Virtual First-Order Structure

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The proof of liveness for various new classes of nets is not intuitive and rather hard to understand. We propose to find the maximum class, called non-virtual-net (NV-net) that are live as long as all minimal siphons never get empty of tokens and the maximum class, called virtual-net (V-net) that may be weakly live if all minimal siphons never get empty of tokens. In the future, when a new system is developed, if it is an NV-net, then it is live as long as no siphons ever get empty. We show that weakly liveness is closely related to a structure called Virtual First Order Structure. We show that both Synchronized Choice Net and Extended Synchronized Choice Net belong to non-virtual-net.

Keywords: Petri nets, synchronized choice nets, siphons, traps, weakly live

1. INTRODUCTION

One of fundamental problems of using Petri nets to model automated systems is the detection of deadlocks which can be characterized by siphons and traps [1-3]. However, for arbitrary large Petri net models, there is hardly any efficient algorithm for checking liveness. A siphon (trap) is a set of places where tokens can leak out (inject in) and may never return (leak out) to make it dead. It is known that in a dead net, the set of places without tokens forms an empty siphon. Hence to avoid being dead, all siphons should never be empty of tokens [4] (the net is called an N_f, see Def. 2).

However, being never empty of tokens does not guarantee liveness as shown in Fig. 1 (a). The set of all places form a siphon that contains a token. t_1 is not live even though both its input places may be marked. There is only one token moving alternatively between the two input places. The net is called *weakly live* and denoted N_w. It is live if there are two tokens in the net. The path (called virtual path, see Def. 7) from p_1 or p_2 to t_1 does not contain any node and both p_1 and p_2 have output transitions other than t_1. Such a structure is called a Virtual First Order Structure (VFOS, see Def. 8). We will show that such a structure makes the net *weakly live*.

As a result, proof of liveness based on siphons only applies to special classes of nets such as bounded Extended Free Choice net (EFC), Systems of Simple Sequential Processes with Resources (S^3PR) [3], Extended Non-Self controlling nets (ENSeC) [4], Extended Resource Control Net (ERCN) [5], Synchronized Choice nets (SNC, see Def. 11) [6, 7], Extended SNC (ESNC, see Def. 12) [2], *etc*.

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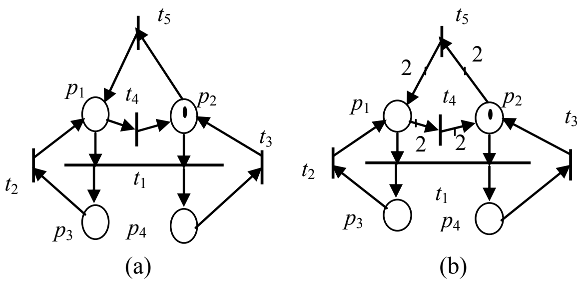


Fig. 1. (a) An example of weakly live net where the only siphon never gets empty of tokens; (b) The GPN version of that in (a).

The above classes of nets have proven to be live as long as they are N_f. But the proof is not intuitive and rather hard to understand. We propose to find the maximal class of nets that are live as long as all minimal siphons never get empty of tokens. We call such nets *non-virtual-net* or *NV-Net* and *they do not have any VFOS*. The rest of nets are called *virtual-net* or *V-net* (Fig. 1 (a)). *This is convenient because if a new class of Petri nets* (*PN*) *lies within the set*, *we don't have to prove its liveness in a hard and non-intuitive way* and it is live if every minimal siphon is always marked.

Section 2 presents the preliminaries. Section 3 presents the problems due to asymmetrical first-order structure (FOS, see Fig. 2). Section 4 proves the above claim on maximal class of nets. Section 5 proves that both SNC (Synchronized Choice nets) and ESNC (Extended SNC) belong to NV-net. Section 6 concludes the paper. We assume the reader knows about Petri nets [8, 9]. However, in order to make the paper as self-contained as possible, an appendix is included with the definition of the main concepts. **For sake of discussion continuity, all proofs are reported in Appendix 2. Appendix 3 lists the index of terms.**

2. PRELIMINARIES

Definition 1 A subnet $N_i = (P_i, T_i, F_i)$ of N is generated by $X = P_i \cup T_i$, if $F_i = F \cap (X \times X)$. It is an *O-subnet* of N if $T_i = P_i\bullet$. The O-subnet of a minimal siphon is denoted O_D.

The net in Fig. 1 (a) has only one siphon: P (all places in N). T is the set of all output transitions of places in P and N is the O_D.

Lemma 1 [1, 10] For a Petri net (N, M_0), if there does not exist any firable transition, then there exists a token-free siphon at M_0.

Definition 2 A net N where all siphons are never token-free (or empty) is denoted N_f. A weakly live net N is denoted N_w. A net N being both N_f and N_w is denoted N_f^w.

Definition 3 (Commoner's Deadlock-Trap Property) [11]: Let (N, M_0) be a marked net. (N, M_0) satisfies the deadlock-trap property, *iff* the following two conditions hold:

(1) Every minimal siphon of N contains a trap.
(2) The maximal trap in each minimal siphon is marked for M_0.

