The Bankruptcy Cost of the Life Insurance Industry under Regulatory Forbearance: An Embedded Option Approach

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Abstract

In this study the Taiwan Insurance Guaranty Fund (TIGF) is introduced to investigate the ex ante assessment insurance guaranty scheme. We study the bankruptcy cost when a financially troubled life insurer is taken over by TIGF. The pricing formula of the fair premium of TIGF incorporating the regulatory forbearance is derived. The embedded Parisian option due to regulatory forbearance on fair premiums is investigated. The numerical results show that leverage ratio, asset volatility, grace period, and intervention criterion influence the default costs. Asset volatility has a significant effect on the default option, while leverage ratio is shown to aggravate the negative influence from the volatility of risky asset. Furthermore, the numerical analysis concludes that the premium for the insurance guaranty fund is risk sensitive and that a risk-based premium scheme could be implemented, hence, to ease the moral hazard.

1. Introduction

Life insurers are highly leveraged firms whose major debt capital is the premiums paid by the policyholders. Thus, life insurers are obligated to pay claims. In order to protect policyholders' rights, the regulatory authority could adopt certain regulatory schemes to deal with the financial problems of insolvent insurers. Either ex ante or ex post assessment scheme is utilized when covering the claim obligations of insolvent insurers. The choice between the two types of funding schemes varies from country to country based on the differences in failure rate and market structure. For example, France, Germany, Japan, and Taiwan adopt the ex ante scheme. No form of funding scheme is better than the other overall, strictly speaking, according to Oxera (2007), whereas Ligon and Thistle (2007) observe that the ex ante scheme gives shareholders less incentive for risk-taking behavior than the actual ex post assessment scheme observed most frequently in practice.

This study fully investigates the Taiwan Insurance Guaranty Fund (TIGF) that adopts the ex ante funding scheme to protect the obligations of policyholders. Because guaranty funds create a put-option-like subsidy to shareholders (Cummins 1988; Lee et al. 1997), a fair premium in a competitive market is an important prerequisite for a guaranty fund (see Eling and Schmeiser 2010). Cummins (1988) extends the work on deposit insurance premiums to price the single-period premiums for insurance guaranty funds. Duan and Yu (2005) develop the multiperiod model to measure the cost of the guaranty fund incorporating the risk-based capital regulations.

However, the recent financial crisis resulting in the massive fall of security prices causes financial institutions to face severe difficulties in asset-liability management. As the major investors in the fi-

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nancial markets, life insurers were, of course, negatively affected by the crisis as the value of many assets in their investment portfolios tumbled. Although most insurers are resistant against the crisis, not all insurance market participants followed a prudent strategy (Eling and Schmeiser 2010). The life insurance companies' financial distress influenced their policyholders' claims. To ease the financial distress of those institutions during the crisis and their capital restructuring schemes, a regulatory authority might consider adopting temporary capital relief plans, for example, by reducing the standard of solvency requirements or providing temporary capital injection.

In this study we perform the impact analysis on the fair premium when the government adopts the regulatory forbearance policy to reduce financial regulatory standards; for example, the regulators extend the grace period of capital injection plans or increase the risk tolerance to insurance companies facing financial difficulties. Although a run on an unhealthy insurance company is not necessarily a bad thing—it can discipline the performance of managers and owners—there is a risk that runs on bad companies can become contagious and spread to good or well-run companies (Saunders and Cornett 2006). Regulatory forbearance employed by the government often induces moral hazard and causes the life insurance industry to face possible contagion risks (for related studies, see Lee et al. 1997; Lee and Smith 1999; Angbazo and Narayanan 1996; Miller and Polonchek 1999; Bernier and Mahfoudhi 2010). To maintain their routine operations when facing financial difficulties, troubled insurers often offer insurance policies with higher guaranteed rates in the market. However, although this risk-taking strategy allows insurers to survive, it significantly worsens their balance sheets and brings out an adverse selection problem. Therefore, the fair premium collected by the guaranty fund is affected by the regulatory forbearance mechanism.

