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Can the identification puzzle of Taiwan's turning points after 1990 be solved?

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Abstract

Although the use of Hamilton's [Hamilton, J.D., 1989. A new approach to the economic analysis of nonstationary time series and the business cycle, Econometrica 54, 357–384.] Markov-switching model to date U.S. business cycles has recently gained respect as a reliable tool, the model failed to date the business cycle of Taiwan, a developing country, since model-dated recession indicated a recession for the whole post-1990 period, which clearly is not true. A lot of effort has been devoted to solve this identification problem of Taiwan's business chronology, but, even so, the puzzle remains. This paper uses an extended multivariate Markov-switching factor model to solve this puzzle. We first consider a second-state variable, the variance, in addition to the conventional one-state variable, the mean. We then employ four variables to assist in the identification of the business cycles. It is determined that the new model successfully dates Taiwan's business cycles.

JEL classification: C22; E32

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1. Introduction

Dating business cycles have recently been the focus of wide attention from academics, government and business, alike. Of particularly growing attention has been the Markov-

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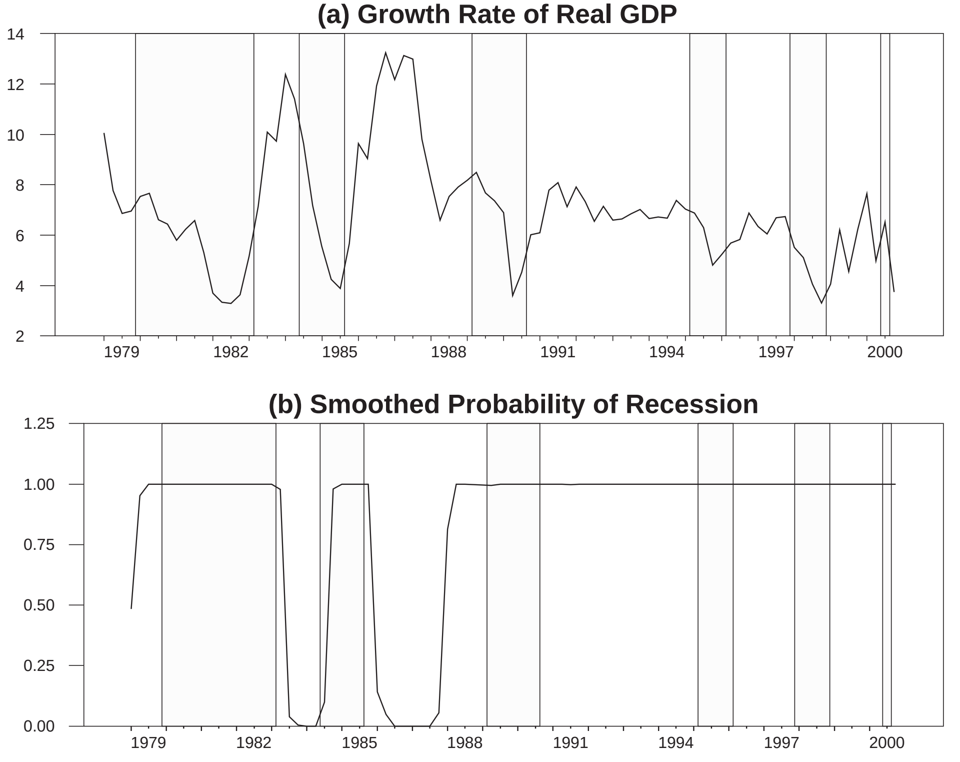

Fig. 1. Hamilton model (1979:Q1–2000:Q4).

switching (hereafter MS) method, as proposed by Hamilton (1989), which defines a business cycle as a discrete, two-state process. Using quarterly GNP growth rates, he demonstrated that the model generates expansionary and recessionary periods which are remarkably consistent with the NBER chronologies of business cycles. Examining the model's robustness, researchers using either the industrial production index, extended data or variations of the MS have also proven that the MS model is equally useful in characterizing business cycle dating.[2]

Applying the MS method in an attempt to date Taiwan's business cycle has also become a focus of interest recently, but not all attempts have been successful. The methodology has been employed by Huang et al. (1998), Huang (1999), Hsu and Kuan (2001), Chen and Lin (2000a,b), and Chen (2001). While these efforts have succeeded using pre-1990 data, they have failed when it came to extending the sample past 1990. In other words, the MS method has successfully identified seven recessionary periods before 1990 in Taiwan, but it has incorrectly identified the entire post-1990 periods as one of recession.[3] Yet, the reason for this misidentification in the post-1990 sample period is pure and simple. In the early stage of Taiwan's economic development, because the economic base was small and not all the capacities were fully utilized,

[2] Examples are Filardo's (1994) and Durland and McCurdy's (1994) models which extend the Hamilton's constant transition probabilities to time-varying and duration-dependent transition probabilities, respectively. See also Lahiri and Wang (1994), Hamilton and Lin (1996), Hamilton and Perez-Quiros (1996), Layton (1998), and Layton and Smith (2000) for more applications on the U.S. business cycles.

[3] "Successful datings" here means the MS's datings are consistent with the official business cycle datings.

the GDP growth can be fast. Hence, prior to 1989, Taiwan enjoyed an averaged 8.5% GDP growth rate during the expansionary periods, and a still very high rate of 5.5% during the recessionary periods. This strikingly fast economic growth, however, decreased year after year on account of the gradually expanding economic base. In other words, with respect to the growth rate, the whole economy slowed down. After 1990, the averaged growth rates in the GDP slumped to become 5.5% and 2.5% in the expansionary and recessionary periods, respectively. From another perspective, the growth rate of the GDP in the recession of the 1980s was almost equivalent to that in the expansion of the 1990s. With the whole sample employed, the MS method mistakenly interpreted the entire post-1990 sample as a recession even though expansion did come into play after that period.

Fig. 1 plots Taiwan's real GDP growth rate and the corresponding smoothed probabilities generated by Hamilton's (1989) MS method from 1979:Q1 to 2000:Q4, which represent the sample periods used in previous studies. A detailed explanation of Hamilton's model is provided in the next section. The shaded areas represent the official recessionary dates as declared by the Taiwan Council for Economic Planning and Development (CEPD). Smoothed probabilities exceeding 0.5 typically denote recessionary periods. In the figure, the smoothed probabilities are "flat" around 0.90 after 1988, indicating that the post-1990 period was identified by the MS method as recessions, a finding which was not actually true. Even if the data available are extended up to the present (2002:Q2) to increase the sample size, the unrealistic results remain.

Chen and Lin (2000b) first tackled this issue by using a multivariate Markov-switching factor approach, which included the GDP, investments, consumption and exports. While they are successful in altering the flat post-1990's smoothed probabilities to be roughly consistent with official business cycle dating, the total fitting is distorted since the number of "false" or "missed" datings actually increase for the early periods of business chronology.[4] Hsu and Kuan (2001) have suggested using only post-1990 data to solve this identification puzzle even at the expense of a quickly decreasing degree of freedom. While this divided sample approach may intuitively seem acceptable, the loss of four business cycles before 1990 should not be ignored. Besides this, it is of extreme academic and professional interest to ascertain whether or not we can consistently estimate the model using the whole sample.

The purpose of this paper, therefore, is to make an attempt to solve this identification puzzle by evaluating the following four models step-by-step. We first employ Hamilton's MS Method to re-state the puzzle and discuss the possible solution. In the second stage, we add a year dummy of 1990 in Hamilton's switching model since earlier studies have shown a structural break at that time. The puzzle, however, remains after the dummy variable is added. Thus, in the third stage, we extend the model so that it becomes a multivariate one by including four variables since business cycles are broadly defined as fluctuations of "aggregate economic activities", rather than as fluctuations of a single variable. This forces us to adopt the Markov-switching factor (MSF hereafter) model of Kim and Nelson (1999). The MSF model, which extracts one common factor from all of the four variables, does indeed improve the fitting of the post-1990 period, but the pre-1990 fitting is still distorted. Simply put, the use of the MSF achieves only partial success in solving the persistent puzzle.

The final extension, the fourth stage of this research, is the use of the multivariate model of "two" two-state MSF. Recently, McConnel and Perez-Quiros (2000) have shown that a structural

[4] "False" signal means that there is no recession but that the model wrongly predicts one. "Missed" signal means that there is a recession but that the model misses it.

break in the volatility of the U.S. GDP growth exist in the first quarter of 1984 and that this may fail the conventional MS model when attempts are made to identify the US business cycle's turning point.[5] This second state-variable, variance, which has not been taken into account in earlier research on Taiwan, is deemed to solve as a potential clue to solving the puzzle. Those authors extended Hamilton's (1989) paper to a two two-state Markov-switching model, but their paper is still only a univariate process of the US GDP growth rate, however. The dating ability substantially improves when we extend their univariate process to the multivariate process. Thus, our conclusion as to how to solve the identification puzzle is to consider a multivariate MS Model with two state variables.

