ELSEVIER

Int. J. Production Economics 63 (2000) 303–317

international journal of
production economics

www.elsevier.com/locate/dsw

Joint determination of process mean, production run size and material order quantity for a container-filling process

Jinshyang Roan[a,*], Linguo Gong[b], Kwei Tang[b]

[a]Department of Business Administration, Soochow University, Taipei, Taiwan, Republic of China
[b]Department of Information Systems and Decision Sciences, Louisiana State University, Baton Rouge, Louisiana, USA

Received 16 April 1997; accepted 5 April 1999

Abstract

Selection of the mean (target value) for a container-filling process is an important decision to a producer especially when material cost is a significant portion of production cost. Because the process mean determines the process conforming rate, it affects other production decisions, including, in particular, production setup and raw material procurement policies. It is evident that these decisions should be made jointly in order to control the production, inventory and raw material costs. In this paper, we incorporate the issues associated with production setup and raw material procurement into the classical process mean problem. The product of interest is assumed to have a lower specification limit, and the items that do not conform to the specification limit are scrapped with no salvage value. The production cost of an item is a linear function of the amount of the raw material used in producing the item. A two-echelon model is formulated for a single-product production process, and an efficient algorithm is developed for finding the optimal solution.

Keywords: Process mean; Production lot size; Order quantity

1. Introduction

Selection of the process mean (target value) for a container-filling process is a classical problem in quality control. This problem can be illustrated by the single-level canning process, in which containers (cans) are filled with a valuable material, and a lower specification limit is set on the amount of the material in a can. A filled can is classified as conforming if its amount of material is larger than or equal to the lower limit. Otherwise, the can is classified as a nonconforming item. The filled cans are weighed, and the nonconforming (underfilled) cans may be sold at reduced prices, reworked, or scrapped. Usually, the producer can control the mean filling amount (process mean), but not the variation among cans because of the inherent variability in the filling process. Consequently, a portion of the cans produced by the process may be nonconforming although the process mean is set higher than the lower specification limit. The issue of determining the process mean is especially important to, but not limited to, the food, drug, and cosmetic industries, which are governed by laws and regulations on net content labeling, which requires that

* Corresponding author. Tel.: + 886-2-23111531, ext. 3423; fax: + 886-2-23718311.
E-mail address: baroan@mail2.scu.edu.tw (J. Roan)

0925-5273/00/$ - see front matter
PII: S0925-5273(99)00023-7

the declaration of net contents accurately express the quantity of contents of the package or container [1]. Studies conducted by federal agencies showed that a common practice used by many producers is to set a high process mean in order to ensure conformance of specification [2]. As pointed out by Kloos and Clark [1], this strategy is unnecessarily conservative and results in a high production cost.

It is evident that a rational choice of the process mean should be based on a balance between production cost and economical consequences associated with conforming items and nonconforming items. Many studies have addressed this issue under different cost/profit structures and production environments. Springer's [3] model is, perhaps, the first one that addressed this issue although his model assumptions are quite different from those used by others. He considered a production situation where upper and lower specification limits are both present and the performance variable follows a gamma distribution. The per-item cost associated with the nonconforming items above the upper specification limit (overfilled items) may be different from those below the lower specification limit (underfilled items). However, these costs are assumed to be constants (independent of the performance variable). The optimal process mean is obtained to minimize the total costs associated with nonconforming items. A nomograph was developed by Nelson [4] for finding solutions to Springer's model.

Hunter and Kartha [5] considered a product with a lower specification limit, and discussed the situation where nonconforming (underfilled) items can be sold at a (constant) reduced price and a penalty (give-away cost) is incurred by the conforming items with excess quality (the difference of the performance variable and the lower limit). A procedure for calculating the optimal process mean is derived. Bisgaard et al. [6] modified Hunter and Kartha's model by assuming that the selling price of the nonconforming items is a linear function of the performance variable and that of the conforming item is constant. Golhar [7] assumed that only the regular market (fixed selling price) is available for the conforming items and that the underfilled items are reprocessed. Schmidt and Pfeifer [8] considered the situation where the process capacity is fixed.

Artificial limits have been proposed to screen out some overfilled items for re-processing in order to reduce the material cost. Bettes [9] studied a situation with a given lower specification limit and an arbitrary upper limit. Underfilled and overfilled items are reprocessed at a fixed cost. Optimal process mean and the upper specification limit are determined simultaneously. Golhar and Pollock [10] extended Golhar's [7] model to include an artificial upper limit so that nonconforming items as well as the items larger than the upper limit are re-processed. Golhar and Pollock [11] also studied the cost savings yielded from a reduction in the process variance.

The models that have been discussed so far assume implicitly or explicitly that a screening (100% inspection) procedure is used to measure the performance variable in order to determine the selling prices and/or the corrective actions. Tang and Lo [12] discussed a situation in which a surrogate variable is used as the screening variable. Carlsson [13] discussed a situation in which the lots produced by a production process are subjected to lot-by-lot acceptance sampling by variables. Boucher and Jafari [14] studied the same problem except that an attributes sampling plan is used to decide whether a lot is accepted. Melloy [15] considered the packaged goods that are subject to regulatory auditing (compliance tests) schemes.

The process mean issue is especially important to the producer when material-related costs are a significant portion of production cost. Because the process mean determines the process conforming and yield rates, it affects other important production decisions, in particular, the production setup policy. These production decisions directly affect the raw material requirement and, thus, its procurement policy. Consequently, process mean, production and raw material procurement policies should be jointly determined. In this paper, we incorporate the issues associated with production setup and raw material procurement into the classical process mean problem for a single-product production process. It is assumed that the product of interest requires one major raw material, which is purchased from outside vendors. The production cost of an item is a linear function of the amount of the raw material used in producing the item. The product has a lower specification limit, and the items that do not conform to the specification limit are scrapped with no salvage value. A two-echelon model is formulated for jointly determining the process mean, production setup and raw material ordering policies.

The concept of formulating the production and inventory structure of the finished product and the raw material is closely related to the single-product, two-echelon inventory model. The focus of the existing literature on this topic is different, however: the product under consideration requires several raw materials and the decision to be made is how to optimally group the raw materials in the procurement process. Goyal [16] first proposed an integrated model that incorporates the inventory problems of the raw materials and the product for a single-product manufacturing system. It was pointed out in the paper that the production run size of the product has to be known before the procurement policy of a raw material can be determined. A search procedure for determining the length of production run was proposed. Based on Goyal's model, Kim and Chandra [17] and Banerjee et al. [18] proposed heuristic procedures to find the strategy for grouping the raw materials. Kim and Chandra considered the situation in which one order of raw materials can cover the need of one or multiple production runs. Banerjee et al. assumed that one order of raw materials can cover the need of at most one production run, and multiple orders can be made within one production run. Hong and Hayya [19] modified Goyal's model regarding planning horizon and the demand for raw material, and developed an exact solution procedure for simultaneously finding the optimal production–inventory policy and grouping the raw materials.

