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ORIGINAL ARTICLE

Technical note—reducing mip iterations for deadlock prevention of flexible manufacturing systems

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Abstract We propose adding monitors to each basic siphon and find conditions for a compound siphon to be already controlled. This (1) relieves the problem of siphon enumeration, which grows exponentially, and (2) reduces the number of subsequent time-consuming mixed integer programming (MIP) iterations.

Keywords Petri nets · Siphons · Deadlocks

1 Introduction

The mixed integer programming (MIP) method proposed have been useful [1, 2] to identify emptiable siphons without time-consuming complete enumeration of siphons which grows exponentially. Each time an 'emptiable' siphon is found, a monitor is added. The process is repeated until there are no more emptiable siphons.

However, MIP is NP-hard and time-consuming. Hence, it is desirable to reduce the number of MIP iterationsas much as possible while making it maximally permissive; i.e., maximizing the number of good states. To do so, the original uncontrolled model should be disturbed as little as possible and each strict minimal siphons (SMS) S be allowed to reach its *limit state*; i.e., min $M(S)$=1.

Our approach is based on the concept of basic (Table 1) and compound siphons (Table 2) in [3, 5] built from elementary and compound circuits. We will show that if we assign monitors to basic siphons first, then many compound siphons are already controlled under certain condition (Theorem 1) and need no monitors.

We further propose to start applying MIP test to add monitors to new problematic siphons (generated from circuits containing control places) after all basic and compound siphon have been controlled. In the sequel, all symbols referred belong to some S^3PR.

Definition 1 $[S] = \|Y_S\|/S$ where $[S]$ is the *complementary siphon* of a siphon S, $\|Y_S\|$ the support of a minimal P-invariant containing S.

Definition 2 (1) Let S_0 (S_1, S_2,…, and S_n) be a *compound siphon* (*basic siphons*) built from compound circuit c_0 (*elementary circuits* c_1, c_2,…, and c_n). S_0 is said to depend on S_1, S_2,…, and S_n, denoted by S_0=S_{12},…, oS_n, if c_0 consists of c_1, c_2,…, and c_n. (2) S_i (i=$1,2, \ldots, n$) is said to reach its *limit state* when $M(S_i)$=1 or $M([S_i])$=$M_0(S_i)$−1; it is *limit-controlled* if it is able to reach its limit state but not able to reach empty state; i.e., *min* $M(S_i)$=1 or max $M([S_i])$= $M_0(S_i)$−1.

Lemma 1 For S_{012},…, and S_n in Def. 2 that satisfies $[S_0] = [S_1] \cup [S_2] \cup \ldots \cup [S_n]$ and $[S_i] \cap [S_j] = \varnothing$, $\forall i,j \in \{1,2,\ldots,n\}$, $i \neq j$, 1) $\max M([S_0]) = \sum_{i=1}^{n} \max M([S_i])$. (2) S_0 is limit-controlled, if $M_0(S_0) - \sum_{i=1}^{n}(M_0(S_i)-1) = 1$.

Proof (1) Obvious. (2) By Def. 2.2, S_0 is limit-controlled, if max $M([S_0])$=$M_0(S_0)$-1. From (1), $\max M([S_0]) = \sum_{i=1}^{n} \max M([S_i]) = \sum_{i=1}^{n}(M_0[S_i]-1) = M_0(S_0)$. Hence, $M_0(S_0) - \sum_{i=1}^{n}(M_0(S_i)-1) = 1$.

Theorem 1 Let S_0=S_{12},…, oS_n, $[S_0]$=$[S_1] \cup [S_2] \cup \ldots \cup [S_n]$ and $[S_0]$=$[S_1] \cup [S_2] \cup \ldots \cup [S_n]$ and $S_i \cap S_j$=$\varnothing$, such that $\forall i \in \{1,2,\ldots,n\}$, S_i is limit-controlled, ($M_0(S_i)$=m_i+b_{i-1},

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Table 1 Basic siphons and resource circuits for the S^3PR in Fig. 2