Our experience in solving this "identification puzzle" is also expected to be a most helpful tool for other developing countries, for it is reasonable to believe that many other developing countries also have fast GDP growth rates during their early economic development but that the rates gradually slow down as their economies mature. The identification problem of misinterpreting a recent expansionary period as a previous recession is, in all likelihood, a general case. This paper, therefore, serves as a benchmark with which the authorities and academics in other developing countries can accurately identify every business cycles.

The remainder of this paper is organized as follows. Section 2 introduces and provides the estimated results of the four models we investigate. Section 3 explains the sensitivity test, and Section 4 concludes the paper.

2. Dating Taiwan's business cycle: a sequential study

This section covers each stage we undertake to solve the persistent identification puzzle.

2.1. Hamilton's Markov-switching model: univariate process

Hamilton's MS Model, which is a univariate process with mean shifting between expansionary and contractionary regimes, is the first attempt to date Taiwan's business cycle. That is, the notion of two-state denotes the expansionary and recessionary states, which are determined by their respective means:

$$y_t - \mu(S_t) = \gamma_1(y_{t-1} - \mu(S_{t-1})) + \ldots + \gamma_p\left(y_{t-p} - \mu\left(S_{t-p}\right)\right) + \varepsilon_t, \text{ and } \varepsilon_t \sim N\left(0, \sigma^2\right), \tag{1}$$

$$\mu(S_t) = \mu_0 + \mu_1 S_t, \tag{2}$$

where y_t is the year-to-year real GDP growth rate; μ is its mean; σ^2 is the variance and S_t is the unobservable binary latent variable. Throughout this paper, S_t takes on the value 1 when the economy is in an expansionary state and 0 when the economy is in contractionary state. Hence, the means of the expansionary and contracted states are $\mu_0 + \mu_1$ and μ_0, respectively. Their variance is assumed to be the same.

Assuming S_t follows a first-order Markov chain, we have:

$$\begin{array}{ll} \Pr[S_t = 0 | S_{t-1} = 0] = p_{00}, & \Pr[S_t = 1 | S_{t-1} = 1] = p_{11}; \\ \Pr[S_t = 1 | S_{t-1} = 0] = 1 - p_{00}, & \Pr[S_t = 0 | S_{t-1} = 1] = 1 - p_{11} \end{array}, \tag{3}$$

[5] Kim and Nelson (1999), Chauvet and Potter (2001), and Kholodilin (2002) have also investigated the same issue in U.S. business fluctuations.

Table 1
Hamilton's model: whole sample, pre-1990 and post-1990

Parameter	Whole sample		Previous studies		Pre-1900 sample		Post-1990 sample	
	1979:Q1–2002:Q4		1979:Q1–2000:Q4		1979:Q1–1989:Q4		1990:Q1–2002:Q4	
	Coefficient	(|t|-value)	Coefficient	(|t|-value)	Coefficient	(|t|-value)	Coefficient	(|t|-value)
μ_0	−0.489	(0.455)	6.231	(36.277)	6.497	(16.830)	−1.527	(2.478)
μ_1	6.978	(28.214)	11.093	(26.033)	10.784	(16.595)	6.084	(30.046)
σ^2	5.290	(6.808)	2.015	(6.128)	3.513	(3.126)	1.837	(4.9410)
p_{00}	0.946	(9.204)	0.967	(45.227)	0.919	(16.375)	0.786	(4.4244)
p_{11}	0.990	(97.748)	0.833	(8.189)	0.792	(6.605)	0.979	(47.363)
log$L$	130.486		88.771		55.207		47.449	

$y_t-\mu(S_t)=\gamma_1(y_{t-1}-\mu(S_{t-1}))+\cdots+\gamma_p(y_{t-p}-\mu(S_{t-p}))+\varepsilon_t$, $\varepsilon_t\sim N(0,\sigma^2)$, $\mu(S_t)=\mu_0+\mu_1 S_t$.

where p_{ij}, i, $j=0,1$ is the transition probability, and denoting the probability of the current state is i given the previous state is j. The transition probabilities are assumed to be constant here.

Three sample periods, namely the whole sample (1979:Q1–2002:Q2), the pre-1990 sample (1979:Q1–1989:Q4) and the post-1990 sample (1990:Q1–2002:Q2), are employed. The dividing date of 1990:Q1 is selected based on Hsu and Kuan's (2001) study, which provides strong evidence that a structural change occurred in 1989:Q4. The three sample observations are 98, 44 and 54, respectively. Quarterly data are used.

Table 1 reports the estimated results. Using the whole sample, the estimated mean growth rates for the expansionary and contracted regimes are 6.98% and −0.50%, respectively. If the sample ends at 2001:Q1, one quarter before 2001:Q2, which was the most severe recession in Taiwan, the real GDP growth rate is −3.45, and the estimated mean growth rates for the expansionary and contracted regime are 11.10% and 6.23%, respectively. Using the first sub-sample (the pre-1990 sample), means of the two regimes are 10.78% and 6.50%, respectively, and they are 6.08% and −1.53%, respectively, in the second sub-sample (the post-1990 periods). Worth noting is that the mean of the contracted regime (6.50%) before 1990 resembles that of the expansionary regime (6.08%) after 1990. Accordingly, the use of the whole sample gives rise to the above-mentioned puzzle.

Fig. 2 plots the smoothed probabilities of contracted periods. Note that the estimated sample period ends in 2002:Q2, instead of 2000:Q4. Not unlike the plot presented in Fig. 1, the sub-plot at the top of Fig. 2 is again the real GDP growth rate for the purpose of comparison. The remaining three sub-plots in Fig. 2 are the smoothed probabilities using the whole, the pre-1990 and the post-1990 samples. Regarding the whole and the post-1990 sample cases, only one recession, that 2001, is determined from the model since the estimation is dominated from the above mentioned most severe recession which occurred in the 2001:Q2.

Turning to the pre-1990 sample, while the model-dating ability is improved, the dated recession in 1987 is slightly too "early" since the CEPD-defined recession date actually started in 1989. We find that this premature dating problem is not caused by the misspecification of Hamilton's model but rather by the inconsistencies between the official recessionary dates and the fluctuations in the real GDP growth rate. The official authority does not date this recession until 1989 though the real GDP growth rate dropped 2 years earlier—in 1987. The official dated recession was later probably because business cycles are broadly defined as fluctuations in "aggregate economic activities", as opposed to fluctuations in a single variable, such as real GDP. Hence, the CEPD, which bases their decision on more than one variable, determined that

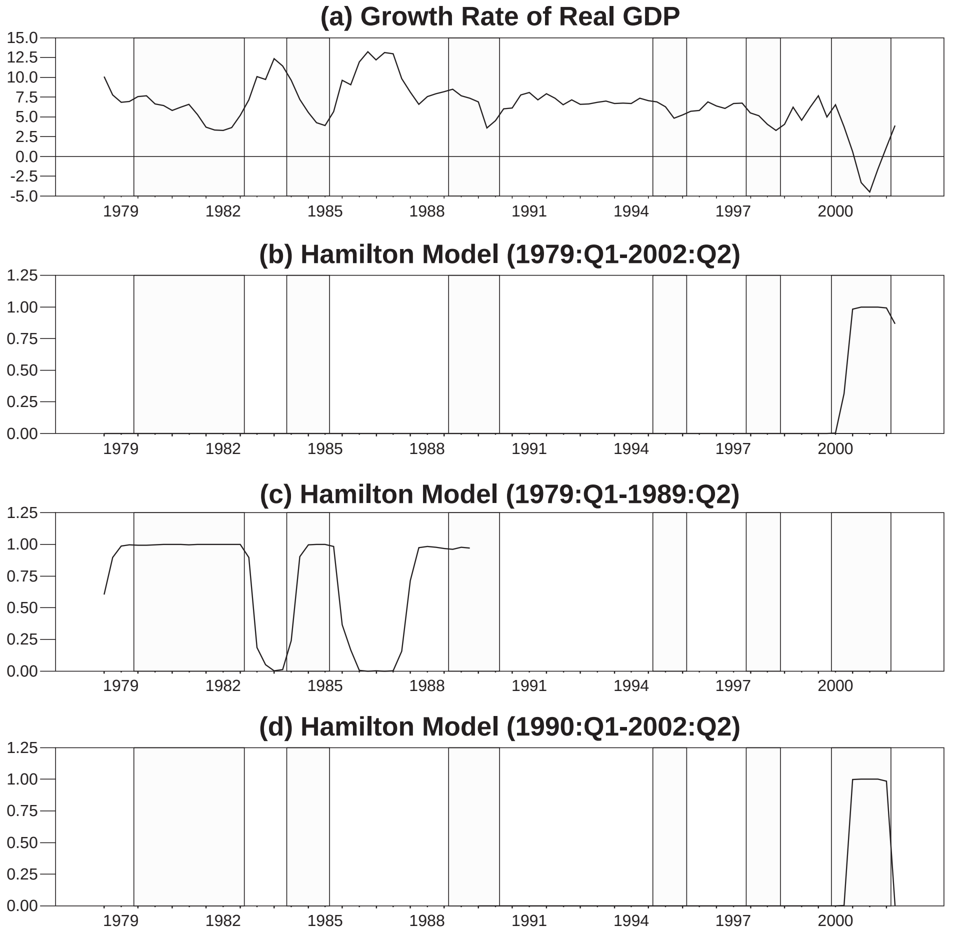

Fig. 2. Hamilton model: different sample periods.

the recession must have started in 1989. Instead, Hamilton's univariate model uses solely real GDP growth rate, thus yielding different results from those given by the CEPD.