In this paper we emphasize the influence of the fair premium of TIGF under regulatory forbearance. To solve the default option problem we have to measure the value of life insurers for policyholders when liquidation happens. The financial literature on the bankruptcy problem has recently extended to insurance issues (see Cummins 1988; Briys and de Varenne 1994, 1997; Grosen and Jøgensen 2002; Bernard et al. 2005a, 2006; Duan and Yu 2005). Chen and Suchanecki (2007) generalized the work in Grosen and Jøgensen (2002) to allow for Chapter 11 bankruptcy, which considered the grace period for the insolvent financial firms. They adopted a Parisian barrier option approach and found that the option values increased as the grace period lengthened.

Several numerical methods are used to determine the value of the Parisian barrier option (see Andersen and Brotherton-Ratcliffe 1996; Chesney et al. 1997; Avellaneda and Wu 1999; Haber et al. 1999; Stokes and Zhu 1999; Costabile 2002; Bernard et al. 2005b). Labart and Lelong's (2009) method introduces the upper bound for the error because of the inversion. In order to improve the numerical accuracy and computational efficiency, the Laplace transformation method and the numerical approximation proposed in Labart and Lelong are employed in this study.

Our numerical results support the idea that TIGF should charge the premium according to the riskiness of the life insurers, because asset volatility is the vital factor under the fair premium pricing. The premium increases with higher asset volatility; moreover, the higher leverage ratio would aggravate the negative influence of the volatility of risky asset. Furthermore, the fair premium goes up with the longer grace period and looser intervention threshold.

This paper is organized as follows: Section 2 introduces the TIGF scheme in Taiwan, Section 3 presents the basic structure of a life insurance company and derives the pricing formula of the fair premium of TIGF, Section 4 performs the numerical study of the premium of TIGF, and Section 5 concludes.

2. OVERVIEW OF TIGF

In this paper we focus on the fair premium of the ex ante insurance guaranty fund, and we take TIGF as an example. According to the Taiwan insurance law article 143-1, in order to preserve the basic interests of the policyholder and to maintain financial stability of the market, the non-life and life insurers are required to make contributions to set up a separated fund supported by contributions from each insurer. The contribution rate of each insurer is determined—taking into account the economy's condition, the financial industry's state of development, and the insurer's ability to pay—and cannot be lower than 0.1% of gross premium income. Originally the life and non-life insurance guarantee funds are separately managed since being founded in 1992. Given that the insurance act was amended in 2009, two funds are merged into a single-entity TIGF for managerial purposes.

The mission of TIGF is to protect the interests of the insured and their beneficiaries, as well as to maintain market stability. Moreover, TIGF provides last-resort protection to policyholders when insurers become insolvent and are not able to fulfill their commitments. In the event that the company fails, the fund covers 90% of the total policy reserves owed to the beneficiary and 20% of the cash values given the policy lapses. The fund also covers 90% of the insured amount for death, disability, and dreaded disease, as well as for matured policy payments—up to the maximum of NT$3 million per policy and per event basis. The maximal payment of annuity per year is limited to NT$200,000 or 90% of the annuity amount, and the maximum medical benefits per year are NT$300,000.

In other words, TIGF can be regarded as a *reinsurer*; therefore, pricing the fair premium becomes crucial. In the current setting, the levies are 0.1% and 0.2% of the total annual written premium income for life and non-life insurers, respectively. According to the information disclosed by the Financial Supervisory Commission (FSC) in 2011, the accumulated fund is NT$14.48 billion for life insurers and NT$1.96 billion for non-life insurers.[1] When the amount of funds accumulated in TIGF is insufficient to safeguard the interests of the insured, the fund may borrow funds from the financial institutions. All insurers in Taiwan are covered by TIGF,[2] and they have the duty to pay the premium to TIGF. There were 30 life insurance companies and branch companies and 20 non-life insurance companies and branch companies covered by TIGF in 2011.[3]

3. MODEL

This section reviews the basic setting and model of life insurers and then defines the regulatory intervention criterion and the default time. Then we derive the pricing formula of the fair premium of TIGF.

