Transactions of the Institute of Measurement and Control 33, 1 (2011) pp. 93–115

A simple modification of deadlock prevention policy of S^3PR based on elementary siphons

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We propose to extend our proprietary approach of siphon synthesis to a new simple deadlock control policy. To prevent each strict minimal siphon (SMS) S from becoming empty of tokens, we often add a control place V_S and associated arcs to form an invariant to control the number of tokens leaking from S. In a disturbanceless approach, the control (called Type-1) arcs are chosen to disturb the original uncontrolled model as little as possible, to reach as many states as possible. However, this policy may generate new SMSs and hence require adding too many control places and arcs to the original Petri net model. Thus, Ezpeleta *et al*. moved all output (called Type-2) arcs of each V_S to the output transition of the entry (called idle place) of input raw materials to limit their rate into the system, called all-sided, or SMSless approach. This may overly constrain the system so that many reachable states are no longer attainable. We hence propose an intermediate, called the one-sided, approach, which does not generate new SMS, based on our siphon-synthesis theory, by appropriately choosing the locations of Type-1 arcs and it can reach more states than the all-sided approach. The same results can be extended to the elementary-siphon approach by Li *et al*. except with no need to fine-tune the locations of Type-1 arcs. Comparison with other approaches has been made.

Key words: deadlock control; flexible manufacturing systems; Petri nets; siphons.

1. Introduction

A flexible manufacturing system (FMS) is a set of working processes (WPs) sharing a number of resources such as robots, machines, AGVs, fixtures, buffers etc.

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*The former name of the author is Yuh Yaw, which appeared in some of his earlier papers.
Figure 5 appears in color online: http://tim.sagepub.com

 10.1177/0142331208095674

(Huang, 2001; Lautenbach and Ridder, 1996; Murata *et al*., 1986). Raw materials enter from one end (modelled by a place p_0 called the idle place) of a WP, moved by some robots and processed by some machines (modelled by operation places) via some pre-established production sequence, and exit as a product from the other end (modelled by the idle place p_0). As the input rate of raw materials into WP increases, so will the production rate, ie, throughput, as well as the competition for resources. The competition eventually reaches such a degree that some WPs cannot proceed to the next stages and mutually wait for each other to release resources, resulting in a deadlock state. To prevent such misshape from happening, it is a necessary requirement for an effective FMS control policy to make sure that deadlock will never occur in the system. Effective handling of deadlocks in various FMSs has turned out to be a major concern in the operation of these systems.

There are two approaches to control deadlocks: deadlock recovery and deadlock prevention/avoidance. Recovery is to restore the system to a normal state and to be able to finish the production. Avoidance (Barkaoui and Abdallah, 1994; Ezpeleta *et al*., 1993; Huang, 2001; Lautenbach, 1987; Lautenbach and Ridder, 1996) determines possible system evolutions at each system state and chooses the correct ones to proceed. Prevention/avoidance is to avoid such situations. Prevention establishes the control policy in a static way (Ezpeleta *et al*., 1995; Huang, 2001) by building freely a Petri net (PN) model first and then adding the necessary control to it such that the controlled model is deadlock-free.

Prevention is preferred to avoidance because the computational effort is carried out offline once. Hence it runs much faster in real-time cases compared with deadlock avoidance algorithms, where much time is consumed by doing this online each time the system ought to change the state. A deadlock prevention control policy is essential when it is unacceptable to have deadlocks, and real time response time is critical.

Deadlock in a PN occurs when a set of places (called siphon S) become empty of tokens. To prevent emptiable siphons, we often add a control place V_S and some control arcs. By controlling the initial number of tokens (denoted by $M_0(V_S)$) in V_S, we can limit the maximal of tokens leaking from S. We say that S is invariant-controlled (Chao, 2006, 2007; Li and Zhou, 2008a). The control arcs are chosen to disturb the original uncontrolled model as little as possible to reach as many states as possible. However, this policy may generate new emptiable siphons and hence require adding too many control places and arcs to the original PN model.

Ezpeleta *et al*. (1995) proposed a class of PN called systems of simple sequential processes with resources (S^3PR). They added a control place V_S – and associated arcs – for each emptiable siphon S (hence also called *all-siphon* approach) to make S invariant-controlled without generating new emptiable siphons. The initial marking of V_S, ie, $M_0(V_S)$, is assigned so that S remains marked under all reachable markings. To prevent new SMS from being generated, Ezpeleta *et al*. moved all output arcs of each V_S to the output of the entry (called *idle place*) of input raw materials to limit their rate into the system – called the *all-sided* approach. This may overly constrain the system so that many reachable states are no longer attainable.

The same problem occurs for the *elementary-siphon* approach proposed by Li and Zhou (2004, 2006a, 2008b), which simplifies the control. S can be divided into two groups: elementary and dependent. They add a control place for each elementary siphon S_e, while controlling all dependent S too so that there is no need to add a control place for S. This leads to much fewer control places so that the method is suitable for large-scale PNs. However, all control arcs remain and it suffers the same problem with fewer reachable states compared with the optimal one in Uzam and Zhou (2006) and Li *et al*. (2008).

We hence propose an intermediate, called *one-sided*, approach based on our earlier innovative search of siphons (Chao, 2007). We will show that the proposed one-sided approach can reach more states than the all-sided approach for both *all-siphon* and *elementary-siphon* approaches. The rest of the paper is organized as follows. Section 2 presents the preliminaries about PNs followed by Section 2A on S^3PR, Section 2B on the controlled model of S^3PR and Section 2C on handle-construction procedure to build SMS, respectively. Section 3 presents the approach and the theory of one-sided control policy. Section 4 presents an example followed by Section 5 to prove the correctness. Section 6 proves the liveness of the one-sided policy and compares with others. Finally Section 7 concludes the paper.

2. Preliminaries

In this paper, we assume that the reader is familiar with the PN basis. Here we present only the definitions that are used in this paper.

Definition 1. An ordinary PN (OPN) is a 4-tuple $PN=(N, M_0)=(P,T,F,M_0)$, *where* $N=(P,T,F)$ *is a net,* $P=\{p_1,p_2,\ldots,p_a\}$ *a set of places,* $T=\{t_1,t_2,\ldots,t_b\}$ *a set of transitions, with* $P\cup T\neq\emptyset$ *and* $P\cap T=\emptyset$, *and* F: $(P\times T)\cup(T\times P)\rightarrow\{0,1\}$ *the flow relation,* M_0: $P\rightarrow\{0,1,2,\ldots\}$ *denotes an initial marking whose ith component,* $M_0(p_i)$, *represents the number of tokens in place* p_i. *A node x in* $N=(P, T, F)$ *is either a* $p\in P$ *or a* $t\in T$. *The post-set of node x is* $x^\bullet=\{y\in P\cup T\mid F(x,y)>0\}$, *and its pre-set* ${}^\bullet x=\{y\in P\cup T\mid F(y,x)>0\}$. *An OPN is called a state machine (SM) if* $\forall t\in T$, $|t^\bullet|=|{}^\bullet t|=1$. *An elementary directed path* Γ *in N is a sequence of nodes:* $\Gamma=[n_1\ n_2\ldots n_k]$, $k\geq 1$, *such that* $n_i\in{}^\bullet n_{i+1}$ $1\leq i<k$ *if* $k>1$, *and* $n_i=n_j$ *which implies that* $i=j$, $\forall 1\leq i, j\leq k$. *A path is (non)-virtual if it contains only (more than) two nodes. An elementary circuit in N is* $\Gamma=[n_1 n_2\ldots n_k]$, $k>1$ *such that* $n_i=n_j$, $1\leq i\leq j\leq k$, *implies that* $i=1$ *and* $j=k$.

Definition 2. A P-vector is a column vector L: $P\rightarrow Z$ *indexed by P and a T-vector is a column vector J:* $T\rightarrow Z$ *indexed by T, where Z is the set of integers. The incidence matrix of N is a matrix [N]:* $P\times T\rightarrow Z$ *indexed by P and T such that* $[N](p, t)=F(t, p)\text{-}F(p, t)$ *where* $F(p, t)$ *is the weight of the arc from place p to its output transition t, and* $F(t, p)$ *is the weight of the arc from transition t to its output place p.*

For economy of space, we use $\Sigma L(p)p(\Sigma J(t)t)$ to denote a $P(T)$-vector.

Definition 3. *t_i is firable or enabled if each place p_j in ${}^\bullet t$ holds no less tokens than the weight $w_j = F(p_j, t_i)$. Firing t_i under M_0 removes w_j tokens from p_j and deposits $w_k = F(t_i, p_k)$ tokens into each place p_k in $t^\bullet$; moving the system state from M_0 to M_1. Repeating this process, it reaches M' by firing a sequence of transitions. M' is said to be reachable from M_0, ie, $M_0[\sigma > M'$. $R(N, M_0)$ is the set of markings reachable from M_0. A transition $t \in T$ is live under M_0 iff $\forall M \in R(N, M_0)$, $\exists M' \in R(N, M)$, t is firable under M'. A transition $t \in T$ is dead under M_0 iff $\nexists M \in R(N, M_0)$ where t is firable. A PN is live under M_0 iff $\forall t \in T$, t is live under M_0. $M(Q) = \sum_{p \in Q} M(p)$, where Q is a set of places.*

Definition 4. *An integer vector Y (with components $Y(p)$, $p \in P$) is called a P-invariant iff $Y \neq 0$ and $Y^T \bullet [N] = 0$, where $[N]$ is the incidence matrix. $\|Y\| = \{p \in P \mid Y(p) \neq 0\}$ is the support of Y. A P-invariant is minimal if there does not exist a P–invariant Y' such that $\|Y'\| \subseteq \|Y\|$. Y_p is a minimal P-invariant whose support contains p. $H(p) = \|Y_p\| \setminus \{p\}$ is the set of holder places that use p. A siphon (trap) D (τ) is a non-empty subset of places such that ${}^\bullet D \subseteq D^\bullet$ ($\tau^\bullet \subseteq {}^\bullet \tau$), ie, every transition having an output (input) place in D (τ) has an input (output) place in D (τ). A minimal siphon does not contain another siphon as a proper subset. A minimal siphon is called a strict minimal siphon (SMS), denoted by S, if it does not contain a trap.*

2.1 S^3PR

The following definitions are adapted from Ezpeleta *et al*. (1995). The reader may refer to this source for more details of the S^3PR model.