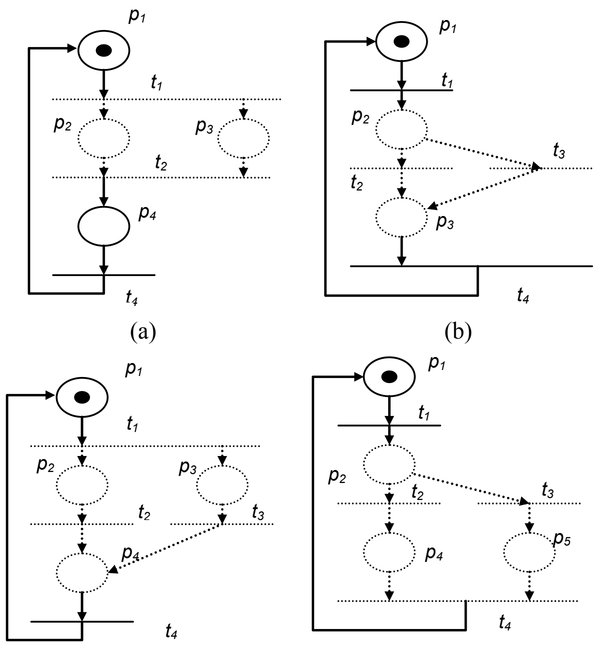


Fig. 2. The four PN containing the four *k*.

Condition 1 is referred to as structural Commoner's property, which is strongly related to the liveness properties [11].

Lemma 2 If a Petri net satisfies the Commoner's Deadlock-Trap Property, then it is an N_f.

Virtual *First-Order Structure* (VFOS) consists of a special type of FOS that contains two handles. We first present the definitions of handles, bridges, AB-handles, and AB-bridges where A and B can be T or P. Roughly speaking, a "handle" is an alternate disjoint path between two nodes. A PT-handle starts with a place and ends with a transition whereas a TP-handle starts with a transition and ends with a place.

Definition 4 Let $N = (P, T, F)$. $H_1 = [n_s n_1 n_2 ... n_k n_e]$ and $H_2 = [n_s n'_1 n'_2 … n'_h n_e]$ are elementary directed paths, $n_i, n'_j \in P \cup T$, $i = 1, 2, …, k$, $j = 1, 2, …, h$. H_1 and H_2 are said to be *mutually complementary*. Each is called a handle in N if $n_i \neq n'_j$ $\forall i, j$ defined above; n_s and n_e are called the start and the end nodes of H_1 and H_2, respectively. Note that n_s and n_e may be identical. An elementary directed path $B = [n_a, n_b, …, n_q]$ is a bridge from H_1 to H_2 if (1) $n_a \in H_1$, $n_q \in H_2$, $n_a \neq n_s$, $n_a \neq n_e$, $n_q \neq n_s$, $n_q \neq n_e$ and (2) $\forall n \in B$, if $n \neq n_a$, $n \neq n_q$, then $n \notin H_1$ and $n \notin H_2$.

In Fig. 3 (a), $H_1 = [p_2\ t_4 p_4\ t_3]$ and $H_2 = [p_2 t_2 p_3\ t_3]$ $n_s = p_2$, $n_e = t_3$. $B_{12} = [t_4 p_3]$ is a bridge from H_1 to H_2 and $B_{21} = [t_2 p_4]$ a bridge from H_2 to H_1.

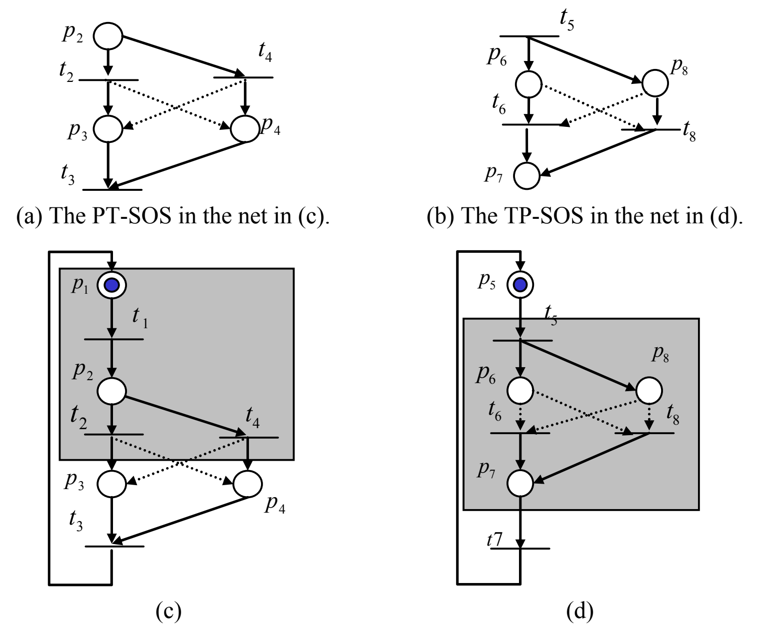


Fig. 3. Examples of SNC in (c) & (d) where the shaded areas cover the structures involving R1 or R2 (Def. 12). Insertion of the dashed lines in (c) & (d) makes all FOS symmetrical respectively.

A handle is an elementary directed path with only its terminal (the start and the end) nodes attached (*i.e.*, common) to a subnet. A bridge is an elementary directed path with only its start node attached to one handle and the end node to another handle.

A *first-order structure* (FOS) contains two handles H_1, H_2 with identical start (called n_s) and end nodes (n_e); there are no paths from H_1 to H_2 and vice versa. Depending on the types of n_s and n_e, there are four kinds of FOS shown dashed in Figs. 2 (a-d). They are called TT-, PP-, TP-, and PT-FOS respectively. A *second-order structure* (SOS) consists of an FOS plus the two bridges between the two handles (with exactly one bridge from one handle to the other).

