

HEDGING AND ARBITRAGE WARRANTS UNDER SMILE EFFECTS: ANALYSIS AND EVIDENCE

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While the Black–Scholes (BS) model and binomial trees assume that the stock price evolves lognormally with constant volatility, volatility smiles are pronounced in almost all the worlds equity markets. To study the effects of volatility smiles on hedging and arbitrage, the method based on the BS model and implemented with observed market volatility smiles is proposed. The empirical results indicate that the proposed model can reduce risk exposure and increase profits on hedging as compared with the BS model, and hence leading to considerable returns on arbitrage-trading in the Taiwan warrant market. The model is proven to be not only useful for warrant issuers who attempt to reduce vega risk, but also practicable for investors to implement as arbitrage strategies under smile effects.

Keywords: Volatility smile, implied volatility, warrant, hedge, arbitrage.

1. Introduction

Due to the Black–Scholes (BS) seminal paper [1], the option markets have become flourished and the option-related products are now the most active instruments in the financial markets worldwide. The model assumes that the stock price evolves lognormally with constant volatility, yet implied volatility is not constant in practice. Despite this, the BS model has become the standard options pricing model in the academics as well as in the derivative industry.

The BS model and binomial trees are perhaps the most commonly used machinery for options pricing. A standard Cox–Ross–Rubinstein (CRR) binomial tree [2] consists of a set of nodes, representing possible future stock prices, with a constant logarithmic spacing between the nodes. The spacing is a measure of future stock price volatility, which is assumed to be constant in the CRR framework. In the continuous-time limit, a CRR tree with an “infinite” number of time steps to expiration represents a continuous risk-neutral evolution of the stock price. Accordingly,

option prices computed using the CRR tree will converge to the BS continuous-time results in the limit.

The effects of volatility smiles on options pricing have been studied by several researchers in recent years. Dupire [3] showed that the BS model can be extended to be compatible with observed market volatility smiles, allowing consistent pricing and hedging of exotic options. Derman and Kani [4] explored how to construct a simple binomial tree model that is consistent with observed volatility smiles. The tree can be used to consistently value and hedge both standard and exotic index options. Rubinstein [5] provided explicit examples of pre- and post-crash volatility smiles for the S&P 500 and investigated how to construct an implied binomial tree. The tree starts with a given stock price distribution for a particular future date and then build a binomial tree whose terminal distribution is equal to the given distribution. Derman, Kani and Chriss [6] introduced a simple method for building “skewed” state spaces that fit typical index option smiles rather well. Although most literature is available on the binomial tree that is consistent with observed volatility smiles, little attention has been paid to hedging and arbitrage with the BS model under smile effects.

The purpose of this paper is to apply volatility smiles to construct a volatility-smile (VS) model via utilizing the relationship between the implied volatility and the moneyness of specific warrants. In addition, the proposed model was tested in the Taiwan warrant market by examining “hedge efficiency”, hedging profits per unit warrant, and annual returns on arbitrage-trading. It is a useful model to reduce vega risk and can be implemented as arbitrage strategies through the BS model that is consistent with observed market volatility smiles. The results of this study will be of interest to those warrant issuers who attempt to develop a more efficient hedging and gainful arbitrage model under smile effects.

The organization of this paper is as follows. In Sec. 2, the basic analysis is presented and the method for deriving the VS model is described. Section 3 describes the simulated method in which the proposed model is tested using Taiwan warrant market data. Section 4 presents the simulation results and confirms the hedging efficiency and arbitrage performance of the VS model. The conclusion is given in the final section.

2. The VS Model

2.1. *Volatility smile*

Volatility smiles have become more pronounced in almost all the world equity markets since the market crash in October 1987. Volatility smile surfaces can be built by plotting the implied volatility against the moneyness (S/K) and the expiration ($\tau = T - t$) of options. Empirically, implied volatility varies with the strike level and the time to expiration. While the surface changes in shape from day to day, the

reasonable implied volatility can be found by using the interpolation and historical data with given moneyness and expiration.

At any fixed expiration, implied volatility varies with the strike level. Almost always, implied volatility increases with the decreasing strike, that is, out-of-the-money puts are traded at higher implied volatility than out-of-the-money calls. In addition, for a fixed strike level, implied volatility varies with the time to expiration. Often, long-term implied volatility exceeds short-term implied volatility.

Furthermore, “the market is right” means that there are no arbitrage opportunities. If this is the case, then the existence of volatility smiles gives a strong indication that the stock price volatility tends to vary with the strike level, despite the constant volatility assumption is used in the BS model.

2.2. Basic analysis

Figure 1 shows that the BS model without adjusting for the smile effects can lead to hedging underperformance. In Fig. 1, the volatility smile curve is the intersection of the volatility smile surface and fixed expiration. Under the smile effects, suppose that the stock price falls from S_1 to S_2 , making the implied volatility upswing from σ_1 to σ_2 , and hence the option market price will move from $C_1 = C(S_1, \sigma_1, \tau)$ to $C_2 = C(S_2, \sigma_2, \tau)$. However, under the BS model with the constant volatility assumption (σ_1), the option model price is given by $C_3 = C(S_2, \sigma_1, \tau)$. Since σ_2 is greater than σ_1 , the BS model price (C_3) will undervalue the true option market price (C_2).

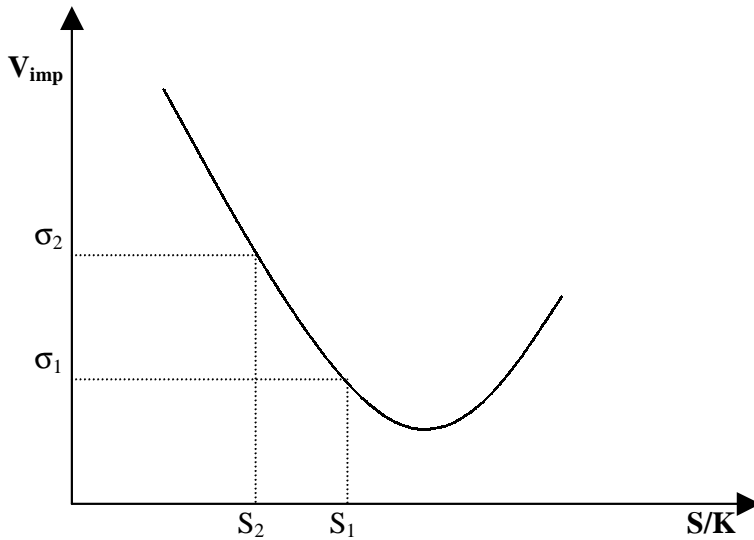


Fig. 1. The volatility smile curve.

Equation (2.1) defines the BS model delta as below:

$$\text{Delta}_{\text{BS}} = \frac{\Delta C}{\Delta S} = \frac{C_3 - C_1}{S_2 - S_1}. \quad (2.1)$$

The market delta due to the smile effects is given by Eq. (2.2):

$$\text{Delta}_{\text{Mkt}} = \frac{\Delta C}{\Delta S} = \frac{C_2 - C_1}{S_2 - S_1}. \quad (2.2)$$

Since C_2 is greater than C_3 , Delta_{BS} is greater than $\text{Delta}_{\text{Mkt}}$. This will cause the BS delta to overestimate the true market delta, and lead to hedging underperformance for those who adopt the BS delta under the smile effects.

2.3. VS model delta

In this subsection, it provides the reasons why VS model delta can be used to reduce vega risk and to provide a profitable arbitrage strategy. Figure 2 depicts the relationship between the stock price and the BS model price, which allows two different volatility (σ_1 and σ_2). Under the BS model, when the stock price falls from S_1 to S_2 , the traveling path of the option model price is on the curve C_1 (from A to B , or from $C(S_1, \sigma_1)$ to $C(S_2, \sigma_1)$), and the model delta is equal to the slope between two points (the slope of line AB). However, under the smile effects, the shift in the stock price will change the moneyness, making the volatility upswing from σ_1 to σ_2 . As a result, the traveling path of the option market price moves from curve C_1 to curve C_2 , (from A to C , or from $C(S_1, \sigma_1)$ to $C(S_2, \sigma_2)$), and the market delta is equal to the slope between the two points (A and C).

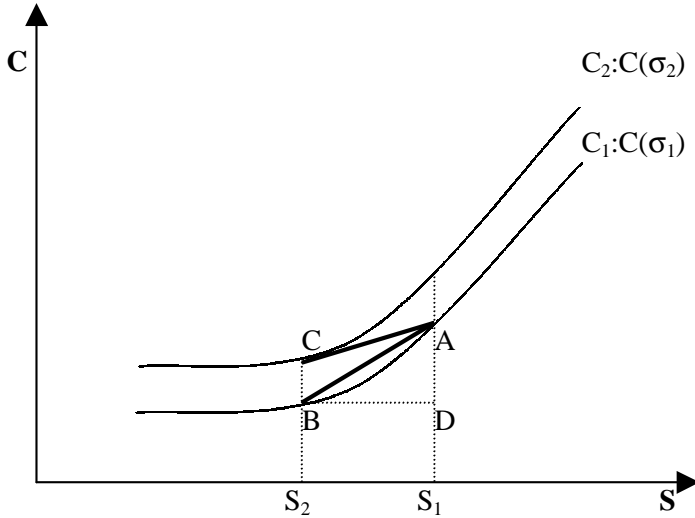


Fig. 2. The BS model price.

The market delta (the VS model delta) can be easily derived and given in Eq. (2.3):

$$\text{Delta}_{\text{Mkt}} = \text{Delta}(\sigma_1) + \text{Vega}(\sigma_1) \times \frac{\Delta\sigma}{\Delta S} = \text{Delta}_{\text{VS}}. \quad (2.3)$$

Since the number of warrants on the same underlying stock is too few to build the volatility smile surface in the Taiwan market, this paper assumes that given a fixed time to maturity, volatility is a function of the moneyness ($\sigma = \sigma(S/K)$).^a With this assumption, the volatility smile curve can be built using one warrant, and leaving K to be constant. Equation (2.4) presents the same contents as Eq. (2.3), which is the partial derivative of the BS model price respect to the stock price at $\sigma = \sigma_1$:

$$\text{Delta}_{\text{VS}} = \left. \frac{\partial C(S, \sigma(S/K))}{\partial S} \right|_{\sigma=\sigma_1} = \left. \frac{\partial C}{\partial S} \right|_{\sigma=\sigma_1} + \left. \frac{\partial C}{\partial \sigma} \right|_{\sigma=\sigma_1} \times \frac{\partial \sigma}{\partial S}. \quad (2.4)$$

The VS model delta has an adjustment factor that the BS model has ignored. The adjustment factor $\Delta\sigma/\Delta S$ accounts for the slop of the volatility smiles and can be estimated by using the interpolation and historical data. In essence, $\text{Vega}(\sigma_1)$ can be represented by $\Delta C/\Delta\sigma$ in the discrete-time concept. Hence, the VS model is expected to be able to reduce the vega risk and to provide a profitable arbitrage model via utilizing the relationship between implied volatility and moneyness under the smile effects.

3. Data and Methodology

3.1. Data

Fifteen expired warrants are drawn from the Taiwan market for the empirical study. Table 1 presents the detailed information.

In the Taiwan warrant market, it always appears that the prices move reversely between the warrants and the underlying stocks at the beginning of the warrants listed in the market. It may be explained as either the investors are not familiar with the derivatives or the warrant issuers as the market makers break down the markup part of the premium at that time. In addition, when the warrants approach expiration or deep out-of-the-money, the warrant market prices always move far away from the model prices. The causes of the difference may be due to that either the liquidity is bad or the lowest quote of the warrants is 0.05. From the above interpretations, this study adopts the period between 24 days after being listed and 24 days before expiration as simulation data for considering the validity of the results.

^aThe impact of expiration will be studied empirically in the later section.

Table 1. The detailed information of the 15 warrants.

Warrant	Underlying	Issue Units	Premium	Strike Price	Listed Date	Expiry Date
0501	2327	22,000	36.04	133.5	9/4/97	9/3/98
0502	1602	22,000	11.34	42	9/4/97	9/3/98
0503	2323	20,000	30.14	98.5	10/22/97	10/21/98
0504	2817	20,000	16.42	69	12/19/97	12/18/98
0505	2327	20,000	22.72	77	1/5/98	1/4/99
0506	2323	20,000	22.61	78.5	1/8/98	1/7/99
0507	1301	20,000	16.44	68.5	2/7/98	2/6/99
0508	1303	20,000	14.875	59.5	2/12/98	2/11/99
0509	2323	20,000	25.97	90.5	2/21/98	2/20/99
0510	2324	20,000	36.42	134.5	2/26/98	2/25/99
0511	2804	20,000	29.43	109	3/5/98	3/4/99
0512	1504	20,000	10.82	45.1	3/16/98	3/15/99
0513	2303	20,000	25.81	91	3/19/98	3/18/99
0514	1602	25,000	8.41	35.8	4/23/98	4/22/99
0515	1602	45,000	7.05	29.4	6/30/98	6/29/99

3.2. Methodology

3.2.1. Estimating the slope of volatility smiles and the implied volatility

Shimko [7] elected a smoother “interpolation” by allowing implied volatility to be represented as a best-fit least-squares parabola. This study adopts this approach by allowing the implied volatility (Y) to be a parabolic function of the moneyness (X), which can be represented by $Y = A_0 + A_1X + A_2X^2$. With this parabolic function and historical data, the slope of volatility smiles and the reasonable implied volatility can be estimated.