Definition 5. (Ezpeleta et al., 1995). *A simple sequential process (S^2P) is a net $N = (P \cup \{p^0\}, T, F)$ where:* (1) *$P \neq \emptyset$, $p^0 \notin P$ (p^0 is called the process idle or initial or final operation place)*; (2) *N is strongly connected state machine (SM) and* (3) *every circuit C of N contains the place p^0.*

Definition 6. (Ezpeleta et al., 1995). *A simple sequential process with resources (S^2PR), also called a WP, is a net $N = (P \cup \{p^0\} \cup P_R, T, F)$ so that* (1) *the subnet generated by $X = P \cup \{p^0\} \cup T$ is an S^2P;* (2) *$P_R \neq \emptyset$ and $P \cup \{p^0\} \cap P_R = \emptyset$;* (3) *$\forall p \in P$, $\forall t \in {}^\bullet p$, $\forall t' \in p^\bullet$, $\exists r_p \in P_R$, ${}^\bullet t \cap P_R = t'^\bullet \cap P_R = \{r_p\}$;* (4) *The two following statements are verified: $\forall r \in P_R$,* (a) *${}^{\bullet\bullet} r \cap P = r^{\bullet\bullet} \cap P \neq \emptyset$;* (b) *${}^\bullet r \cap r^\bullet = \emptyset$.* (5) *${}^{\bullet\bullet}(p^0) \cap P_R = (p^0)^{\bullet\bullet} \cap P_R = \emptyset$. $\forall p \in P$, p is called an operation place. $\forall r \in P_R$, r is called a resource place. $H(r) = {}^{\bullet\bullet} r \cap P$ denotes the set of holders of r (operation places that use r). A path (circuit, subnet) $\Gamma(c, N')$ in N is called a resource path (circuit, subnet) if $\forall p \in \Gamma$ (c, N'), $p \in P_R$. A strongly connected resource subnet of N is briefed as SCRS. Transitions in ${}^\bullet P^0$and $P^{0\bullet}$ are called sink and source transitions of an S^3PR, respectively.*

Definition 7. (Ezpeleta et al., 1995). *A system of S^2PR (S^3PR) is defined recursively as follows:* (1) *An S^2PR is defined as an S^3PR;* (2) *Let $N_i = (P_i \cup P_i^0 \cup P_{Ri}, T_i, F_i)$, $i \in \{1, 2\}$ be two S^3PR so that $(P_1 \cup P_1^0) \cap (P_2 \cup P_2^0) = \emptyset$. $P_{R1} \cap P_{R2} = P_C (\neq \emptyset)$ and $T_1 \cap T_2 = \emptyset$.*

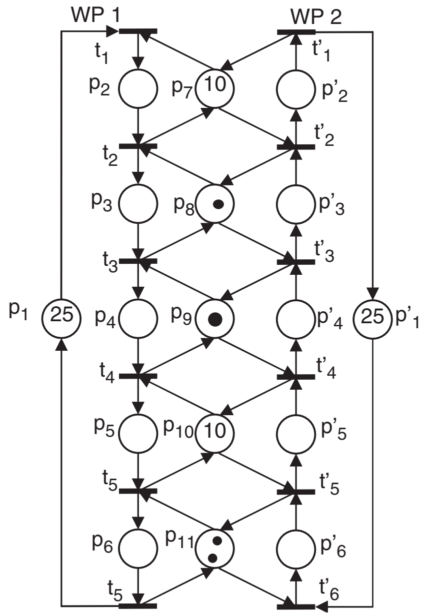

Figure 1 Example S^3PR

The net $N=(P\cup P^0\cup P_R, T, F)$ *resulting from the composition of* N_1 *and* N_2 *via* P_C *(denoted by* N_1 *o* N_2*) defined as follows:* (1) $P=P_1\cup P_2$; (2) $P^0=P_1^0\cup P_2^0$; (3) $P_R=P_{R1}\cup P_{R2}$; (4) $T=T_1\cup T_2$ *and* (5) $F=F_1\cup F_2$ *is also an* S^3PR.

An example of S^3PR is shown in Figure 1.

2.2 The controlled model of S^3PR

For each SMS, we add a control place V_S and the control arcs exactly the same as in Ezpeleta *et al*. (1995). The following definitions are from Ezpeleta *et al*. (1995) and the reader may refer to this source for more details of the S^3PR controlled model.

Definition 8. (Ezpeleta et al., 1995). *Let* $N=O_{i=1}{}^k N_i=(P\cup P^0\cup P_R, T, F)$ *be an* S^3PR *and* Π *be the set of SMS in N. Given* $S\in\Pi$, $[S]=(\cup_{r\in S\cap P_R}H(r))\backslash S$ *denotes the set of holders, corresponding to resources in S, which do not belong to S.* $[S^i]=[S]\cap P_i$, $i\in I_k=\{1, 2, \ldots, k\}$. $[S]$ *is called S's complementary set.*

Definition 9. (Ezpeleta et al., 1995). *Let* $N=(P, T, F)$ *be an* S^2P: (1) *let C be a circuit of N,* $\|C\|$ *the set of nodes in it,* $|\,\|C\|\,|$ *the length of C and x, y two nodes of C, We say that x is previous to y in C iff there exists a path in from x to y, the length of which is greater than* 1 *and which does not pass* p^0. *This fact is denoted by* $x\rightarrow_C y$; (2) *Let x and y be two nodes of N. We say that x is previous to y in N iff there exists a circuit C such that* $x\rightarrow_C y$. *This fact is denoted as* $x\rightarrow_N y$; (3) *Let x and* $A\subseteq(P\cup T)$ *be a node and a set of nodes of N, respectively.*

Then $x \to_N A$ iff there exists a node $y \in A$ such that $x \to_N y$. $A \to_N x$ iff there exists a node $y \in A$ such that $y \to_N x$.

Definition 10. (Ezpeleta et al., 1995). *Let $N = O_{i=1}^{k} N_i = (P \cup P^0 \cup P_R, T, F)$ be an S^3PR, $\bar{N}_i$ the S^2P of N_i, and Π be the set of SMS in N. $\Pi^+ : T \to \wp(\Pi)$ ($\wp(\Pi)$ is the power set of Π) is a mapping where $\Pi^+(t) = \{S \in \Pi | t \to_{\bar{N}_i} [S^i]\}$. $\Pi^- : T \to \wp(\Pi)$ is a mapping where $\Pi^-(t) = \{S \in \Pi | [S^i] \to_{\bar{N}_i} t\}$. $\forall i \in \{1, 2, \ldots, k\}$, $\forall S \in \Pi$, $P_S = \cup_{i=1}^{k} P_s^i$, $P_s^i = [S^i] \cup \{p \in P_i \,|\, p \to_{\bar{N}_i} [S^i]\}$.*

Definition 11. (Ezpeleta et al., 1995). *Let (N, M_0) be a marked $S^3PR = (P \cup P^0 \cup P_R, T, F)$. The net $(N_A, M_{0A}) = (P \cup P^0 \cup P_R \cup P_A, T, F \cup F_A, M_{0A})$ is the SMSless controlled system of (N, M_0) iff* (1) *$P_S = \{V_S | S \in \Pi\}$ is the set of additional control places such that there exists a bijective mapping from Π into it;* (2) *$F_A = F_A^1 \cup F_A^2 \cup F_A^3$ where $F_A^1 = \{(V_S, t) | t \in P^{0\bullet}$, $S \in \Pi^+(t)\}$, $F_A^1 = \{(V_S, t) | t \in P^{0\bullet}$, $S \notin \Pi^+(t)\}$, $F_A^3 = \cup_{i=1}^{k} \{(t, V_S) | t \in T_i \backslash P^{0\bullet}$, $S \notin \Pi^-(t)$, ${}^\bullet t \cap P_i \subseteq P_s^i$, $t \not\to_{\bar{N}_i} [S^i]\}$, and* (3) *$M_{0A}$ is defined as follows:*

$$\text{(a) } \forall p \in P \cup P^0 \cup P_R, M_{0A}(p) = M_0(p); \forall V_S, M_{0A}(V_S) = M_0(S) - 1$$

2.3 Synthesis of SMS

Definition 12. (Chao, 2007). *Let $N = (P, T, F)$ be a net. $H_1 = [n_s n_1 n_2 \ldots n_k n_e]$ and $H_2 = [n_s n_1' n_2' \ldots n_h' n_e]$ are elementary directed paths, n_i, $n_j' \in P \cup T$, $i = 1, 2, \ldots, k$, $j = 1, 2, \ldots, h$. H_1 and H_2 are said to be mutually complementary (ie, $H_1^c = H_2$ and $H_2^c = H_1$) since H_1 and H_2 have the same terminal nodes:start node n_s and end node n_e. Each of H_1 and H_2 is called a handle in N if $n_i \neq n_j'$, $\forall i, j$ defined above; n_s (n_e) is called a terminal node or the start (the end node) of H_1 and H_2. n_i and n_j' ($1 \le i \le k$, $1 \le j \le h$) are called the interior nodes of H_1 and H_2 respectively. Note that n_s and n_e may be identical. H_1 is a resource handle if all places in H_1 are resource places.*