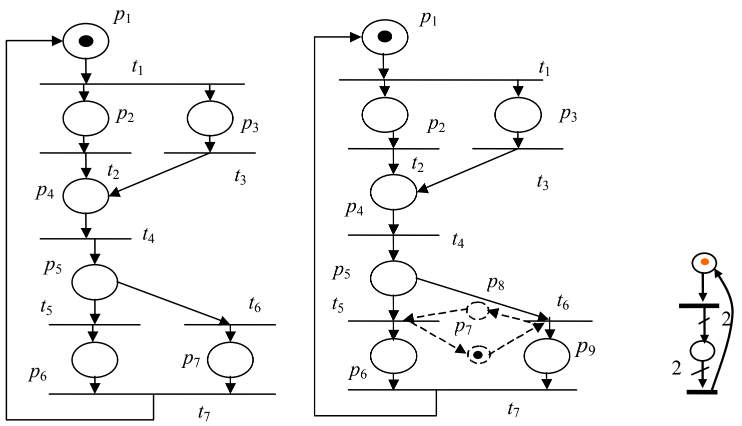
Definition 5 (1) Let $\Psi = H_1 \cup H_2$ denote the union of two graphical objects H_1 and H_2. H_1 is a *prime handle* to H_2, if there are no bridges B between H_1 and H_2 and Ψ is defined to be a first-order structure (FOS). Each H_i (i = 1, 2) is called a *leg* of Ψ. (2) Let ω be an FOS (or handle, bridge, path), if its $n_s \in T$, $n_e \in P$, then ω is called a TP-ω. PT-ω, TT-ω and PP-ω can be defined similarly. If n_s and n_e are of the same type; *i.e.*, both are transitions or places, then ω is said to be *symmetrical*; otherwise it is *asymmetrical*. (3) If B_{12} and B_{21} are the only bridge from H_1 to H_2 and from H_2 to H_1 respectively, then $\varphi = H_1 \cup H_2 \cup B_{12} \cup B_{21}$ is defined to be a second-order structure (SOS) (Figs. 3 (a) and (b)).

[p_2 t_2 p_3] and [p_2 t_3 p_3] in Fig. 2 (b) are two *prime handles*; $n_s = p_2$ and $n_e = p_3$. Note that there are no bridges interconnecting them; hence, they together form an FOS. Since $n_s \in P$, $n_e \in P$, it is *symmetrical.*

3. PROBLEMS DUE TO ASYMMETRICAL FOS

To discover the maximal class of weakly live nets with nonempty siphons, we discuss the problem (and its solutions) induced by asymmetrical FOS (AFOS): An AFOS may result in unboundedness (or nonliveness). For TP-FOS in Fig. 2 (c), firing n_s creates two tokens, which will flow to n_e; after an infinite number of n_s firings, n_e becomes unbounded (see Fig. 2 (c)). For PT-FOS in Fig. 2 (d), tokens in n_s may always flow along a handle and become trapped at an input place of n_e, causing n_e never firable and not live. On the contrary, there are no such problems for symmetrical FOS of TT-FOS (Fig. 2 (a)) and PP-FOS (Fig. 2 (b)) where the n_s and n_e are of the same type.

To consume the extra token, a PT-FOS should follow the above TP-FOS as shown in Fig. 4 (a). Note that n_{e1} (p_4) and n_{s2} (p_5) must be in a circuit to be sequential to each other. Otherwise, the PT-FOS cannot consume the extra token from the TP-FOS. The net is live if the two output transitions t_5 and t_6 of the n_s of the PT FOS are synchronized so that t_5 and t_6 fire alternatively (Fig. 4 (b)).



(a) (b) (c)

Fig. 4. (a) Combing TP and PT FOS; (b) Adding a regulation circuit (RC) to synchronize t_5 and t_6.

Another way to fix the problem is to add a PP-handle (dashed line in Fig. 5 (a)) for each leg (or handle) of the PT-FOS. This remobilizes the trapped tokens by returning them to the siphon via the PP-handle. Such an action is called *detrapping* and the PT-FOS or the PP-handle is a *detrapping* one. Note that in general the PP-handle may be replaced by a live subnet N^l (Fig. 5 (h)).

Definition 6 Let $H = [n_s n_1 n_2 \ldots n_k n_e]$ be a leg of a PT-FOS. Ψ and $H' = [n_k n'_1 n'_2 \ldots n'_q n_s]$ a PP-handle to Ψ. Both Ψ and H' are called detrapping.

Note that in general H' may take different forms than a PP-handle (to simplify the presentation in this paper) to make the net an N_f^w. For instance, it may be two or more PP-handles to H. The detail will be discussed in a future paper.