3.2.2. Measuring hedging efficiency

The most intuitive method of measuring the hedging efficiency is to compare the profits of hedging portfolios, but it can’t represent the degree of risk exposure. It will be more convincible to measure hedging efficiency by using the method as given in Eq. (3.1). However, this study employs both “hedge efficiency” (H.E.) and the profits per unit warrant to determine hedging performance. “Hedge efficiency” is defined below:

$$\text{H.E.} = 1 - \frac{\text{Var}(\text{DPL}_H)}{\text{Var}(\text{DPL}_{NH})} \times 100\%, \quad (3.1)$$

where DPL_H : Daily profits with hedging; DPL_{NH} : Daily profits without hedging; $\text{Var}(\)$: Variance.

3.2.3. Arbitrage-trading

The reasonable implied volatility can be estimated by using the interpolation and historical data with given moneyness, and then the VS model price can be obtained with this volatility. Since this price is considered to be reasonable under the smile effects, the arbitrage-trading can be built based on the concept “buy low and sell high” whenever the market price moves far away from the VS model price. However, it is not permitted by law to short warrant in the Taiwan market. As a result, when the warrant market price falls through the lower critical price, the arbitrage position can be built by longing warrants and shorting Δ_{VS} shares of the underlying stocks per unit warrant simultaneously, and then trade reversely when the warrant market price upswings through the higher critical price.

4. Empirical Results

4.1. Hedge

This empirical study applies the BS and VS model delta to proceed “daily dynamic delta hedge” simulations^b via using Taiwan warrant market data (daily close prices) and employing ex-day’s implied volatility to the hedge volatility. Moreover, in order to study the impact of expiration, the data is divided into three periods including nine, six and three months to maturity, respectively. Tables 2 to 9 display hedge efficiency and the profits per unit warrant simulated by the two models. The VS(n) in the tables represents using n -day historical data to estimate the slope of volatility smiles, and VS(All) uses all data available from listed to ex-day.

By observing Table 5, as a whole, the average H.E. simulated by the VS model increases as the number of days employed to estimate the slope increases. Increasing the number of days employed cannot only stabilize the “Delta”, but also make the daily PL (DPL_H in Eq. (3.1)) more unvary, thereby leading to better hedge efficiency. In addition, the VS model is better in average H.E. than the BS delta hedge, either employing long or short period historical data. Moreover, the VS model with any number of days employed also shows better in H.E. than the BS delta hedge, except for few warrants (with fairly small differences).

By observing Table 9, although the impact of the number of days employed on the total profits per unit warrant is not significant, the VS model has higher average total profits than the BS delta hedge, either employing long or short period historical data. Moreover, 11 of the 15 warrants hedged with the VS model show higher average total profits than the BS delta hedge.

The simulation results reported in Table 5 exhibit the same conclusions as shown by Tables 2, 3 and 4, and the VS model is superior to the BS delta hedge with approaching expiration. The impact of expiration may seem unremarkable. But when

^bThe volatility smile curves of respective warrants are shown in Appendix A, in which the X -coordinate is moneyness and the Y -coordinate is implied volatility.

Table 2. The “hedge efficiency” in the first period simulated by the BS and VS model.

H.E.	BS	VS(6)	VS(12)	VS(24)	VS(All)	VS Average
0501	45.49%	56.39%	56.42%	55.78%	55.62%	56.05%
0502	53.84%	50.20%	53.38%	54.83%	16.25%	43.66%
0503	-4.96%	16.48%	17.48%	27.39%	44.27%	26.40%
0504	46.35%	44.14%	53.25%	50.50%	58.54%	51.61%
0505	42.62%	71.67%	69.69%	72.21%	69.94%	70.88%
0506	76.51%	78.74%	68.89%	75.01%	78.08%	75.18%
0507	57.14%	47.63%	53.72%	59.53%	62.23%	55.78%
0508	21.27%	31.45%	26.75%	39.78%	42.75%	35.18%
0509	40.22%	37.25%	39.12%	44.82%	45.18%	41.59%
0510	58.32%	72.46%	75.33%	78.08%	79.22%	76.27%
0511	18.03%	-22.68%	-13.26%	18.52%	14.99%	-0.61%
0512	59.46%	53.52%	58.52%	57.87%	56.27%	56.54%
0513	57.05%	49.36%	56.98%	59.77%	58.41%	56.13%
0514	65.41%	65.76%	69.52%	67.77%	69.10%	68.04%
0515	60.66%	48.27%	53.42%	50.43%	38.78%	47.73%
Average	46.49%	46.71%	49.28%	54.15%	52.64%	50.70%

Table 3. The “hedge efficiency” in the second period simulated by the BS and VS model.

H.E.	BS	VS(6)	VS(12)	VS(24)	VS(All)	VS Average
0501	-87.11%	-0.76%	19.28%	29.84%	14.68%	15.76%
0502	49.74%	47.42%	55.69%	59.30%	52.09%	53.62%
0503	40.72%	44.59%	49.98%	61.86%	61.32%	54.44%
0504	51.59%	50.37%	56.26%	62.88%	54.31%	55.96%
0505	33.35%	31.59%	19.32%	39.42%	45.60%	33.98%
0506	65.67%	70.41%	73.82%	72.92%	69.04%	71.55%
0507	39.21%	46.33%	58.77%	57.88%	58.01%	55.24%
0508	40.40%	34.19%	51.22%	49.85%	50.02%	46.32%
0509	14.42%	23.91%	26.23%	21.79%	19.93%	22.97%
0510	68.78%	68.90%	67.61%	74.49%	78.01%	72.25%
0511	22.28%	46.28%	49.73%	40.99%	21.82%	39.71%
0512	-2.86%	36.56%	54.54%	52.07%	28.75%	42.98%
0513	8.72%	38.35%	43.98%	19.49%	28.63%	32.61%
0514	46.33%	52.04%	44.28%	42.39%	57.19%	48.98%
0515	17.27%	54.50%	63.34%	60.82%	7.67%	46.58%
Average	27.23%	42.98%	48.94%	49.73%	43.14%	46.20%

Table 4. The “hedge efficiency” in the third period simulated by the BS and VS model.

H.E.	BS	VS(6)	VS(12)	VS(24)	VS(All)	VS Average
0501	-91.27%	-42.41%	-34.78%	-25.33%	-16.12%	-29.66%
0502	10.28%	-36.08%	-6.33%	3.54%	14.29%	-6.14%
0503	-42.83%	24.13%	22.93%	10.59%	-9.42%	12.06%
0504	-12.89%	12.73%	11.69%	8.85%	8.16%	10.36%
0505	22.74%	56.12%	61.39%	59.54%	43.26%	55.08%
0506	46.24%	63.49%	55.48%	56.59%	52.32%	56.97%
0507	39.01%	61.59%	69.39%	72.90%	68.26%	68.04%
0508	65.18%	77.22%	76.74%	66.79%	70.66%	72.85%
0509	75.10%	77.23%	76.67%	77.53%	75.69%	76.78%
0510	51.41%	61.21%	70.59%	69.49%	63.71%	66.25%
0511	35.58%	75.36%	72.62%	65.66%	26.86%	60.13%
0512	43.82%	57.42%	63.39%	63.74%	60.97%	61.38%
0513	10.27%	35.50%	42.51%	38.03%	31.15%	36.80%
0514	1.25%	50.64%	54.19%	45.49%	32.04%	45.59%
0515	53.89%	73.41%	73.94%	61.31%	61.93%	67.65%
Average	20.52%	43.17%	47.36%	44.98%	38.92%	43.61%

Table 5. The “hedge efficiency” results simulated by the BS and VS model.

H.E.	BS	VS(6)	VS(12)	VS(24)	VS(All)	VS Average
0501	28.45%	48.18%	50.73%	51.59%	50.02%	50.13%
0502	39.91%	24.10%	36.74%	41.20%	26.38%	32.10%
0503	-0.74%	25.28%	28.06%	33.11%	36.49%	30.73%
0504	43.06%	44.32%	50.69%	52.97%	51.30%	49.82%
0505	37.94%	53.59%	47.45%	57.60%	58.36%	54.25%
0506	65.83%	72.07%	68.67%	70.57%	69.05%	70.09%
0507	50.94%	49.99%	57.28%	61.67%	62.63%	57.89%
0508	40.92%	48.90%	48.84%	51.53%	54.52%	50.95%
0509	42.67%	44.50%	45.84%	47.41%	46.57%	46.08%
0510	60.77%	69.30%	71.90%	75.38%	76.09%	73.17%
0511	21.54%	5.78%	12.38%	29.44%	18.71%	16.58%
0512	41.80%	50.41%	58.64%	57.78%	50.95%	54.45%
0513	51.05%	48.11%	55.53%	56.07%	54.98%	53.67%
0514	53.73%	59.58%	59.42%	56.93%	62.03%	59.49%
0515	50.78%	54.32%	59.52%	54.85%	37.04%	51.43%
Average	41.91%	46.56%	50.11%	53.21%	50.34%	50.05%

Table 6. The profits per unit warrant in the first period simulated by the BS and VS model.

Profits/Unit	BS	VS(6)	VS(12)	VS(24)	VS(All)	VS Average
0501	0.21	7.80	6.41	6.54	7.01	6.94
0502	1.71	0.29	1.90	1.35	1.94	1.37
0503	5.07	-10.67	1.42	0.13	-1.90	-2.76
0504	3.78	1.65	3.23	1.50	2.97	2.34
0505	-1.35	-1.29	-0.32	2.52	2.02	0.73
0506	4.08	3.77	4.67	3.53	5.21	4.30
0507	0.53	2.25	4.18	0.61	0.91	1.99
0508	1.29	8.31	6.21	4.78	5.33	6.16
0509	5.37	6.80	2.06	6.60	7.23	5.67
0510	1.11	-0.67	-0.57	-0.56	-2.99	-1.20
0511	4.59	3.41	4.21	6.85	4.09	4.64
0512	1.12	0.66	0.77	0.35	0.97	0.69
0513	1.18	3.54	3.24	2.63	2.43	2.96
0514	1.26	1.60	1.61	2.40	1.90	1.88
0515	-0.12	-0.82	0.74	1.02	0.59	0.38
Average	1.99	1.77	2.65	2.68	2.51	2.41

Table 7. The profits per unit warrant in the second period simulated by the BS and VS model.

Profits/Unit	BS	VS(6)	VS(12)	VS(24)	VS(All)	VS Average
0501	6.11	0.97	3.58	5.10	5.44	3.77
0502	3.04	3.69	2.29	2.07	3.03	2.77
0503	3.50	9.67	7.57	7.55	5.12	7.48
0504	-0.11	4.49	0.46	1.61	0.82	1.84
0505	0.53	7.88	10.67	8.12	3.61	7.57
0506	4.76	4.82	4.83	5.67	5.59	5.23
0507	1.91	1.91	2.41	2.29	2.20	2.20
0508	1.67	1.53	2.07	2.31	1.94	1.96
0509	3.01	2.68	5.80	4.09	3.58	4.04
0510	0.10	-0.97	-1.91	1.13	0.30	-0.36
0511	3.37	4.90	5.93	5.17	3.35	4.84
0512	0.54	0.81	1.15	0.80	0.79	0.89
0513	1.29	2.06	2.64	2.34	1.45	2.12
0514	1.04	-0.37	1.19	2.50	0.91	1.06
0515	1.21	1.85	2.14	2.61	1.62	2.06
Average	2.13	3.06	3.39	3.56	2.65	3.16

Table 8. The profits per unit warrant in the third period simulated by the BS and VS model.