This premature dating problem suggests that if the objective is to be consistent with the official recessionary dates, then more macro variables, as opposed to only real GDP growth rate, are required to identify recessions. That is, while the real GDP alone is typically used to analyze business fluctuations, it may not fully capture all of the activity of business. As a result, we also introduce more macroeconomic variables to characterize Taiwan's business fluctuations.

2.2. Hamilton's Markov-switching model: adding dummy 1990

An intuitive concept to solve the identification problem is to modify Hamilton's Eq. (2) by adding a dummy D_t, which is equal to unity before 1990 and zero afterwards (Table 2). Hence, separating the samples is no longer required. The mean of Eq. (3) is modified as Eq. (4); that is:

$$\mu(S_t) = (\mu_0 + \mu_1 S_t)D_t + (\mu_0^* + \mu_1^* S_t)(1 - D_t). \tag{4}$$

Table 2
Hamilton's model with dummy in 1990

Parameter	Coefficient	S.E.	t-statistics
p_{00}	0.958	0.024	38.816
p_{11}	0.916	0.059	15.331
γ_1	0.990	0.104	9.435
γ_2	−0.117	0.150	−0.782
γ_3	−0.024	0.151	−0.164
γ_4	−0.363	0.112	−3.231
σ^2	0.981	0.080	12.243
μ_0	−6.104	0.512	−11.900
μ_1	8.656	0.377	22.946
μ_0^*	−3.398	0.777	−4.373
μ_1^*	−2.458	0.481	−5.106
log$L$	142.474		

$y_t-\mu(S_t)=\gamma_1(y_{t-1}-\mu(S_{t-1}))+\cdots+\gamma_p(y_{t-p}-\mu(S_{t-p}))+\varepsilon_t$, $\varepsilon_t\sim N(0,\sigma^2)$, $\mu(S_t)=(\mu_0+\mu_1 S_t)(1-D_t)+(\mu_0^*+\mu_1^* S_t)(1-D_t)$.

Based on this equation, we have four means. Before 1990, the means of recessionary and contracted states are μ_0 and $\mu_0+\mu_1$, respectively, whereas after 1990, the means of the two regimes are μ_1^* and $\mu_0^*+\mu_1^*$, respectively.

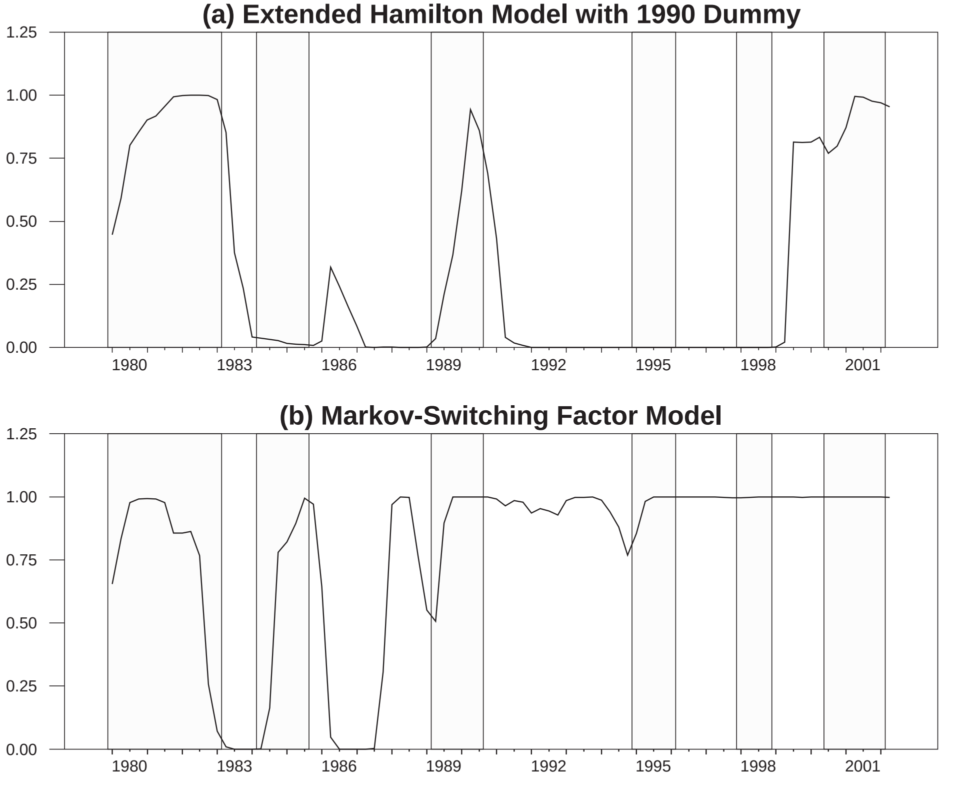

Fig. 3. Extended Hamilton and Markov-switching factor models.

The estimated results are valuable indeed since they provide the first clue as to how to solve the puzzle. Fig. 3, which presents the smoothed probabilities using the whole sample, suggests that, based on their amplitudes, there are "large" versus "small" business cycles. In other words, while there are peaks and troughs in each cycle, the amplitude of each cycle is different. Interesting to note is that this model only captures the "large" cycles during the 1979 to 1989 and 2000 to 2001 periods, thus skipping the "small" business cycles during the 1990 to 1999 period. Hence, the amplitudes (fluctuations) of the real GDP growth rate may also affect the estimations, highly indicative of a non-constant variance.

Two clues are now available for the solution to the puzzle. For one, the premature dating problem that appeared in Fig. 2 suggests a multivariate model. The second clue is that given that the variance of the business cycle may not be constant suggests the need for a second state variable, the variance. The next two sub-sections investigate these two clues step-by-step.

2.3. Conventional Markov-switching factor model

This sub-section extends Hamilton's (1989) univariate model to a multivariate one. Since there is a tradeoff when selecting macro variables to characterize business fluctuations, it is preferable to select more variables in order to fully reflect these fluctuations. However, using

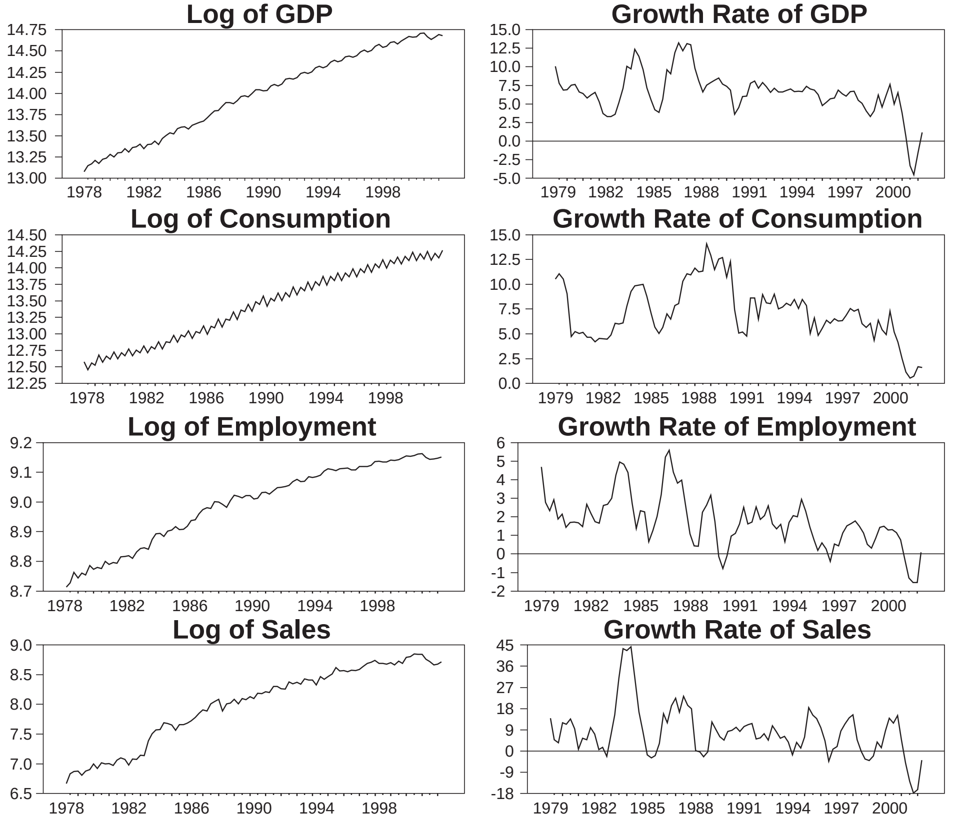

Fig. 4. Scatter plots of empirical series.