Definition 13. (Chao, 2007). *The handle H to a subnet N' (similar to the handle of a tea pot) is an elementary directed path from n_s in N' to another node n_e in N'; any other node in H is not in N'. H is said to be a handle in $N'' = N' \cup H$. A handle H $[n_s n_1 n_2 \ldots n_k n_e]$ is a XY-handle where X and Y can be T or P. X is T (P) if $n_s \in T$ ($n_s \in P$). Y is T (P) if $n_e \in T$ ($n_e \in P$). H is a resource handle if all places in H are resource places. A handle is called resourceless iff all its interior nodes are not resource places. H is virtual if $H = [n_s n_e]$, ie, it contains only two nodes. If $n_s \in P$, $n_e \in P$, $n_s = n_e$ ($n_s \neq n_e$), H is called a PP-circuit (PP-handle). H^{TP} (H^{PT}, H^{PP}, H^{TT}) denotes a TP-handle (PT-handle, PP-handle, TT-handle). H^{TP*} (H^{PT*}, H^{PP*}, H^{TT*}) denotes resourceless H^{TP} (H^{PT}, H^{PP}, H^{TT}). A (non)PP′-handle is a PP-handle (not) of the form [r t r′], r, $r' \in P_R$. N_i^e denotes an expanded subnet N_i by adding all PP′-handles to N_i.*

Definition 14. (Chao, 2007). *A subnet $N_i = (P_i, T_i, F_i)$ of N is generated by $X = P_i \cup T_i$, if $F_i = F \cap (X \times X)$. It is an I–subnet, denoted by I, of N if $T_i = {}^\bullet P_i$. I_S is the I–subnet (the subnet derived from (S, ${}^\bullet$S)) of an SMS S. Note that $S = P(I_S)$; S is the set of places in I_S. A resource subnet υ of N is a subnet of N and all places in υ are resource places.*

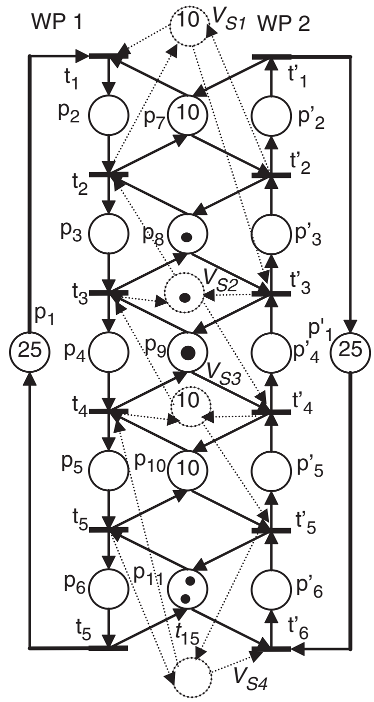

Figure 2 Disturbanceless control (not all control elements are shown)

Definition 15. A resource (control) circuit is an elementary circuit where all places are resource (non-resource) places. An impure circuit is an elementary but not a control circuit where some places are operation places. A mixture circuit is an elementary circuit that contains both resource and control places. An α-mixture circuit is due to a resource TP handle to a control circuit. A β-mixture circuit is of the form $[V_{S1}\Gamma V_{S1}]$, where Γ is a resource path. An γ-mixture circuit is due to a resource TT-handle to a core circuit.

$[V_{S1}\ t'_3\ V_{S2}\ t_2\ V_{S1}]$ is a control circuit in the disturbanceless model in Figure 2. $[V_{S2}\ t_1\ p_2\ t_2\ V_{S1}\ t'_3\ V_{S2}]$ in Figure 2 is an impure circuit. $[V_{S2}\ t_2\ V_{S1}\ t'_3 p_9\ t_3\ V_{S2}]$ is a α-mixture circuit due to the resource TP handle $[t'_3 p_9\ t_3\ V_{S2}]$ to control circuit $[V_{S1}\ t'_3\ V_{S2}\ t_2\ V_{S1}]$. $[V_{S1}\ t'_3\ p_9\ t_3\ p_8\ t_2\ V_{S1}]$ in Figure 2 is a β-mixture circuit. There are no *γ-mixture circuits* in Figure 2 and we will illustrate it in Section 5.

Lemma 1. (*Chao, 2007*)**.** (1) *I_S is strongly connected;* (2) *A subnet N' is an I-subnet* (*see Definition 14*) *of a minimal siphon iff N' is maximal in the sense that each handle H in N' is a PP- or TP- or virtual PT-handle and there are none of PP-, TP-, and virtual PT-handles to N';* (3) *$P(N')$ is an SMS iff there is a non-virtual PT-handle to N'', which is a subnet of N' without any TP-handles.*

In Chao (2007), we construct an SMS by building handles upon a resource subnet υ as follows.

Definition 16. Handle construction procedure (*Chao, 2007*). *Given a strongly connected resource subnet* (*SCRS*) υ: (1) *add all PP*-handles of the form* $[r_1\ t'\ r_2]$, $r_1 \in \upsilon$ *and* $r_2 \in \upsilon$, *to* υ *to form an expanded resource subnet* υ'; (2) *add all PP*-or TP*-or virtual PT*-handles to* υ' *to form* υ''; (3) $P(\upsilon'')$ *is an SMS if it does not contain a* $\rho(r)$, $r \in P(\upsilon)$.

Example. For the net in Figure 1, first find resource circuit $c = [p_7\ t'_2\ p_8\ t_2\ p_7]$ (a circuit is strongly connected; hence it is an SCRS). Second, add TP-handles $[t_2\ p_3\ t_3\ p_8]$ and $[t'_2 p'_2\ t'_1\ p_7]$ to get *IS* and $S = P(IS) = \{p_3, p_7, p_8, p'_2\}$.

An example of disturbanceless control is shown in Figure 2 where the control subnet contains operation places.

3. Theory

This section develops the theory. First we explain the idea of invariant-controlled siphons and the approach in Subsections A and B respectively. We then show different control policies, defined based on two types of control arcs, with simple examples

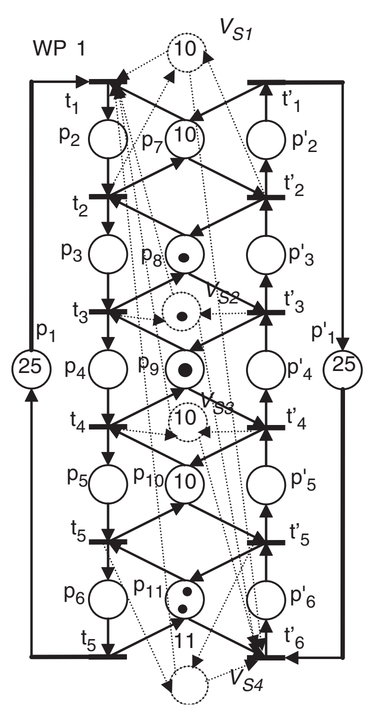

Figure 3 SMSless control (not all control elements are shown)

in Figures 2–4. Finally, we define the one-sided policy and prove that the associated control places do not generate new SMSs and that it reaches more states than the traditional control model proposed by Ezpeleta *et al*. To avoid the clustering of graphical objects and simplify the presentation, we show control places and arcs only for SMS generated from elementary resource circuits in Figures 2–4.

3.1 Controlled siphons

As mentioned earlier, SMS in an S^3PR can be synthesized from resource subnets such as resource circuits, which arise often due to the sharing of a set of resources R ($\{p_7, p_8, p_9, p_{10}, p_{11}\}$ in Figure 1) between adjacent WPs WP_i (WP1) and WP_j (WP2). In order to form resource circuits, R must be used by WP_i in a top to bottom manner and by WP_j in a reverse, ie, from bottom to top, manner.

In Figure 2, we add a control place V_{S1} and the associated arcs for S_1; the corresponding support of the new P-invariant is $\{V_{S1}, p_2, p'_3\} = \{V_{S1}\} \cup [S_1]$ (S_1's *complementary set*); hence

$$M_0(V_{S1}) = M(V_{S1}) + M([S_1]) \qquad (1)$$

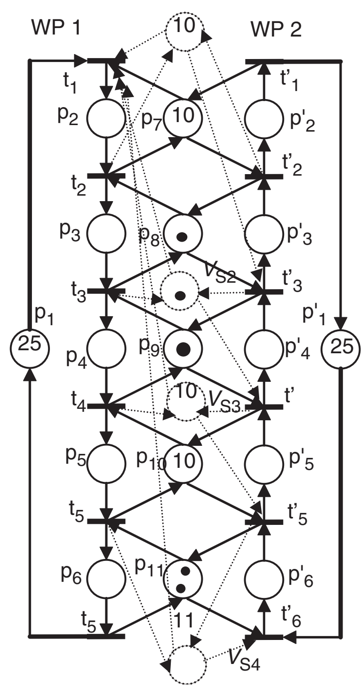

Figure 4 One-sided policy (not all control elements are shown)

Note that $S_1 \cup [S_1] = (\cup_{r \in S}(\{r\} \cup H(r))$ is also the support of a P-invariant; hence

$$M_0(S_1) = M(S_1) + M([S_1]) \tag{2}$$

Substituting $M_0(V_{S1}) = (M_0(S_1) - 1)$ into Equation (1), we have

$$M([S_1]) = M_0(V_{S1}) - M(V_{S1}) = (M_0(S_1) - 1) - M(V_{S1}) \tag{3}$$

Substituting Equation (3) into Equation (2), we have $M(S_1) = 1 + M(V_{S1}) \geq 1$. Thus, the least number of tokens S_1 can hold equals 1. We say that S_1 is controlled by the new P-invariant.