3.1 Financial Structure of Life Insurers

For simplicity, we assume that the policyholder is the only debt holder of the insurance company, and the liability is the policy reserves denoted by L_0, which is the aggregated liability portfolio with n lines of business. Moreover, we assume the leverage ratio at time $t = 0$ is α, $\alpha \in [0, 1]$; then $E_0 = (1 - \alpha)A_0$ represents the initial assets financed by the equity holder at time $t = 0$. At the beginning, a life insurer has assets A_0 to invest in equities, corporate bonds, real estate, or other areas, and A_t is the value of the firm's assets at time t.

We assume the financial markets are in a continuous frictionless world and ignore any market imperfections. Using the equivalent martingale measures, the asset dynamics $\{A_t\}_{t\in[0,T]}$ on the insurer's balance sheet follow a geometric Brownian motion:

$$dA_t = A_t(rdt + \sigma dW_t),$$

where r denotes the deterministic interest rate, σ represents the deterministic volatility of $\{A_t\}_{t\in[0,T]}$, and $\{W_t\}_{t\in[0,T]}$ is an equivalent Q-martingale process.

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[1] The assets are approximately NT$13,063 billion and NT$276 billion for life and non-life insurers, respectively, in 2011.

[2] "Insurer" means any of various organizations engaged in the insurance business that has the right to claim a premium upon entering into an insurance contract and is liable for indemnification, in accordance with the contracted insurance obligations, when an insured peril occurs.

[3] In 2009 FSC had taken over financially troubled Kuo Hua Life insurance company because of its weak finances. Kuo Hua's total assets amounted to NT$246 billion but had run into a net deficit of NT$57.9 billion in June 2009. Kuo Hua became the first life insurance firm in Taiwan taken over by the government since Kuo Kuang Life Insurance was placed under government management in 1970. Two non-life insurance companies were taken over by FSC: Kuo Hua non-life insurance company in 2005 and Walsun Insurance Limited in 2009. Under the current low-interest rate environment, many Taiwanese life insurers face a significant negative-interest rate gap. Hence the issue of how to improve the solvency regulation framework and the consumer protection against insurance company failures becomes vital.

Table 1
Balance Sheet of Life Insurer

Asset	Liability and Ownership Equity
A_0	$\sum_{i=1}^{n} L_0^i = L_0 = \alpha A_0$ $E_0 = (1 - \alpha)A_0$

The insurance market has recently offered a significant amount of with-profit life insurance policies that contain an interest rate (see Briys and de Varenne 1994, 1997; Grosen and Jøgensen 2002). The insurer is required to provide the policyholder a minimal compounded return g.[4] The guaranteed payment to the policyholder at maturity is $L_T = L_0 e^{gT}$, where T is the maturity date. Moreover, we assume δ is the participation rate of the policyholder; then the payoff to the policyholder at maturity, $\psi_L(A_T)$, is

$$\psi_L(A_T) = \delta[\alpha A_T - L_T]^+ + \min(L_T, A_T) = \delta[\alpha A_T - L_T]^+ + L_T - [L_T - A_T]^+. \tag{1}$$

This payment consists of two components. The first component, on the right-hand side, is a bonus option. The second component is the minimal value of the firm and guaranteed insurance benefits at maturity. This second component of payoff at maturity consists of two parts, a guaranteed fixed payment, which includes the accumulated premiums compounded at the credit rate, and a short position of a put option due to the limited liability of the shareholder.

3.2 Regulatory Intervention Criterion and the Default Time

According to Chen and Suchanecki (2007), an insurance company facing liquidation returns a rebate payment $\Theta_L(\tau) = \min(L_\tau, A_\tau)$ to the policyholder at time τ. In other words, the payoff to the policyholder is only the second component in equation (1) when a life insurer is insolvent. The rebate term implicitly depends on the parameter η in triggering the intervention. The trigger condition is formulated in the following inequality:

$$A_\tau \le B_\tau = \eta L_\tau, \tag{2}$$

where B_τ is the regulatory barrier. In this study we adopt the relative intervention criterion (RIC)[5] in equation (2). Under the RIC, the regulatory authority determines a given constant multiplier η of the liability of the insurer, that is, ηL_0, as a benchmark to trigger intervention. Note that the rebate corresponds to the asset value where $\eta < 1$. A regulatory barrier is used to place the insurance company under conservatorship or receivership.