The organization of the paper is as follows. The assumptions and model formulation are given in the next section. Then, in Section 3, analytical properties of the optimal solution are derived and a solution algorithm is proposed. A numerical example and a sensitivity analysis on the effects of model parameters on the optimal solution and the benefit of using the proposed model are presented, in Sections 4 and 5, respectively. The last section is a brief summary of the results given in this paper and possible future extensions.

2. Model formulation

Consider a product with a constant demand rate of D items per unit time. A production process with a production rate of r items per unit time is used to satisfy the demand. Let X denote the performance variable of interest. As discussed in the last section, X is a measure of the raw material used in the production, such as weight and volume. Assume X is a "larger-is-better" variable so that only a lower specification limit is specified. An example of X is the volume (or weight) of a certain wine in a bottle. Let L denote the lower specification limit of X, so that an item is conforming if its X value is larger than or equal to L. Assume that the production process is stable and X follows a normal distribution with an adjustable mean μ and a constant variance σ^2. Note that this distribution assumption is valid in many production environments when the process under study is in control. However, this assumption may not be appropriate for every process. Nevertheless, the model formulated based on the normality assumption can be modified easily for other distributions.

For given μ, the conforming rate of the production process is

$$p = \int_L^\infty f(x)\,\mathrm{d}x = 1 - \Phi\left(\frac{L-\mu}{\sigma}\right),$$

where $f(x)$ is the probability density function of X and $\Phi(\cdot)$ is the standard normal distribution function.

Assume that nonconforming items are scrapped with no salvage value. Note that the model developed in this paper can be easily modified for the situation where the salvage value is not zero. Consequently, for given μ, the yield rate of the production process is $\lambda = rp$. It is assumed that all the demand will be satisfied in such a way that the expected total number of conforming items produced is equal to the total demand and no backlog is allowed. Note that λ has to be greater than or equal to D to ensure that the production capacity is large enough to meet the demand.

The expected amount of the raw material required to produce one conforming item is μ/p. Let c denote the unit cost of the raw material; thus, cx is the material cost required for producing an item of the finished product. We further assume that the direct cost of producing an item is a linear function of the item's material cost:

$$g(x) = b + \alpha c x,$$

where b is the fixed production cost, and α is a constant larger than or equal to 1. This cost function implies that the production cost consists of a fixed cost and a variable cost which is proportional to the raw material used in production. Note that $\alpha - 1$ is the relative value (cost) added on the raw material during the production process. It can be verified that, for given μ, the expected cost of yielding a conforming item is $(b + \alpha c\mu)/p$. Let h be the cost of holding each unit of the raw material for a unit time. In other words, the cost of holding a monetary unit of raw material is $h_1 = h/c$ per unit time. Assume that the costs of holding a monetary unit of raw material and finished product are the same. Then, the cost of holding a conforming item for a unit time is

$$H = \frac{h}{p}\left(\alpha\mu + \frac{b}{c}\right).$$

Let q be the production run size, which is the number of items (including both conforming and nonconforming items) produced in a production run. The inventory level as a function of time is described in part (a) of Fig. 1. Assume that a production run begins at time 0. Until q items are produced, the finished product inventory increases at a rate of $\lambda - D$ items per unit time. At time q/r, the production run is complete, and, then, the inventory decreases at a rate of D items per unit time until time qp/D when the inventory level reaches 0 and the second production run starts. Let S be the production setup cost. Since the total number of setups required per unit time is D/qp, the total setup cost is SD/qp. It can be verified that the average inventory level for the finished product is $(q/2r)(\lambda - D)$. As a result, the total holding cost for finished products is $H(q/2r)(\lambda - D)$ per unit time. Furthermore, because the expected cost of yielding a conforming item is $(b + \alpha c\mu)/p$, the per-unit-time direct production cost is $D(b + \alpha c\mu)/p$.

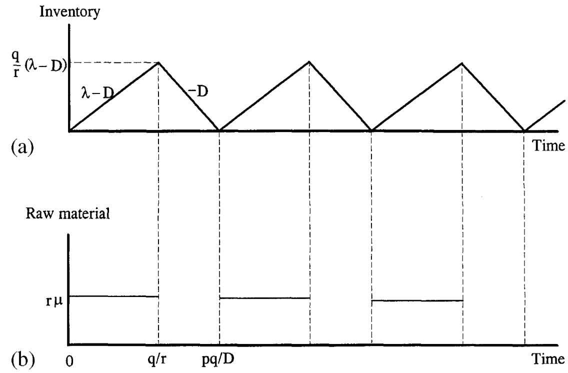

Fig. 1. (a) Inventory level of finished product; (b) demand rate of raw material.

We define the cost associated with the finished product as the sum of the production cost, the process setup cost and the inventory holding cost:

$$\mathrm{FPC}(\mu, q) = \frac{D(b + c\alpha\mu)}{p} + \frac{DS}{pq} + H\frac{q}{2r}(\lambda - D).$$

For given process mean μ and production run size q, the requirement for raw material as a function of time is illustrated in part (b) of Fig. 1: the requirement is a constant rate $r\mu$ during production, and is zero when the production process is idle. We assume instantaneous delivery leadtime and constant order quantity for the raw material procurement. Let Q denote the raw material order quantity. To determine the setup and holding costs of the raw material, we consider the following two ordering policies:

Case A: Each order quantity of the raw material satisfies the requirement of one or multiple production runs; that is, $Q = nq\mu$, where n is an integer larger than or equal to 1.

Case B: Multiple orders are made in a production run; that is, $Q = q\mu/m$, where m is an integer larger than or equal to 1.