Table 3
Means and variances of variables

Variable	[1] 1979:Q1–2002:Q2	[2] 1979:Q1–1989:Q4	[3] 1990:Q1–2002:Q2	[4] 1990:Q1–2000:Q4	[5] 2001:Q1–2002:Q2	H_0: $\mu_1=\mu_2$	H_0': $\sigma_1^2=\sigma_2^2$
	Mean (S.D.)	Mean (S.D.)	Mean (S.D.)	Mean (S.D.)	Mean (S.D.)		
GDP	6.516 (2.971)	7.821 (2.768)	5.480 (2.645)	6.218 (1.161)	−0.645 (3.071)	3.661 [0.000]	5.147 [0.000]
CP	7.138 (2.822)	8.147 (2.944)	6.365 (2.349)	6.939 (1.676)	1.365 (0.705)	2.645 [0.004]	2.704 [0.001]
EMP	1.762 (1.414)	2.642 (1.309)	1.039 (1.038)	1.245 (0.827)	−0.629 (0.965)	5.562 [0.000]	2.739 [0.001]
SALE	8.103 (10.613)	11.710 (12.288)	5.181 (7.772)	6.831 (5.644)	−8.418 (8.745)	2.358 [0.009]	4.764 [0.000]

(i) Terms μ_1 and μ_2 denote the means of the sample size of 1979:Q1–1989:Q4 and 1990:Q1–2000:Q4, respectively.
(ii) Terms σ_1^2 and σ_2^2 denote the variances of sample size of 1979:Q1–1989:Q4 and 1990:Q1–2000:Q4, respectively.
(iii) Numbers inside the parentheses and brackets are the standard errors and p-values, respectively.

more variables adds to the estimation burden because it increases computation time and the difficulty of convergence. Hence, as a compromise, only four variables are selected.[6] These four variables are the real gross domestic product (GDP), real private consumption expenditures (CP), the labor force employed (EMP) and real manufacturing sales (SALE).[7] These four series are all coincident indicators and pro-cyclical (see Farmer, 2002 for the meaning of pro-cyclical). All series are taken from the AREMOS data bank published by the Taiwan Ministry of Education. The same sample period and frequency as those in Fig. 1 are used.

Fig. 4 presents the time series of the four variables after logarithm transformations and their corresponding annual growth rates. An upward trend among the four series is evident, and a commonalty seemingly exists among their annual growth rates. The volatilities of these series in the post-1990 periods appear to be smaller than those in the pre-1990 periods.

The means and volatilities of the four macro variables can be examined in Table 3. The mean growth rates of the real GDP decline from 7.821% during the pre-1990 sample period to 5.480% during the post-1990 period. The other three series also exhibit similar patterns. The variations in the four series, however, are similar across two periods. Take GDP growth rate as an example; the standard deviations are 2.768 and 2.645 for two periods, respectively. This similar variation is attributed to the most severe recent recession which occurred in the second quarter of 2001. Thus, we divide the sample into three subperiods, namely 1979:Q1–1989:Q4, 1990:Q1–2000:Q4 and 2001:Q1–2002:Q2, where the variances of these three periods are 2.768, 1.161 and 3.071, respectively. These, of course, are substantially different.

The statistical tests for the null hypotheses of H_0: $\mu_{\text{pre-1990}}=\mu_{\text{post-1990}}$ and H_0': $\sigma^2_{\text{pre-1990}}=\sigma^2_{\text{post-1990}}$ are summarized at the bottom of Table 1. The terms $\sigma_{\text{pre-1990}}$ and $\sigma_{\text{post-1990}}$ ($\sigma^2_{\text{pre-1990}}$ and $\sigma^2_{\text{post-1990}}$) denote the means (variances) of the sample size of 1979:Q1–1989:Q4 and 1990:Q1–2000:Q4, respectively. The results show that both hypotheses are rejected, therefore, providing solid evidence that not only the mean changes, but also volatility changes across periods.

Our first multivariate MSF Model is based on Kim and Nelson's (1998) model, which does not take changing variance into account. Their model assumes that there is a common factor

[6] It is worth noting that, in compiling their composite coincident indicator in order to monitor the trends in U.S. economy, the National Bureau of Economic and Research (NBER) in the US does pick up four real variables, i.e., employees on non-agricultural payrolls, personal income less transfer payments, index of industrial production and manufacturing and trade sales.

[7] Manufacturing sales is deflated by the wholesale price index (WPI) in advance.

Table 4
Conventional Markov-switching factor model: four variables

Parameter	Coefficient	S.E.	t-statistics
μ_0	−0.682	0.132	−5.167
μ_1	1.682	0.132	12.742
p_{00}	0.945	0.032	29.531
p_{11}	0.843	0.088	9.579
σ_1	1.953	0.156	12.192
σ_2	2.883	0.233	12.373
σ_3	0.000	0.068	0.000
σ_4	10.380	0.761	13.639
γ_1	1.066	0.139	7.669
γ_2	−0.066	0.139	0.474
γ_3	0.894	0.063	14.190
γ_4	0.105	0.063	1.667
log$L$	588.566		

$\mathbf{y}_t=\gamma_i c_t+\mathbf{z}_t$, $t=1,\ldots,T$.
$\phi(B)c_t=\mu(S_t)+\eta_t$, $\eta_t\sim$i.i.d.N(0,1).
$\mu(S_t)=\mu_0(1-S_t)+\mu_1 S_t$, $S_t=0,1$.
σ_c^2 is normalized to be one.

which can be extracted from the four macro variables. Furthermore, this common factor follows a Markov-switching process with a state variable, the mean.

We define a vector of macro variables $\mathbf{Y}_t=$[GDP, CP, EMP, SALE], as a function of a common factor c_t and individual idiosyncratic noises $\mathbf{z}_{it}$. All variables are transformed into annual growth rates, i.e., $\mathbf{y}_t=(\ln(\mathbf{Y}_t)-\ln(\mathbf{Y}_{t-4}))\times 100$. The common factor captures the co-movement and asymmetry among the empirical variables. The specifications of this MSF model are:

$$\mathbf{y}_t=\gamma c_t+\mathbf{z}_t, t=1,\ldots T, \tag{5}$$

$$\phi(B)c_t=\mu(S_t)+\eta_t, \eta_t \sim \text{i.i.d.N}(0,1), \tag{6}$$

$$\mu(S_t)=\mu_0(1-S_t)+\mu_1 S_t, S_t=0,1, \tag{7}$$

$$\theta(B)\mathbf{z}_t=\varepsilon_t, \varepsilon_t \sim \text{MVN}\left(0, \sum\right). \tag{8}$$

The terms ε_{it}, $i=1,\ldots,n$, represent a stationary series with a mean of zero and a variance of σ_i^2. Function $\phi(B)=(1-\phi_1 B-\cdots-\phi_k B^k)$, while $\theta(B)=1-\theta_1 B-\cdots-\theta_r B^r$ are both scalar polynomial, with B denoting the backward operator.

Table 4 presents the estimated results, and Fig. 5 graphs the smoothed probability using the whole sample.[8] When the CEPD-defined business chronology is taken as the benchmark,

[8] We do not report estimated results using the pre-1990 and post-1990 periods since their results are similar to those plotted in Fig. 2.