Profits/Unit	BS	VS(6)	VS(12)	VS(24)	VS(All)	VS Average
0501	3.63	3.05	2.66	4.05	3.57	3.33
0502	1.02	1.37	2.30	-0.12	1.15	1.17
0503	0.52	2.02	1.54	-0.84	0.00	0.68
0504	1.67	1.84	2.05	2.15	1.84	1.97
0505	2.73	2.01	2.58	2.44	2.56	2.40
0506	-0.59	-0.91	2.41	-0.56	-0.86	0.02
0507	2.42	2.19	1.92	1.51	2.30	1.98
0508	2.31	1.65	1.39	0.86	2.04	1.48
0509	4.41	2.63	2.62	1.99	4.41	2.91
0510	5.35	4.18	1.01	1.91	4.04	2.78
0511	1.61	1.66	2.39	2.10	1.66	1.95
0512	0.34	1.55	1.03	0.82	0.66	1.02
0513	2.64	1.95	2.26	1.58	2.40	2.05
0514	-0.07	1.47	1.12	0.99	0.33	0.98
0515	0.30	1.25	1.31	0.99	0.30	0.96
Average	1.89	1.86	1.91	1.33	1.76	1.71

Table 9. The profits per unit warrant simulated by the BS and VS model.

Profits/Unit	BS	VS(6)	VS(12)	VS(24)	VS(All)	VS Average
0501	9.94	11.81	12.66	15.69	16.02	14.04
0502	5.78	5.35	6.49	3.29	6.11	5.31
0503	9.08	1.02	10.53	6.84	3.22	5.40
0504	5.34	7.98	5.74	5.26	5.64	6.16
0505	1.92	8.60	12.93	13.08	8.19	10.70
0506	8.25	7.68	11.91	8.64	9.95	9.54
0507	4.86	6.35	8.51	4.42	5.40	6.17
0508	5.26	11.48	9.67	7.95	9.32	9.60
0509	12.80	12.11	10.49	12.69	15.22	12.63
0510	6.55	2.54	-1.47	2.47	1.35	1.22
0511	9.57	9.97	12.52	14.12	9.10	11.43
0512	2.00	3.02	2.96	1.97	2.43	2.59
0513	5.12	7.56	8.14	6.55	6.28	7.13
0514	2.23	2.70	3.92	5.89	3.15	3.91
0515	1.38	2.29	4.19	4.63	2.51	3.40
Average	6.01	6.70	7.94	7.57	6.92	7.28

Table 10. The annual returns and the number of arbitrage opportunities simulated by the VS model.

	VS(6)		VS(12)		VS(24)		VS(All)		Average	
	Returns	Num.	Returns	Num.	Returns	Num.	Returns	Num.	Returns	Num.
0%–40%	1687.64%	9.00	977.93%	12.00	521.50%	13.00	322.16%	17.00	877.31%	12.75
10%–40%	1476.84%	10.00	955.27%	12.00	781.13%	15.00	457.48%	22.00	917.68%	14.75
0%–30%	716.15%	19.00	744.64%	16.00	431.00%	26.00	403.52%	27.00	573.83%	22.00
10%–30%	631.92%	20.00	701.36%	16.00	611.16%	30.00	459.65%	33.00	601.02%	24.75
Average	1128.14%	14.50	844.80%	14.00	586.20%	21.00	410.70%	24.75	742.46%	18.56

the expiration date approaches, the adjustment factor will be far more pronounced due to the sensitivity of “Vega”.

The main difference in the deltas between the two models is that the VS model has an adjustment term represented by “Vega” multiplied by the slope of volatility smiles. Because the expiration affects “Vega” and the slope depends on the shape of volatility smiles, the adjustment will become more significant when the time to expiration is shorter or volatility smiles are steeper. In this circumstance, the VS model will stand out to be far better than the BS delta hedge.

4.2. Arbitrage-trading

This empirical study employs two critical prices and the VS model delta to proceed arbitrage-trading simulations^c using Taiwan warrant market quote data (daily close prices) and employing ex-day’s implied volatility. The study adopts the period between 24 days after being listed and 24 days before expiry as simulation data for considering the liquidity of warrants. Since employing (0%, 10%) and (30%, 40%) as the high and low critical prices, respectively, there are four combinations of critical prices. Table 10 displays the annual returns and the number of arbitrage opportunities for different combinations and the number of days employed to estimate the reasonable implied volatility simulated by the VS model.

Observing Table 10, regarding to the average annual returns and the number of arbitrage opportunities, the annual average returns increase as the number of days employed decreases. Decreasing the number of days employed can reveal the more recent status of the market, and becomes more suitable to the highly volatile Taiwan market. Moreover, the number of arbitrage opportunities increases as the number of days employed increases. This can be explained as follows: long period historical data make the shape of volatility smile curves static, and the market price can fall through the low critical price effortlessly.

^cThe detailed arbitrage-trading simulation results of respective warrants are shown in Appendix B.

In light of the combinations of critical prices, the annual average returns increase and the number of arbitrage opportunities decreases as the low critical price decreases (from 30% to 40%), and the impact of the high critical price is unremarkable. However, the VS model can make considerable returns on arbitrage-trading as a whole with all combinations of the critical prices and the number of days employed.

5. Conclusion

This paper has presented a new model for simulating the effects of volatility smiles on warrant hedging and arbitrage. On the basis of these results, it can be concluded that the risk exposure can be reduced by using the VS model as compared with the BS delta hedge. The technique may be useful for reducing the vega risk of warrant issuers and help arbitrageurs make more informed arbitrage-trading decisions in the Taiwan warrant market. The approach proposed in this paper can also be applied to the other active derivatives such as stock indexes, interest rates, currency and futures options.

Appendix A.

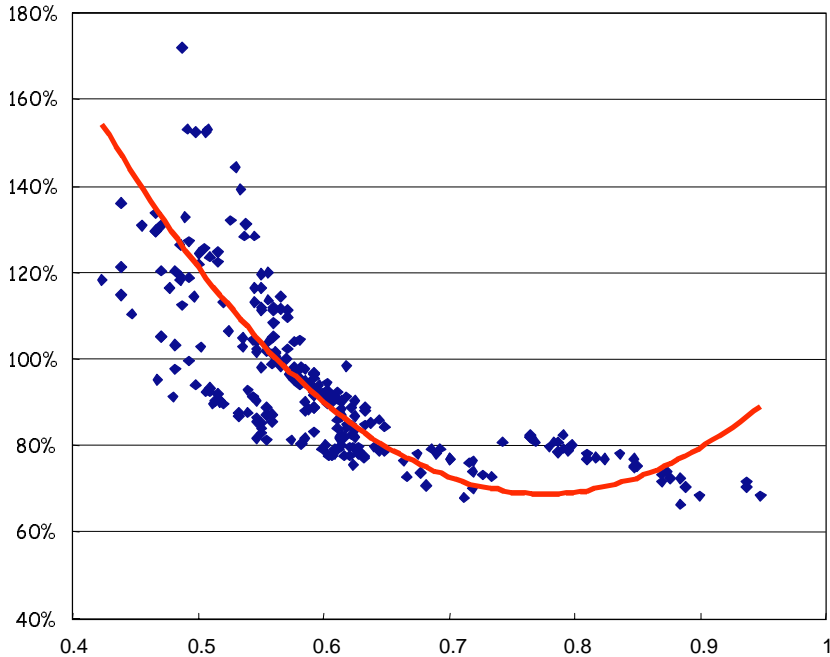


Fig. A.1. The volatility smile curve of 0501.

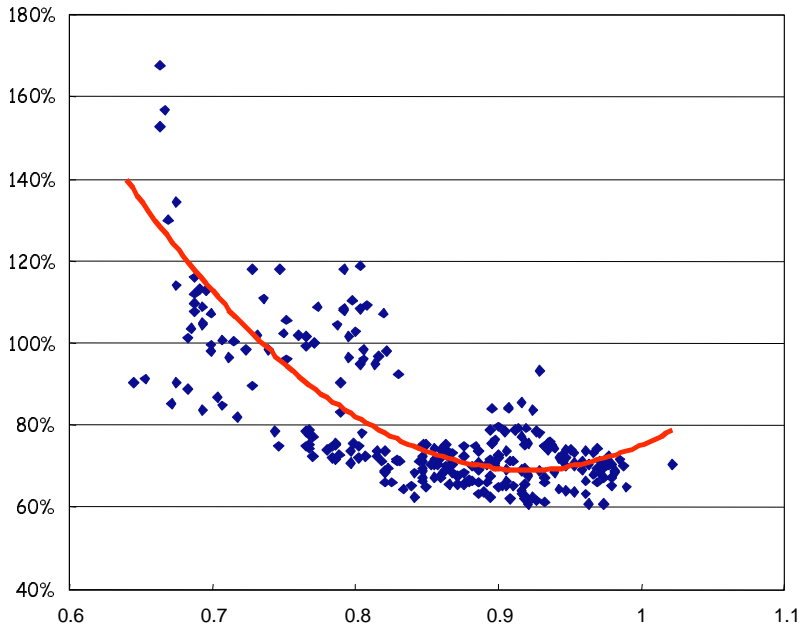


Fig. A.2. The volatility smile curve of 0502.

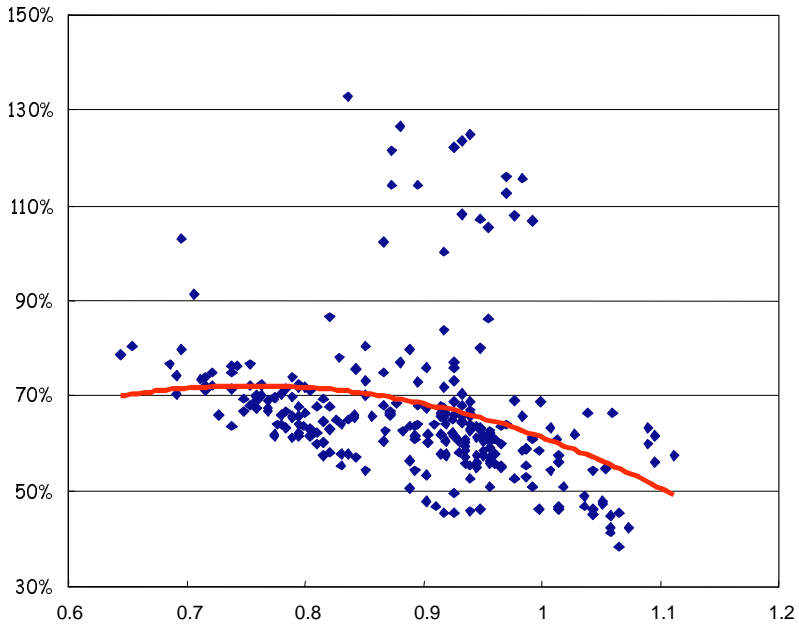


Fig. A.3. The volatility smile curve of 0503.

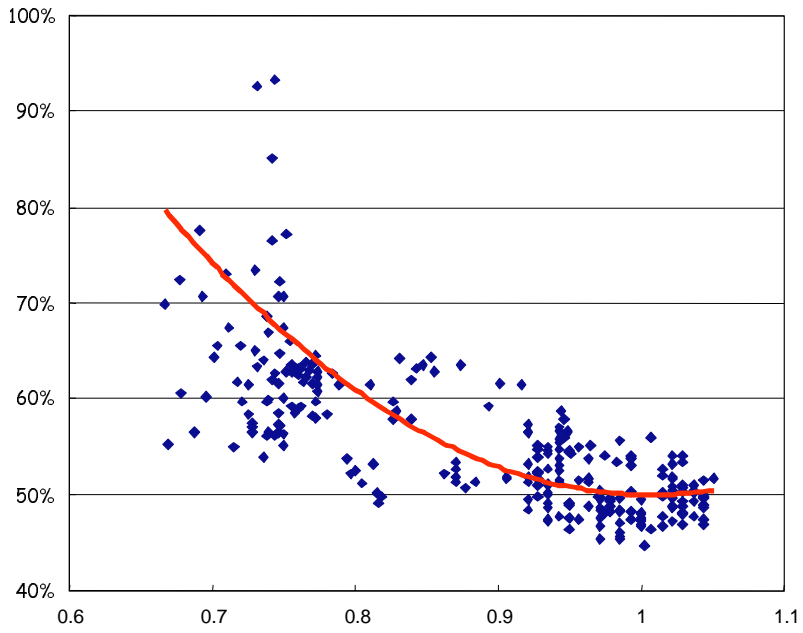


Fig. A.4. The volatility smile curve of 0504.