In case A, since the raw material requirement is $D\mu/p$ per unit time, the raw material should be ordered $D\mu/(Qp)$ times in a unit time. Let K denote the setup cost per order of the raw material. Then, the total setup cost associated with the raw

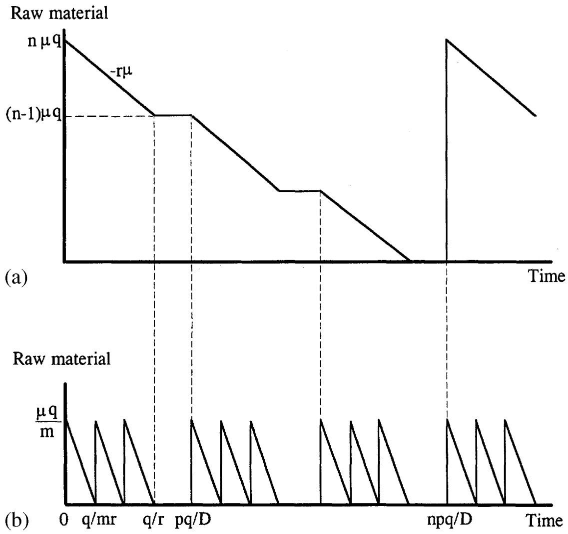

Fig. 2. (a) Inventory level of raw material (case A); (b) Inventory level of raw material (case B).

material is $KD\mu/(pQ)$ per unit time. Since $Q = nq\mu$, the total setup cost can also be expressed as $KD/(npq)$. The inventory level as a function of time for the raw material is described in part (a) of Fig. 2, from which we can find that the average raw material inventory level is

$$\frac{(n-1)}{2}\mu q + \frac{\mu q D}{2rp}.$$

Consequently, the total material setup and holding cost per unit time is

$$\mathrm{MC_A}(\mu, q, n) = \frac{KD}{npq} + h\left[\frac{n-1}{2}\mu q + \frac{\mu q D}{2rp}\right].$$

The total expected cost per unit time for case A is the sum of $\mathrm{FPC}(\mu, q)$ and $\mathrm{MC_A}(\mu, q, n)$:

$$\mathrm{TC_A}(\mu, q, n) = H\left(\frac{q}{2r}\right)(\lambda - D) + \frac{DS}{pq} + \frac{D(c\alpha\mu + b)}{p} + \frac{KD}{npq} + h\frac{(n-1)}{2}\mu q + h\frac{\mu q D}{2rp}. \quad (1)$$

In case B, because m orders are made in one production run, the setup cost per unit time is KmD/pq, and the average raw material inventory level is $q\mu D/2rpm$. The inventory level as a function of time is shown in part (b) of Fig. 2. As a result, the total setup and holding costs per unit time for the raw materials procurement is

$$\mathrm{MC_B}(\mu, q, m) = \frac{KDm}{pq} + h\frac{q\mu D}{2rpm},$$

and the total cost per unit time is the sum of $\mathrm{FPC}(\mu, q)$ and $\mathrm{MC_B}(\mu, q, m)$:

$$\mathrm{TC_B}(\mu, q, m) = H\left(\frac{q}{2r}\right)(\lambda - D) + \frac{DS}{pq} + \frac{D(c\alpha\mu + b)}{p} + \frac{KDm}{pq} + h\frac{q\mu D}{2rpm}. \quad (2)$$

It is easy to verify that expressions (1) and (2) are equivalent when $n = m = 1$. In the next section, we propose a procedure to find the optimal process mean, production run size and the raw material ordering policy, which minimize the total cost.

3. Optimal solution

In this section, we first derive several important analytical properties for the optimal solution, based on which, we propose an efficient solution algorithm.

3.1. Analytical properties

We first discuss case A. Using (1), the optimal q for given μ and n can be found by solving $\partial \mathrm{TC_A}(\mu, q, n)/\partial q = 0$, leading to the following result:

Result 1. *For given μ and n, the optimal value for production run size is given by*

$$q_n = \sqrt{\frac{2rD(S + (K/n))}{Hp(\lambda - D) + h\mu(\lambda(n-1) + D)}}. \quad (3)$$

Furthermore, it can be verified that, for given μ and q, $\mathrm{TC_A}(\mu, q, n)$ is a convex function of n. As a result, if there is an integer n° such that

$$\mathrm{TC_A}(\mu, q, n^{\circ} - 1) \geqslant \mathrm{TC_A}(\mu, q, n^{\circ}) < \mathrm{TC_A}(\mu, q, n^{\circ} + 1), \quad (4)$$

then n^o is the optimal n value for given μ and q. Using straightforward algebraic manipulation, (4) can be translated into the following explicit condition for n^o:

$$\tfrac{1}{2}(\sqrt{1+4a}-1) < n^o \leqslant \tfrac{1}{2}(\sqrt{1+4a}+1), \tag{5}$$

where

$$a = 2DK/h\mu pq^2 > 0.$$

Substituting the production run size q_n given by (3) into (5), we obtain the following result:

Result 2. *For given μ, the optimal n value, denoted by n^*, satisfies the following condition:*

$$n_L < n^* \leqslant \frac{1}{2}\left\{\sqrt{1+4\frac{K}{\mu\lambda S}\left\{\left[(\alpha-1)\mu+\frac{b}{c}\right](\lambda-D)+\mu\lambda\right\}}+1\right\},$$

where

$$n_L = \frac{1}{2}\left\{\sqrt{1+4\frac{K}{\mu\lambda S}\left\{\left[(\alpha-1)\mu+\frac{b}{c}\right](\lambda-D)-\mu\lambda\right\}}-1\right\}, \tag{6}$$

or $n_L = 0$ if n_L given by (6) is not a real number.

We can obtain similar results for case B. First, the optimal q for given μ and m can be found by solving $\partial \mathrm{TC_B}(\mu, q, m)/\partial q = 0$. The result is stated as follows.

Result 3. *For given μ and m, the optimal value for production run size is given by*

$$q_m = \sqrt{\frac{2rD(S+Km)}{Hp(\lambda-D)+h\mu(D/m)}}. \tag{7}$$

Let m^o be the optimal m value for given μ and q. Then,

$$\tfrac{1}{2}(\sqrt{1+4a'}-1) < m^o \leqslant \tfrac{1}{2}(\sqrt{1+4a'}+1), \tag{8}$$

where

$$a' = h\mu q^2/2rK.$$

Substituting q_m given by (7) into (8), the following result is obtained.

Result 4. *The optimal m value for given μ, denoted by m^*, satisfies the following condition:*

$$m_L < m^* \leqslant \frac{1}{2}\left\{\sqrt{1+\frac{4\mu D(S+K)}{K(\alpha\mu+b/c)(\lambda-D)}}+1\right\},$$

where

$$m_L = \frac{1}{2}\left\{\sqrt{1+\frac{4\mu D(S-K)}{K(\alpha\mu+b/c)(\lambda-D)}}-1\right\}, \tag{9}$$

or $m_L = 0$ if m_L given by expression (9) is not a real number.