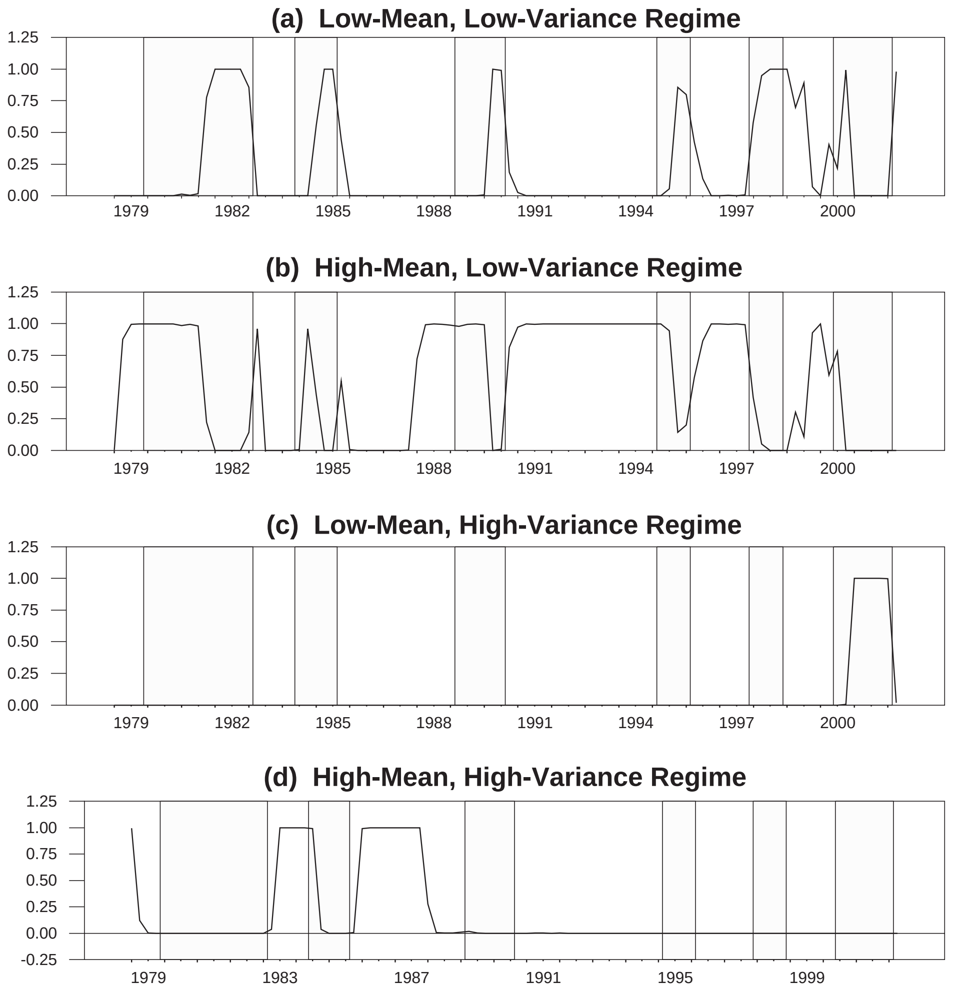

Fig. 5. Extended Markov-switching factor models.

the model-defined business chronology is not consistent with the official-defined chronology. Employing the 0.5 rule as a criterion to assess the false and missed signals, as suggested by Hamilton (1989), four false signals obviously occurred in 1988, 1991–1994, 1996 and 1999. From this, it is concluded that the MSF's smoothed probabilities do not solve the identification problem since the post-1990 period is still identified as a recession.

2.4. Extended factor Markov-switching model

Recall that the variances may not be constant as shown in Fig. 3. In that figure, we find that except for the period from 1990 to 2000, which is a period of small fluctuations, large fluctuations in the real GDP growth rate are typically found. This non-constant variance embedded in Taiwan's economic growth, however, is not taken into account in the previous three models. This subsection examines this possibility by adding a second state variable, i.e., the variance, into account. Thus, the extended MSF model is the same as the MSF except that there are now the two state variables.

$$\mathbf{y}_t = \gamma c_t + \mathbf{z}_t, t = 1,\ldots, T, \tag{9}$$

$$\phi(B)c_t = \mu(S_t, S_t^*) + \eta_t, \eta_t \sim \text{i.i.d.N}\big(0, \sigma_c^2(S_t^*)\big), \tag{10}$$

$$\mu(S_t, S_t^*) = \mu_{00}(1 - S_t)(1 - S_t^*) + \mu_{01}S_t(1 - S_t^*) + \mu_{10}(1 - S_t)S_t^* + \mu_{11}S_tS_t^*, \quad S_t = 0, 1, \tag{11}$$

and

$$\theta(B)\mathbf{z}_t = \varepsilon_t, \varepsilon_t \sim \text{MVN}(0, \Sigma). \tag{12}$$

This extended model has two features. First, the variance (volatility) of the common factor is modified so that it is dependent upon an unobserved state variable S_t^*, a modification which is designed to capture the reduction in amplitude in the post-1990s. Second, the intercept of the common factor not only depends on an unobserved state variable S_t but is also affected by the state variable S_t^*.

The addition of an extra state variable, however, increases the complexity considerably. The new state variable, S_t^*, takes the value 1 when the economic state is in a high-variance regime and the value 0 when it is in a low-variance regime. Taking the two state variables together, four regimes are created by the extended MSF model, viz, the low- and high-mean states of S_t as well as the low-variance and high-variance states of S_t^*. These four possible regimes are denoted as 1, 2, 3, and 4, respectively, and can be summarized into a new latent variable $S_t^\dagger$ for exposition purposes. Thus:

$$\begin{aligned} S_t^\dagger &= 1 \text{ for } \mu(0,0), \quad \text{if } S_t = 0 \text{ and } S_t^* = 0,\\ S_t^\dagger &= 2 \text{ for } \mu(0,1), \quad \text{if } S_t = 0 \text{ and } S_t^* = 1,\\ S_t^\dagger &= 3 \text{ for } \mu(1,0), \quad \text{if } S_t = 1 \text{ and } S_t^* = 0, \text{ and}\\ S_t^\dagger &= 4 \text{ for } \mu(1,1), \quad \text{if } S_t = 1 \text{ and } S_t^* = 1. \end{aligned} \tag{13}$$

To interpret Eq. (13), if value of $S_t^\dagger$ is equal to 1, then the mean rate of economic growth is in a low-mean and low-variance regime. Similarly, if $S_t^\dagger = 3$, then the mean economic growth rate is in a high-mean and low-variance regime. The four new regimes are thus mutations of S_t and S_t^*.

Also assume that the two state variables follow a first-order Markov chain such that $p_{ij}^{\mu} = \text{Prob}(S_t = j | S_{t-1} = i)$, and $p_{ij}^{\sigma^2} = \text{Prob}(S_t^* = j | S_{t-1}^* = i)$, with $\sum_{j=0}^{1} p_{ij}^{\mu} = \sum_{j=0}^{1} p_{ij}^{\sigma^2} = 1$, $i,j = 0,1$. In this case, we have a new transition probability; that is, the probability law that causes the economy to switch between these four states, which is governed by a new transition probability, is a 4×4 matrix $\mathbf{P}^\dagger$ with each element:

$$p_{ij}^\dagger = \text{Prob}\left(S_t^\dagger = j | S_{t-1}^\dagger = i\right), i,j = 1, 2, 3, 4, \tag{14}$$

Table 5
Extended Markov-switching factor model: four variables

Parameter	Coefficient	S.E.	t-statistics
p_{00}^{μ}	0.789	0.089	8.81
p_{11}^{μ}	0.907	0.038	23.80
$p_{00}^{\sigma^2}$	0.958	0.023	40.10
$p_{11}^{\sigma^2}$	0.803	0.088	9.04
$\mu(0,0)$	4.427	0.249	17.80
$\mu(1,0)$	6.876	0.141	48.90
$\mu(0,1)$	−1.497	0.806	1.86
$\mu(1,1)$	10.963	0.495	22.10
γ_2	1.022	0.032	31.10
γ_3	0.287	0.013	21.70
γ_4	1.483	0.116	12.80
$\sigma_c^2(0)$	0.615	0.122	5.05
$\sigma_c^2(1)$	3.090	1.07	2.88
σ_1	7.19E−11	2.16E−06	3.33E−05
σ_2	5.160	0.752	6.86
σ_3	0.842	0.123	6.86
σ_4	64.000	9.340	6.86
log$L$	838.986		

$\mathbf{y}_t=\gamma_i c_t+\mathbf{z}_t$, $t=1,\ldots,T$.
$\phi(B)c_t=\mu(S_t,S_t^*)+\eta_t$, $\eta_t\sim$i.i.d.N$(0,\sigma_c^2(S_t^*))$.
$\theta(B)\mathbf{z}_t=\varepsilon_t$, $\varepsilon_t\sim$MVN$(0,\Sigma)$.
γ_1 is normalized to be one.

and $\sum_{j=1}^{4} p_{ij}^{\dagger} = 1$. Thus, while the model is expanded, the meaning of this new transition probability remains the same as the conventional one. For example,

$$p_{11}^{\dagger} = \text{Prob}\left(S_t^{\dagger} = 1|S_{t-1}^{\dagger} = 1\right)$$

$$= \text{Prob}(S_t = 0, S_t{*} = 0|S_{t-1} = 0, S_{t-1}{*} = 0)$$

$$= \text{Prob}(S_t = 0|S_{t-1} = 0)\text{Prob}(S_t{*} = 0|S_{t-1}{*} = 0)$$

$$= p_{00}^{\mu}p_{00}^{\sigma^2}$$

meaning that when the economic status is in a low-mean growth and low-volatility regime in period $t-1$, the probability that the economic status will stay in the same regime is $p_{11}\dagger$ in period t.[9]

Table 5 reports the parameter estimates of the extended multivariate MSF Model. The notation $\mu(i,j)$, $i,\ j=0,1$ denotes that the mean growth rate of the economy is in the i-state of the mean and the j-state of volatility. For example, $\mu(0,1)$ denotes that the mean is in a low-mean and high-volatility regime, whereas $\mu(1,0)$ denotes that it is in a high-mean and low-volatility

[9] Likewise, $p_{12}^{\dagger}=\text{Prob}(S_t^{\dagger}=2|S_{t-1}^{\dagger}=1)=p_{00}^{\mu}p_{01}^{\sigma^2}$ which means that given the economic status is in a low-mean growth and a low-volatility regime in period $t-1$, the probability that the economic status will switch to a low-mean growth and high-volatility regime is $p_{12}^{\dagger}$ in period t.