Note that we can consider V_{S1} as another kind of resource place shared by WP_1 (via operation place p_2) and WP_2 (via operation place p'_3). The only difference is that the set of holder places (ie, $H(V_{S1}) = \{p_2, p'_3\}$) of V_{S1} is not an $H(r)$. As a result, we can derive a circuit (SMS) from adjacent control places similar to a resource circuit (SMS) from adjacent resource places.

3.2 Approach

We developed theory (Chao, 2007) to efficiently extract SMS and specialized it for S^3PR (Systems of Simple Sequential Processes) proposed by Ezpeleta *et al*. in an incremental fashion using an algorithm rather than the traditional global approach.

Each time we add a resource place we find its neighbouring resource places such that there is a circuit (called basic siphon circuit c_b in Chao, 2007) containing the resource place and a neighbouring resource place. Then we add some handles onto the circuit (like handles to a teapot). The resulting place set is an SMS or a bad minimal siphon (Chao, 2007).

Only linear numbers of SMS need to be searched. The rest can be found by adding and deleting common sets of places from existing ones with search time significantly reduced (Chao, 2007).

In the PN model of an S^3PR, a token in the idle place p_0 moves via a number of operation places before returning to p_0. An operation place with a token models the state of a resource (robotics or machine) being used. Firing the input (output) transition of an operation place indicates the acquiring (releasing) of a resource. Every transition in an S^3PR except those in $p_0^\bullet$ (output transitions of idle place) indicates the action of releasing a resource. Thus, only transitions in $p_0^\bullet$ do not have resource output places. Thus, every circuit c containing transitions in $p_0^\bullet$ must contain operation places in $p_0^{\bullet\bullet}$. The siphon synthesized from c is not minimal since it contains a minimal siphon, which is the support (including places in $p_0^{\bullet\bullet}$ and a V_S) of a P-invariant.

Consider a control circuit $V_{S1} \to t_1 \to V_{S2} \to t_2 \ldots V_{Sn} \to t_n \to V_{S1}$, where '$\to$' indicates an output arc from a node. The control policy by Ezpeleta *et al*. is to break the output arc from each V_{Si}, $i = 1, 2, \ldots, n$, in the above circuit. It is not necessary to break all such arcs, rather only one such arc is sufficient to disrupt the circuit to avoid the generation of new control circuits.

Consider a set of resources shared by two simple processes i and j with idle places p_i^0 and p_j^0, respectively, and an associated control place V_S with output control arcs (V_S, t_i) and (V_S, t_j), where $t_i(t_j)$ is a transition on process $i(j)$. We either move t_i to the output transition of p_i^0 on process i (ie, $t_j^0 \in p_i^{0\bullet}$) or move t_i to the output transition of p_j^0 on process j (ie, $t_j^0 \in p_j^{0\bullet}$), but not both as in the all-sided approach.

However, there may be mixture circuits that contain both control and resource places. We will show the condition to have mixture circuits and provide a method to avoid such.

3.3 Two types of output control arcs

The above approach, called a *disturbanceless* one, disturbs the original or uncontrolled model less than the traditional one in Ezpeleta *et al*. (1995), where the support of the new P-invariant associated with V_S covers [S] $\cup$ V_S as a proper subset. Therefore it may reach more states since $M(p_3') + M(p_4') + M(p_5') + M(p_6') \leq 10$ (Figure 3) for the SMSless approach versus $M(p_3') + M(p_4') + M(p_5') + M(p_6') \leq 14$ (Figure 2) for the disturbanceless approach. However, it may create new SMS (eg, $\{V_{S1}, V_{S2}, p_3, p_3'\}$ in Figure 2) while the traditional one (called SMSless approach) does not. Type-1 and Type-2 control arcs refer to those for the SMSless and the disturbanceless approach respectively.

The following defines WP_{ia} (used in Definition 18), $R_{ia,jb}$ and $D_{ia,jb}$.

*Definition 17. Let WP_{ia} be circuit C_a of the S^2P of WP_i, $R_{ia,jb}$ ($D_{ia,jb}$) the set of resource (*control*) places that are shared between WP_{ia} and WP_{jb}. WPi and WPj are adjacent if $\exists C_a$ of WP_i and circuit C_b of WP_j such that $R_{ia,jb} \neq \emptyset$. If C_a is the only one circuit in the $S^2P$of WP_i, then WP_{ia} is briefed as WP_i. $R_{ia,jb}(S)$ denotes the set of resources in S that are shared between WPai and WPj_b.*

The following definition formalizes Type-1 and 2 control arcs.

Definition 18. Let $T(WP_{ij})$ be the set of transitions in WP_{ij}. Type-1 control arcs from (input to) V_S to (from) a transition in WP_{ij}: Type-1(S, WP_{ij}, V_S): $F_A = F_A^1 \cup F_A^2 \cup F_A^3$ where $F_A^1 = \{(V_S,\ t) | t \in P^{0\bullet} \cap T(WP_{ij}),\ S \in \Pi^+(t)\}$, $F_A^2 = \{(t,\ V_S) | t \in [S]^\bullet \cap T(WP_{ij}),\ S \notin \Pi^+(t)\}$, $F_A^3 = \cup_{i=1}^{k} \{(t,\ V_S) | t \in T_i \backslash P^{0\bullet},\ S \notin \Pi^-(t),\ {}^\bullet t \cap P_i \subseteq p_S^i,\ t \not\to_{\bar{N}_i} [S^i]\}$. Type-2 control arcs from (input to) V_S to (from) a transition in WP_{ij}: Type-2(S, WP_{ij}, V_S): $F_B = F_B^1 \cup F_B^2$ where $F_B^1 = \{(V_S, t) | t \in {}^\bullet[S] \cap T(WP_{ij}),\ S \notin \Pi^-(t)\}$, $F_B^2 = \{(t,\ V_S) | t \in [S]^\bullet \cap T(WP_{ij}),\ S \notin \Pi^+(t)\}$.

Note that the above Type-1 control arcs are exactly the same as those in the traditional one in Ezpeleta *et al*. (1995) or Definition 11, and Type-2 (1) control arcs do not (do) have F_B^3 (F_A^3). The presence of F_A^3 is to block tokens leakage by firing transitions at the end of F_A^3 without returning to [S]. In Figure 3, if p_2 has another output transition t_x

in addition to t_2, By firing t_x, tokens at p_2 may leak out from $\|Y_{VS2}\|$, the support $\{V_{S2}, p_2, p_3, p'_4, p'_5, p'_6\}$ of a minimal *P-invariant* Y_{VS2} and the sum of tokens in $\|Y_{VS2}\|$ may no longer be conserved. Such leakage would not occur for Type-2 control arcs in Figure 2, where $\|Y_{VS2}\| = \{V_{S2}, p_3, p'_4\}$. Note that the absence of F_B^3 reduces the number of control arcs.

We propose to adopt an intermediate approach to employ both Type-1 and Type-2 control arcs to reach more states than the traditional SMSless one while avoiding new SMS generation. We will prove this in Section 5.

3.4 Formal definitions of control systems

Based on the above definitions of Type-1 and 2 control arcs, we redefine (define) SMSless (one-sided, disturbanceless) controlled system as follows.

Definition 19. Let (N, M_0) *be a marked* $S^3PR = (P \cup P^0 \cup P_R, T, F)$ *and* $V_{ia,jb} = \{V_S | [S] \cap P(C_a) \neq \emptyset, [S] \cap P(C_b) \neq \emptyset\}$, *where* $P(C_a)$ $(P(C_b))$ *is the set of places in* $C_a(C_b)$. *The net* $(N_A, M_{0A}) = (P \cup P^0 \cup P_R \cup P_A, T, F \cup F_X, M_{0A})$ *is a controlled system of* (N, M_0) *iff* (1) $P_A = \{V_S \mid S \in \Pi\}$ *is the set of additional control places such that there exists a bijective mapping from* Π *into it;* (2) M_{0A} *is defined as follows:* $\forall p \in P \cup P^0 \cup P_R$, $M_{0A}(p) = M_0(p)$; $\forall V_S \in P_A$, $M_{0A}(V_S) = M_0(S) - 1$; (3) *Control arcs* F_X *is defined as follows:* $\forall V_{ia,jb} \neq \emptyset$, $\forall V_S \in V_{ia,jb}$, *the control arcs between* V_S *and transitions on* C_a (C_b) *belong to Type a* (b), *where* $a, b \in \{1, 2\}$. *If* $a \neq b$ $(a = b = 1, a = b = 2)$, *then the net* (N_A, M_{0A}) *is a one-sided* (*SMSless, disturbanceless*) *controlled system.*

Based on the above definition, we will prove in Section 5 that any one-sided system achieves more reachable systems (has the same number of SMS) than (as) that of the corresponding traditional SMSless system.

