects of knitting technique – invariants and a new class of Petri net," *IEEE Transactions on Systems, Man, Cybernetics – Part A*, Vol. 27, 1997, pp. 962-977.
17. K. Lautenbach, "Linear algebraic calculation of deadlocks and traps," in K. Voss, H. Genrich, and G. Rozenberg, (eds.), *Concurrency and Nets*, Springer-Verlag, New York, 1987, pp. 315-336.
18. K. Barkaoui and J. Pradat-Peyre, "On liveness and controlled siphons in Petri nets," in W. Reisig, (ed.), *Theory and Application of Petri Nets*, LNCS 1091, Springer-Verlag, Berlin, 1996, pp. 57-72.
19. F. S. Hsieh, "Robustness analysis of Petri nets for assembly/disassembly processes with unreliable resources," *Automatica*, Vol. 42, 2006, pp. 1159-1166.
20. F. S. Hsieh, "Fault tolerant deadlock avoidance algorithm for assembly processes," *IEEE Transaction on System, Man and Cybernetics – Part A*, Vol. 34, 2004, pp. 65-79.
21. D. Y. Chao, "A graphic-algebraic computation of elementary siphons of BS^3PR," *Journal of Information Science and Engineering*, Vol. 23, 2007, pp. 1817-1831.
22. D. Y. Chao, "Computation of elementary siphons in Petri nets for deadlock control," *Computer Journal*, Vol. 49, 2006, pp. 470-479.

APPENDIX 1: PETRI NET-RELATED DEFINITIONS

A Petri net is a 4-tuple $PN = (P, T, F, M_0)$ where $P = \{p_1, p_2, \ldots, p_a\}$ is a set of places, $T = \{t_1, t_2, \ldots, t_b\}$ a set of transitions, with $P \cup T \neq \emptyset$ and $P \cap T = \emptyset$, F: $(P \times T) \cup (T \times P) \rightarrow \{0, 1, 2, \ldots\}$ is the *flow relation* and a marking of N is a mapping M: $P \rightarrow$ **IN**, where **IN** $= \{0, 1, 2, \ldots\}$. The ith component of M, $M(p_i)$, represents the number of tokens in place p_i under M. A node x in $N = (P, T, F)$ is either a $p \in P$ or a $t \in T$. The post-set of node x is $x\bullet = \{y \in P \cup T \mid F(x, y) > 0\}$, and its pre-set $\bullet x = \{y \in P \cup T \mid F(y, x) > 0\}$. The preset (postset) of a set is defined as the union of the presets (postsets) of its elements. A directed path $\Gamma = [n_1, n_2, \ldots, n_k]$, $k \geq 1$, is a graphical object containing a sequence of nodes and the single arc between each two successive nodes in the sequence. $N' = (P', T', F')$ is called a subnet of N where $P' \subseteq P$, $T' \subseteq T$, and $F' = F \cap ((P' \times T') \cup (T' \times P'))$.

The incidence matrix of N is a matrix A: $P \times T \rightarrow Z$ indexed by P and T such that $A(p_i, t_j) = a_{ij}^+ - a_{ij}^-$ where $a_{ij}^- = F(p_i, t_j)$ is the weight of the arc from place p_i to its output transition t_j, and $a_{ij}^+ = F(t_i, p_j)$ is the weight of the arc from transition t_i to its output place p_j.

t_i is *firable* or enabled if each place p_j in $\bullet t$ holds no less tokens than the weight $w_j = F(p_j, t_i)$. Firing t_i under M_0 removes w_j tokens from p_j and deposits $w_k = F(t_i, p_k)$ tokens into each place p_k in $t\bullet$; moving the system state from M_0 to M_1. Repeating this process, it reaches M' by firing a sequence of transitions. M' is said to be reachable from M_0; *i.e.*, $M_0[\sigma\rangle M'$.