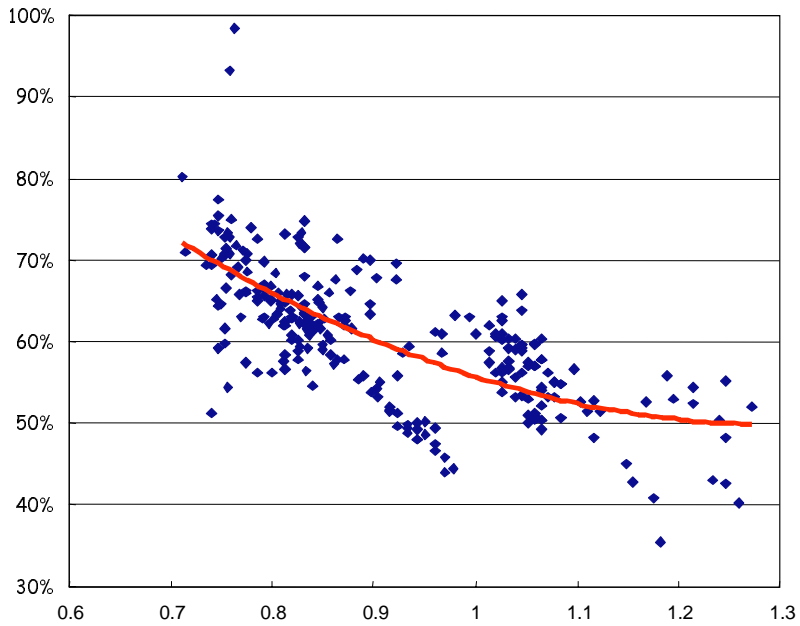


Fig. A.5. The volatility smile curve of 0505.

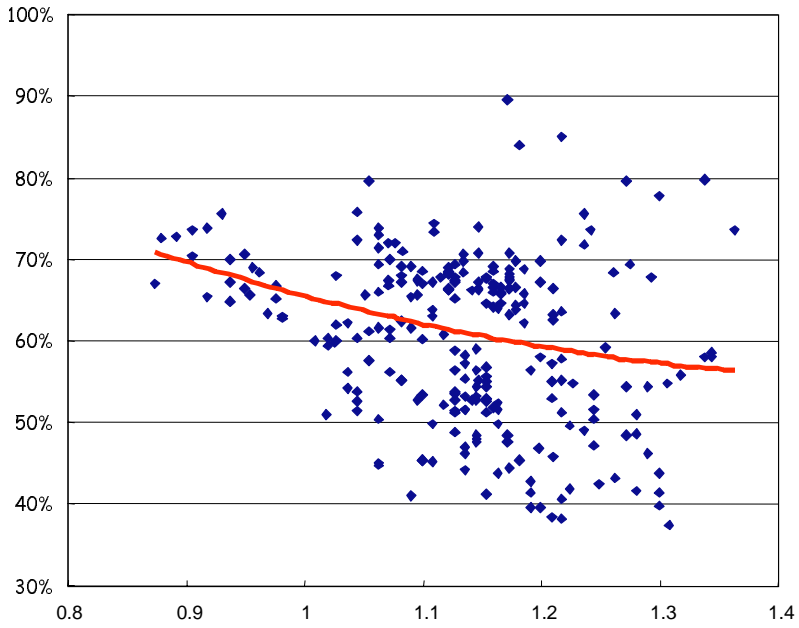


Fig. A.6. The volatility smile curve of 0506.

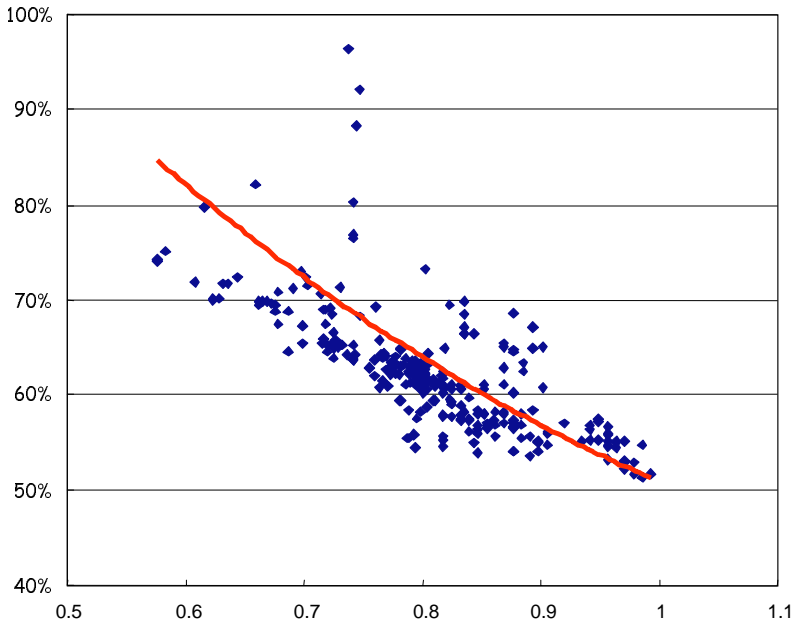


Fig. A.7. The volatility smile curve of 0507.

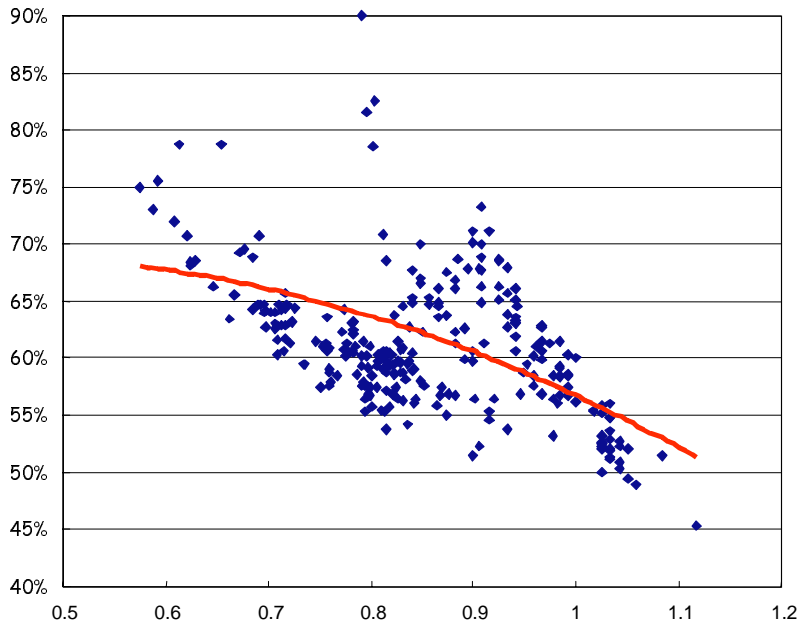


Fig. A.8. The volatility smile curve of 0508.

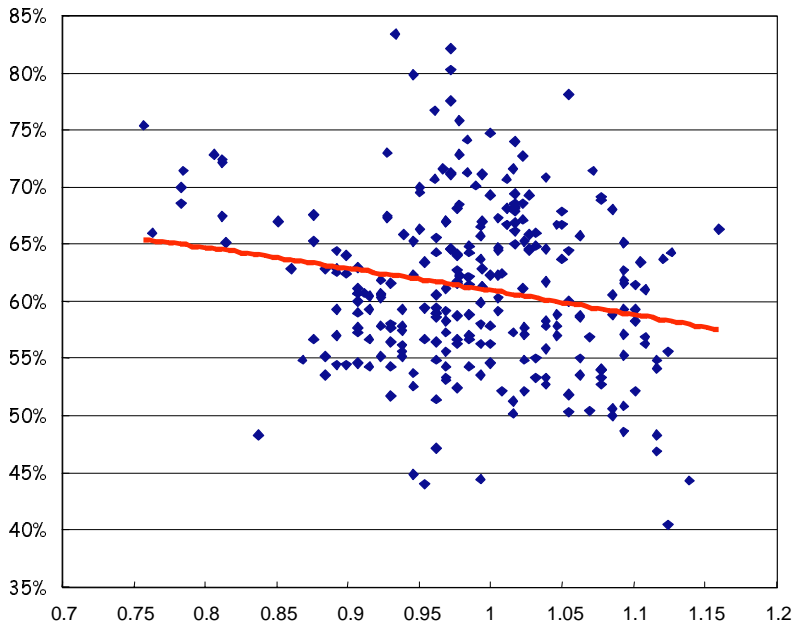


Fig. A.9. The volatility smile curve of 0509.

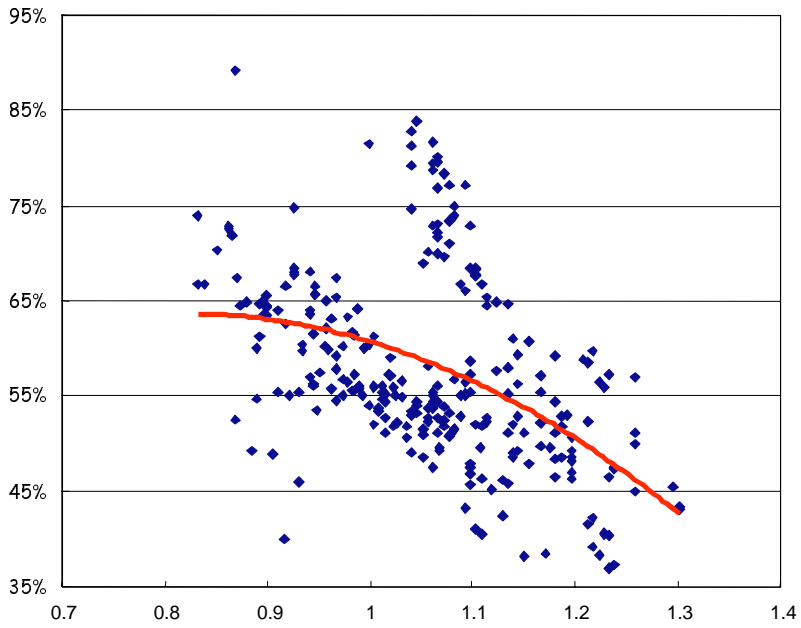


Fig. A.10. The volatility smile curve of 0510.

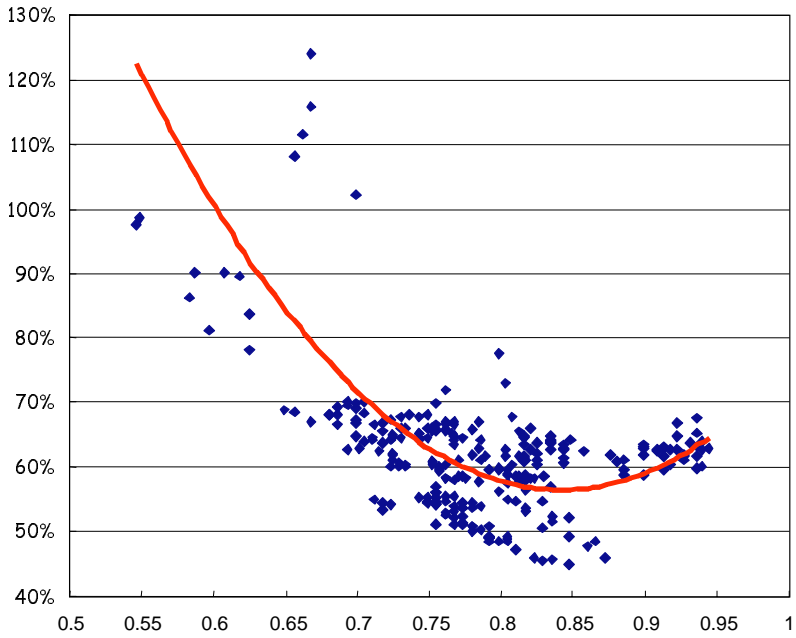


Fig. A.11. The volatility smile curve of 0511.

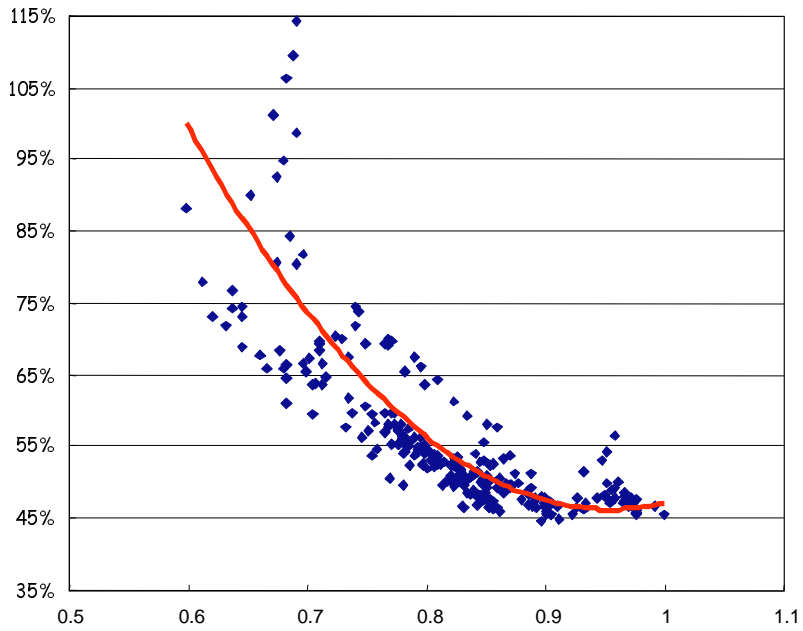


Fig. A.12. The volatility smile curve of 0512.