From (5) and (8), we find that $n = m = 1$ if and only if

$$a < 2 \quad \text{and} \quad a' < 2,$$

which is equivalent to the following condition:

$$\mu q^2/4r < K/h < \mu pq^2/D. \tag{10}$$

Based on (10), we obtain the following result.

Result 5. *For given μ and q,*

(1) *Case A should be used if K/h is higher than $\mu pq^2/D$;*
(2) *Case B should be used if K/h is less than $\mu q^2/4r$.*

It is also worthwhile to note that the raw material order quantity is directly dependent on the production run size, but does not have an explicit relationship with the value-added factor α.

3.2. Solution algorithm

The analytical results presented in the last section provide the basis for developing an efficient solution procedure to find the optimal production run size and the raw material ordering policy for a given process mean. Specifically, if case A is considered, for given μ, the optimal value for n has to satisfy the condition given by Result 2. Then, for the given μ and n, the optimal production run size is obtained by using Result 1. If case B is considered, the optimal values for m and q are found by using Results 4 and 3, respectively. Then, the optimal solutions associated with two cases are compared, and the one with the smaller cost is selected. The procedure is summarized as follows.

Step 1: Find integer sets $\boldsymbol{N} = \{n\colon n$ satisfies the condition given in Result 2$\}$ and $\boldsymbol{M} = \{m\colon m$ satisfies the condition given in Result 4$\}$.

Step 2: Obtain the production run size, q_n, for all $n \in \boldsymbol{N}$, using Result 1, and q_m for all $m \in \boldsymbol{M}$, using Result 3.

Step 3: Compute total costs: $TC_A(\mu, q_n, n)$ for $n \in \boldsymbol{N}$, and $TC_B(\mu, q_m, m)$ for $m \in \boldsymbol{M}$, using expressions (1) and (2), respectively. Then, find the optimal value for q and the optimal material ordering policy (i.e., the optimal value for n or m), which give the minimum total cost.

As a result, the total cost becomes a function of single variable μ. Therefore, a one-dimensional, direct-search procedure, such as the Fibonacci search method and the golden-section search method, can be used to search for the optimal process mean μ^*. In this paper, the range for the search is $[L, L + z\sigma]$, where z is a predetermined real number. In most applications, $z = 4$ is large enough to include the possible optimal solution. The reason L is used as the lower bound is that, when μ equals L, the process conforming rate is 50%, which is very low in most realistic applications. Note that the Fibonacci search method provides the optimal solution if the objective function is unimodal, which was found to be true in all the examples that we tested. In general, multiple-starting points can always be used in the search procedure to ensure that the global minimum is found.

4. An example

In this section, an example is used to illustrate the solution procedure given in the last section. This example will also be used in the sensitivity analysis in the next section.

Consider a product that requires at least 1.6 mg of main content in each item. The item that is less than 1.6 mg is considered nonconforming and is scrapped without salvage value. Because of the variation in the production process, the content of an item produced by the process follows a normal distribution with an adjustable process mean and a constant standard deviation of 0.7 mg. Assume that the product demand rate and the production rate are 5000 items and 7500 items per unit time, respectively. The setup cost per production run is \$500, the fixed production cost is \$0.05 per item, and α is 2. The raw material is purchased from a vendor. Suppose the material cost is \$0.1/mg, and the setup cost per order is \$130. Furthermore, the cost for holding \$1 of inventory (finished product or raw material) is \$0.08 per unit time.

A FORTRAN program has been written to implement the solution procedure given in the last section. The method used for searching for the optimal process mean is the golden-section search method. The program was run on a Pentium II personal computer. The running time for solving this problem was just several seconds, suggesting that the proposed algorithm is computationally efficient. The optimal process mean μ^* was found to be 2.2335 mg.

To demonstrate the three steps for finding the optimal material ordering policy for a given process mean, we give the process used to find the optimal material ordering policy associated with the optimal process mean as follows:

Step 1: For $\mu^* = 2.2335$, using Results 2 and 4, the optimal n and m are found to satisfy $0 < n^* < 1.254$ and $1 < m^* \leqslant 3.646$. Therefore, $\boldsymbol{N} = \{1\}$ and $\boldsymbol{M} = \{1, 2, 3\}$.

Step 2: Using Result 1, $q_n = 18\,762$ for $n = 1$; using result 3, $q_m = 25\,229$ and $q_m = 29\,900$ for $m = 2, 3$, respectively.

Step 3: Using expressions (1) and (2), $TC_A(\mu^*, q_n, 1) = 3449.64$; $TC_B(\mu^*, q_m, 2) = 3407.38$, and $TC_B(\mu^*, q_m, 3) = 3402.99$.

From Step 3, we found that total cost is the minimum when $m = 3$. Consequently, the optimal process mean is 2.2335 mg, resulting in a process conforming rate of 81.73%. The optimal production run size $q_m = 29\,900$, and the order for the raw material should be placed three times within each production run. The order quantity for the raw material is 22 261 mg.

5. Sensitivity analysis

In this section, a sensitivity analysis is performed to study the effects of the following model parameters on the optimal solution: (1) the product demand rate D, (2) the production rate r, (3) the process standard deviation σ, (4) the value-added factor α, (5) the production setup cost S, (6) the material ordering setup cost K, (7) the unit material cost c, and (8) the holding cost h_1. The sensitivity analysis is based on the example given in the last section.

In the model formulation, the cost components are evaluated in terms of unit time. It is also possible to formulate the model based on per-item costs. Because the demand rate is constant, the results of these two formulation methods are the same. However, in the sensitivity analysis, the demand rate is changed to observe its effects on the optimal solution. For comparing these results, per-item costs may be more appropriate. Consequently, three per-item costs are used to report the results in this section. The first one is the per-item finished product related cost given by

$$\mathrm{PFPC} = \mathrm{FPC}(\mu, q)/D.$$

The second per-item cost is the total cost of material setup and handling costs given by

$$\mathrm{PMC} = \mathrm{MC_A}(\mu, q, n)/D \quad \text{or} \quad \mathrm{MC_B}(\mu, q, n)/D,$$

depending on which policy, A or B, is used for purchasing the raw material. Similarly, the per-item total cost is given by

$$\mathrm{PTC} = \mathrm{TC_A}(\mu, q, n)/D \quad \text{or} \quad \mathrm{TC_B}(\mu, q, n)/D.$$

In addition to studying the effects of the model parameters on the optimal solution, we also study the benefit of using the proposed model by comparing its performance with that of a hierarchical model, where the process mean is determined first and the production run size and material order quantity are determined accordingly. Note that the obvious drawback of the hierarchical model is that the demand rate is not considered in determining the process mean, and consequently, it may result in infeasible solutions if the production yield rate is smaller than the demand rate.