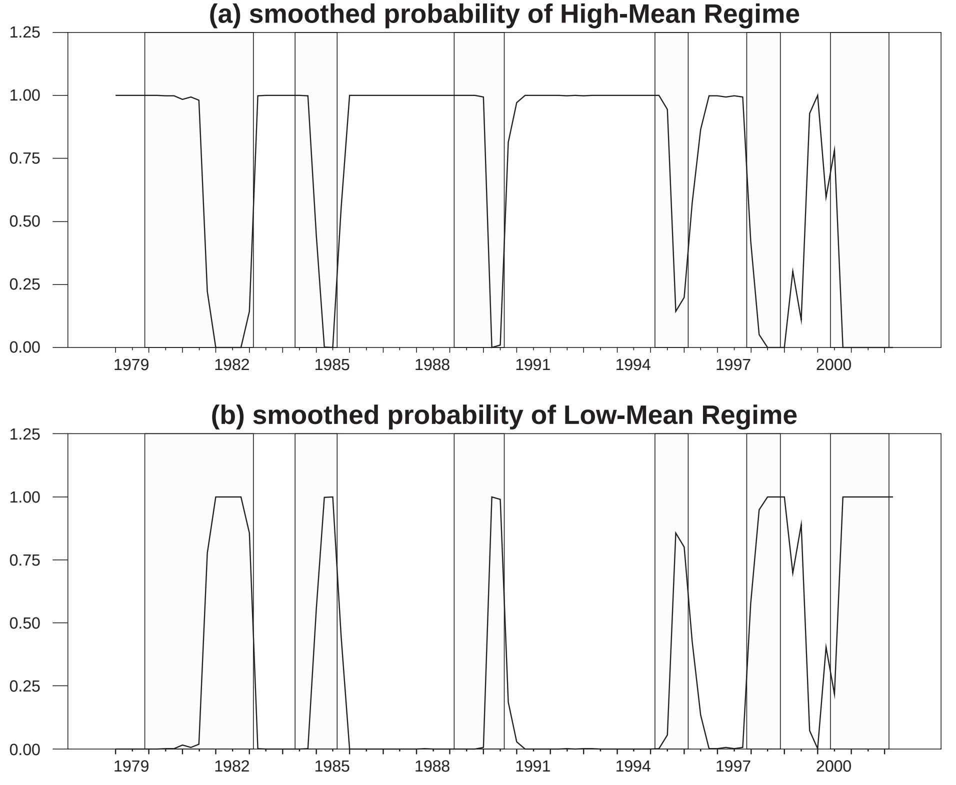

Fig. 6. Extended Markov-switching factor: mean regime.

regime. The empirical results show that $\mu(0,0)=4.427\%$, $\mu(0,1)=-1.497\%$, $\mu(1,0)=6.876\%$, and that $\mu(1,1)=10.963\%$, respectively. These values are also statistically significant using either the individual t-test or the pairwise F-test, suggesting that four means may indeed be classified by both the mean and the volatility. Also, the results show that a low mean growth rate is typically accompanied by the high volatility and vice versa.

Fig. 5a–d summarize the corresponding graphs of the smoothed probabilities, including the smoothed probabilities of the low-mean and low variance, the high-mean and low variance, the low-mean and high variance and the high-mean and high variance. The shaded areas are again the CEPD-defined contractionary dates. The performance of the low-mean and low-volatility regimes seems to correctly date most recessionary periods. Nevertheless, it misses the recent 2001 recession, which is categorized as a low-mean and high-volatility state.

We further sum the subplots (a) and (c) in Fig. 5 to "sum out" the effect of variance since the purpose of dating recession takes only the mean into account. Similarly, we add subplots (b) and (d) together to yield expansionary periods without considering the state of variance. The subplots at the top and bottom of Fig. 6 present the resulting smoothed probabilities of the low and high mean states, respectively. The subplot of the smoothed probabilities at the top corresponds to the model-dated business cycle which successfully matches most official-dated recessions, including the recent one. Accordingly, the extended MFS model can clearly identify turning points in Taiwan's economy, and these are consistent with the CEPD-defined recessionary dates (Fig. 7).

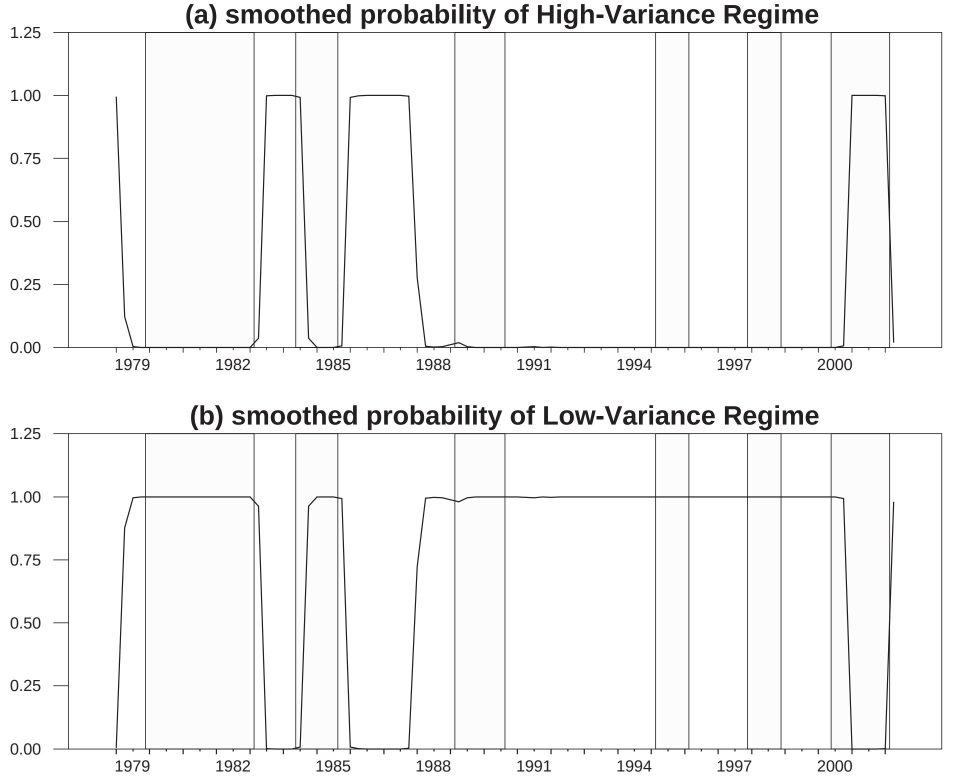

Fig. 7. Extended Markov-switching factor: variance regime.

Recall that p_{00}^{μ} and p_{11}^{μ} denote the transition probabilities of a low-mean and high-mean growth regime, respectively. The estimate of p_{00}^{μ} then is 0.789 which is smaller than the estimate of $p_{11}^{\mu}=0.907$, suggesting that the duration periods for the high-growth state (expansion) are longer than those for the low-growth state (contraction). The estimates of the transition probability for volatility, i.e., $p_{00}^{\sigma^2}$ and $p_{11}^{\sigma^2}$, are 0.958 and 0.803 for the low-volatility and high-volatility regimes, respectively. The results also provide evidence that the duration periods for the low-volatility state are longer than those for the high-volatility state.

To further confirm the success of using the extended MSF Model, we next consider the quadratic probability score (QPS) as a criterion to evaluate the prediction performance (see Diebold and Rudebusch, 1989 or Hamilton and Perez-Quiros, 1996). The QPS is defined as:

$$\mathrm{QPS} = K^{-1}\sum_{t=1}^{K}\{\mathrm{prob}(S_t = 0|\psi_T) - d_t\}^2,$$

where $d_t=1$ if dated as a period of the CEPD-defined contraction. The smaller the QPS is, the better the prediction is.