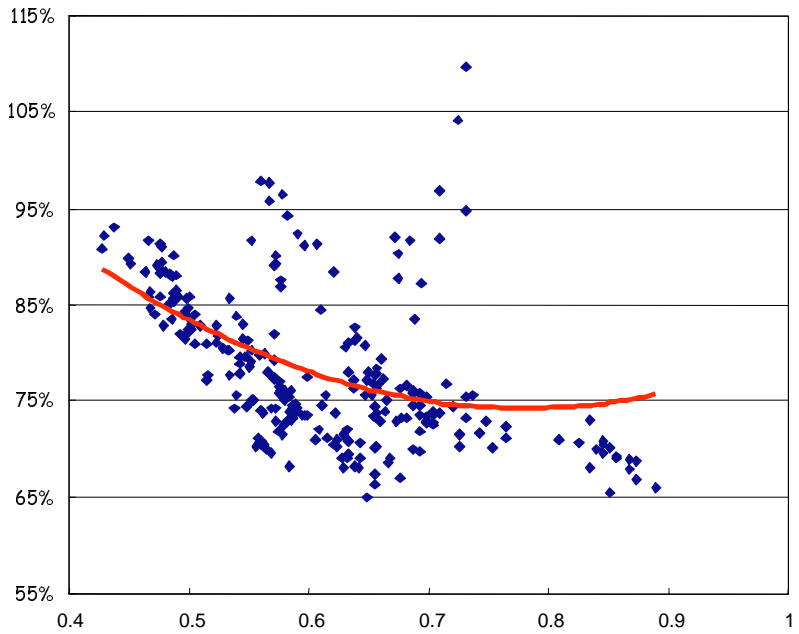


Fig. A.13. The volatility smile curve of 0513.

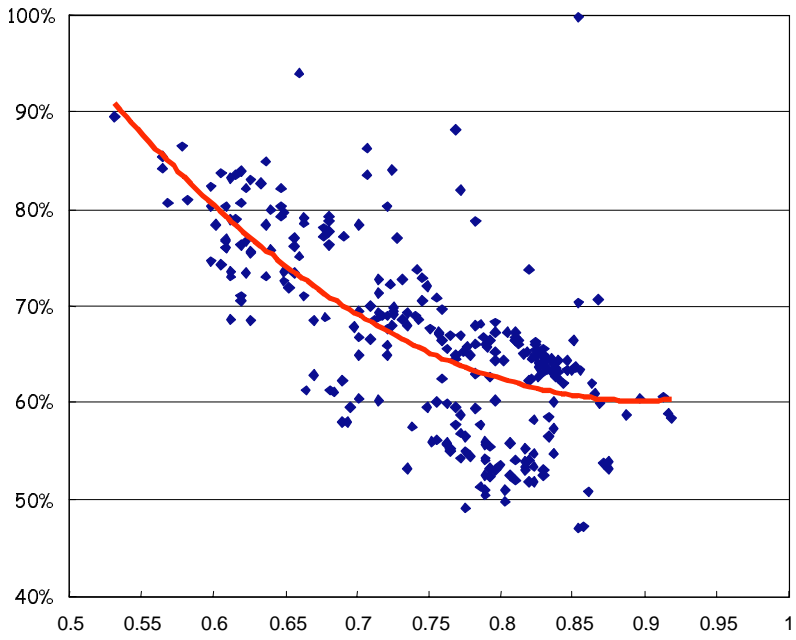


Fig. A.14. The volatility smile curve of 0514.

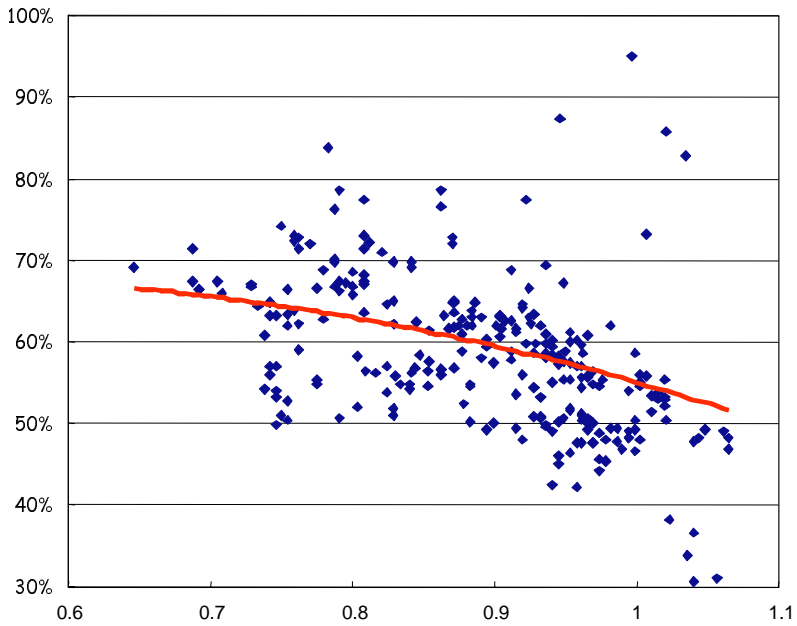


Fig. A.15. The volatility smile curve of 0515.

Appendix B.

Table B.1. The detailed arbitrage-trading results simulated by the combination of critical prices 0% and 40%.

Warrant ID	Model	Begin Date	Warrant Price	Stock Price	Model Price	Delta	Exercis Ratio	End Date	Warrant Price	Stock Price	Model Price	Annual Returns
0501	VS(6)	06/18/98	3.00	65.00	4.43	0.41	1.0770	06/25/98	2.00	62.50	1.88	17.41%
0502	VS(6)	06/18/98	4.95	30.50	7.00	2.58	1.2590	06/19/98	4.65	29.90	4.38	639.63%
0502	VS(6)	07/29/98	0.50	26.30	0.91	0.16	1.2590	08/01/98	0.85	26.90	0.75	547.09%
0504	VS(6)	08/26/98	2.00	37.00	2.83	0.30	1.3980	08/27/98	2.90	36.50	2.49	2534.76%
0505	VS(6)	06/30/98	3.40	57.00	6.87	0.09	1.0000	07/02/98	6.90	60.50	6.29	7060.06%
0509	VS(6)	06/15/98	8.75	54.00	13.28	0.38	1.4270	06/16/98	12.25	57.00	8.55	1968.46%
0512	VS(6)	01/13/99	0.35	25.70	0.57	0.18	1.2360	01/14/99	0.35	24.90	0.21	1182.85%
0512	VS(6)	01/27/99	0.15	24.90	0.24	0.05	1.2360	01/28/99	0.20	24.80	0.20	1271.52%
0514	VS(6)	02/24/99	0.15	17.60	0.22	0.05	1.2100	03/02/99	0.15	17.70	0.13	-33.06%
0502	VS(12)	06/25/98	3.80	30.90	6.11	1.42	1.2590	07/06/98	3.30	30.40	3.22	24.44%
0502	VS(12)	07/28/98	1.00	26.60	1.41	0.05	1.2590	08/01/98	0.85	26.90	0.72	-587.44%
0505	VS(12)	06/30/98	3.40	57.00	6.86	0.16	1.0000	07/02/98	6.90	60.50	6.65	4554.60%
0505	VS(12)	10/26/98	1.50	41.50	2.14	0.30	1.4000	11/02/98	2.30	46.00	1.94	328.72%
0509	VS(12)	06/15/98	8.75	54.00	13.56	0.41	1.4270	06/17/98	15.50	57.50	12.29	2330.57%
0512	VS(12)	01/13/99	0.35	25.70	0.63	0.14	1.2360	01/14/99	0.35	24.90	0.34	1157.84%
0512	VS(12)	01/27/99	0.15	24.90	0.23	0.06	1.2360	01/28/99	0.20	24.80	0.19	1166.30%
0514	VS(12)	02/25/99	0.15	18.20	0.21	0.03	1.2100	03/02/99	0.15	17.70	0.15	181.56%
0514	VS(12)	03/03/99	0.10	18.40	0.15	0.03	1.2100	03/08/99	0.10	17.90	0.10	185.47%
0515	VS(12)	07/18/98	6.80	28.70	9.83	1.69	1.0000	07/31/98	6.10	27.10	6.04	111.43%
0515	VS(12)	03/16/99	0.55	18.00	0.83	0.12	1.2100	03/19/99	1.00	19.40	0.73	1092.00%
0515	VS(12)	05/18/99	0.40	19.90	0.64	0.15	1.2100	05/19/99	0.60	20.00	0.50	1847.10%
0501	VS(24)	12/27/97	7.50	73.00	10.66	0.23	1.0000	01/06/98	7.70	71.00	7.51	106.54%
0501	VS(24)	04/27/98	3.75	81.00	5.31	0.12	1.0000	05/16/98	3.80	81.00	3.70	7.81%
0504	VS(24)	10/01/98	0.75	35.30	1.25	0.07	1.3980	10/06/98	0.60	33.50	0.57	62.23%
0505	VS(24)	06/30/98	3.40	57.00	6.43	0.22	1.0000	07/09/98	8.45	65.50	8.40	891.24%
0505	VS(24)	10/21/98	1.80	41.20	2.58	0.15	1.4000	11/02/98	2.30	46.00	2.05	-156.67%
0509	VS(24)	06/15/98	8.75	54.00	13.91	0.48	1.4270	06/24/98	20.30	63.50	19.17	481.79%
0512	VS(24)	01/13/99	0.35	25.70	0.70	0.10	1.2360	01/22/99	0.35	25.40	0.34	46.71%
0512	VS(24)	01/27/99	0.15	24.90	0.22	0.10	1.2360	01/28/99	0.20	24.80	0.18	808.84%
0513	VS(24)	01/13/99	0.85	47.00	1.25	0.05	1.2900	01/20/99	0.80	46.20	0.76	3.68%
0514	VS(24)	02/24/99	0.15	17.60	0.22	0.05	1.2100	03/09/99	0.10	17.90	0.10	-172.16%
0514	VS(24)	03/10/99	0.05	18.00	0.09	0.02	1.2100	03/11/99	0.10	18.20	0.08	4741.62%
0515	VS(24)	03/10/99	0.75	18.00	1.08	0.15	1.2100	03/19/99	1.00	19.40	0.90	-4.77%
0515	VS(24)	05/17/99	0.85	21.30	1.22	0.13	1.2100	05/21/99	0.55	19.50	0.51	-37.34%
0501	VS(All)	12/27/97	7.50	73.00	11.31	0.21	1.0000	03/06/98	8.80	80.50	8.46	-7.13%
0501	VS(All)	04/27/98	3.75	81.00	5.55	0.15	1.0000	06/04/98	1.50	62.50	1.40	36.86%
0503	VS(All)	06/18/98	7.90	61.00	11.12	0.40	1.5810	06/30/98	12.50	63.00	11.39	237.48%
0504	VS(All)	10/01/98	0.75	35.30	1.20	0.12	1.3980	10/20/98	0.65	34.60	0.58	4.88%
0504	VS(All)	10/26/98	1.00	40.40	1.48	0.17	1.3980	11/03/98	0.80	38.20	0.72	155.56%
0505	VS(All)	06/30/98	3.40	57.00	6.39	0.30	1.0000	07/13/98	9.10	63.50	7.89	561.00%
0505	VS(All)	10/28/98	1.20	41.20	1.75	0.15	1.4000	11/06/98	1.95	43.30	1.87	140.56%
0508	VS(All)	06/12/98	4.60	43.70	6.86	0.24	1.0000	08/26/98	2.10	32.00	1.98	10.75%
0509	VS(All)	06/15/98	8.75	54.00	14.36	0.50	1.4270	07/20/98	24.10	68.50	23.79	118.87%
0511	VS(All)	10/12/98	4.45	63.00	6.45	0.33	1.3480	12/14/98	3.40	64.00	3.32	-29.40%
0514	VS(All)	08/27/98	2.05	24.80	2.88	0.27	1.0000	12/07/98	2.70	24.60	2.65	31.34%
0514	VS(All)	02/25/99	0.15	18.20	0.21	0.05	1.2100	03/02/99	0.15	17.70	0.14	194.19%
0514	VS(All)	03/03/99	0.10	18.40	0.17	0.05	1.2100	03/09/99	0.10	17.90	0.10	168.28%
0514	VS(All)	03/10/99	0.05	18.00	0.09	0.03	1.2100	03/11/99	0.10	18.20	0.09	2225.18%
0515	VS(All)	09/09/98	5.50	23.40	7.74	1.01	1.2100	10/23/98	6.30	23.70	6.28	11.42%
0515	VS(All)	03/09/99	0.80	17.90	1.17	0.21	1.2100	05/05/99	2.05	23.00	2.03	-5.97%
0515	VS(All)	05/18/99	0.40	19.90	0.56	0.17	1.2100	05/19/99	0.60	20.00	0.56	1622.91%

Table B.2. The detailed arbitrage-trading results simulated by the combination of critical prices 10% and 40%.