We adopt the method suggested by Golhar and Pollock [10] in our comparisons. We first consider an ideal situation where the process has zero variation; i.e., $\sigma = 0$. Under this ideal situation, every item is filled with exactly L units in every filling attempt. The cost associated with the ideal situation represents the minimum cost of the system that cannot be reduced further by the producer's decision. Let PTC_1 and PTC_2 denote the optimal costs associated with the ideal situation and the hierarchical model, respectively.

We define the percent benefit of the integrated model over the hierarchical model as

$$\varepsilon = \frac{\mathrm{PTC}_2 - \mathrm{PTC}}{\mathrm{PTC} - \mathrm{PTC}_1} \times 100.$$

Note that in the example, the optimal process mean for the hierarchical model is 2.3314, and its cost, PTC_2, is 0.6821. The unit cost associated with the ideal situation is 0.4356, resulting in $\varepsilon = 0.49$. Although the benefit of the integrated model in the example is very moderate, significant ε values are found in the sensitivity analysis that follows.

5.1. Effect of demand rate

To study the effects of the demand rate, we obtained optimal solutions for selected values of D ranging from 1500 to 7000 per unit time with an increment of 500. The results are summarized in Table 1.

As the demand rate increases, one would expect that the process mean should be set higher in order to meet the demand. The results show, however, that the process mean actually decreases until the demand rate reaches 5500 per unit time. The main reason for this result is that when the demand rate is low, the production rate (capacity) is too high. To avoid costs incurred because of frequent production setups and excess finished product inventory, the process mean is set lower to reduce the process yield rate. When the demand rate is closer to the production rate (D is larger than 5500), we observe more reasonable results in which the production run size, the process mean, and the process yield rate increase as the demand rate increases. Furthermore, when the demand rate is close to the production rate, the production run size becomes very sensitive to the demand rate. In particular, significant changes in the production run size are observed when the demand rate is larger than 5000.

Table 1
Effect of demand rate

D	μ^*	p	λ	q_m	m^*	Q	PFPC	PMC	PTC	ε
1500	2.3163	0.8469	6352	7902	1	18 303	0.7634	0.0310	0.7943	0.015
2000	2.3133	0.8459	6344	9372	1	21 680	0.7349	0.0301	0.7650	0.023
2500	2.3104	0.8449	6337	10 778	1	24 900	0.7144	0.0300	0.7444	0.031
3000	2.3079	0.8441	6331	12 161	1	28 066	0.6984	0.0304	0.7288	0.041
3500	2.2855	0.8363	6272	16 620	2	18 991	0.6849	0.0308	0.7157	0.144
4000	2.2782	0.8337	6253	18 906	2	21 535	0.6727	0.0303	0.7029	0.193
4500	2.2696	0.8306	6230	21 539	2	24 442	0.6614	0.0302	0.6916	0.257
5000	2.2335	0.8173	6130	29 900	3	22 260	0.6501	0.0305	0.6806	0.597
5500	2.0361	0.7334	5500	2 494 440	228	22 276	0.6240	0.0324	0.6564	6.385
6000	2.1892	0.8000	6000	2 458 143	233	23 095	0.6103	0.0308	0.6411	7.759
6500	2.3776	0.8667	6500	2 925 849	289	24 070	0.6068	0.0296	0.6364	—[a]
7000	2.6508	0.9333	7000	4 564 231	476	25 417	0.6218	0.0291	0.6509	—[a]

[a] — denotes that the solution given by the hierarchical model is not feasible because the production yield rate is smaller than the demand rate.

This is because the inventory accumulation rate $(\lambda - D)$ during production is so low that a process setup is economical only after a long period of production. For example, when the demand rate is 5500, the process yield rate is 5500.2, resulting an average inventory accumulation rate of 0.2 item per unit time. Consequently, the production run size becomes very large, resulting in a continuous production situation with very few stops (setups). The results also indicate that the benefit of the integrated model is larger as λ is closer to D. Note that when the demand rate is 6500 and 7000, the solution given by the hierarchical model is not feasible, because the production yield rate is smaller than the demand rate. Furthermore, by comparing m and Q, when ordering frequency m keeps the same, material order quantity Q increases when demand rate increases. The material-related costs resulting from the ordering policy do not show a clear pattern, however.

Another important observation is that per-item total cost is not a decreasing function of demand rate. It first decreases, then starts to increase when the demand is larger than 6500. The increase in the total cost suggests that the production capacity is not large enough to effectively satisfy the demand rate. These results suggest that a carefully designed production capacity is very important to controlling production cost. In most manufacturing systems, the same facility is used to produce several different products. The result implies that pooling too many products for production in a single, fast machine may not be a good production design, since regardless of the change over cost from one product to the another, the inventory cost for each product may also arise.

5.2. *Effect of production rate*

Table 2 gives the results for selected values of r. As was argued in the last section, a larger production rate does not necessarily lead to the most economical situation. The per-item total cost has its lowest value when the production rate is 6000. Although the general pattern of the process mean in response to the change in r is not found, the process mean increases as r increases when the material ordering frequency is at the same level. Furthermore, the production yield rate increases as r increases, which is generally caused by a decreasing production run size in order to reduce the cost of holding finished product inventory.