Table 6 summarizes Taiwan's business cycle chronology as determined by the officials and our four Markov-switching models. Simply judging the beginning and ending periods of the business cycle turning points, the extended MSF seems to perform the best among the four. The QPS for Hamilton's, for the extended Hamilton's, the MSF and the extended MSF are 0.484,

Table 6
Business chronology: dates by four models

	Official-dated		Early begin (false)	Late begin (miss)	Overlapping (correct)	Early end (miss)	Late end (false)
(1) Hamilton's model							
V	80:Q1–83:Q1	79:Q1–83:Q2	4		13		1
VI	84:Q3–85:Q3	84:Q4–85:Q4		1	4		2
VII	89:Q3–90:Q3	89:Q3–02:Q2			5		19/2*
VIII	95:Q2–96:Q1	89:Q3–02:Q2	19/2*		4		8/2*
IX	98:Q1–98:Q4	89:Q3–02:Q2	8/2*		4		8/2*
X	00:Q3–02:Q2	89:Q3–02:Q2	8/2*		8		
QPS = 0.484; Correct–False–Missing = –5							
(2) Extended Hamilton's Model							
V	80:Q1–83:Q1	80:Q2–83:Q2		1	12		1
V	80:Q1–83:Q1	80:Q2–83:Q2		1	12		1
VI	84:Q3–85:Q3	No dating					
VII	89:Q3–90:Q3	90:Q1–90:Q4		2	3		1
VIII	95:Q2–96:Q1	No dating					
IX	98:Q1–98:Q4	No dating					
X	00:Q3–02:Q2	99:Q3–02:Q2	4		6		
QPS = 0.212; Correct–False–Missing = 10							
(3) Markov-Switching Factor Model							
V	80:Q1–83:Q1	80:Q1–82:Q3			11	2	
VI	84:Q3–85:Q3	84:Q4–86:Q1		1	5	2	
VII	89:Q3–90:Q3	89:Q3–02:Q2			5		19/2*
VIII	95:Q2–96:Q1	89:Q3–02:Q2	19/2*		4		8/2*
IX	98:Q1–98:Q4	89:Q3–02:Q2	8/2*		4		8/2*
X	00:Q3–02:Q2	89:Q3–02:Q2	8/2*		8		
QPS = 0.414; Correct–False–Missing = -3							
(4) Extended Markov-switching factor model							
V	80:Q1–83:Q1	81:Q1–83:Q1		4	10		
VI	84:Q3–85:Q3	85:Q1–85:Q3		2	3		
VII	89:Q3–90:Q3	90:Q2–90:Q3		3	2		
VIII	95:Q2–96:Q1	95:Q4–96:Q1		2	2		
IX	98:Q1–98:Q4	98:Q1–99:Q3			4		3
X	00:Q3–02:Q2	00:Q4–02:Q2		1	7		
QPS = 0.189; Correct–False–Missing = 12							

*: Although the whole post-1990 period is identified as a recession, it is not easy to define early or late.
*: Beginning and ending. Here, we divide it by 2 to attribute an incorrect signal to the two neighboring cycles.
The QPS criterion is defined as follows.

$$\mathrm{QPS} = K^{-1} \sum_{t=1}^{K} \{\mathrm{prob}(S_t = 0|\psi_T) - d_t\}^2,$$

where $d_t = 1$ if dated as a period of the CEPD-defined contraction.

0.212, 0.414 and 0.189, respectively. It can be stated with certainty that the extended MSF indeed improves the dating of Taiwan's business cycle turning points.

A rule of thumb regarding the forecasts of the beginning and ending of the business cycle turning points is also employed. We define the signals of early/late "beginning" of the business cycle turning points if the model predicts a peak earlier/later than those of the official peak.

Table 7
Extended Markov-switching factor model: three variables

Parameter	Coefficient	S.E.	t-statistics
p_{00}^{μ}	0.801	0.082	9.70
p_{11}^{μ}	0.905	0.037	24.3
$p_{00}^{\sigma^2}$	0.958	0.023	40.3
$p_{11}^{\sigma^2}$	0.802	0.089	8.97
$\mu(0,0)$	4.460	0.223	20.00
$\mu(1,0)$	6.826	0.151	45.20
$\mu(0,1)$	−1.663	0.824	−2.020
$\mu(1,1)$	11.199	0.536	20.90
γ_2	0.290	0.013	21.50
γ_3	1.503	0.117	12.90
$\sigma_c^2(0)$	0.200	0.382	0.52
$\sigma_c^2(1)$	2.710	1.130	2.39
σ_1	0.451	0.428	1.05
σ_2	0.800	0.124	6.45
σ_3	61.900	9.290	6.67
$\log L$	628.025		

$\mathbf{y}_t = \gamma_i c_t + \mathbf{z}_t$, $t = 1,\ldots,T$.
$\phi(B)c_t = \mu(S_t, S_t^*) + \eta_t$, $\eta_t \sim$ i.i.d.N$(0, \sigma_c^2(S_t^*))$.
$\theta(B)\mathbf{z}_t = \varepsilon_t$, $\varepsilon_t \sim$ MVN$(0, \Sigma)$.
γ_1 is normalized to be one.

Hence, the earlier/later beginnings are equivalent to the false/missed signals, respectively. Similarly, the signals of the earlier/later "endings" of business cycle turning points are defined if the model predicts a trough earlier/later than that of official troughs. Hence, these earlier/later endings are equivalent to the missed/false signals, respectively. We refer to the "correct" signal if model-dating overlaps with the official-dating. Then, we count the number of correct, false and missed signals. The performances of the four models are then compared by counting the number of correct minus the number of false and missed signals. The right-hand side of Table 6 presents the counting processes of the three types of signals. The higher number denotes the better forecasting performance, of which the four models are −5, 10, −3 and 12, respectively. Again the extended MSF is by far the best of the four.

3. Sensitivity analysis

One possible critique of the above empirical results may lie in the selection of the variables. However, we find the results do not change very much when we consider only three variables. Table 7, for example, shows the results of the extended MSF model when only the GDP, EMP, and SALE are used. The results do not change since the estimates of the transition probabilities and the mean growth rate for the four different regimes are almost identical to those reported in Table 6. The smoothed probabilities are also very similar, and hence are not reported. As a consequence, our results are rather robust.

4. Concluding remarks

The purpose of this paper is to solve the identification puzzle of Taiwan's business chronology. The puzzle is a product of the fact that Hamilton's (1989) Markov-switching model incorrectly shows a recession for the whole post-1990 period in Taiwan. Though a great deal of effort has been made to solve this problem, the puzzle has long remained.

This paper finds that previous studies have considered the mean of the real GDP growth rate as the only state variable and have ignored the fact that the variance also switches. Furthermore, employing only the GDP growth rate may ignore the certain features of the business cycle because the cycle describes aggregate economic activities. Hence, our extended Markov-switching factor model solves the puzzle into two directions. First, two state variables, the mean and variance, are incorporated into the model. Next, four variables, i.e., the real GDP growth rate, consumption expenditures, labor force employed and manufacturing sales are used. Our empirical results clearly show that, in identifying Taiwan's turning points, our extended multivariate MSF model outperforms other Markov-switching models, including Hamilton's, the extended Hamilton's and conventional Markov-switching factor models. All of this aside and most importantly, our model successfully identifies Taiwan's turning points in post-1990 periods.

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Appendix A. Estimation procedure

In order to find parameter estimates of the dynamic Markov-switching factor model, we rewrite Eqs. (9)–(12) as a state-space representation. While either Gibbs sampler approach or Kim's (1994) approximate maximum likelihood method can be employed to do the estimation work, and therefore, we adopt the latter approach for its simplicity in estimation. Basically, the algorithm for the state-space model with a Markov-switching mechanism must apply the Kalman filter and the Hamilton filter, along with an approximation proposed by Kim (1994).

Performing the estimations consists of the following several steps. First, the ergodic probability must be calculated as the initial value, and then the Kalman and the Hamilton filters are applied to this model. The most innovative aspect of the Hamilton filter is its ability to objectively date state of the economy by determining the so-called filtered and smoothed probabilities. The filtered probabilities, collected in a $(T\times 1)$ vector denoted as $\xi_{t|t}$ ($\xi_{t|t}=p(S_t=j|\Psi^t)$, $t=1, \ldots, T$, Ψ^t is the information set) denote the conditional probability that the analyst's inference about the value of S_t is based on information obtained through date t. It is also possible to calculate smoothed probabilities, $\xi_{t|T}=p(S_t=j|\Psi^T)$, which are based on the full sample.[10] Finally, an approximation, as proposed by Kim (1994), must be made in order to write the log-likelihood function as:

$$\log L = \ln f\left(\mathbf{Y}_T, \mathbf{Y}_{T-1},\ldots|\Psi^0\right) = \sum_{t=1}^{T} \ln f\left(\mathbf{Y}_t|\Psi^{t-1}\right). \tag{15}$$

The unknown parameter estimates of the model can be obtained by maximizing the log-likelihood with respect to the unknown parameters by using the numerical method.

[10] Hamilton (1989) describes how to make an inference about the particular state an economy is in at date t.