Warrant ID	Model	Begin Date	Warrant Price	Stock Price	Model Price	Delta	Exercis Ratio	End Date	Warrant Price	Stock Price	Model Price	Annual Returns
0501	VS(6)	06/18/98	3.00	65.00	4.43	0.41	1.0770	06/24/98	2.00	63.00	2.14	-26.24%
0501	VS(6)	07/28/98	0.55	53.50	0.83	0.02	1.5070	07/29/98	0.55	54.00	0.60	-290.69%
0502	VS(6)	06/18/98	4.95	30.50	7.00	2.58	1.2590	06/19/98	4.65	29.90	4.38	639.63%
0502	VS(6)	07/29/98	0.50	26.30	0.91	0.16	1.2590	08/01/98	0.85	26.90	0.75	547.09%
0504	VS(6)	08/26/98	2.00	37.00	2.83	0.30	1.3980	08/27/98	2.90	36.50	2.49	2534.76%
0505	VS(6)	06/30/98	3.40	57.00	6.87	0.09	1.0000	07/02/98	6.90	60.50	6.29	7060.06%
0509	VS(6)	06/15/98	8.75	54.00	13.28	0.38	1.4270	06/16/98	12.25	57.00	8.55	1968.46%
0512	VS(6)	01/13/99	0.35	25.70	0.57	0.18	1.2360	01/14/99	0.35	24.90	0.21	1182.85%
0512	VS(6)	01/27/99	0.15	24.90	0.24	0.05	1.2360	01/28/99	0.20	24.80	0.20	1271.52%
0514	VS(6)	02/24/99	0.15	17.60	0.22	0.05	1.2100	03/01/99	0.15	17.90	0.15	-119.01%
0502	VS(12)	06/25/98	3.80	30.90	6.11	1.42	1.2590	06/29/98	3.70	29.80	3.78	318.27%
0502	VS(12)	07/28/98	1.00	26.60	1.41	0.05	1.2590	08/01/98	0.85	26.90	0.72	-587.44%
0505	VS(12)	06/30/98	3.40	57.00	6.86	0.16	1.0000	07/02/98	6.90	60.50	6.65	4554.60%
0505	VS(12)	10/26/98	1.50	41.50	2.14	0.30	1.4000	11/02/98	2.30	46.00	1.94	-328.72%
0509	VS(12)	06/15/98	8.75	54.00	13.56	0.41	1.4270	06/16/98	12.25	57.00	13.47	1737.55%
0512	VS(12)	01/13/99	0.35	25.70	0.63	0.14	1.2360	01/14/99	0.35	24.90	0.34	1157.84%
0512	VS(12)	01/27/99	0.15	24.90	0.23	0.06	1.2360	01/28/99	0.20	24.80	0.19	1166.30%
0514	VS(12)	02/25/99	0.15	18.20	0.21	0.03	1.2100	03/01/99	0.15	17.90	0.16	136.17%
0514	VS(12)	03/03/99	0.10	18.40	0.15	0.03	1.2100	03/08/99	0.10	17.90	0.10	185.47%
0515	VS(12)	07/18/98	6.80	28.70	9.83	1.69	1.0000	07/24/98	6.30	27.50	6.65	184.09%
0515	VS(12)	03/16/99	0.55	18.00	0.83	0.12	1.2100	03/19/99	1.00	19.40	0.73	1092.00%
0515	VS(12)	05/18/99	0.40	19.90	0.64	0.15	1.2100	05/19/99	0.60	20.00	0.50	1847.10%
0501	VS(24)	12/27/97	7.50	73.00	10.66	0.23	1.0000	01/05/98	7.60	71.00	7.61	100.41%
0501	VS(24)	04/27/98	3.75	81.00	5.31	0.12	1.0000	05/13/98	3.85	81.50	4.16	7.66%
0502	VS(24)	07/29/98	0.50	26.30	1.12	0.14	1.2590	08/01/98	0.85	26.90	0.89	656.69%
0504	VS(24)	10/01/98	0.75	35.30	1.25	0.07	1.3980	10/06/98	0.60	33.50	0.57	62.23%
0505	VS(24)	06/30/98	3.40	57.00	6.43	0.22	1.0000	07/02/98	6.90	60.50	6.96	3431.64%
0505	VS(24)	10/21/98	1.80	41.20	2.58	0.15	1.4000	11/02/98	2.30	46.00	2.05	-156.67%
0509	VS(24)	06/15/98	8.75	54.00	13.91	0.48	1.4270	06/17/98	15.50	57.50	15.91	1877.80%
0512	VS(24)	01/13/99	0.35	25.70	0.70	0.10	1.2360	01/21/99	0.35	25.50	0.37	35.03%
0512	VS(24)	01/27/99	0.15	24.90	0.22	0.10	1.2360	01/28/99	0.20	24.80	0.18	808.84%
0513	VS(24)	01/13/99	0.85	47.00	1.25	0.05	1.2900	01/18/99	0.80	46.10	0.86	18.37%
0514	VS(24)	02/24/99	0.15	17.60	0.22	0.05	1.2100	03/02/99	0.15	17.70	0.16	-33.23%
0514	VS(24)	03/03/99	0.10	18.40	0.17	0.04	1.2100	03/08/99	0.10	17.90	0.11	196.27%
0514	VS(24)	03/10/99	0.05	18.00	0.09	0.02	1.2100	03/11/99	0.10	18.20	0.08	4741.62%
0515	VS(24)	03/10/99	0.75	18.00	1.08	0.15	1.2100	03/19/99	1.00	19.40	0.90	-4.77%
0515	VS(24)	05/17/99	0.85	21.30	1.22	0.13	1.2100	05/20/99	0.70	20.40	0.72	-24.89%
0501	VS(All)	12/27/97	7.50	73.00	11.31	0.21	1.0000	02/02/98	9.45	79.00	10.18	31.28%
0501	VS(All)	02/13/98	8.85	91.00	12.68	0.24	1.0000	03/04/98	8.90	84.50	9.53	108.94%
0501	VS(All)	04/27/98	3.75	81.00	5.55	0.15	1.0000	06/02/98	1.65	66.00	1.81	12.74%
0503	VS(All)	06/18/98	7.90	61.00	11.12	0.40	1.5810	06/30/98	12.50	63.00	11.39	237.48%
0504	VS(All)	10/01/98	0.75	35.30	1.20	0.12	1.3980	10/19/98	0.70	35.40	0.71	-22.54%
0504	VS(All)	10/26/98	1.00	40.40	1.48	0.17	1.3980	11/03/98	0.80	38.20	0.72	155.56%
0505	VS(All)	06/30/98	3.40	57.00	6.39	0.30	1.0000	07/02/98	6.90	60.50	7.32	2382.19%
0505	VS(All)	10/28/98	1.20	41.20	1.75	0.15	1.4000	11/06/98	1.95	43.30	1.87	140.56%
0508	VS(All)	06/12/98	4.60	43.70	6.86	0.24	1.0000	06/24/98	6.00	46.00	6.49	183.52%
0509	VS(All)	06/15/98	8.75	54.00	14.36	0.50	1.4270	06/17/98	15.50	57.50	16.56	1777.88%
0511	VS(All)	10/12/98	4.45	63.00	6.45	0.33	1.3480	12/10/98	3.70	65.00	3.93	-34.32%
0513	VS(All)	01/13/99	0.85	47.00	1.22	0.11	1.2900	01/19/99	0.80	45.60	0.83	132.44%
0514	VS(All)	08/27/98	2.05	24.80	2.88	0.27	1.0000	09/05/98	3.50	27.30	3.51	393.59%
0514	VS(All)	09/24/98	2.60	23.60	3.68	0.35	1.2100	11/04/98	2.25	22.60	2.46	5.99%
0514	VS(All)	11/18/98	2.30	25.10	3.30	0.38	1.2100	12/01/98	2.50	24.50	2.72	105.30%
0514	VS(All)	02/25/99	0.15	18.30	0.21	0.05	1.2100	03/01/99	0.15	17.90	0.16	145.64%
0514	VS(All)	03/03/99	0.10	18.40	0.17	0.05	1.2100	03/08/99	0.10	17.90	0.10	201.93%
0514	VS(All)	03/10/99	0.05	18.00	0.09	0.03	1.2100	03/11/99	0.10	18.20	0.09	2225.18%
0515	VS(All)	09/09/98	5.50	23.40	7.74	1.01	1.2100	10/23/98	6.30	23.70	6.28	11.42%
0515	VS(All)	03/09/99	0.80	17.90	1.17	0.21	1.2100	04/09/99	2.35	22.70	2.55	79.56%
0515	VS(All)	04/15/99	2.70	25.00	3.87	0.61	1.2100	04/29/99	1.50	21.70	1.56	167.34%
0515	VS(All)	05/18/99	0.40	19.90	0.56	0.17	1.2100	05/19/99	0.60	20.00	0.56	1622.91%

Table B.3. The detailed arbitrage-trading results simulated by the combination of critical prices 0% and 30%.