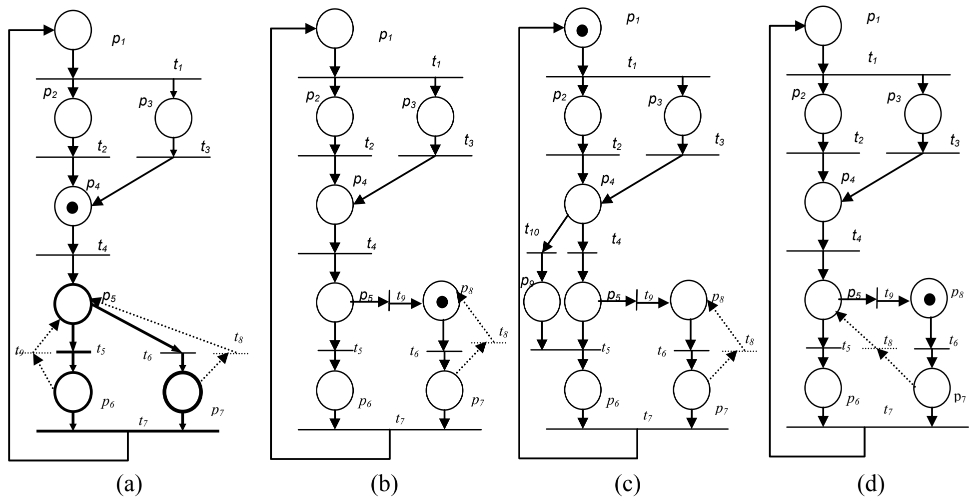


Fig. 5. (a) The O-subnet of D_m, $O(D_m)$ (the whole net), has virtual PT-handles [p_6 t_7] and [p_7 t_7]. [p_7 t_8 p_9] and [p_6 t_9 p_5] are detrapping PP-handles (dashed) of the full detrapping PT-FOS. N is an N_f^w. (b) N is not an N_f but is an N_w (weakly live). The $D_m = \{p_1, p_2, \ldots, p_6\}$ is always empty of tokens; (c) Neither N_f nor N_w; (d) The only PT-FOS is not full detrapping. Neither N_f nor N_w.

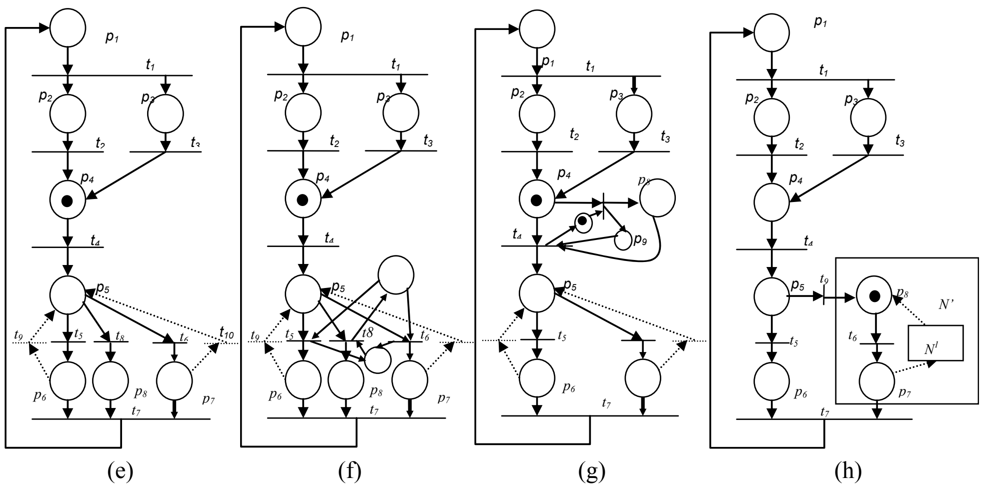


Fig. 5. (e) Neither N_f nor N_w despite the presence of the full-detrapping PT-FOS; (f) Neither N_f nor N_w despite the presence of the RC; (g) N is an N_f^w. $D_m = \{p_1, p_2, \ldots, p_7\}$; (h) N is an N_w, but not an N_f. N' is a maximum live subnet.

Definition 7 A path is *virtual* if it contains only two nodes.

Definition 8 Ψ is called *fully detrapping* and *Virtual First-Order Structure* (*VFOS*) if either (1) both legs are *detrapping and nonvirtual* or (2) one leg is virtual and another is detrapping and nonvirtual. *V-net* is the set of nets where there exists *VFOS*. Otherwise, it is *NV-net*.

The net in Fig. 5 (a) satisfies Def. 8-(1), while that in Fig. 5 (d) is detrapping but not a fully one; it turns fully by replacing the leg [p_5 t_5 p_6 t_7] with a virtual [p_5 t_7] satisfying Def. 8-(1). The bold part in Fig. 5 (a) is a VOS and it is a V-net.

Definition 9 The handle H to a strongly connected subnet N' in N is a directed path from n_s in N' to another node n_e in N'; any other node in H is not in N'. H is said to be a handle in $N' \cup H$.

In Fig. 5 (a), $H = [p_6\ t_7]$, $N' = N\backslash[p_6\ t_7]$, and H is a handle to N' and a handle in N (a V-net). Note that to avoid being trapped again, the n_s of the PP-handle must be the input place (p_6, p_7) of n_e (t_7) of the PT-FOS. This creates virtual paths ([p_6 t_7], [p_7 t_7]) (*i.e.*, only two nodes in each. see Def. 7). We say that the O-subnet (N in Fig. 5 (a)) of a minimal siphon D_m, O_D has virtual PT-handles (VPTH).

Thus the O-subnet of D_m, O_D, has *VFOS*. This triggers the idea that V-net is the maximal class of nets that may be weakly live if all minimal siphons are never empty**.** This is proved by developing a set of lemmas and by looking first at a maximum live subnet in a weakly live net.

Lemma 3 Any handle H in an O-subnet O_D of a minimal siphon D_m must be a PP- or a TP- or a virtual PT-handle.

In Fig. 1, N is an O_D where there are neither nonvirtual PT- nor TT-handles; [p_2 t_5 p_1] is a PP-handle, [t_1 p_4 t_3 p_2] a TP-handle, and [p_2 t_1] a virtual PT-handle. The net N in Fig. 3 (d) is an NV-net. The set P is a siphon and N is its O-subnet which has four VPTH (dashed). Thus, by Definition 8, N seems to be a V-net. This is wrong since the siphon P is not minimal and it contains two minimal siphons, $\{p_5, p_6, p_7\}$ and $\{p_5, p_7, p_8\}$ respectively.

4. THEORY

This section shows that *NV-net* is the maximal class of nets that as long as all siphons of an NV-net N are never empty, N is live by looking first at a maximal live subnet in a weakly live net.