Basic siphons	Places	c
S_1	$p_{10}, p_{18}, p_{22}, p_{26}$	$[p_{22}\ t_{10}\ p_{26}\ t_{16}\ p_{22}]$
S_4	$p_4, p_{10}, p_{17}, p_{21}, p_{22}, p_{24}, p_{26}$	$[p_{21}\ t_{17}\ p_{26}\ t_{16}\ p_{22}\ t_5\ p_{24}\ t_4\ p_{21}]$
S_{10}	$p_4, p_9, p_{12}, p_{17}, p_{21}, p_{24}$	$[p_{21}\ t_{13}\ p_{24}\ t_4\ p_{21}]$
S_{16}	$p_2, p_4, p_8, p_{13}, p_{17}, p_{21}, p_{26}$	$[p_{21}\ t_{17}\ p_{26}\ t_9\ p_{21}]$
S_{17}	$p_2, p_4, p_8, p_{12}, p_{15}, p_{20}, p_{21}, p_{23}, p_{25}$	$[p_{21}\ t_3\ p_{23}\ t_2\ p_{20}\ t_{19}\ p_{25}\ t_{18}\ p_{21}]$
S_{18}	$p_2, p_4, p_8, p_{12}, p_{16}, p_{21}, p_{25}$	$[p_{21}\ t_8\ p_{25}\ t_{18}\ p_{21}]$

$(i\neq 1)$), ($b_i=M_0(r_i)$, $S_i \cap S_{i+1}=\{r_i\}$ $(i\neq n)$). S_0 is limit-controlled if $b_i=M_0(r_i)=1$, $\forall i\in\{1,2,\ ...,n-1\}$.

Proof First, $max\ M([S_i])=M_0(S_i)-1$ by Def. 2.2, and the fact that S_i is limit-controlled, then based on Lemma 1, S_0 is limit-controlled, *if* $M_0(S_0) - \sum_{i=1}^{n}(M_0(S_i)-1) = 1 \Rightarrow M_0(S_0) - \left(M_0(S_1) - 1 + \sum_{i=1}^{n-1}(M_0(S_{i+1}) - b_i) + \sum_{i=1}^{n-1}(b_i - 1)\right) = 1 \Rightarrow 0 = \sum_{i=1}^{n-1}(b_i-1) = 0 \Rightarrow b_i = 1,\ \forall i.$

In Fig. 1, S_0 (S_1 and S_2) is a compound siphon (basic siphons). $S_1=\{p_9, p_{10}, p_3, p_6\}$, $[S_1]=\{p_2, p_7\}$; $S_2=\{p_{10}, p_{11}, p_4, p_7\}$, $[S_2]=\{p_3, p_8\}$; $S_0=\{p_9, p_{10}, p_6, p_{11}, p_4\}$, $[S_0]=\{p_2, p_3, p_7, p_8\}$. $[S_0]=[S_1]\cup[S_2]$. S_1 and S_2 (S_0) are basic (compound) siphons; both are limit controlled since $M_0(V_{S1})=M_0(p_{12})=a+b-1$ and $M_0(V_{S2})=M_0(p_{13})=b+c-1$. $M_0(S_0)=a+b+c$ and $A=((M_0(S_1)-1)+M_0(S_2)-1))=a+b+c-1+(b-1)$. $M_0(S_0)-A=1-(b-1)$. Thus, S_0 is limit-controlled (no need for control elements) *if* $b=1$.

On the other hand, if $b>1$, we need add control elements for S_0 to be limit-controlled. This is because S_0 can be emptied under M, if $M(p_2)=a$, $M(p_8)=c$ and $M(p_3)+M(p_8)=b$. Note that both S_1 and S_2 are controlled if $M(p_3)>0$ and $M(p_8)>0$—possible if $b>1$.

Table 2 Compound siphons and their dependency

Compound siphons	Places	Dependency
S_2	$p_4,p_{10},p_{15},p_{20},p_{21},p_{22},p_{23},p_{24},p_{25},p_{26}$	$S_2=S_4oS_{17}$
S_3	$p_4,p_{10},p_{16},p_{21},p_{22},p_{24},p_{25},p_{26}$	$S_3=S_4oS_{18}$
S_5	$p_4,p_9,p_{13},p_{15},p_{20},p_{21},p_{23},p_{24},p_{25},p_{26}$	$S_5=S_{10}oS_{16}oS_{17}$
S_6	$p_4,p_9,p_{13},p_{16},p_{21},p_{24},p_{25},p_{26}$	$S_6=S_{10}oS_{16}oS_{18}$
S_7	$p_4,p_9,p_{13},p_{17},p_{21},p_{24},p_{26}$	$S_7=S_{10}oS_{16}$
S_8	$p_4,p_9,p_{12},p_{15},p_{20},p_{21},p_{23},p_{24},p_{25}$	$S_8=S_{10}oS_{17}$
S_9	$p_4,p_9,p_{12},p_{16},p_{21},p_{24},p_{25}$	$S_9=S_{10}oS_{18}$
S_{11}	$p_2,p_4,p_8,p_{10},p_{15},p_{20},p_{21},p_{22},p_{23},p_{25},p_{26}$	$S_{11}=S_1oS_{16}oS_{17}$
S_{12}	$p_2,p_4,p_8,p_{13},p_{15},p_{20},p_{21},p_{23},p_{25},p_{26}$	$S_{12}=S_{16}oS_{17}$
S_{13}	$p_2,p_4,p_8,p_{10},p_{16},p_{21},p_{22},p_{25},p_{26}$	$S_{13}=S_1oS_{16}oS_{18}$
S_{14}	$p_2,p_4,p_8,p_{13},p_{16},p_{21},p_{25},p_{26}$	$S_{14}=S_{16}oS_{18}$
S_{15}	$p_2,p_4,p_8,p_{10},p_{17},p_{21},p_{22},p_{26}$	$S_{15}=S_1oS_{16}$