Appendix B. State-space representation and algorithm

In this appendix, we briefly describe how to re-write the two-factor model with regime switching into a state-space representation and how to apply Kim's (1994) algorithm of approximate maximum likelihood method to get unknown parameter estimates. Basically, Kim's algorithm is a synthesis of Kalman's and Hamilton's filters. Eqs. (9)–(12) can be transformed into the measurement Eq. (16) and the transition Eq. (17) as follows:

$$\mathbf{Y}_t = \mathbf{H}_t \boldsymbol{\xi}_t \tag{16}$$

$$\boldsymbol{\xi}_t = \beta_{S_t^\dagger} + \mathbf{T}_t \boldsymbol{\xi}_{t-1} + \mathbf{u}_t \tag{17}$$

with

$$\mathbf{H}_t = \begin{bmatrix} \gamma_1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \gamma_2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \gamma_3 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ \gamma_4 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\mathbf{T}_t = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\boldsymbol{\xi}_t = [\, c_t \quad c_{t-1} \quad c_{t-2} \quad c_{t-3} \quad z_{1,t} \quad z_{2,t} \quad z_{3,t} \quad z_{4,t} \,]',$$

$$\beta_{S_t^\dagger} = \beta_{S_t, S_t^*} = [\, \mu(S_t, S_t^*) \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \,]',$$

$$\mathbf{u}_t = [\, \eta_t(S_t^*) \quad 0 \quad 0 \quad 0 \quad \varepsilon_{1t} \quad \varepsilon_{2t} \quad \varepsilon_{3t} \quad \varepsilon_{4t} \,]',$$

$$\mathbf{Q}_t = \begin{bmatrix} \sigma^2(S_t^*) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma^2_{\varepsilon_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma^2_{\varepsilon_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma^2_{\varepsilon_3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma^2_{\varepsilon_4} \end{bmatrix}$$

where $\mathbf{Q}=E(\mathbf{u}_t\mathbf{u}'_t)$. Given a realization of the state variables at time t and $t-1$ ($S_t^\dagger=j$ and $S_{t-1}^\dagger=i$, where $i,\ j=0$ or 1) and using the notation $Z_{t|t-1}^{(i,\,j)}$ to denote the variable Z conditional on the information available up to $t-1$ and the realized states j and i, the Kalman filter can be represented as:

$$\boldsymbol{\xi}_{t|t-1}^{(i,j)} = \mathbf{T}_t \boldsymbol{\xi}_{t-1|t-1}^{(i)} + \beta_{S_t^{\dagger}}^{(j)} \tag{18}$$

$$\mathbf{P}_{t|t-1}^{(i,j)} = \mathbf{T}_t \mathbf{P}_{t-1|t-1}^{(i)} \mathbf{T}_t' + \mathbf{Q}_t \tag{19}$$

$$\boldsymbol{\xi}_{t|t}^{(i,j)} = \boldsymbol{\xi}_{t|t-1}^{i,j} + \mathbf{K}_t^{(i,j)} \mathbf{g}_{t|t-1}^{(i,j)} \tag{20}$$

$$\mathbf{P}_{t|t}^{(i,j)} = \left(\mathbf{I} - \mathbf{K}_t^{(i,j)} \mathbf{H}_t\right) \mathbf{P}_{t|t-1}^{(i,j)} \tag{21}$$

$$\mathbf{g}_{t|t-1}^{(i,j)} = \mathbf{Y}_t - \mathbf{H}_t \boldsymbol{\xi}_{t|t-1}^{i,j} \tag{22}$$

$$\mathbf{W}_{t|t-1}^{(i,j)} = \mathbf{H}_t \mathbf{P}_{t|t-1}^{(i,j)} \mathbf{H}_t' \tag{23}$$

$$\mathbf{K}_t^{(i,j)} = \mathbf{P}_{t|t-1}^{(i,j)} \mathbf{H}_t' \left(\mathbf{W}_{t|t-1}^{(i,j)}\right)^{-1} \tag{24}$$

where Eqs. (18) and (19) are the prediction formulae, Eqs. (20) and (21) are the updating formulae and Eq. (24) is the Kalman gain. $\boldsymbol{\eta}_{t|t-1}^{(i,j)}$ is the conditional forecast error of $\mathbf{Y}_t$ based on information up to $t-1$; and $\mathbf{W}_{t|t-1}^{(i,j)}$ is the conditional variance of the forecast error $\boldsymbol{\eta}_{t|t-1}^{(i,j)}$. As noted by Harrison and Stevens (1976), each iteration of the above Kalman filtering produces a 4-fold increase in the number of cases to consider. Kim (1994) provided a fast approximation algorithm applicable to this problem. The idea is to collapse the dimension of the (4×4) posteriors ($\boldsymbol{\xi}_{t|t}^{(i,j)}$ and $\mathbf{P}_{t|t}^{(i,j)}$) to two posteriors ($\boldsymbol{\xi}_{t|t}^{(i,j)}$ and $\mathbf{P}_{t|t}^{(i,j)}$) by taking the weighted averages over all states at $t-1$. That is:

$$\boldsymbol{\xi}_{t|t}^{(j)} = \frac{\sum_{S_{t-1}^{\dagger}=1}^{4} \Pr\left[S_t^{\dagger} = j, S_{t-1}^{\dagger} = i | \Psi_t\right] \times \boldsymbol{\xi}_{t|t}^{(i,j)}}{\Pr\left[S_t^{\dagger} = j | \Psi_t\right]} \tag{25}$$

$$\mathbf{P}_{t|t}^{(j)} = \frac{\sum_{S_{t-1}^{\dagger}=1}^{4} \Pr\left[S_t^{\dagger} = j, S_{t-1}^{\dagger} = i | \Psi_t\right] \times \{\mathbf{P}_{t|t}^{(i,j)} + \left(\boldsymbol{\xi}_{t|t}^{(j)} - \boldsymbol{\xi}_{t|t}^{(i,j)}\right)\left(\boldsymbol{\xi}_{t|t}^{(j)} - \boldsymbol{\xi}_{t|t}^{(i,j)}\right)'\}}{\Pr\left[S_t^{\dagger} = j | \Psi_t\right]} \tag{26}$$

where Ψ_t refers to information available at time t. Following Hamilton (1989, 1990), the filter can be obtained by the Bayes's theorem.

$$\Pr\left[S_t = j, S_{t-1}^{\dagger} = i | \Psi_t\right] = \frac{\Pr\left[\mathbf{Y}_t, S_t^{\dagger} = j, S_{t-1}^{\dagger} = i | \Psi_{t-1}\right]}{\Pr[\mathbf{Y}_t | \Psi_{t-1}]}$$

$$= \frac{f\left[\mathbf{Y}_t | S_t^{\dagger} = j, S_{t-1}^{\dagger} = i, \Psi_{t-1}\right] \times \Pr\left[S_t^{\dagger} = j, S_{t-1}^{\dagger} = i | \Psi_{t-1}\right]}{\Pr[\mathbf{Y}_t | \Psi_{t-1}]} \tag{27}$$

where

$$f\left[\mathbf{Y}_t|S_t^\dagger=j,S_{t-1}^\dagger=i,\Psi_{t-1}\right]=(2\pi)^{-N/2}|\mathbf{W}_{t|t-1}^{(i,j)}|^{-1/2}\times\exp\left\{-\frac{1}{2}\boldsymbol{\eta}_{t|t-1}^{(i,j)\prime}\left(\mathbf{W}_{t|t-1}^{(i,j)}\right)^{-1}\boldsymbol{\eta}_{t|t-1}^{(i,j)}\right\}. \tag{28}$$

The smoothed probabilities $p(S_t^\dagger|\Psi_T)$, on the other hand, are the conditional probabilities which are based on data available through the whole sample at future date T, which amounts to:

$$\Pr\left[S_{t+1}^\dagger=k,S_t^\dagger=j|\Psi_T\right]\approx\frac{\Pr\left[S_{t+1}^\dagger=k|\Psi_T\right]\times\Pr\left[S_t^\dagger=j|\Psi_t\right]\times\Pr\left[S_{t+1}^\dagger=k|S_t^\dagger=j\right]}{\Pr\left[S_{t+1}^\dagger=k|\Psi_t\right]} \tag{29}$$

$$\Pr\left[S_t^\dagger=j|\Psi_T\right]=\sum_{S_{t+1}^\dagger}^{4}\Pr\left[S_{t+1}^\dagger=k,S_t^\dagger=j|\Psi_T\right]. \tag{30}$$

The approximate sample conditional log-likelihood is:

$$\log L=\ln f(\mathbf{Y}_T,\mathbf{Y}_{T-1},\ldots|\Psi_0)=\sum_{t=1}^{T}\ln f(\mathbf{Y}_t|\Psi_{t-1}). \tag{31}$$

The approximate maximum likelihood estimates of the model can be obtained by maximizing the log-likelihood with respect to the unknown parameters.

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