When the production rate is between 6000 and 7000, the production yield rate is just slightly larger than the demand rate. As a result, the inventory accumulation rate $(\lambda - D)$ is very small, and, thus, the production run size is very sensitive to a change in the production rate. Similar to the situation in

Table 2
Effect of production rate

r	μ^*	p	λ	q_m	m^*	Q	PFPC	PMC	PTC	ε
6000	2.7720	0.8333	5000	8 679 269	938	21 070	0.6067	0.0337	0.6404	6.286
6500	2.1155	0.7693	5000	1 978 327	198	21 137	0.6157	0.0338	0.6495	8.762
7000	1.9662	0.7143	5000	2 860 602	268	21 307	0.6294	0.0341	0.6635	5.620
7500	2.2335	0.8173	6130	29 900	3	22 260	0.6501	0.0305	0.6806	0.597
8000	2.2682	0.8301	6641	23 354	2	26 485	0.6558	0.0294	0.6851	0.268
8500	2.2752	0.8326	7077	22 341	2	25 414	0.6599	0.0283	0.6883	0.215
9000	2.2810	0.8347	7512	21 545	2	24 572	0.6634	0.0275	0.6910	0.177
9500	2.2853	0.8362	7944	20 910	2	23 893	0.6664	0.0269	0.6933	0.148
10 000	2.2888	0.8374	8374	20 387	2	23 331	0.6690	0.0264	0.6954	0.126

Table 3
Effect of process variation

σ	μ^*	p	λ	q_m	m^*	Q	PFPC	PMC	PTC	ε
0.1	1.8015	0.9780	7335	22 491	2	20 258	0.4657	0.0229	0.4886	0.086
0.2	1.9379	0.9544	7158	22 330	2	21 636	0.5055	0.0243	0.5298	0.111
0.3	2.0435	0.9303	6978	22 415	2	22 903	0.5403	0.0256	0.5659	0.140
0.4	2.1264	0.9059	6794	22 686	2	24 119	0.5718	0.0269	0.5986	0.176
0.5	2.1899	0.8810	6607	23 130	2	25 326	0.6004	0.0281	0.6285	0.219
0.6	2.2171	0.8481	6361	28 310	3	20 921	0.6265	0.0294	0.6559	0.339
0.7	2.2335	0.8173	6130	29 900	3	22 260	0.6501	0.0305	0.6808	0.602
0.8	1.9446	0.6667	5000	4 186 924	374	21 769	0.6587	0.0348	0.6936	4.470
0.9	1.9877	0.6667	5000	2 790 148	252	22 007	0.6718	0.0352	0.7071	6.974
1.0	2.0308	0.6667	5000	2 278 747	208	22 248	0.6849	0.0356	0.7205	8.465

the last section, using the integrated model is especially beneficial when the production run size is very close to the demand rate.

When the production rate is greater than 7000, the process mean and the yield rate increase as r increases. The reason for this is a decreasing production run size will reduce the cost of holding finished product inventory. When r is less than 7000, since the demand has to be satisfied, the production yield rate is limited to a certain level, as is the process mean. The process mean, therefore, does not follow the pattern (decrease) in response to the decrease of r.

When the material ordering frequency is the same, the material order quantity is affected by an increasing process mean but a smaller production run size. The result shows that when the material ordering frequency is the same, the material order quantity decreases as the production rate increases.

5.3. *Effect of process variation*

It is well known that the performance of a process can be improved by reducing its inherent variation [20,21]. For a given process mean, a small process standard deviation implies a higher process yield rate. On the other hand, to maintain the same yield rate, the process mean can be set lower when σ is smaller. In this situation, the material requirement is reduced and thus the material ordering policy may be also affected. To study the effect of the process standard deviation on the optimal solution, the optimal solutions for some selected values of σ ranging from 0.1 to 1.0 are reported in Table 3.

As expected, the per-item total cost increases as σ increases. When σ increases, the process mean increases until σ is equal to 0.7. Then, the value of the process mean is set so that the production yield rate is very close to the demand rate when σ is larger than 0.7. The process conforming rate follows a similar pattern. The decrease in the conforming rate is mainly because of process variation and excess capacity. The conforming rate becomes very stable, however, when the process yield rate is close to the demand rate. The production run size is relatively stable when σ is small, and becomes very sensitive to σ when the production yield rate is close to the demand rate. The material ordering policy is relatively less sensitive to σ. It shows, however, that material order quantity increases when σ increases as long as order frequency remains the same. As a result, it is also found that production-related costs are more sensitive to σ than material-related costs are. The benefit of the integrated model is higher when σ is larger and becomes significant when the production rate and the demand rate are close.

5.4. Effect of value-added factor

A larger α implies a larger cost of producing an item. The holding cost H also becomes larger. Table 4 gives the results for selected values of α. From the table, it is clear that when m remains the same, the process mean decreases as α increases. The main reason is that the cost of raw material is reduced relatively in the model because of the increase in the value of the finished product, which, in turn, increases the importance of reducing the holding of finished products. A low process mean can help to achieve this objective by reducing the per unit production cost and cumulative speed of finished product. A lower process mean reduces the production cost and thus the unit holding cost. At the same time, the material ordering policy is very stable.

In general, the benefit of the integrated model decreases as α increases. This is because the finished product related costs become more important than the setup costs and material holding cost. As the result, when α is larger, the optimal process mean is closer to that given by the hierarchical model. However, the benefit of the integrated model is moderate and stable as α changes.

5.5. Effects of production setup cost

When the production setup cost increases, the production run size is expected to increase in order to reduce the number of production setups. This result is observed in Table 5. As a result, the inventory holding cost associated with the finished product decreases when the setup cost is less than or equal to 600. When the setup cost is larger than 600, the process mean cannot be lowered because the production yield rate is very close to the demand rate. The production run size shifts to a very high level and the process mean drops significantly to the lowest possible level, resulting in a very low inventory level. It is also observed that the process mean decreases as the production setup

Table 4
Effect of value-added factor

α	μ^*	p	λ	q_m	m^*	Q	PFPC	PMC	PTC	ε
1	1.9016	0.6667	5000	2 365 925	209	21 526	0.3609	0.0344	0.3953	1.225
2	2.2335	0.8173	6130	29 900	3	22 260	0.6501	0.0305	0.6806	0.597
3	2.2336	0.8173	6130	22 789	2	25 450	0.9321	0.0306	0.9627	0.364
4	2.2229	0.8132	6099	21 182	2	23 542	1.2126	0.0305	1.2431	0.348
5	2.2185	0.8115	6087	19 818	2	21 982	1.4924	0.0306	1.5230	0.327
6	2.2155	0.8104	6078	18 686	2	20 699	1.7716	0.0308	1.8024	0.302
7	2.2147	0.8101	6076	17 698	2	19 597	2.0505	0.0310	2.0815	0.281
8	2.2142	0.8099	6074	16 849	2	18 653	2.3290	0.0313	2.3603	0.202
9	2.2297	0.8158	6119	13 150	1	29 319	2.6072	0.0313	2.6385	0.145
10	2.2285	0.8154	6115	12 711	1	28 327	2.8849	0.0311	2.9160	0.139