4. FMS example

This section compares our one-sided approach with the traditional one based on the well-known S^3PR example in Li and Zhou (2004). To further convince the reader, we have included the data using the elementary-siphon approach. For the sake of completeness, definitions (examples) of elementary, dependent siphons and characteristic T-vectors are provided in the Appendix I. The reader may refer to Li and Zhou (2004, 2006a) for more details.

In the sequel, we will refer to the method by Ezpeleta *et al*. as the *all-siphon control model*, since a control place is added for every SMS.

The flexible manufacturing cell as shown in Figure 5(a) has four machines M1, M2, M3 and M4, each holding two units at the same time. Also the cell contains three robots R1, R2 and R3, one unit at the same time. Parts enter the cell through three loading buffers I1, I2 and I3, and leave the cell through three unloading buffers O1, O2 and O3. The robots deal with the movements of parts. R1 handles part movements from I3 to M1, I3 to M3 and M3 to O3 respectively. R2 handles part movements from M1 to M2, M4 to M3, M3 to M4, I1 to M2 and M2 to O1. R3 handles parts movements from I2 to M4, M2 to O3 and M4 to O3. Three part types P1, P2 and P3 are produced. Their respective production routes are also shown in Figure 5(a) and the PN model of the system is shown in Figure 5(b). Table 1 lists the $R_{ia,jb}$ and $V_{ia,jb}$ associated for the net in Figure 5(b).

The net system is an S^3PR and contains deadlocks. There are six resource circuits corresponding to six elementary or basic siphons and 12 dependent siphons. For example, S_3 is a dependent SMS w.r.t. to S_4 and S_{18}. To apply the elementary-siphon approach to this net system, we first add six control places V_{S1}, V_{S4}, V_{S10}, V_{S16}, V_{S17} and V_{S18} which correspond to six elementary siphons S_1, S_4, S_{10}, S_{16}, S_{17} and S_{18}, respectively.

For the sake of completeness, we have reproduced the controlled model for the elementary-siphon (all-siphon) approach in Figure 5(c) (Table 2). The proposed one-sided controlled model for the elementary-siphon (all-siphon) approach is shown in Figure 5(d) (Table 3).

Note that we use the method proposed in Li and Zhou (2004) to add a control place for each elementary siphon and no new SMS will be generated because of these new additional places.

Note that Type-1 control arcs apply only to WP3 side, while Type-2 control arcs apply to the rest WP (WP1 and WP2). Compared with that in Ezpeleta *et al*. (1995) and Li and Zhou (2008a), some control arcs are eliminated (eg., (V_{S3} t_1)) while some new control arcs (eg., (t_3 V_{S3}) and (t_7 V_{S3})) are added due to the presence of operation places with more than one output transition. There are 18 control places and 103 control arcs, compared with 18 control places and 106 control arcs of that in Li and Zhou (2004).

Using the integrated net analyzer (INA), we confirm that the one-sided approach produces no new SMS for the controlled models in Figure 5(d) and Table 3. Table 4 lists the results for four controlled models: (1) all-sided-elementary (ie, SMSless and elementary siphon approach); (2) one-sided-elementary (ie, one-sided and elementary siphon approach); (3) all-sided (SMSless and all-siphon approach); (4) one-sided (one-sided and all-siphon approach). The proposed approach takes the same number of control places as those of traditional ones, but with three fewer control arcs for both the elementary-siphon and the all-siphon approaches. The total number of reachable states roughly doubles using the proposed one-sided approach, compared with the traditional approaches. This confirms that our approach is more permissive.

It is interesting that the proposed one-sided-elementary approach reaches more states than that of one-sided ($15999 > 9722$) but with fewer monitors. This is because: (1) the additional monitors are redundant (ie, the net remains live after removing them), and (2) the net gets disturbed more due to some redundant

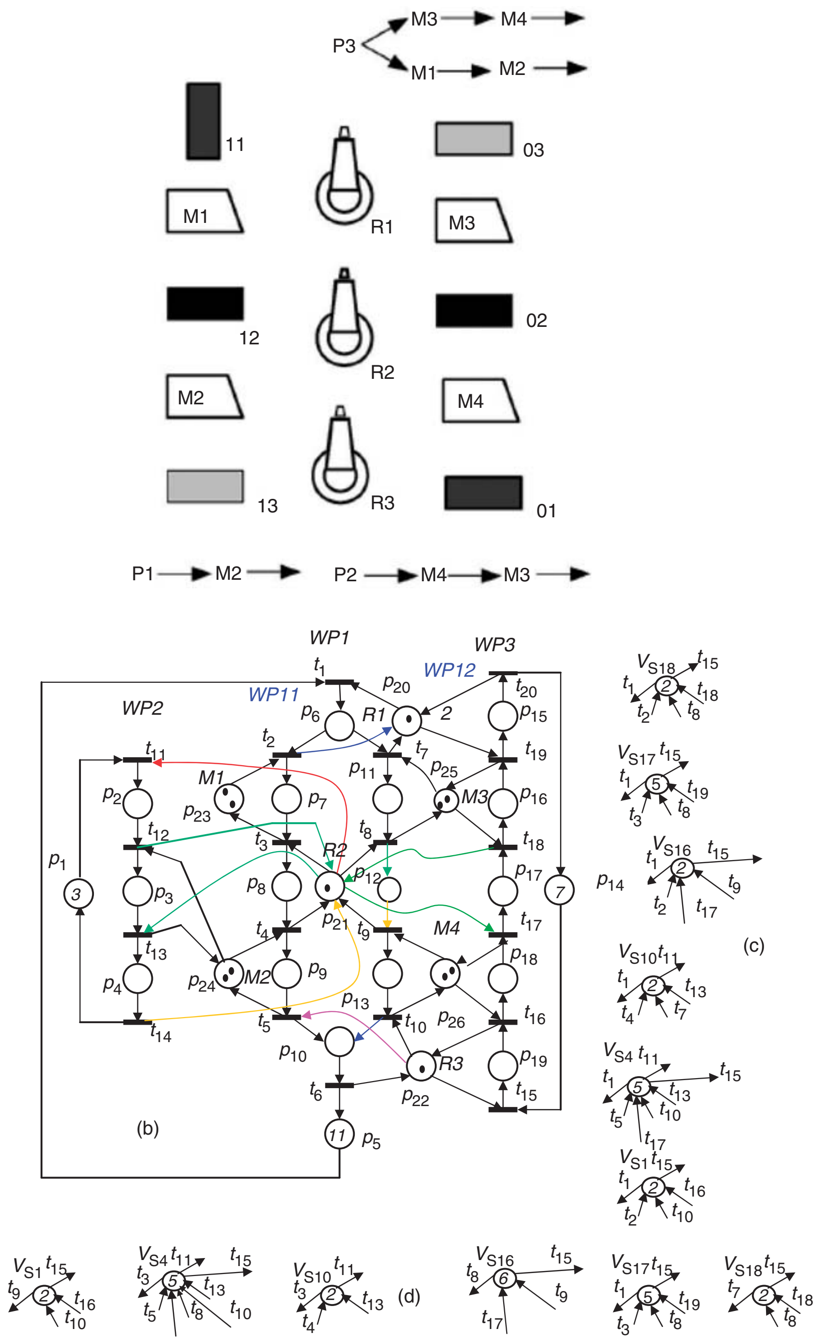

Figure 5 (a) Flexible manufacturing cell [7,10]; (b) an S3PR in [7]; (c) all-sided elementary control model; (d) one-sided elementary control model

Table 1 $R_{ia,jb}$ and $V_{ia,jb}$ for the net in Figure 5(b)

$R_{ia,jb}$	*WP*2		*WP*3		$V_{ia,jb}$	*WP*2	*WP*3
*WP*11	p_{21}		p_{20}		*WP*11	V_{S10}, V_{S5},	V_{S17}, V_{S2},
		S_{10}		S_{17}		V_{S6}, V_{S7},	V_{S5}, V_{S8},
	p_{24}		p_{21}			V_{S8}, V_{S9},	V_{S11}, V_{S12},
		S_{10}		S_4			V_{S4}, V_{S3}
	p_{21}		p_{22}				
*WP*12	p_{21}		p_{20}		*WP*12	V_{S10}, V_{S5},	V_{S18}, V_{S3},
		S_{10}				V_{S6}, V_{S7},	V_{S6}, V_{S9},
	p_{24}		p_{25}			V_{S8}, V_{S9},	V_{S13},
		S_{10}		S_{18}			V_{S14},
	p_{21}		p_{21}				V_{S16}, V_{S5},
				S_{16}			V_{S7}, V_{S11},
			p_{26}				V_{S12},
				S_1			V_{S15},
			p_{22}				V_{S1}

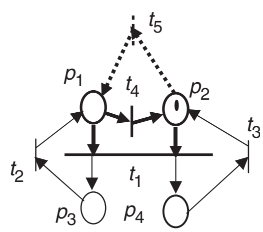

Figure 6 An example of VFOS; the only siphon never gets empty of tokens

monitors' control arcs ending at source transitions of WPs rather than at sink transitions of the SMS.