Warrant ID	Model	Begin Date	Warrant Price	Stock Price	Model Price	Delta	Exercis Ratio	End Date	Warrant Price	Stock Price	Model Price	Annual Returns
0501	VS(6)	05/27/98	2.25	75.50	2.94	0.03	1.0770	05/28/98	2.15	74.50	2.06	-652.78%
0501	VS(6)	06/18/98	3.00	65.00	4.43	0.41	1.0770	06/25/98	2.00	62.50	1.88	17.41%
0502	VS(6)	06/18/98	4.95	30.50	7.00	2.58	1.2590	06/19/98	4.65	29.90	4.38	639.63%
0502	VS(6)	06/23/98	3.50	29.60	4.56	0.20	1.2590	07/02/98	3.40	29.60	3.21	-39.14%
0502	VS(6)	07/29/98	0.50	26.30	0.91	0.16	1.2590	08/01/98	0.85	26.90	0.75	547.09%
0504	VS(6)	08/26/98	2.00	37.00	2.83	0.30	1.3980	08/27/98	2.90	36.50	2.49	2534.76%
0505	VS(6)	06/30/98	3.40	57.00	6.87	0.09	1.0000	07/02/98	6.90	60.50	6.29	7060.06%
0505	VS(6)	10/21/98	1.80	41.20	2.40	0.25	1.4000	10/30/98	1.40	42.70	0.64	-253.82%
0507	VS(6)	09/07/98	3.80	42.10	5.18	0.51	1.1330	09/14/98	3.70	43.10	3.55	-137.56%
0508	VS(6)	09/07/98	3.10	35.20	4.08	0.44	1.1700	09/11/98	3.00	36.50	2.99	-361.60%
0509	VS(6)	06/15/98	8.75	54.00	13.28	0.38	1.4270	06/16/98	12.25	57.00	8.55	1968.46%
0512	VS(6)	01/13/99	0.35	25.70	0.57	0.18	1.2360	01/14/99	0.35	24.90	0.21	1182.85%
0512	VS(6)	01/27/99	0.15	24.90	0.24	0.05	1.2360	01/28/99	0.20	24.80	0.20	1271.52%
0513	VS(6)	12/28/98	1.30	40.30	1.70	0.07	1.2900	12/31/98	1.25	40.30	1.14	-129.04%
0513	VS(6)	01/13/99	0.85	47.00	1.12	0.06	1.2900	01/19/99	0.80	45.60	0.77	82.09%
0514	VS(6)	01/20/99	0.65	18.10	0.85	0.04	1.2100	01/27/99	0.65	19.30	0.61	-214.01%
0514	VS(6)	02/24/99	0.15	17.60	0.22	0.05	1.2100	03/02/99	0.15	17.70	0.13	-33.06%
0515	VS(6)	03/11/99	0.75	18.20	0.99	0.38	1.2100	03/19/99	1.00	19.40	0.69	-167.16%
0515	VS(6)	05/17/99	0.85	21.30	1.13	0.22	1.2100	05/19/99	0.60	20.00	0.43	291.10%
0502	VS(12)	06/25/98	3.80	30.90	6.11	1.42	1.2590	07/06/98	3.30	30.40	3.22	24.44%
0502	VS(12)	07/23/98	1.45	28.00	1.93	0.17	1.2590	08/01/98	0.85	26.90	0.72	-213.36%
0504	VS(12)	08/26/98	2.00	37.00	2.79	0.27	1.3980	08/27/98	2.90	36.50	2.50	2701.19%
0505	VS(12)	06/30/98	3.40	57.00	6.86	0.16	1.0000	07/02/98	6.90	60.50	6.65	4554.60%
0505	VS(12)	10/21/98	1.80	41.20	2.48	0.20	1.4000	11/02/98	2.30	46.00	1.94	-213.51%
0509	VS(12)	06/15/98	8.75	54.00	13.56	0.41	1.4270	06/17/98	15.50	57.50	12.29	2330.57%
0512	VS(12)	01/13/99	0.35	25.70	0.63	0.14	1.2360	01/14/99	0.35	24.90	0.34	1157.84%
0512	VS(12)	01/27/99	0.15	24.90	0.23	0.06	1.2360	01/28/99	0.20	24.80	0.19	1166.30%
0513	VS(12)	12/28/98	1.30	40.30	1.71	0.05	1.2900	01/18/99	0.80	46.10	0.79	-414.33%
0514	VS(12)	01/22/99	0.65	19.10	0.85	0.15	1.2100	01/28/99	0.65	18.70	0.65	117.45%
0514	VS(12)	02/02/99	0.40	18.30	0.53	0.05	1.2100	02/05/99	0.30	15.60	0.25	503.97%
0514	VS(12)	02/24/99	0.15	17.60	0.21	0.04	1.2100	03/02/99	0.15	17.70	0.15	-32.61%
0514	VS(12)	03/03/99	0.10	18.40	0.15	0.03	1.2100	03/08/99	0.10	17.90	0.10	185.47%
0515	VS(12)	07/15/98	6.70	27.40	9.29	0.74	1.0000	07/31/98	6.10	27.10	6.04	-34.67%
0515	VS(12)	03/08/99	0.85	17.90	1.11	0.13	1.2100	03/19/99	1.00	19.40	0.73	-83.97%
0515	VS(12)	05/17/99	0.85	21.30	1.15	0.19	1.2100	05/19/99	0.60	20.00	0.50	164.89%
0501	VS(24)	12/27/97	7.50	73.00	10.66	0.23	1.0000	01/06/98	7.70	71.00	7.51	106.54%
0501	VS(24)	04/21/98	4.40	79.00	5.82	0.07	1.0000	05/16/98	3.80	81.00	3.70	-116.77%
0503	VS(24)	06/18/98	7.90	61.00	10.83	0.39	1.5810	06/29/98	8.50	61.00	8.16	48.03%
0504	VS(24)	08/26/98	2.00	37.00	2.76	0.28	1.3980	08/27/98	2.90	36.50	2.49	2631.99%
0504	VS(24)	09/28/98	1.10	35.80	1.44	0.07	1.3980	10/06/98	0.60	33.50	0.57	-269.74%
0504	VS(24)	10/08/98	0.75	36.30	1.00	0.17	1.3980	10/17/98	0.70	35.60	0.60	54.78%
0505	VS(24)	06/30/98	3.40	57.00	6.43	0.22	1.0000	07/09/98	8.45	65.50	8.40	891.24%
0505	VS(24)	10/21/98	1.80	41.20	2.58	0.15	1.4000	11/02/98	2.30	46.00	2.05	-156.67%
0508	VS(24)	10/02/98	1.50	33.70	1.95	0.10	1.1700	10/07/98	2.45	38.80	2.37	523.30%
0509	VS(24)	06/15/98	8.75	54.00	13.91	0.48	1.4270	06/24/98	20.30	63.50	19.17	481.79%
0511	VS(24)	10/02/98	3.40	58.00	4.60	0.14	1.3480	10/14/98	4.05	60.50	4.02	38.51%
0511	VS(24)	01/14/99	0.65	55.50	0.85	0.06	1.3480	01/18/99	0.75	56.50	0.72	33.95%
0512	VS(24)	01/05/99	0.80	25.90	1.10	0.07	1.2360	01/22/99	0.35	25.40	0.34	-293.72%
0512	VS(24)	01/27/99	0.15	24.90	0.22	0.10	1.2360	01/28/99	0.20	24.80	0.18	808.84%
0513	VS(24)	12/28/98	1.30	40.30	1.72	0.04	1.2900	01/20/99	0.80	46.20	0.76	-395.29%
0514	VS(24)	08/25/98	2.35	25.10	3.09	0.28	1.0000	09/01/98	1.90	23.80	1.64	-51.96%
0514	VS(24)	10/01/98	2.00	22.80	2.61	0.26	1.2100	10/09/98	2.35	22.80	2.33	189.56%
0514	VS(24)	01/06/99	0.80	18.30	1.11	0.12	1.2100	01/11/99	1.10	19.00	1.03	465.50%
0514	VS(24)	01/25/99	0.65	19.50	0.86	0.13	1.2100	01/29/99	0.60	18.20	0.56	417.63%
0514	VS(24)	02/02/99	0.40	18.30	0.54	0.11	1.2100	02/05/99	0.30	15.60	0.22	1219.32%
0514	VS(24)	02/24/99	0.15	17.60	0.22	0.05	1.2100	03/09/99	0.10	17.90	0.10	-172.16%
0514	VS(24)	03/10/99	0.05	18.00	0.09	0.02	1.2100	03/11/99	0.10	18.20	0.08	4741.62%
0515	VS(24)	07/15/98	6.70	27.40	9.29	0.74	1.0000	07/27/98	6.45	27.20	6.25	-12.50%
0515	VS(24)	03/06/99	0.90	18.00	1.21	0.22	1.2100	03/19/99	1.00	19.40	0.90	-145.36%
0515	VS(24)	04/15/99	2.70	25.00	3.61	0.63	1.2100	04/21/99	2.15	23.40	1.97	205.04%
0515	VS(24)	05/17/99	0.85	21.30	1.22	0.13	1.2100	05/21/99	0.55	19.50	0.51	-37.34%

Table B.3. (Continued)

Warrant ID	Model	Begin Date	Warrant Price	Stock Price	Model Price	Delta	Exercis Ratio	End Date	Warrant Price	Stock Price	Model Price	Annual Returns
0501	VS(All)	12/26/97	8.60	73.00	11.42	0.21	1.0000	03/06/98	8.80	80.50	8.46	-32.33%
0501	VS(All)	04/24/98	4.50	82.00	5.93	0.17	1.0000	06/04/98	1.50	62.50	1.40	14.51%
0502	VS(All)	07/29/98	0.50	26.30	0.67	0.13	1.2590	08/01/98	0.85	26.90	0.64	708.55%
0503	VS(All)	06/17/98	7.00	57.50	9.19	0.34	1.5810	06/30/98	12.50	63.00	11.39	199.40%
0504	VS(All)	08/26/98	2.00	37.00	2.73	0.25	1.3980	08/27/98	2.90	36.50	2.52	2911.21%
0504	VS(All)	09/19/98	2.00	39.20	2.61	0.24	1.3980	10/20/98	0.65	34.60	0.58	15.22%
0504	VS(All)	10/23/98	0.95	39.30	1.32	0.17	1.3980	11/03/98	0.80	38.20	0.72	40.99%
0505	VS(All)	06/30/98	3.40	57.00	6.39	0.30	1.0000	07/13/98	9.10	63.50	7.89	561.00%
0505	VS(All)	07/27/98	9.00	53.50	11.90	0.50	1.4000	08/25/98	4.25	41.60	4.14	105.10%
0505	VS(All)	10/27/98	1.35	41.50	1.86	0.17	1.4000	11/06/98	1.95	43.30	1.87	66.31%
0508	VS(All)	06/10/98	6.00	45.90	7.93	0.25	1.0000	08/26/98	2.10	32.00	1.98	-12.27%
0508	VS(All)	10/09/98	3.00	41.50	3.94	0.32	1.1700	12/01/98	5.00	48.30	4.93	-21.89%
0509	VS(All)	06/15/98	8.75	54.00	14.36	0.50	1.4270	07/20/98	24.10	68.50	23.79	118.87%
0509	VS(All)	08/28/98	9.00	56.00	11.80	0.44	1.4270	10/27/98	8.50	56.50	8.22	-12.15%
0511	VS(All)	09/17/98	5.80	62.50	7.63	0.34	1.3480	12/14/98	3.40	64.00	3.32	-40.36%
0512	VS(All)	01/27/99	0.15	24.90	0.20	0.06	1.2360	01/28/99	0.20	24.80	0.18	1148.64%
0513	VS(All)	07/02/98	4.30	41.10	5.68	0.27	1.2900	07/27/98	4.70	40.20	4.42	60.68%
0513	VS(All)	11/04/98	1.30	37.90	1.74	0.13	1.2900	11/11/98	1.40	36.80	1.40	211.69%
0513	VS(All)	11/20/98	1.95	45.00	2.66	0.17	1.2900	11/24/98	2.95	46.10	2.77	653.74%
0514	VS(All)	08/25/98	2.35	25.10	3.06	0.29	1.0000	12/07/98	2.70	24.60	2.65	19.51%
0514	VS(All)	02/25/99	0.15	18.20	0.21	0.05	1.2100	03/02/99	0.15	17.70	0.14	194.19%
0514	VS(All)	03/03/99	0.10	18.40	0.17	0.05	1.2100	03/09/99	0.10	17.90	0.10	168.28%
0514	VS(All)	03/10/99	0.05	18.00	0.09	0.03	1.2100	03/11/99	0.10	18.20	0.09	2225.18%
0515	VS(All)	07/15/98	6.70	27.40	9.29	0.74	1.0000	07/27/98	6.45	27.20	6.25	-12.50%
0515	VS(All)	09/09/98	5.50	23.40	7.74	1.01	1.2100	10/23/98	6.30	23.70	6.28	11.42%
0515	VS(All)	03/06/99	0.90	18.00	1.25	0.24	1.2100	05/05/99	2.05	23.00	2.03	-30.77%
0515	VS(All)	05/18/99	0.40	19.90	0.56	0.17	1.2100	05/19/99	0.60	20.00	0.56	1622.91%

Table B.4. The detailed arbitrage-trading results simulated by the combination of critical prices 10% and 30%.

Warrant ID	Model	Begin Date	Warrant Price	Stock Price	Model Price	Delta	Exercis Ratio	End Date	Warrant Price	Stock Price	Model Price	Annual Returns
0501	VS(6)	05/27/98	2.25	75.50	2.94	0.03	1.0770	05/28/98	2.15	74.50	2.06	-652.78%
0501	VS(6)	06/18/98	3.00	65.00	4.43	0.41	1.0770	06/24/98	2.00	63.00	2.14	-26.24%
0501	VS(6)	07/28/98	0.55	53.50	0.83	0.02	1.5070	07/29/98	0.55	54.00	0.60	-290.69%
0502	VS(6)	06/18/98	4.95	30.50	7.00	2.58	1.2590	06/19/98	4.65	29.90	4.38	639.63%
0502	VS(6)	06/23/98	3.50	29.60	4.56	0.20	1.2590	06/26/98	3.60	30.30	3.85	-94.29%
0502	VS(6)	07/29/98	0.50	26.30	0.91	0.16	1.2590	08/01/98	0.85	26.90	0.75	547.09%
0504	VS(6)	08/26/98	2.00	37.00	2.83	0.30	1.3980	08/27/98	2.90	36.50	2.49	2534.76%
0505	VS(6)	06/30/98	3.40	57.00	6.87	0.09	1.0000	07/02/98	6.90	60.50	6.29	7060.06%
0505	VS(6)	10/21/98	1.80	41.20	2.40	0.25	1.4000	10/30/98	1.40	42.70	0.64	-253.82%
0507	VS(6)	09/07/98	3.80	42.10	5.18	0.51	1.1330	09/11/98	3.80	43.60	3.84	-307.45%
0508	VS(6)	09/07/98	3.10	35.20	4.08	0.44	1.1700	09/09/98	2.70	34.50	2.97	-39.27%
0509	VS(6)	06/15/98	8.75	54.00	13.28	0.38	1.4270	06/16/98	12.25	57.00	8.55	1968.46%
0512	VS(6)	01/13/99	0.35	25.70	0.57	0.18	1.2360	01/14/99	0.35	24.90	0.21	1182.85%
0512	VS(6)	01/27/99	0.15	24.90	0.24	0.05	1.2360	01/28/99	0.20	24.80	0.20	1271.52%
0513	VS(6)	12/28/98	1.30	40.30	1.70	0.07	1.2900	1 2/31/98	1.25	40.30	1.14	-129.04%
0513	VS(6)	01/13/99	0.85	47.00	1.12	0.06	1.2900	01/15/99	0.80	47.60	0.84	-432.60%
0514	VS(6)	01/20/99	0.65	18.10	0.85	0.04	1.2100	01/26/99	0.60	19.20	0.65	-436.10%
0514	VS(6)	02/24/99	0.15	17.60	0.22	0.05	1.2100	03/01/99	0.15	17.90	0.15	-119.01%
0515	VS(6)	03/11/99	0.75	18.20	0.99	0.38	1.2100	03/17/99	0.60	18.10	0.66	-75.79%
0515	VS(6)	05/17/99	0.85	21.30	1.13	0.22	1.2100	05/19/99	0.60	20.00	0.43	291.10%
0502	VS(12)	06/25/98	3.80	30.90	6.11	1.42	1.2590	06/29/98	3.70	29.80	3.78	318.27%
0502	VS(12)	07/23/98	1.45	28.00	1.93	0.17	1.2590	08/01/98	0.85	26.90	0.72	-213.36%
0504	VS(12)	08/26/98	2.00	37.00	2.79	0.27	1.3980	08/27/98	2.90	36.50	2.50	2701.19%
0505	VS(12)	06/30/98	3.40	57.00	6.86	0.16	1.0000	07/02/98	6.90	60.50	6.65	4554.60%
0505	VS(12)	10/21/98	1.80	41.20	2.48	0.20	1.4000	11/02/98	2.30	46.00	1.94	-213.51%
0509	VS(12)	06/15/98	8.75	54.00	13.56	0.41	1.4270	06/16/98	12.25	57.00	13.47	1737.55%
0512	VS(12)	01/13/99	0.35	25.70	0.63	0.14	1.2360	01/14/99	0.35	24.90	0.34	1157.84%