Consider another well-known S^3PR in Fig. 2 with basic and compound siphons shown in Tables 1 and 2. Let $S_{i,j}=S_ioS_jb_{i,j}=M_0(r),S_i\cap S_j=\{r\}$. We have $b_{i,j}=1$, $i\neq j$ for all basic siphons S_i and S_j except for $S_{1,16}=S_{15}$, $b_{1,16}=2$; we add a monitor V_{S15}. Now for all the rest compound siphons, let $S_{i,j,k}=S_ioS_joS_k=S_{i,j}oS_k$, $b_{i,j,k}=M_0(r)$, $S_{i,j}\cap S_j=\{r\}$. We have $b_{i,j,k}=1$, for all basic siphons S_i, S_j and S_k, $i\neq j\neq k$, $i\neq k$. Thus, they are already controlled and need no control elements. Afterwards we perform MIP tests as in [1] to find emptiable siphons and add control elements.

Definition 1 An n-compound siphon is a compound siphon depending on n basic siphons.

In general, we add monitor for each basic siphon. Then we add monitor for each two-compound siphon if it does not satisfy the condition in Theorem 1. Repeat this until all compound siphons have been controlled. If no k-compound siphon needs monitor, neither will be any g-compound siphon, where g>k.

Note that we have directed output arcs of a control place related to a minimal siphon S to the sink transitions of S as in [1] to maximize the number of good states. The introduction of control places may create new unmarked siphons, and the process continues until no siphon can become unmarked.

The resulting model is shown in Table 3 with 1020 (>>28 in the original model) minimal siphons where we