The net in Fig. 1 is a V-net and an N_f^w (both N_f and N_w). A V-net (Figs. 5 (a, e-g)) may not be weakly live (Figs. 5 (e) and (f) since some siphons can become empty). The V-nets in Figs. 5 (a) and (g) are N_f (also N_f^w); no minimal siphons can get empty. The rest are not N_f since they have empty siphons. A weakly live (Figs. 5 (a, b), and (g)) net may not be a V-net. The nets in Figs. 5 (c, d) and (e, f) are neither live nor weakly live. The following lemma is useful to prove Lemma 4.

Definition 10 Let N' be a maximum strongly connected live subnet in a weakly live N, a node with inputs in N' is called an output of N' (denoted Ω_o) and a node with outputs in N' is called an input of N' (denoted Ω_i).

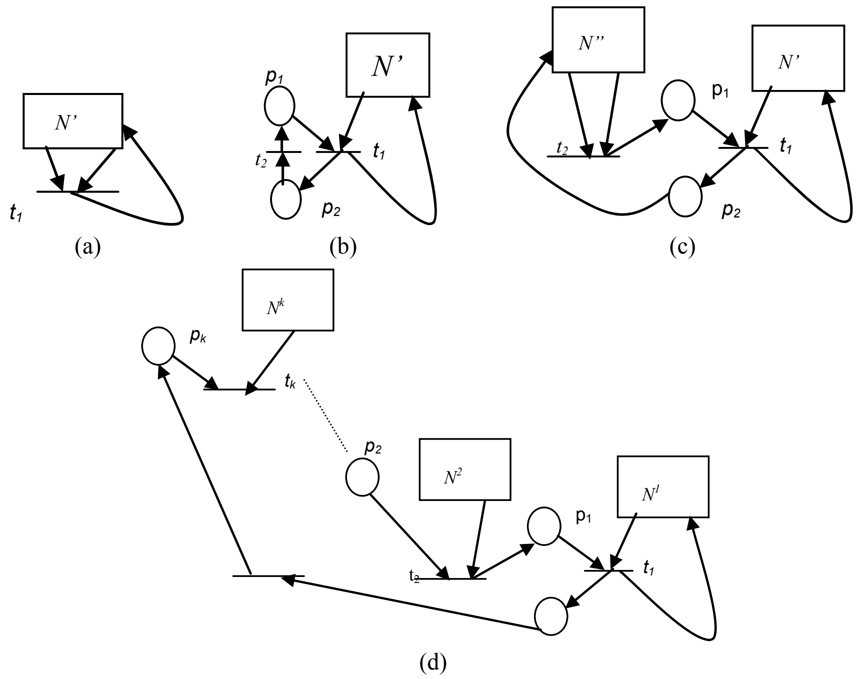


Fig. 6. (a) t_1 is not live N' is a maximum strongly connected live subnet; (b) $\{p_1, p_2\}$ is an empty siphon; (c) t_1 and t_2 are output transitions (Ω_o) of N' and N'' respectively and not live. There are no empty siphons; (d) Here is an net with no detrapping PT-FOS or VFOS. $t_1, t_2, \ldots, t_k$ are dead. $\{p_1, p_2, \ldots, p_k\}$ is an empty siphon; hence it is not an N_f. $N^1, N^2, \ldots, N_k$ are maximum strongly live subnets.

In Fig. 6 (c), t_1 and t_2 are both Ω_o and t_1 is an Ω_i.

Lemma 4 There exists a maximum strongly live subnet N' in an N_f^w, $\exists t \in p\bullet, p \in N'$, t is an Ω_o and the n_e of a VFOS.

Corollary 1 An N_f^w cannot be an NV-net.

The nets in Figs. 5 (a) and (g) are both N_f^w and V-net.

Lemma 5 A V-net may be weakly live if it is an N_f.

The net in Fig. 1 is both an N_f^w and a V-net.

Theorem 1 Any NV-net N is live if all minimal siphons are never empty.

Theorem 2 (1) V-net is the maximal class of nets that may be weakly live if all minimal siphons are never empty. (2) NV-net is the maximal class of nets that are live as long as all minimal siphons are never empty.

Lemma 6 In an N_f^w, there exists a minimal siphon whose O-subnet O_D has at least two virtual PT-handles (VPTH) with the same end node n_e.

5. PROOF OF SNC AND ESNC BEING NV

This section shows that both Synchronized Choice Nets (SNC) and Extended Synchronized Choice Nets (ESNC) belong to NV-net. Hence they are live *iff* they are N_f. First we define SNC and ESNC.

Definition 11 A strongly connected net is SNC (Synchronized Choice net) if it satisfies the two requirements $R1$ and $R2$ where $R1$: every PT-handle to a certain circuit has a TP-bridge from its complementary PT-handle to itself and $R2$: every TP-handle to a certain circuit has a PT-bridge from its complementary TP-handle to itself.

Figs. 3 (c) and (d) are examples of SNC where the shaded areas cover the structures involving $R1$ or $R2$. In Fig. 3 (c), the only two PT-handles $H_1 = [p_2\, t_4 p_4\, t_3]$ and $H_2 = [p_2\, t_2\, p_3\, t_3]$ start from the same place p_2 but they join at a transition t_3. To satisfy $R1$**,** there is a TP-bridge $B_{12} = [t_4 p_3]$ from H_1 to H_2 and a TP-bridge $B_{21} = [t_2 p_4]$ from H_2 to H_1. In Fig. 3 (d), the only two TP-handles $H_1 = [t_5 p_6\, t_6\, p_7]$ and $H_2 = [t_5 p_8 t_8 p_7]$ start from the same transition t_5 but they join at a place p_7. To satisfy $R2$**,** there is a PT-bridge $B_{12} = [p_6\, t_8]$ from H_1 to H_2 and a PT-bridge $B_{21} = [p_8\, t_6]$ from H_2 to H_1.