In reality, the regulatory authority has to monitor the liability dynamics in controlling the bankruptcy cost within the grace period; that is, the regulatory authority would not trigger intervention once an insurer's assets are below their liabilities. On the other hand, the regulatory authority would set regulatory barriers within a grace period. Even if the life insurer's assets are below liabilities at any time t, as long as remaining higher than the monitoring point, the life insurer would not be taken over immediately and can continue to operate until time $t + d$. In this grace period d, the life insurer has to try to recover its financial shortage $(A(t) - L(t))$, or it will be forced to face intervention at $t + d$. This is a kind of Parisian option problem, and we have to define the default time. In the standard Parisian down-and-out option framework, the final payoff $\psi_L(A_T)$ is paid only if the following technical condition is satisfied:

[4] For simplicity, we assume the minimal guaranteed rate is constant.

[5] The absolute intervention criterion (AIC) is another intervention criterion. Under the AIC, the regulatory authority chooses a positive tolerance value Y and sets up a liability lower boundary $L_0 - Y$ as a benchmark to trigger intervention.

$$T_B^- = \inf\{t > 0 \,|\, (t - g_{B,t}^A) 1_{\{A_t < B_t\}} > d\} > T, \tag{3}$$

where $g_{B,t}^A = \sup\{s \leq t | A_s < B_s\}$; here $g_{B,t}^A$ denotes the last time before t at which the value of the assets A_s reach the barrier B_s. The term T_B^- represents the first time at which an excursion below the regulatory barrier lasts more than d units of time, which means the period of the insurer's asset less the regulation criterion is longer than the grace period d. In fact, T_B^- is the liquidation date of the life insurer if $T_B^- < T$. Note that the above condition is equivalent to[6]

$$T_b^- = \inf\{t > 0 \,|\, (t - g_{b,t}) 1_{\{Z_t < b\}} > d\} > T, \tag{4}$$

where $g_{b,t} = \sup\{s \leq t | Z_s = b\}$ and $b = 1/\sigma \ln(\eta L_0/A_0)$. $\{Z_t\}_{t \in [0,T]}$ is a martingale under a new probability measure P defined by the Radon-Nikodym density:[7]

$$\frac{dQ}{dP}\bigg| F_t = \exp\left\{ m Z_T - \frac{m^2}{2} T \right\},\ m = \frac{1}{\sigma}\left(r - g - \frac{1}{2}\sigma^2 \right), \text{ and } Z_t = W_t + mt.$$

According to the explanation above, the excursion of the value of the assets below the exponential barrier $B_t = \eta L_0 e^{gt}$ is an event in which the excursion of the Brownian motion Z_t is below a constant barrier $b = 1/\sigma \ln(\eta L_0/A_0)$. Therefore, under the Q and P measures, the maturity benefit $\psi_L(A_T)$ can be expressed as follows:

$$E_Q[e^{-rT} \psi_L(A_T) 1_{\{T_B^- > T\}}] = e^{-(r+(1/2)m^2)T} E_P[1_{\{T_b^- > T\}} \psi_L(A_0 \exp\{\sigma Z_T\} \exp\{gT\} \exp\{m Z_T\})]. \tag{5}$$