Fewer control arcs leads to fewer types of mixture circuits and generates fewer core circuits. For instance, in the disturbanceless control model of the S^3PR in Figure 5(b), there are three new SMS:

(1) $\{p_2, p_3, p_8, p_{11}, p_{15}, p_{20}, p_{23}, p_{25}, V_{S9}\}$ associated with V_{S9}
(2) $\{p_2, p_3, p_8, p_{11}, p_{12}, p_{15}, p_{20}, p_{23}, p_{25}, V_{S6}\}$ associated with V_{S6}
(3) $\{p_2, p_3, p_8, p_9, p_{11}, p_{12}, p_{13}, p_{15}, p_{20}, p_{23}, p_{25}, V_{S3}\}$ associated with V_{S3}

All these are due to TT-handles (of the *I-subnets* of the supports of minimal *P-invariants* associated with three dependent siphons V_{S9}, V_{S6} and V_{S3}) to resource circuit $[p_{21}\ t_3\ p_{23}\ t_2\ p_{20}\ t_{19}\ p_{25}\ t_{18}\ p_{21}]$. However, for the elementary-siphon approach,

Table 2 The complementary sets [*S*], the control *PN* elements added by, and $M_0(V_S)$ associated with the all-sided all-siphon control policy for the net in Figure 5(b)

	[*S*]	V_S	$V_S^\bullet$	$^\bullet V_S$	M_0 (*S*)	$M_0(V_S)$
1	$\{p_{13}, p_{19}\}$	P_{27}	$\{t_1, t_{15}\}$	$\{t_{16}, t_{10}, t_2\}$	3	2
2	$\{p_2, p_3, p_6, p_7, p_8, p_9, p_{11}, p_{12}, p_{13}, p_{16}, p_{17}, p_{18}, p_{19}\}$		$\{t_1, t_{11}, t_{15}\}$	$\{t_5, t_{10}, t_{13}, t_{19}\}$	11	10
3	$\{p_2, p_3, p_8, p_9, p_{11}, p_{12}, p_{13}, p_{17}, p_{18}, p_{19}\}$		$\{t_1, t_{11}, t_{15}\}$	$\{t_5, t_{10}, t_{13}, t_{18}\}$	8	7
4	$\{p_2, p_3, p_8, p_9, p_{12}, p_{13}, p_{18}, p_{19}\}$		$\{t_1, t_{11}, t_{15}\}$	$\{t_5, t_{10}, t_{13}, t_{17}\}$	6	5
5	$\{p_2, p_3, p_6, p_7, p_8, p_{11}, p_{12}, p_{16}, p_{17}, p_{18}\}$		$\{t_1, t_{11}, t_{15}\}$	$\{t_4, t_9, t_{13}, t_{19}\}$	10	9
6	$\{p_2, p_3, p_8, p_{11}, p_{12}, p_{17}, p_{18}\}$		$\{t_1, t_{11}, t_{15}\}$	$\{t_4, t_9, t_{13}, t_{18}\}$	7	6
7	$\{p_2, p_3, p_8, p_{12}, p_{18}\}$		$\{t_1, t_{11}, t_{15}\}$	$\{t_4, t_9, t_{13}, t_{17}\}$	5	4
8	$\{p_2, p_3, p_8, p_6, p_7, p_{11}, p_{16}, p_{17}\}$		$\{t_1, t_{11}, t_{15}\}$	$\{t_4, t_8, t_{13}, t_{19}\}$	5	4
9	$\{p_2, p_3, p_8, p_{11}, p_{17}\}$		$\{t_1, t_{11}, t_{15}\}$	$\{t_4, t_8, t_{13}, t_{18}\}$	9	8
10	$\{p_2, p_3, p_8\}$		$\{t_1, t_{11}\}$	$\{t_4, t_7, t_{13}\}$	3	2
11	$\{p_6, p_7, p_{11}, p_{12}, p_{13}, p_{16}, p_{17}, p_{18}, p_{19}\}$		$\{t_1, t_{15}\}$	$\{t_3, t_{10}, t_{19}\}$	10	9
12	$\{p_6, p_7, p_{11}, p_{12}, p_{16}, p_{17}, p_{18}\}$		$\{t_1, t_{15}\}$	$\{t_3, t_9, t_{19}\}$	8	7
13	$\{p_{11}, p_{17}, p_{12}, p_{18}, p_{13}, p_{19}\}$		$\{t_1, t_{15}\}$	$\{t_2, t_{10}, t_{18}\}$	10	9
14	$\{p_{11}, p_{12}, p_{17}, p_{18}\}$		$\{t_1, t_{15}\}$	$\{t_2, t_9, t_{18}\}$	5	4
15	$\{p_{12}, p_{13}, p_{18}, p_{19}\}$		$\{t_1, t_{15}\}$	$\{t_2, t_{10}, t_{17}\}$	4	3
16	$\{p_{12}, p_{18}\}$		$\{t_1, t_{15}\}$	$\{t_2, t_9, t_{17}\}$	3	2
17	$\{p_6, p_7, p_{11}, p_{16}, p_{17}\}$		$\{t_1, t_{15}\}$	$\{t_3, t_8, t_{19}\}$	6	5
18	$\{p_{11}, p_{17}\}$		$\{t_1, t_{15}\}$	$\{t_2, t_8, t_{18}\}$	3	2

these mixture circuits can never occur since we do not need to add monitors to dependent siphons V_{S9}, V_{S6} and V_{S3}.

5. Proof of absence of new SMS

Because the controlled model is no longer an S^3PR, the I_S of some SMS may be constructed from a so-called core subnet containing only control places, or control and resource places, or even operation places. We can add resourceless handles upon an SCRS (core subnet) of an S^3PR (controlled model) to find I-subnets of an SMS. For the controlled model, the core subnet may no longer be an SCRS and it may contain operation places.

[V_{S1} t_3' V_{S2} t_2 V_{S1}] is a control circuit in the disturbanceless model in Figure 2. In the one-sided model in Figure 4, the terminal end of arc (V_{S2} t_2) is moved to $t_1 \in p_1^\bullet$ so that there is no longer a control circuit containing both V_{S1} and V_{S2}. In the

Table 3 The complementary sets [*S*], the control *PN* elements added by, and $M_0(V_S)$ associated with the one-sided all-siphon control policy for the net in Figure 5(b)

	[S]	$V_S^\bullet$	$^\bullet V_S$	M_0 (S)	$M_0(V_S)$
1	$\{p_{13}, p_{19}\}$	$\{t_9, t_{15}\}$	$\{t_{16}, t_{10}\}$	3	2
2	$\{p_2, p_3, p_6, p_7, p_8, p_9, p_{11}, p_{12}, p_{13}, p_{16}, p_{17}, p_{18}, p_{19}\}$	$\{t_1, t_{11}, t_{15}\}$	$\{t_5, t_{10}, t_{13}, t_{19}\}$	11	10
3	$\{p_2, p_3, p_8, p_9, p_{11}, p_{12}, p_{13}, p_{17}, p_{18}, p_{19}\}$	$\{t_3, t_7, t_{11}, t_{15}\}$	$\{t_5, t_{10}, t_{13}, t_{18}\}$	8	7
4	$\{p_2, p_3, p_8, p_9, p_{12}, p_{13}, p_{18}, p_{19}\}$	$\{t_3, t_8, t_{11}, t_{15}\}$	$\{t_5, t_{10}, t_{13}, t_{17}\}$	6	5
5	$\{p_2, p_3, p_6, p_7, p_8, p_{11}, p_{12}, p_{16}, p_{17}, p_{18}\}$	$\{t_1, t_{11}, t_{16}\}$	$\{t_4, t_9, t_{13}, t_{19}\}$	10	9
6	$\{p_2, p_3, p_8, p_{11}, p_{12}, p_{17}, p_{18}\}$	$\{t_3, t_7, t_{11}, t_{15}\}$	$\{t_4, t_9, t_{13}, t_{18}\}$	7	6
7	$\{p_2, p_3, p_8, p_{12}, p_{18}\}$	$\{t_3, t_8, t_{11}, t_{15}\}$	$\{t_4, t_9, t_{13}, t_{17}\}$	5	4
8	$\{p_2, p_3, p_8, p_6, p_7, p_{11}, p_{16}, p_{17}\}$	$\{t_1, t_{11}, t_{15}\}$	$\{t_4, t_8, t_{13}, t_{19}\}$	5	4
9	$\{p_2, p_3, p_8, p_{11}, p_{17}\}$	$\{t_3, t_7, t_{11}, t_{15}\}$	$\{t_4, t_8, t_{13}, t_{18}\}$	9	8
10	$\{p_2, p_3, p_8\}$	$\{t_3, t_{11}\}$	$\{t_4, t_{13}\}$	3	2
11	$\{p_6, p_7, p_{11}, p_{12}, p_{13}, p_{16}, p_{17}, p_{18}, p_{19}\}$	$\{t_1, t_{15}\}$	$\{t_3, t_{10}, t_{19}\}$	10	9
12	$\{p_6, p_7, p_{11}, p_{12}, p_{16}, p_{17}, p_{18}\}$	$\{t_1, t_{15}\}$	$\{t_3, t_9, t_{19}\}$	8	7
13	$\{p_{11}, p_{17}, p_{12}, p_{18}, p_{13}, p_{19}\}$	$\{t_7, t_{15}\}$	$\{t_{10}, t_{18}\}$	10	9
14	$\{p_{11}, p_{12}, p_{17}, p_{18}\}$	$\{t_7, t_{15}\}$	$\{t_9, t_{18}\}$	5	4
15	$\{p_{12}, p_{13}, p_{18}, p_{19}\}$	$\{t_8, t_{15}\}$	$\{t_{10}, t_{17}\}$	4	3
16	$\{p_{12}, p_{18}\}$	$\{t_8, t_{15}\}$	$\{t_9, t_{17}\}$	3	2
17	$\{p_6, p_7, p_{11}, p_{16}, p_{17}\}$	$\{t_1, t_{15}\}$	$\{t_3, t_8, t_{19}\}$	6	5
18	$\{p_{11}, p_{17}\}$	$\{t_7, t_{15}\}$	$\{t_8, t_{18}\}$	3	2

Table 4 Comparison of the number of control places, control arcs and reachable states of different control policies for the net in Figure 5(b)

No	All-sided-elementary	One-sided-elementary	All-sided	One-sided
Control places	6	6	18	18
Control arcs	32	29	106	103
Reachable states	6287	15 999	5382	9722

two-sided model in Figure 3, the terminal end of arc (V_{S1} t'_3) is further moved to $t'_6 \in p'^{\bullet}_1$ without changing the fact that there is no longer a control circuit containing both V_{S1} and V_{S2}. If there are no elementary control circuits, then there are no compound control circuits.