Before we prove Theorem 3, we have

Lemma 7 All first-order structures (FOS) in an SNC are symmetrical.

Theorem 3 SNC belongs to NV.

We now turn to ESNC. An example is shown in Fig. 4 (b).

Definition 12 [4] (1) A *composite first-order structure* (*CFOS*) $Z = \Psi_1 \cup \Psi_2 \cup \ldots \cup \Psi_k$ is a set of first-order structures $\Psi_1, \Psi_2, \ldots, \Psi_k$, $k \geq 2$, that are (a) interconnected; that is, $\forall\, \Psi_i, \exists\, \Psi_j$ such that $\Psi_i \cap \Psi_j \neq \phi$, if $i \neq j$ and (b) $\forall$ CFOS Z_i, Z_j, $1 \geq |Z_i \cap Z_j|$, if $i \neq j$. (2) If all Ψ_i is of TP (PT) type, then it is a *TP* (*PT*) *composite first-order structure* with symbol Z^T (Z^P).

A CFOS is the largest subnet that can be reduced to a weighted arc with two end nodes. To this end, any Z must intersect with any other by at most one node; *i.e.*, $\forall Z_i, Z_j$, $1 \geq |Z_i \cap Z_j|$, if $i \neq j$. Examples of CFOS are shown in Fig. 4 (b) where Z_1 and Z_2 are a Z^T and Z^P, respectively. In a CFOS, no Ψ_i is followed completely by another Ψ_j.

A PT-CFOS will make the net not live. One way to make it live is to add a regulation circuit (RC, Fig. 4 (b)). Such a structure is no longer a CFOS; however, for brevity, we shall still refer to it as CFOS.

The net (Fig. 4 (b)) can be transformed into a General Petri net (GPN, Fig. 4 (c)) according to the following rule.

Rule of Transformation to GPN [4]: Replace every CFOS by a single arc with two ends being n_s and n_e and the arc weight being $|Z|$ which is the number of handles from n_s to n_e of Z.

Definition 13 [4] *Weighted SNC* (*WSNC*) is a General Petri net (GPN), whose OPN (ordinary PN) version; *i.e.*, by making all arc weights unity, is an Synchronized Choice Nets (SNC) (see Def. 11). *ESNC* (*Extended SNC*) is the class of nets that can be transformed into WSNC.

One can consider WSNC the reduced representation of ESNC and the ESNC the expanded one of the WSNC.

Theorem 4 [4] (1) An ESNC, whose transformed net WSNC is a Weighted Marked Graph (WMG), is well-behaved (WB); *i.e.*, bounded and live, *iff* the WSNC is WB. (2) ESNC belongs to NV.

The following lemma offers an alternative proof that asymmetric choice net (AC) satisfying the Commoner's deadlock trap property is live by showing that any AC is an NV-net.

Lemma 8 (1) Any V-net is not an AC. (2) Any AC is live if it satisfies the Commoner's deadlock trap property.

However, if N is a live AC, it is not necessary that it satisfy the Commoner's deadlock trap property. For instance, the net in Fig. 4 (b) is both an AC and an NV-net; $D_m = \{p_1, p_2, p_3, p_4, p_5, p_6\}$ does not contain a trap.

Definition 14 [1] A path ($\Gamma = [n_1\ n_2 \ldots n_k]$, $k \geq 1$) is *conflict-free iff* $\forall n_i \in T, j \neq i-1, n_j \notin \bullet n_i$.

Definition 15 A Petri net PN is *Extended Non-self Controlling* (*ENSeC*) *iff*, for every pair of transitions t_1 and t_2 such that $\bullet t_1 \cap \bullet t_2 \neq \phi$, there does not exist a conflict-free path leading from t_1 to t_2.

Note that the weakly V-net in Fig. 5 (a) is an Extended Non Self-Controlling Net because every directed path between the only two output transitions of p_5 passes p_5 and hence is not conflict-free. But according to [1], any ENSec is live if it is also an N_f – contradiction.

6. CONCLUSION

We have proved that NV-net is the maximal class of nets that are live as long as all siphons are never empty by first studying the properties of a maximal live subnet in a weakly live net. We found that weakly liveness is closely related to a structure called virtual first order structure (VFOS). We have also derived the necessary condition for an N_f (*i.e.*, all siphons never token-free) to be also an N_w (*i.e.*, weakly live); that is, the net must be a V-net that has VFOS.

APPENDIX 1. PETRI NET-RELATED DEFINITIONS

A Petri net (or Place/Transition net) is a 3–tuple $N = (P, T, F)$ is defined as a net where $P = \{p_1, p_2, \ldots, p_a\}$ be a set of places, $T = \{t_1, t_2, \ldots, t_b\}$ a set of transitions, with $P \cup T \neq \varnothing$ and $P \cap T = \varnothing$; F a mapping from $(P \times T) \cup (T \times P)$ to nonnegative integers indicating the weight of directed arcs between places and transitions. M_0: $P \rightarrow \{0, 1, 2, \ldots\}$ denotes an initial marking whose ith component, $m_0(p_i)$, represents the number of tokens in place p_i.