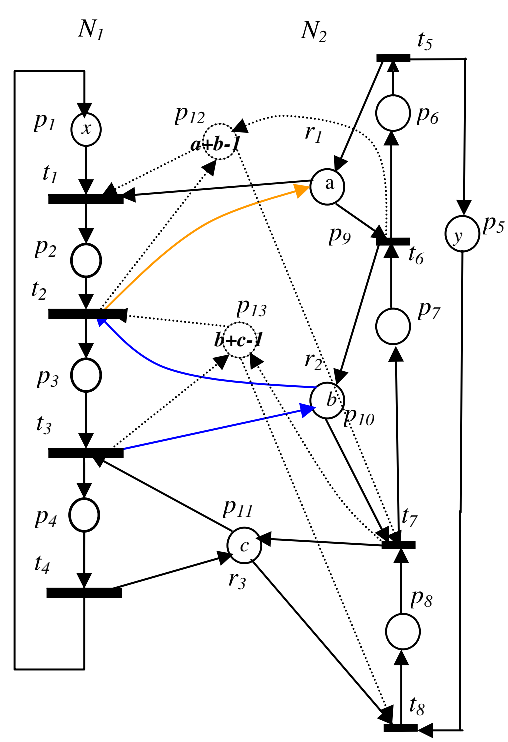

Fig. 1 Example 1

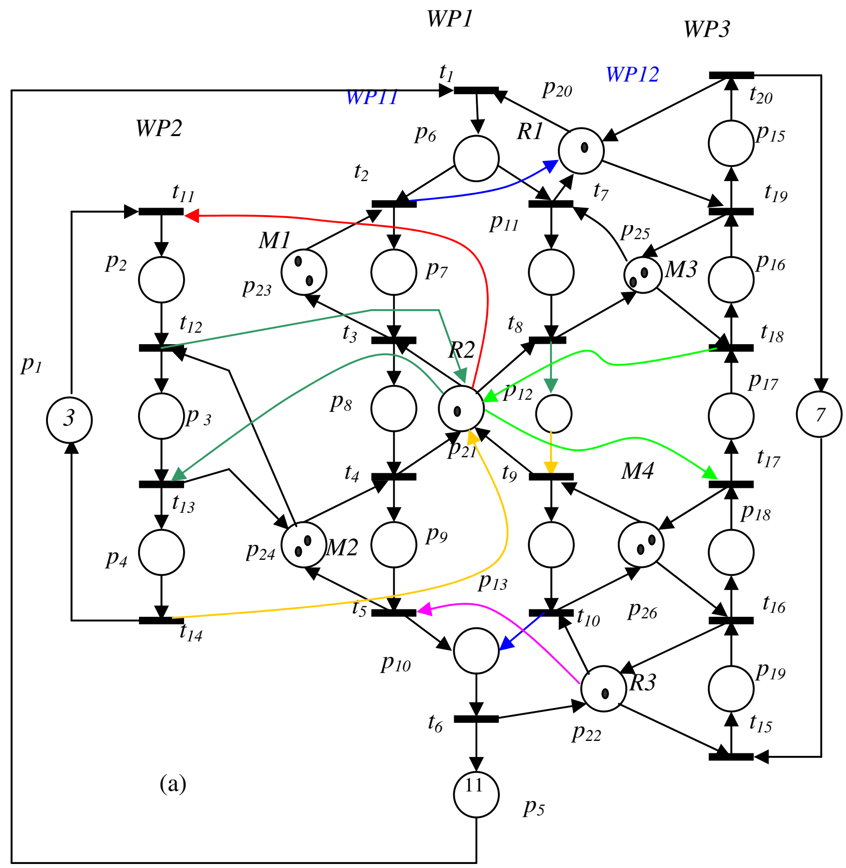

Fig. 2 [2]. Example 2. An S^3PR in [2]

have added 14 monitors and 78 control arcs vs. 19 monitors and 120 control arcs in [4] but with the same 21,562 good states.

Note that among so many new SMS generated, we only need to add control elements to seven of them (Table 3).

It is easy to see that an S^3PR cannot be controlled with monitors fewer than the number of elementary siphons since some dependent siphons may not be controlled.

We have applied it to the nets in Figs. 3 and 6 in [1]. After all basic and compound siphons have been controlled; the FMS are live and maximally permissive. The rest monitors with weighted control arcs are redundant and can be removed.

In the worst case, all basic and compound siphons need control elements and the problem of siphon enumeration cannot be avoided. However, the MIP-test

Table 3 Control elements for the S^3PR in Fig. 2*

V_i	V_i	V_i	$M_0(V_i)$	S
1	t10 t16	t9 t15	2	Elementary(S_1)
2	t5 t10 t13 t17	t3 t9 t11 t15	5	Elementary(S_4)
3	t4 t13	t3 t11	2	Elementary(S_{10})
4	t10 t17	t8 t15	3	Dependent(S_{15})
5	t9 t17	t8 t16	2	Elementary(S_{16})
6	t3 t8 t19	t1 t17	5	Elementary(S_{17})
7	t8 t18	t7 t17	2	Elementary(S_{18})
8	t8 t17	t7 t16	3	New SMS
9	t9 t17	t7 t15	4	New SMS
10	t8 t10 t17	t7 t9 t15	4	New SMS
11	t5 t8 t13 t17	t7 t3 t11 t15	6	New SMS
12	t5 t8 t10 t18	t1 t9 t15	9	New SMS
13	t5 t8 t10 t17 t19	t1 t9 t15 t18	9	New SMS
14	t5 t9 t17 t19	t1 t15 t18	9	New SMS

*Baisc (S_1, S_4, S_{10}, S_{16}, S_{17}, S_{18}) and compound (S_{15}) siphons are shown in Tables 1 and 2.

iterations must be more than the total number of basic and compound siphons equivalent to complete siphon enumeration.

2 Conclusion

We have proposed to avoid complete siphon enumeration and reduce the number of iterations of MIP-test by adding monitors to all basic siphons and only those compound siphons violating the derived condition. The result can be extended to other models such as ES^3PR, S^2LSPR, and S^3PMR [1] as well. Although we have presented the condition assuming $S_{i,j} \cap S_j = \{r\}$, the condition otherwise will be derived in a future paper.

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