Table 5
Effect of production setup cost

S	μ^*	p	λ	q_m	m^*	Q	PFPC	PMC	PCT	ε
100	2.3084	0.8442	6332	10 838	1	25 018	0.6264	0.0300	0.6564	0.036
200	2.2763	0.8330	6248	18 962	2	21 581	0.6345	0.0303	0.6648	0.204
300	2.2707	0.8310	6233	21 013	2	23 856	0.6406	0.0302	0.6708	0.252
400	2.2646	0.8288	6216	22 921	2	25 935	0.6459	0.0304	0.6763	0.298
500	2.2335	0.8173	6130	29 900	3	22 260	0.6501	0.0305	0.6806	0.597
600	1.9016	0.6667	5000	1 935 651	171	21 525	0.6464	0.0344	0.6808	2.243
700	1.9016	0.6667	5000	2 083 001	184	21 527	0.6464	0.0344	0.6809	3.915
800	1.9016	0.6667	5000	2 219 077	196	21 529	0.6465	0.0344	0.6810	5.564

Table 6
Effect of raw material ordering setup cost

K	μ^*	p	λ	q_m	m^*	Q	PFPC	PMC	PTC	ε
30	1.9016	0.6667	5000	1 761 802	324	10 340	0.6463	0.0165	0.6628	1.438
80	1.9016	0.6667	5000	1 758 421	198	16 887	0.6463	0.0270	0.6733	0.845
130	2.2335	0.8173	6130	29 900	3	22 260	0.6501	0.0305	0.6806	0.597
180	2.2542	0.8250	6188	26 383	2	29 736	0.6503	0.0358	0.6801	0.393
230	2.2489	0.8230	6173	27 995	2	31 479	0.6502	0.0404	0.6906	0.440
280	2.2443	0.8213	6160	29 528	2	33 135	0.6502	0.0446	0.6948	0.485
330	2.2400	0.8197	6148	31 000	2	34 719	0.6503	0.0486	0.6988	0.531
380	2.2352	0.8179	6134	32 438	2	36 252	0.6504	0.0523	0.7027	0.577
430	2.2312	0.8164	6123	33 815	2	37 723	0.6506	0.0558	0.7064	0.429
480	2.2847	0.8360	6270	22 680	1	51 816	0.6514	0.0584	0.7098	0.152
530	2.2831	0.8354	6266	23 273	1	53 134	0.6512	0.0612	0.7124	0.159

cost increases until a further reduction will result in unsatisfied demand. A general pattern is found in material order quantity: as long as the ordering frequency (m^*) remains the same, the material order quantity increases when setup cost increases. If the material order frequency increases because of the increase in production run size, the material ordering quantity decreases. The benefit of the integrated model increases as the setup cost increases and becomes significantly large when the production run size is very large.

5.6. *Effect of material ordering setup cost*

In Table 6, we find that the material ordering frequency decreases as the setup cost associated with raw material ordering increases. On the other hand, the material order quantity increases. The production run size and the process mean have an interesting relationship with the material ordering frequency: under the same material ordering frequency, the production run size increases, but the process mean decreases as the setup cost increases. For example, when m^* is 1, the process mean decreases from 2.2847 to 2.2831, and the production run size increases from 22 680 to 23 273. The increase of q is due to the increase of material ordering quantity, which is in response to the increase of material ordering setup cost. On the other hand, in order to reduce the holding cost, which is raised by a larger q, the process mean decreases. The benefit of the integrated model is moderate and not sensitive to the changes in the material ordering setup cost.

Table 7
Effect of material unit cost

c	μ^*	p	λ	q_m	m^*	Q	PFPC	PMC	PCT	ε
0.01	2.2335	0.8173	6130	29 900	3	22 260	0.6501	0.0305	0.6806	0.597
0.02	2.2257	0.8143	6107	21 632	3	16 048	1.2126	0.0423	1.2558	0.331
0.03	2.2257	0.8143	6107	17 755	3	13 172	1.7716	0.0529	1.8245	0.228
0.04	2.2268	0.8147	6110	15 400	3	11 430	2.2880	0.0610	2.3898	0.174
0.05	2.2285	0.8154	6115	13 773	3	10 231	2.8847	0.0682	2.9529	0.141
0.06	2.2301	0.8160	6120	12 567	3	9342	3.4398	0.0747	3.5145	0.118
0.07	2.2317	0.8166	6124	11 626	3	8648	3.9943	0.0806	4.0749	0.101
0.08	2.2322	0.8168	6126	10 876	3	8092	4.5481	0.0862	4.6343	0.089
0.09	2.2333	0.8172	6129	10 248	3	7629	5.1016	0.0914	5.1929	0.078
0.10	2.2343	0.8176	6132	9716	3	7236	5.6546	0.0963	5.7509	0.070
0.11	2.2350	0.8178	6134	9261	3	6899	6.2074	0.1010	6.3084	0.064
0.12	2.2357	0.8181	6136	8863	3	6604	6.7599	0.1055	6.8653	0.058
0.13	2.2368	0.8185	6139	8508	3	6343	7.3121	0.1097	7.4219	0.056
0.14	2.2372	0.8187	6140	8197	3	6112	7.7641	0.1139	7.9780	0.051
0.15	2.2375	0.8188	6141	7918	3	5905	8.4160	0.1179	8.5338	0.047

Table 8
Effect of inventory holding cost

h_1	μ^*	p	λ	q_m	m^*	Q	PFPC	PMC	PCT	ε
0.02	2.2863	0.8366	6274	56 995	3	43 436	0.6286	0.0151	0.6437	0.139
0.05	2.2573	0.8261	6196	36 985	3	27 828	0.6413	0.0240	0.6653	0.361
0.08	2.2335	0.8173	6129	29 900	3	22 260	0.6501	0.0305	0.6806	0.597
0.11	1.9016	0.6667	5000	1 505 975	156	18 357	0.6464	0.0404	0.6868	3.486
0.14	1.9016	0.6667	5000	1 326 554	155	16 274	0.6466	0.0456	0.6921	6.156
0.17	1.9016	0.6667	5000	1 211 406	156	14 766	0.6467	0.0502	0.6969	8.641
0.20	1.9016	0.6667	5000	1 109 875	155	13 616	0.6468	0.0545	0.7012	10.993
0.23	1.9016	0.6667	5000	1 034 964	155	12 697	0.6469	0.0584	0.7053	13.245
0.26	1.9016	0.6667	5000	973 425	155	11 942	0.6470	0.0621	0.7091	15.417
0.29	1.9016	0.6667	5000	927 503	156	11 306	0.6471	0.0656	0.7126	17.527
0.32	1.9016	0.6667	5000	877 433	155	10 764	0.6471	0.0689	0.7160	19.586
0.35	1.9016	0.6667	5000	844 268	156	10 291	0.6472	0.0720	0.7193	21.603

5.7. *Effect of unit material cost*

An increase in c implies that the per-item production cost and holding cost are larger. We observe from Table 7 that, as c increases, the process mean generally increases except $c = 0.01$. However, we find that the process mean is not sensitive to the change in c. Because of a higher holding cost, the production run size and the raw material order quantity decrease as c increases. As expected, both the material and total costs increase as c increases. The benefit of the integrated model is moderate and not sensitive to the change in c.