A node x in $N = (P, T, F)$ is either a $p \in P$ or a $t \in T$. The post-set of node x is $x\bullet = \{y \in P \cup T \mid F(x, y) > 0\}$, and its pre-set $\bullet x = \{y \in P \cup T \mid F(y, x) > 0\}$. A is the incidence matrix of a net with m places and n transitions: $A = [a_{ij}]$; a $b \times a$ matrix of integers and its typical entry is given by $a_{ij} = a_{ij}^+ - a_{ij}^-$ where $a_{ij}^- = F(t_i, p_j)$ is the weight of the arc from transition t_i to its output place p_j, and $a_{ij}^+ = F(p_j, t_i)$ is the weight of the arc to transition t_i from its input place p_j.

t_i is firable if each place p_j in $\bullet t$ holds no less tokens than the weight $w_j = F(p_j, t_i)$. Firing t_i under M_0 removes w_j tokens from p_j and deposits $w_k = F(t_i, p_k)$ tokens into each place p_k in $t\bullet$; moving the system state from M_0 to M_1. Repeating this process, it reaches M' by firing a sequence of transitions. M' is said to be reachable from M_0; *i.e.*, $M_0[\sigma > M'$.

Ordinary Petri nets (*OPN*) are those for which F: $(P \times T) \cup (T \times P) \rightarrow \{0, 1\}$. An OPN is called a marked graph (MG) if $\forall p \in P$, $|p\bullet| = |\bullet p| = 1$. It is an extended free choice net (EFC) if $\forall p_1, p_2 \in P$, $p_1\bullet \cap p_2\bullet \neq \phi \Rightarrow p_1\bullet = p_2\bullet$. It is an asymmetric choice net (AC) if $\forall p_1 \cap p_2\bullet \neq \phi \Rightarrow p_1\bullet \subseteq p_2\bullet$ or $p_1\bullet \supseteq p_2\bullet$. *General Petri Nets* (*GPN*) are those for which $\exists j$, $w_j > 1$, or $\exists k$, $w_k > 1$. A Weighted Marked Graph (WMG) is a GPN and a MG if all arc weights are reduced to one.

$R(M_0)$ is the set of markings reachable from M_0. A transition $t \in T$ is live under M_0 *iff* $\forall M \in R(M_0)$, $\exists M' \in R(M)$, t is firable under M'. A transition $t \in T$ is dead under M_0 *iff* $\nexists M \in R(M_0)$ where t is firable. A net PN is live under M_0 *iff* $\forall t \in T$, t is live under M_0. It is weakly live under M_0 *iff* $\exists t \in T$ and PN is not live. It is bounded if $\forall M \in R(M_0)$, $\forall p \in P$, $\exists k$, *a* positive integer, the marking at p, $m(p) \leq k$.

An elementary directed path Γ in N is a graphical object containing a sequence of nodes $n_1 n_2 \ldots n_i \ldots n_k$ and the single arc between each two successive nodes (*i.e.*, $n_{i+1} \in n_i\bullet$) in the sequence with the notation: $\Gamma = [n_1\ n_2 \ldots n_k]$, $k \geq 1$, such that $n_i \neq n_j$ for $i \neq j$. An *elementary circuit* c in N is $\Gamma = [n_1\ n_2 \ldots n_k]$, $k > 1$ such that $n_i = n_j$, $1 \leq i \leq j \leq k$, implies that $i = 1$ and $j = k$.

For a Petri net (N, M_0), a non-empty subset $D(\tau)$ of places is called a siphon (trap) if $\bullet D \subseteq D\bullet$ ($\tau\bullet \subseteq \bullet\tau$), *i.e.*, every transition having an output (input) place in $D(\tau)$ has an input (output) place in $D(\tau)$. If $M_0(D) = \sum_{p \in D} m_0(p) = 0$, D is called a t*oken-free* or *empty siphon* at M_0. A minimal siphon D_m does not contain a siphon as a proper subset.

An integer vector Y (X, respectively) is called an S- (T-, *respectively*) invariant *iff* Y (X, respectively) $\neq 0$ and $AY = 0$ ($A^T X = 0$, respectively) where A is the incidence matrix. When $Y > 0$, the set of places p such that $Y(p) > 0$ is called the *support* of the S-invariant and is defined as $P(Y)$. If there is a firing sequence containing all the transitions $t \in T$, such that M_0 can be recovered, the Petri net is said to be *consistent*.

APPENDIX 2. PROOFS

Proof of Lemma 2: The two conditions in Definition 3 imply that all minimal siphons never become empty of tokens; hence it is an N_f. ❑

Proof of Lemma 3: Assume contrary and then H must be non-virtual PT- or TT-handles. All places in H can be deleted from the siphon; the rest of the places still form a siphon violating the fact that D_m is a minimal siphon. ❑

Proof of Lemma 4: There exists a dead transition $t_1 \in T$ whose two input places are in N'. The net in Fig. 6 (a) has a single N' and t_1 is the n_e (called n_e^d) of a VFOS since both of its input places in N' are in live circuits in N'. To challenge the lemma, we make t_1 not an n_e^d and not live by an empty circuit $c = [p_1\ t_1\ p_2\ t_2\ p_1]$ as in Fig. 6 (b). But $\{p_1, p_2\}$ is an empty siphon and N is not an N_f. To make it an N_f, we should insert another N' in c. A variant of this action is shown in Fig. 6 (c). Note that both t_1 and t_2 are not live. This action cannot be continued indefinitely since the net size is finite. It terminates by (1) making t_2 an n_e^d to discontinue the challenge as shown in Fig. 6 (c). Note that both N' and N" are maximum strongly live subnets. t_1 and t_2 are output dead transitions of N' and N" respectively. But only t_2 is the n_e of a VFOS. (2) returning to an earlier N' as in Fig. 6 (d). In this case, there exists an empty D_m and violates the fact that N is an N_f. ❑