$c=[V_{S3}\ t_3\ p_{23}\ t_2\ p_{20}\ t_{19}\ p_{25}\ t_{18}V_{S3}]$ is an γ-mixture circuit due to a TT-handle $H=[t_{18}V_{S3}\ t_3]$ to the resource one $[p_{21}\ t_3\ p_{23}\ t_2\ p_{20}\ t_{19}\ p_{25}\ t_{18}\ p_{21}]$. Let $c_m=(c\backslash H^c)\cup H$ be a mixture circuit, where $H^c=[t_{18}\ p_{21}\ t_3]$. If the SMS synthesized from c is controlled, so is that from c_m.

Lemma 2. Let $c_m=(c\backslash H^c)\cup H$ be an γ-mixture circuit If $[t_1\ V_S\ t_2]$ is a TT-handle H to a core circuit c, then: (1) $R(H^c)\subset R(S)$ where S is an SMS synthesized from a control or resource circuit c' containing H^c; (2) $M_0(V_S)\geq M_0(R(H^c))$ where $R(H^c)=\{r\mid r\in P_R\cap H^c\}$; (3) let S' (S'') be the SMS synthesized from c_m (c), then if S'' can never be empty, so will be S'.

Proof. (1) Otherwise, $t_2\in{}^\bullet S$, but $t_2\notin S^\bullet$ and S is not an SMS—contradiction; (2) $M_0(V_S)=M_0(S)-1=M_0(R(H^c))+M_0(S\backslash R(H^c))-1\geq M_0(R(H^c))$ since $R(H^c)\subset R(S)$; (3) It follows from the fact that $M_0(S')\geq M_0(S'')$ since $M_0(V_S)\geq M_0(R(H^c))$ and $c_m=(c\backslash H^c)\cup H$. ■

Lemma 3. Let c' be a β^1-mixture circuit with transitions on WP_{ia} and WP_{jb} respectively. Then the siphon D synthesized on c' contains an SMS S and is not minimal.

Proof. Suppose $\{r_{k+1},\ldots,r_n\}\subset R_{ia,jb}(S)$ and $\{r_k, r_{k+1},\ldots,r_n\}\not\subset R_{ia,jb}(S)$, ie, r_k is used by WP_{jb} but not by WP_{ia}, while each of $r_{k+1},\ldots,r_n$ is used by both WP_{jb} and WP_{ia}. Then $c^*=[r_k\ t_k r_{k+1} t_{k+1}\ldots r_n\ t'_{n-1}\ r_{n-1}\ t'_{n-2}\ r_{n-2}\ldots r_k]$ is a resource circuit. Adding TP*-paths, PP*-paths and virtual PT-paths upon c^* allows us to find an SMS S. These TP*-paths, PP*-paths and virtual PT-paths remain to be TP*-paths, PP*-paths, and virtual PT-paths to c'. Thus D contains S as a proper subset and is not minimal. ■

β-impure circuit (eg, $[V_{S15}\ t_8\ p_{12}\ t_9\ V_{S14}\ t_{16}\ p_{18}\ t_{17}V_{S15}]$ for the control model in Figure 5(d)) corresponds to a dependent siphon, which needs no control elements since the synthesized siphon can never be emptied. Note that S_{14} and S_{15} are dependent siphons, $V_{S14}{}^\bullet=\{t_7, t_{16}\}$, ${}^\bullet V_{S14}=\{t_9, t_{18}\}$, $V_{S15}{}^\bullet=\{t_8, t_{15}\}$, and ${}^\bullet V_{S14}=\{t_{10}, t_{17}\}$.

β^2-*mixture* circuit $c=[V_{S18}\ t_{17}\ p_{26}\ t_{16}\ p_{22}\ t_5\ p_{24}\ t_4\ p_{21} t_8\ V_{S18}]$ in Figure 5(d), however, contain transitions on WP_{12}, WP_{11}, and WP_3 and hence does not meet the condition (called β^1-*mixture* circuit) in the above lemma and it can generate an emptiable siphon.

For an example of Lemma 3, in Figure 2, $c'=[V_{S1}\ t'_3\ p_9\ t'_4\ p_{10}\ t_4\ p_9\ t_3\ p_8\ t_2\ V_{S1}]=V_{S1}\Gamma V_{S1}$ is a β^1-*mixture* circuit. $\{r_9, r_{10}\}\subset R_{12}(S_1)$ and $\{r_8, r_9, r_{10}\}\not\subset R_{12}(S)$. $c^*=[p_8\ t'_3\ p_9\ t'_4\ p_{10}\ t_4\ p_9\ t_3\ p_8]$ is a resource compound circuit from which S can be synthesized. $D=\{V_{S1}, p'_3, p_5, p_8, p_9, p_{10}\}$ synthesized from mixture circuit c' contains $S=\{p'_3, p_5, p_8, p_9, p_{10}\}$ and is not minimal.

Note that all circuits mentioned so far occur between two WP_{ij}. $[V_{S2}\ t_1\ p_2\ t_2\ V_{S1}\ t'_3\ V_{S2}]$ in Figure 4 is an impure circuit; the generated siphon $\{V_{S2}, p_2, V_{S1}, p'_3\}$ contains the support $\|Y_{VS1}\|$ of a minimal P-invariant Y_{VS1}. $[V_{S1}\ t'_3\ p_9\ t_3\ p_8\ t_2\ V_{S1}]$ in Figure 4 is a mixture circuit; the generated I_S contains resource circuit $[p_8\ t'_3\ p_9\ t_3\ p_8]$ and the generated siphon $\{V_{S1}, p_8, p_9, p'_3, p_4\}$ contains SMS $\{p_8, p_9, p'_3, p_4\}$ generated from resource circuit $[p_8\ t'_3\ p_9\ t_3\ p_8]$.

Theorem 1. Let $(N_B, M_{0B}) = (P \cup P^0 \cup R \cup P_A, T, F \cup F_B, M_{0A})$ *be a one-sided control system of an* S^3PR *and* Π (Π_b) *the set of SMS in the uncontrolled original (one-sided control) system. Then* $\Pi = \Pi_b$*, ie, there is no new SMS generation.*

Proof. Assume contrarily that $\Pi \neq \Pi_b$. Let the new SMS and its I-subnet be S and I_S respectively. By Property 1, I_S must contain a circuit c. c must contain at least a control place. Otherwise, $S \in \Pi$ is not new. Different types of c are as follows: (1) control circuits; (2) impure circuits; and (3) mixture circuits. Control circuits containing only control places do not exist by the one-sided policy as discussed earlier. Siphons synthesized on impure circuits are not minimal as discussed earlier. α-mixture circuits form by adding resource TP-handles upon control circuits, which do not exist. Hence, there are also no α-mixture circuits. By Lemma 3, SMS synthesized from β^1-mixture circuits are not minimal. β^2-mixture circuits can be broken by the one-sided policy; hence, they, similar to control circuits, do not exist. By Lemma 2, SMS synthesized from γ-mixture circuits are redundant. Thus, new emptiable SMS does exist and we have $\Pi = \Pi_b$. ■

Unlike the β^1-*mixture* circuit with two sides to choose, the β^2-*mixture* circuit $c = [V_{S18}\ t_{17}\ p_{26}\ t_{16}\ p_{22}\ t_5\ p_{24}\ t_4\ p_{21} t_8\ V_{S18}]$ in Figure 5(c) can only be broken (ie, applying Type-1 control arcs) on the WP_3 side. Any control circuit spanning between WP_3 and other WP' must (not) break the WP_3 (WP') side.

The following defines the degree of disturbance.

Definition 20. The degree of disturbance is defined as $\gamma = |H(V_S)| / |[S]|$, which is the ratio of the number of holders of control places to that of holders of resources in SMS S.

The larger the value of γ, the more disturbed the original uncontrolled model is, and hence the more reachable states are eliminated. We will show that the γ of the one-sided control model is less that of the SMSless control model, and hence can reach more states.

Theorem 2. Let γ_o (γ_s, γ_d) *be the* γ *of the one-sided (SMSless, disturbanceless). We have* $\gamma_s > \gamma_o > \gamma_d = 1$.

Proof. For disturbanceless control, we have $H(V_S) = [S]$ and $\gamma_d = 1$. For the one-sided control, we have $[S] \subset H(V_S)$ and $\gamma_o > 1$. By the definitions of one-sided and SMSless controls, we have $H(V_S)_o \subset H(V_S)_S$; hence we have $\gamma_s > \gamma_o > 1$, where $H(V_S)_o$ ($H(V_S)_S$) is the $H(V_S)$ for one-sided (SMSless). ■

5.1 Discussions

Note that there may be a number of implementations of one-sided control policy for an S^3PR with different degrees of disturbance. Here we develop two lemmas to help to select a policy with less disturbance to the original uncontrolled model.