Ordinary Petri nets (*OPN*) are those for which F: $(P \times T) \cup (T \times P) \rightarrow \{0, 1\}$. An OPN is called a *state machine* (*SM*) if $\forall t \in T$, $|t\bullet| = |\bullet t| = 1$. It is a *free choice net* (*FC*) if $\forall p_1, p_2 \in P$, $p_1\bullet \cap p_2\bullet \neq \emptyset \Rightarrow |p_1\bullet| = |p_2\bullet| = 1$. It is an *asymmetric choice net* (*AC*) if $\forall p_1\bullet \cap p_2\bullet \neq \emptyset \Rightarrow p_1\bullet \subseteq p_2\bullet$ or $p_1\bullet \supseteq p_2\bullet$.

$R(N, M_0)$ is the set of markings reachable from M_0. A transition $t \in T$ is live under M_0 *iff* $\forall M \in R(N, M_0)$, $\exists M' \in R(N, M)$, t is firable under M'. A transition $t \in T$ is dead under M_0 *iff* $\nexists M \in R(N, M_0)$ where t is firable. A PN is *live* under M_0 *iff* $\forall t \in T$, t is live under M_0. It is *quasi-live* iff $\forall t \in T$, $\exists M \in R(N, M_0)$ s.t. $M[t\rangle$; *i.e.*, t is potentially firable under M_0. It is *weakly live* under M_0 *iff* N is not live and $\exists t \in T$, t is live under M_0. It is *bounded* if $\forall M \in R(N, M_0)$, $\forall p \in P$, the marking at p, $M(p) \leq k$, where k is a positive integer. It is reversible *iff* if $M_0 \subset R(N, M)$, $\forall M \subset R(N, M_0)$.

$\Gamma = [n_1, n_2, \ldots, n_k]$, $k \geq 1$, is an *elementary directed path* in N if $\forall (i, j)$, $1 \leq i < j \leq k$, $n_i \neq n_j$. Γ is (non) *virtual* if it contains only (more than) two nodes. Γ is an *elementary-circuit c* in N if $\forall (i, j)$ $1 \leq i \leq j \leq k$, $n_i = n_j$ implies that $i = 1$ and $j = k$.

For a Petri net (N, M_0), a non-empty subset $D(\tau)$ of places is called a *siphon* (*trap*) if $\bullet D \subseteq D\bullet$ ($\tau\bullet \subseteq \bullet\tau$), *i.e.*, every transition having an output (input) place in $D(\tau)$ has an input (output) place in $D(\tau)$. If $M_0(D) = \sum_{p \in D} M_0(p) = 0$, D is called a *empty siphon* at M_0. A *minimal siphon* does not contain a siphon as a proper subset. It is called a *strict minimal siphon* (*SMS*), denoted by S, if it does not contain a trap.

An integer vector Y is called an S-invariant *iff* $Y \neq 0$ and $A^T \bullet Y = 0$, where A is the incidence matrix. $||Y|| = \{p \in P | Y(p) \neq 0\}$ is the *support* of Y. A Petri net N is called *conservative iff* there exists a positive integer vector $Y > 0$ such that $M^T \bullet Y = M_0^T \bullet Y$, $\forall M \in R(N, M_0)$.

Daniel Yuh Chao (趙玉) received the Ph.D. degree from in Electrical Engineering and Computer Science from the University of California, Berkeley in 1987. From 1987-1988, he worked at Bell Laboratories. Since 1988, he joined the Computer and Information Science Department of New Jersey Institute. Since 1994, he joined the Department of Management and Information Science, National Cheng Chi University as an Associate Professor. Since February, 1997, he has been promoted to a Full Professor. His research interest was in the application of Petri nets to the design and synthesis of communication protocols and the CAD implementation of a multi-function Petri net graphic tool. He is now working on the optimal control of flexible manufacturing systems. He has published 110 (including 45 journal) papers in the area of communication protocols, Petri nets, DQDB, networks, FMS, data flow graphs and neural networks.