5.8. *Effects of holding cost*

As h_1 increases, the costs of holding finished items and raw materials are higher. This should result in lower inventory levels for both finished items and raw material. In Table 8, we find that the process mean and the material order quantity decrease as h_1 increases. We also find that when h_1 reaches 0.11, the process mean stays at 1.9016, and the production yield rate is almost identical to the demand rate, resulting in near zero inventory accumulation. Under this situation, the production run size is very large. Although the material order quantity decreases as h_1 increases, the changes are relatively small. As discussed, the benefit of the integrated model is very significant under the situation. Furthermore, the production run size decreases as h_1 increases because the material order quantity decreases. Comparing to other model parameters, the optimal solution and the benefit of the integrated model are more sensitive to the holding cost.

5.9. Summary

In the sensitivity analysis, we found that the optimal solution is generally not very sensitive to the model parameters. However, under the situation in which it is economical to set the production rate close to the demand rate, the optimal solution becomes sensitive to the model parameters. In this situation, the production run size is very large because the inventory accumulation rate is very small. We also find that, under the same situation, the benefit of using the integrated model is significant. Furthermore, in the results presented in this section, the raw material order policy is always based on Case B. We find from our computation experience that Case B should be used when S is larger than K, although it is difficult to prove it analytically. On the other hand, when K is significantly larger than S and r is large, Case A may be the optimal order policy.

6. Conclusion

In this paper, a two-echelon model is used to incorporate the issues associated with production run size and raw material procurement policy into the classical process mean problem for a single-product production process. The performance variable of the product has a lower specification limit, and the items that do not conform to the specification limit are scrapped with no salvage value. The production cost of an item is a linear function of the amount of raw material used in producing the item. An efficient solution procedure has been developed for a joint determination of process mean, production run size and material order quantity for minimizing the total cost incurred by production, inventory holding and raw material procurement. A sensitivity analysis reveals the effects of the model parameters on the optimal solution and the benefit of using the proposed model.

The model structure presented in this paper provides a useful framework for future research on several interesting issues related to this classical problem in quality control. In particular, the following four extensions are possible. First, the model can be easily modified to consider the situation in which quantity discounts are available for raw material purchasing. Second, one can consider perishable raw material and finished product. The issue becomes very interesting and important when the deterioration speeds of the raw material and the finished product are different. The third extension is to incorporate production process deterioration into the model. The fourth extension is to consider a multiple-level filling process in which several raw materials are added in different stages. However, this issue could be very complicated when the product conformance is jointly determined by the amounts of several raw materials. These issues have been included in the authors' future research plans.

Acknowledgements

We wish to thank an anonymous referee for many helpful suggestions, which improved this paper.

References

[1] C. Kloos, W.L. Clark, Statistical provisions of federal net content control programs, Food Technology 35 (1981) 50–57.
[2] T.M. Quinn, Net weight labeling regulations, Report of the 63rd National Conference on Weights and Measures, National Bureau of Standards Spec. Publication 532, 1978, pp. 138–141.
[3] C.H. Springer, A method of determining the most economic position of a process mean, Industrial Quality Control 8 (1951) 36–39.
[4] L.S. Nelson, Nomograph for setting process to minimize scrap cost, Journal of Quality Technology 11 (1979) 48–49.
[5] W.G. Hunter, C.P. Kartha, Determining the most profitable target value for a production process, Journal of Quality Technology 9 (1977) 176–181.

[6] S. Bisgaard, W.G. Hunter, L. Pallesen, Economic selection of quality of manufactured product, Technometrics 26 (1984) 9–18.
[7] D.Y. Golhar, Determination of the best mean contents for a 'canning problem', Journal of Quality Technology 19 (1987) 82–84.
[8] R.L. Schmidt, P.E. Pfeifer, Economic selection of the mean and upper limit for a canning problem with limited capacity, Journal of Quality Technology 23 (1991) 312–317.
[9] D.C. Bettes, Finding an optimal target value in relation to a fixed lower limit and an arbitrary upper limit, Applied Statistics 11 (1962) 202–210.
[10] D.Y. Golhar, S.M. Pollock, Determination of the optimal process mean and the upper limit for a canning problem, Journal of Quality Technology 20 (1988) 188–192.
[11] D.Y. Golhar, S.M. Pollock, Cost savings due to variance reduction in a canning process, IIE Transactions 24 (1) (1992) 89–92.
[12] K. Tang, J. Lo, Determination of the optimal process mean when inspection is based on a correlated variable, IIE Transactions 25 (1993) 66–72.
[13] O. Carlsson, Economic selection of a process level under acceptance sampling by variables, Engineering Costs and Production Economics 16 (1989) 69–78.
[14] T.O. Boucher, M. Jafari, The optimum target value for single filling operations with quality sampling plans, Journal of Quality Technology 23 (1991) 44–47.
[15] B.J. Melloy, Determining the optimal process mean and screening limits for packages subject to compliance testing, Journal of Quality Technology 23 (1991) 318–323.
[16] S.K. Goyal, An integrated inventory model for a single product system, Operations Research Quarterly 28 (1977) 539–545.
[17] S.H. Kim, J. Chandra, An integrated inventory model for a single product and its raw materials, International Journal of Production Research 25 (1987) 627–634.
[18] A. Banerjee, C. Sylla, C. Eiamkanchanalai, Input/output lot sizing in single stage batch production systems under constant demand, Computers and Engineering Proceedings 19 (1990) 37–41.
[19] J.D. Hong, J.C. Hayya, An optimal algorithm for integrating raw materials inventory in a single-product manufacturing system, European Journal of Operational Research 59 (1992) 313–318.
[20] W.E. Deming, Out of the Crisis, MIT Press Cambridge, MA, 1986.
[21] G. Taguchi, Introduction to Quality Evaluation and Quality Control, Japanese Standards Association, Tokyo, Japan, 1978.