Table B.4. (Continued)

Warrant ID	Model	Begin Date	Warrant Price	Stock Price	Model Price	Delta	Exercis Ratio	End Date	Warrant Price	Stock Price	Model Price	Annual Returns
0512	VS(12)	01/27/99	0.15	24.90	0.23	0.06	1.2360	01/28/99	0.20	24.80	0.19	1166.30%
0513	VS(12)	12/28/98	1.30	40.30	1.71	0.05	1.2900	01/16/99	0.80	47.10	0.87	-492.42%
0514	VS(12)	01/22/99	0.65	19.10	0.85	0.15	1.2100	01/27/99	0.65	19.30	0.70	-70.47%
0514	VS(12)	02/02/99	0.40	18.30	0.53	0.05	1.2100	02/05/99	0.30	15.60	0.25	503.97%
0514	VS(12)	02/24/99	0.15	17.60	0.21	0.04	1.2100	03/01/99	0.15	17.90	0.16	-117.39%
0514	VS(12)	03/03/99	0.10	18.40	0.15	0.03	1.2100	03/08/99	0.10	17.90	0.10	185.47%
0515	VS(12)	07/15/98	6.70	27.40	9.29	0.74	1.0000	07/24/98	6.30	27.50	6.65	-77.16%
0515	VS(12)	03/08/99	0.85	17.90	1.11	0.13	1.2100	03/19/99	1.00	19.40	0.73	-83.97%
0515	VS(12)	05/17/99	0.85	21.30	1.15	0.19	1.2100	05/19/99	0.60	20.00	0.50	164.89%
0501	VS(24)	12/27/97	7.50	73.00	10.66	0.23	1.0000	01/05/98	7.60	71.00	7.61	100.41%
0501	VS(24)	04/21/98	4.40	79.00	5.82	0.07	1.0000	05/13/98	3.85	81.50	4.16	-129.74%
0502	VS(24)	07/23/98	1.45	28.00	1.94	0.14	1.2590	08/01/98	0.85	26.90	0.89	-282.84%
0503	VS(24)	06/18/98	7.90	61.00	10.83	0.39	1.5810	06/25/98	9.00	62.50	9.71	23.05%
0504	VS(24)	08/26/98	2.00	37.00	2.76	0.28	1.3980	08/27/98	2.90	36.50	2.49	2631.99%
0504	VS(24)	09/28/98	1.10	35.80	1.44	0.07	1.3980	10/06/98	0.60	33.50	0.57	-269.74%
0504	VS(24)	10/08/98	0.75	36.30	1.00	0.17	1.3980	10/15/98	0.65	35.90	0.70	-4.38%
0505	VS(24)	06/30/98	3.40	57.00	6.43	0.22	1.0000	07/02/98	6.90	60.50	6.96	3431.64%
0505	VS(24)	10/21/98	1.80	41.20	2.58	0.15	1.4000	11/02/98	2.30	46.00	2.05	-156.67%
0508	VS(24)	10/02/98	1.50	33.70	1.95	0.10	1.1700	10/03/98	2.05	36.00	2.19	2065.20%
0509	VS(24)	06/15/98	8.75	54.00	13.91	0.48	1.4270	06/17/98	15.50	57.50	15.91	1877.80%
0511	VS(24)	10/02/98	3.40	58.00	4.60	0.14	1.3480	10/06/98	3.35	57.50	3.60	31.28%
0511	VS(24)	01/14/99	0.65	55.50	0.85	0.06	1.3480	01/18/99	0.75	56.50	0.72	33.95%
0512	VS(24)	01/05/99	0.80	25.90	1.10	0.07	1.2360	01/07/99	0.95	26.60	1.03	527.81%
0512	VS(24)	01/12/99	0.60	26.00	0.79	0.11	1.2360	01/21/99	0.35	25.50	0.37	-193.12%
0512	VS(24)	01/27/99	0.15	24.90	0.22	0.10	1.2360	01/28/99	0.20	24.80	0.18	808.84%
0513	VS(24)	12/28/98	1.30	40.30	1.72	0.04	1.2900	01/07/99	1.20	43.00	1.32	-275.24%
0513	VS(24)	01/12/99	1.00	46.30	1.32	0.06	1.2900	01/18/99	0.80	46.10	0.86	-260.83%
0514	VS(24)	08/25/98	2.35	25.10	3.09	0.28	1.0000	08/29/98	2.30	24.70	2.36	65.14%
0514	VS(24)	10/01/98	2.00	22.80	2.61	0.26	1.2100	10/03/98	2.20	22.10	2.33	908.04%
0514	VS(24)	01/06/99	0.80	18.30	1.11	0.12	1.2100	01/08/99	1.15	19.50	1.16	1041.07%
0514	VS(24)	01/25/99	0.65	19.50	0.86	0.13	1.2100	01/28/99	0.65	18.70	0.66	450.64%
0514	VS(24)	02/02/99	0.40	18.30	0.54	0.11	1.2100	02/05/99	0.30	15.60	0.22	1219.32%
0514	VS(24)	02/24/99	0.15	17.60	0.22	0.05	1.2100	03/02/99	0.15	17.70	0.16	-33.23%
0514	VS(24)	03/03/99	0.10	18.40	0.17	0.04	1.2100	03/08/99	0.10	17.90	0.11	196.27%
0514	VS(24)	03/10/99	0.05	18.00	0.09	0.02	1.2100	03/11/99	0.10	18.20	0.08	4741.62%
0515	VS(24)	07/15/98	6.70	27.40	9.29	0.74	1.0000	07/24/98	6.30	27.50	6.83	-77.16%
0515	VS(24)	03/06/99	0.90	18.00	1.21	0.22	1.2100	03/19/99	1.00	19.40	0.90	-145.36%
0515	VS(24)	04/15/99	2.70	25.00	3.61	0.63	1.2100	04/19/99	2.85	25.10	2.99	33.97%
0515	VS(24)	05/17/99	0.85	21.30	1.22	0.13	1.2100	05/20/99	0.70	20.40	0.72	-24.89%
0501	VS(All)	12/26/97	8.60	73.00	11.42	0.21	1.0000	02/02/98	9.45	79.00	10.18	-18.39%
0501	VS(All)	02/12/98	9.45	90.50	12.63	0.25	1.0000	03/04/98	8.90	84.50	9.53	58.70%
0501	VS(All)	04/24/98	4.50	82.00	5.93	0.17	1.0000	06/02/98	1.65	66.00	1.81	-9.00%
0502	VS(All)	07/29/98	0.50	26.30	0.67	0.13	1.2590	07/31/98	0.70	27.10	0.70	295.48%
0503	VS(All)	06/17/98	7.00	57.50	9.19	0.34	1.5810	06/30/98	12.50	63.00	11.39	199.40%
0504	VS(All)	08/26/98	2.00	37.00	2.73	0.25	1.3980	08/27/98	2.90	36.50	2.52	2911.21%
0504	VS(All)	09/19/98	2.00	39.20	2.61	0.24	1.3980	10/19/98	0.70	35.40	0.71	-3.41%
0504	VS(All)	10/23/98	0.95	39.30	1.32	0.17	1.3980	11/03/98	0.80	38.20	0.72	40.99%
0505	VS(All)	06/30/98	3.40	57.00	6.39	0.30	1.0000	07/02/98	6.90	60.50	7.32	2382.19%
0505	VS(All)	07/27/98	9.00	53.50	11.90	0.50	1.4000	08/17/98	6.70	47.10	6.94	88.44%
0505	VS(All)	10/27/98	1.35	41.50	1.86	0.17	1.4000	11/06/98	1.95	43.30	1.87	66.31%
0508	VS(All)	06/10/98	6.00	45.90	7.93	0.25	1.0000	06/24/98	6.00	46.00	6.49	-3.99%
0508	VS(All)	10/09/98	3.00	41.50	3.94	0.32	1.1700	10/28/98	2.50	39.60	2.66	23.73%
0509	VS(All)	06/15/98	8.75	54.00	14.36	0.50	1.4270	06/17/98	15.50	57.50	16.56	1777.88%
0509	VS(All)	08/28/98	9.00	56.00	11.80	0.44	1.4270	08/31/98	12.50	58.50	13.29	572.35%
0509	VS(All)	09/01/98	11.00	60.00	14.34	0.53	1.4270	09/22/98	14.00	63.00	15.26	24.01%
0511	VS(All)	09/17/98	5.80	62.50	7.63	0.34	1.3480	12/10/98	3.70	65.00	3.93	-44.49%
0512	VS(All)	01/27/99	0.15	24.90	0.20	0.06	1.2360	01/28/99	0.20	24.80	0.18	1148.64%
0513	VS(All)	07/02/98	4.30	41.10	5.68	0.27	1.2900	07/13/98	4.30	39.60	4.63	100.80%
0513	VS(All)	11/04/98	1.30	37.90	1.74	0.13	1.2900	11/10/98	1.45	37.20	1.48	232.69%
0513	VS(All)	11/20/98	1.95	45.00	2.66	0.17	1.2900	11/24/98	2.95	46.10	2.77	653.74%
0513	VS(All)	01/13/99	0.85	47.00	1.22	0.11	1.2900	01/19/99	0.80	45.60	0.83	132.44%
0514	VS(All)	08/25/98	2.35	25.10	3.06	0.29	1.0000	09/05/98	3.50	27.30	3.51	190.55%

Table B.4. (*Continued*)

Warrant ID	Model	Begin Date	Warrant Price	Stock Price	Model Price	Delta	Exercis Ratio	End Date	Warrant Price	Stock Price	Model Price	Annual Returns
0514	VS(All)	09/23/98	2.80	23.80	3.80	0.37	1.2100	11/04/98	2.25	22.60	2.46	-0.56%
0514	VS(All)	11/17/98	2.20	24.10	2.87	0.34	1.2100	12/01/98	2.50	24.50	2.72	31.00%
0514	VS(All)	02/25/99	0.15	18.20	0.21	0.05	1.2100	03/01/99	0.15	17.90	0.16	145.64%
0514	VS(All)	03/03/99	0.10	18.40	0.17	0.05	1.2100	03/08/99	0.10	17.90	0.10	201.93%
0514	VS(All)	03/10/99	0.05	18.00	0.09	0.03	1.2100	03/11/99	0.10	18.20	0.09	2225.18%
0515	VS(All)	07/15/98	6.70	27.40	9.29	0.74	1.0000	07/24/98	6.30	27.50	6.83	-77.16%
0515	VS(All)	09/09/98	5.50	23.40	7.74	1.01	1.2100	10/23/98	6.30	23.70	6.28	11.42%
0515	VS(All)	03/06/99	0.90	18.00	1.25	0.24	1.2100	04/09/99	2.35	22.70	2.55	20.55%
0515	VS(All)	04/15/99	2.70	25.00	3.87	0.61	1.2100	04/29/99	1.50	21.70	1.56	167.34%
0515	VS(All)	05/18/99	0.40	19.90	0.56	0.17	1.2100	05/19/99	0.60	20.00	0.56	1622.91%

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