Proof of Corollary 1: It follows from Lemma 4. ❑

Proof of Lemma 5: There exists a virtual first order structure (VFOS) in N that can be marked such that its n_e is not live. Since all siphons are marked, it cannot be dead by Lemma 1 and hence it is weakly live, an N_w. ❑

Proof of Theorem 1: Let N be an NV-net. By Corollary 1, N is not an N_f^w. By Lemma 1, N is not dead if it is an N_f. Now N is neither dead nor weakly live, it must be live. ❑

Proof of Theorem 2: Both (1) and (2) follow from Theorem 1 that any NV-net must be live if it is an N_f (*i.e.*, all minimal siphons are never empty). For an N_f, it is either (a) weakly live or (b) live. For (a), it must have VFOS and hence is a V-net. If it is not a V-net, then it is an NV-net and can never be weakly live. Hence V-net is the maximal class of nets that may be weakly live if it is an N_f. For (b), it can be either a V-net or an NV-net. But there exists a M_0 such that a V-net is weakly live. Hence it does not satisfy the condition that it is live as long as all minimal siphons are never empty. And only NV-net (the maximal class) meets the condition. ❑

Proof of Lemma 6: Assume contrary and then O_D has none or one VPTH. If there is only one virtual PT-handle H in O_D, then $O_D \backslash H$ is not strongly connected contradicting Def. 9. Thus O_D has no VPTH and it is an NV-net and it cannot be weakly live by Theorem 1 – contradiction. ❑

Proof of Lemma 7: The action of $R1$ and $R2$ repairs the PT- and TP-asymmetry respectively so that all FOS involved becomes symmetrical. ❑

Proof of Theorem 3: Since all FOS in an SNC are symmetrical, there is no virtual first order structure (VFOS) and it is an NV-net. ❑

Proof of Theorem 4: (1) See the proof of Theorem 1 in [2]. (2) The reduced ESNC is an SNC. A set of PT-handles can be reduced to a single but weighted PT-arc. There are no virtual first order structures (VFOS) by the definition of Z^P and it is an NV-net. ❑

Proof of Lemma 8: (1) Any V-net contains a VFOS. For the VFOS in Fig. 5 (a), $p_6\bullet \cap p_7\bullet = \{t_7\} \neq \phi$, $p_6\bullet = \{t_7, t_9\}$, $p_7\bullet = \{t_7, t_8\}$, neither $p_6\bullet \subset p_7\bullet$ nor $p_7\bullet \subset p_6\bullet$; hence the net is not an AC. (2) By (1), any AC is an NV-net. Hence it is live if it satisfies the Commoner's deadlock trap property. ❑

APPENDIX 3. INDEX OF TERMS

Ψ: FOS (Def. 5)
Ω_i: Input transition of a maximum strongly connected live subnet N' (Def. 10)
Ω_o: Output transition of a maximum strongly connected live subnet N'(Def. 10)
AC: Asymmetric choice net (Appendix 1)
AFOS: Asymmetric first-order structure (Def. 5)
B_{12} (B_{21}): The bridge from H_1 to H_2 (H_2 to H_1) (Def. 4)
CFOS: Composite first-order structure (Def. 12)
D: A siphon (Appendix 1)
D_m: A minimal siphon (Appendix 1)
ENSeC: Extended Non-self Controlling net (Def. 15)
ESNC: Extended SNC (Def. 13)
EFC: Extended free choice net (Appendix 1)
FOS: First-order structure (Def. 5)
GPN: General Petri net (Appendix 1)
H: Handle (Def. 4)
M_0: Initial marking (Appendix 1)
N: A Petri net (Appendix 1)
n_e: The end node of a handle H or bridge B or FOS or SOS
n_s: The start node of a handle H or bridge B or FOS or SOS
N_f: Net with all siphons never empty (Def. 2)
N_w: Weakly live net (Def. 2)
N_f^w: Both N_f and N_w (Def. 2)
NV-net: Non-virtual-net (Def. 2)
O_D: O-subnet of a siphon D (Def. 1)
OPN: Ordinary Petri net (Appendix 1)
P: The set of places in N (Appendix 1)
PN: Petri net (Appendix 1)
PP-handle: A handle from a place to a place
PT-handle: A handle from a place to a transition
PP-FOS: A FOS with $n_s \in P$ and $n_e \in P$. (Def. 5)
PT-FOS: A FOS with $n_s \in P$ and $n_e \in T$ (Def. 5)
PT-SOS: SOS with $n_s \in P$ and $n_e \in T$ (Def. 5)

$R1$ ($R2$): Rule 1 (2) (Def. 11)
RC: Regulation circuit (RC)
SNC: Synchronized Choice Nets (Def. 11)
S^3PR: Systems of Simple Sequential Processes with Resources
SOS: Second-order structure (Def. 5)
TP-FOS: FOS with $n_s \in T$ and $n_e \in P$ (Def. 5)
TP-handle: Handle from a transition to a place (Def. 5)
TP-SOS: SOS with $n_s \in T$ and $n_e \in P$ (Def. 5)
TT-FOS: FOS with $n_s \in T$ and $n_e \in T$ (Def. 5)
TT-handle: Handle from a transition to a transition (Def. 5)
VFOS: First order structure (Def. 8)
V-net: Virtual-net (Def. 2)
VPTH: Virtual PT-handle
WMG: Weighted Marked Graph (Appendix 1)
WSNC: Weighted SNC (Def. 13)
Y: S-invariant (Appendix 1)
Z: CFOS (Def. 12)
Z^T: TP composite first-order structure (Def. 12)
Z^P: PT composite first-order structure (Def. 12)

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