Lemma 4. If $|R_{ia,jb}|=2$, ie, there are only two resource places in $R_{ia,jb}$, then applying Type-1 control arcs to both sides $WP_{ia,jb}$ does not create new SMS.

Proof. There is only resource circuit in the resource subnet υ that contains only places in $R_{ia,jb}$; hence there is only one SMS corresponding to only one control place that can be synthesized from υ. No control circuits can be generated from a single control place. This leads to the conclusion that there is no new SMS generation. ■

For instance, in Figure 5(b), we have $|R_{2,11}|=2$ since $R_{2,11}=\{p_{21}, p_{24}\}$ (corresponding to S_{10}) in Table 1, and for each V_S in $V_{2,11}=\{V_{S10}, V_{S5}, V_{S6}, V_{S7}, V_{S8}, V_{S9}\}$, Type-2 control arcs are applied to *WP*2 and *WP*11 as in Figure 5(d) and Table 3.

Lemma 5. Let $V_{X,A}\neq\emptyset$, $V_{X,B}\neq\emptyset, \ldots, V_{X,K}\neq\emptyset$, where each of $X, A, B, \ldots, K$ is a one-digit or two-digit index. Then if $R_{I,J}\subseteq R_{X,I}\cup R_{X,J}$, we have $V_{I,J}=\emptyset$, $\forall I, J\in\{A, B, \ldots, K\}$, $I\neq J$.

Proof. The places of $R_{X,Q}$, $Q\in\{A, B, \ldots, K\}$, in WP_X are arranged either: (1) from top to bottom, or (2) from bottom to top. For Case 1, the places of $R_{X,Q}$ are arranged in WP_Q from bottom to top manner. No resource circuits (hence SMS) with places in $R_{X,I}\cup R_{X,J}$ can form between WP_I and WP_J. Similar conclusion applies to Case 2. There is no need to add control places accordingly; hence if $R_{I,J}\subseteq R_{X,I}\cup R_{X,J}$, we have $V_{I,J}=\emptyset$, $\forall I, J\in\{A, B, \ldots, K\}$, $I\neq J$. ■

For instance, in Table 1, $V_{X,A}\neq\emptyset$ and $V_{X,B}\neq\emptyset$, where $X=3$, $A=12$, and $B=11$. Resource places in *WP*3 (*WP*11, *WP*12) are used from bottom to top (top to bottom). Thus, there are no resource circuits between *WP*11 and *WP*12. To reduce the degree of disturbance, we have applied Type-1 (Type-2) control arcs to *WP*3 (*WP*11, *WP*11) as in Figure 5(d) and Table 3. There is another reason for the above application: there is one type of mixture circuit which can generate emptiable siphons. An example mentioned earlier is shown in Figure 5(c), mixture circuit $c=[V_{S18}\, t_{17}\, V_{S16}\, t_{16}\, p_{22}\, t_5\, p_{24}\, t_4\, p_{21}\, t_8\, V_{S18}]$ contain transitions on WP_{12}, WP_{11}, and WP_3 can generate an emptiable siphon. To avoid such, the only way is to break c (ie, applying Type-1 control arcs) on the WP_3 side where all places in c are control ones.

6. Liveness and comparison

We now prove the liveness of the controlled model associated with the proposed control policy. The idea of the proof is based on the results in Chao (2006) where it shows that if the I_S of every P-invariant does not contain any VFOS (virtual first-order structure, to be explained below), then deadlock-freeness is equivalent to liveness and an ordinary PN is live if all minimal siphons never become empty of tokens.

An example of VFOS is shown in Figure 6. The set of all places form a siphon S that contains a token. t_1 is not live (but deadlock-free) even though both its input places may be marked.

There is only one token moving alternatively between the two input places of t_1. The net is weakly live. It is live if there are two tokens in the net. The path from p_1 or p_2

to t_1 does not contain any node and both p_1 and p_2 have output transitions other than t_1. Such a structure (bold parts) is called a VFOS (see Chao, 2006, Definition 8), which contains two directed paths [p_1 t_1] and [p_1 t_4 p_2 t_1] (each is called a handle in Chao, 2006, Definition 4) with identical start and end nodes p_1 and t_1. The net is deadlocked without the dashed PP-path [p_2 t_5 p_1] (called detrapping in Chao, 2006, Definition 6), which serves to de-trap the token at p_2. It is a special kind of first-order structure (FOS, see Chao, 2006, Definition 5), which has detrapping PP-handles. It has been shown in Chao (2006) that such a structure makes the net weakly live (ie, deadlock-free) when all siphons are never empty of tokens.

Note that the net itself is the I_S of the only problematic siphon S and it is not a state machine (*SM*). This is generally true for any I_S that contains a VFOS. It is easy to see that I_S of any P-invariant of an S^3PR is an SM and hence does not contain any VFOS, implying that the S^3PR is live as long as any minimal siphon never gets empty of tokens. Further, the I_S of any P-invariant associated with a control place in any of the control models, including the proposed one, in this paper is an SM. Thus, we conclude that they are live if no siphons ever become empty.

To prevent deadlocks in a PN, the recent work of Huang *et al*. (2006) has made some efforts on controlling fewer SMS, where a mixed-integer programming (MIP) technique is employed to find a maximal siphon under a given marking. Then all SMS, which may possibly be emptied, can be obtained from the maximal siphon (Huang *et al*., 2006). In fact, the SMS obtained by MIP method exclude those that can never be emptied. However, much simpler PN controllers with liveness can be expected if the concept of elementary siphons, as is used in Li and Zhou (2004). Further, the MIP test is an NP-complete problem and there are redundant monitors with weighted control arcs.

However, their method suffers from the expensive computation of siphons since the number of siphons is theoretically exponential w.r.t. the number of place elements in it (Ezpeleta *et al*., 1993; Lautenbach, 1987). Hence, their prevention algorithm takes exponential time. In addition, they have to make extra effort to extract elementary siphons from all SMS. Also the behaviour of the modelled system is restricted (ie, more reachable states are forbidden) when both the number of tokens in the top and bottom part of the PN model are small. This arises because some control arcs always connect to the output transition of the initial states, thereby restricting the entry of raw materials into the system.

To avoid the above expensive computation, Li and Zhou (2006b) further proposed a two-stage approach to synthesizing liveness-enforcing supervisors for systems of simple sequential processes with resources (S^3PR), one type of FMS. First, they find siphons (and add monitors) that need to be controlled using a mixed-integer programming (MIP) method to avoid time-consuming complete siphon enumeration. Second, they rearrange the output arcs of the monitors providing that liveness is still preserved. For the PN model in Figure 5(a), there are 15999 good states using six monitors, the same as our one-sided elementary approach. Yet, they have wasted time to rearrange the control arcs and perform MIP test, which is an NP-complete problem. Our approach, however, takes linear time to find six elementary siphons, and add monitors and control arcs.

7. Conclusion

We have proposed a so-called one-sided approach. We show that the proposed one-sided approach does not generate new emptiable SMS with fewer control arcs and it can reach more states than the traditional all-sided approach. This approach is by no means optimal and how to optimize should be a future research endeavour. The time complexity is similar to that in Li and Zhou (2004), since the only difference is the location of end nodes of some control arcs. We have improved in Chao (2008) by proposing a polynomial time algorithm to find elementary siphons. Future work is to study controllability (Li and Zhao, 2008) of siphons of the proposed one-sided control policy.

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Appendix I: Definitions of elementary siphons and characteristic *T*-vectors

Definition I.1. (Li and Zhou, 2004). *Let* $\Omega \subseteq P$ *be a subset of places of N. P–vector* λ_Ω *is called the characteristic P-vector of* Ω *iff* $\forall p \in \Omega$, $\lambda_\Omega(p) = 1$; *otherwise* $\lambda_\Omega(p) = 0$. η *is called the characteristic T-vector of* Ω, *if* $\eta^{T} = \lambda_\Omega{}^{T} \bullet [N]$, *where* $[N]$ *is the incidence matrix.*

Definition I.2. (Li and Zhou, 2004). *Let* $N = (P, T, F)$ *be a net with* $|P| = m$, *which has k siphons* $S_1, S_2, \ldots, S_k, m, k \in N^+$. *Define* $[\lambda]_{k\times m} = [\lambda_1 | \lambda_2 | \ldots | \lambda_k]^T$ *and* $[\eta]_{k\times n} = [\eta_1 | \eta_2 | \ldots | \eta_k]^T$. $[\lambda]$ $([\eta])$ *is called the characteristic P(T)-vector matrix* $[\lambda]$ $([\eta])$ *of the siphons in N. Let* $\eta_{S\alpha}$, $\eta_{S\beta}, \ldots,$ *and* $\eta_{S\gamma}$ $(\{\alpha, \beta, \ldots, \gamma\}\{1, 2, \ldots, k\})$ *be a linear independent maximal set of matrix* $[\eta]$. *Then* $\Pi_E = \{S_\alpha, S_\beta, \ldots, S_\gamma\}$ *is called a set of elementary siphons.* $S \notin \Pi_E$ *is called a strongly dependent siphon if* $\eta_S = \sum_{Si \in \Pi E} a_i \eta_{Si}$ *where* $a_i \geq 0$. $S \notin \Pi_E$ *is called a weakly dependent siphon if* $\exists$ *non-empty A,* $B \subset \Pi_E$, *such that* $A \cap B = \emptyset$ *and* $\eta_S = \sum_{Si \in A} a_i \eta_{Si} - \sum_{Si \in B} a_i \eta_{Si}$ *where* $a_i > 0$.