3.3 The Fair Premium of TIGF

The TIGF provides last-resort protection to policyholders when insurers become insolvent and are not able to fulfill their commitments. We assume that the insurance guaranty fund covers λ of the total amount owed to the beneficiary in the event that the company fails. The fair premium $C(0)$ constructed by the bankruptcy cost is when the time of the insurer becoming insolvent is within the grace period and the bankruptcy cost at maturity. $C(0)$ is formulated as

$$C(0) = E_Q[e^{-rT_B^-}[\lambda L_{T_B^-} - \min\{L_{T_B^-}, A_{T_B^-}\}]^+ 1_{\{T_B^- \leq T\}}] + E_Q[e^{-rT}[\lambda L_T - A_T]^+ 1_{\{T_B^- > T\}}], \tag{6}$$

where λ is the coverage ratio. In equation (6), the first term on the right-hand side represents the cost of bankruptcy when the time of the insurer becoming insolvent is less than the given grace period. The second term on the right-hand side means the cost of bankruptcy at maturity.

We can represent the above equation as

$$\begin{aligned} C(0) &= E_Q[e^{-rT_B^-}[[L_{T_B^-} - A_{T_B^-}]^+ - (1-\lambda) L_{T_B^-}]^+ 1_{\{T_B^- \leq T\}}] + \mathrm{PDOP}[A_0, T, B_0, \lambda L_0, r, g] \\ &= E_Q[e^{-rT_B^-}[\lambda L_{T_B^-} - A_{T_B^-}]^+ 1_{\{T_B^- \leq T\}}] + \mathrm{PDOP}[A_0, T, B_0, \lambda L_0, r, g]. \end{aligned} \tag{7}$$

Note that the fair premium of TIGF consists of two parts: (1) a Parisian down-and-in put option with strike $\lambda L_{T_B^-}$ when insurers have defaulted before maturity and (2) a Parisian down-and-out put option with strike λL_T.

The solution of the Parisian down-and-out put option (PDOP) in equation (7) can be found in Labart and Lelong (2009).[8] In this study we derive the first term on the right-hand side in equation (7). It can be derived by the following equation:

$$E_Q[e^{-rT_B^-}[\lambda L_{T_B^-} - A_{T_B^-}]^+ 1_{\{T_B^- \leq T\}}] = E_Q[e^{-rT_B^-}[\lambda L_{T_B^-}] 1_{\{A_{T_B^-} < \lambda L_{T_B^-}\}} 1_{\{T_B^- \leq T\}}] - E_Q[e^{-rT_B^-}[A_{T_B^-}] 1_{\{A_{T_B^-} < \lambda L_{T_B^-}\}} 1_{\{T_B^- \leq T\}}]. \tag{8}$$

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[6] The equivalence derivation is according to Chen and Suchanecki (2007).

[7] P measure is an equivalent measure of Q, but is not a real-world probability measure.

[8] Labart and Lelong (2009) derive the Laplace transform of the Parisian down-and-in call option (PDIC) and the Parisian up-and-in call option (PUIC) and utilize the put-call parity and in-out parity of the Parisian-type option so that the Parisian down-and-in put option (PDOP) can be derived.

The first term on the right-hand side in equation (8) can be derived by the following equation:

$$\begin{aligned} E_Q[e^{-rT_{\bar{B}}}[\lambda L_{T_{\bar{B}}}]1_{\{A_{T_{\bar{B}}^-}<\lambda L_{T_{\bar{B}}}\}}1_{\{T_{\bar{B}}\le T\}}] &= \lambda L_0 E_P[e^{-(r-g+(1/2)m^2)T_b^-}\ \exp(mZ_{T_b^-})1_{\{Z_{T_b^-}<1/\sigma \ln(\lambda\alpha)\}}1_{\{T_b^-\le T\}}] \\ &= \lambda L_0 E_P[e^{-(r-g+(1/2)m^2)T_b^-}1_{\{T_b^-\le T\}}]\ E_P[\exp(mZ_{T_b^-})1_{\{Z_{T_b^-}<b^*\}}]. \end{aligned}$$

The last equality follows the idea that $Z_{T_b^-}$ and T_b^- are independent, shown in the appendix of Chesney et al. (1997), where $b^* = 1/\sigma \ln(\lambda\alpha)$. Furthermore, the corresponding laws for these random variables are also given in Chesney et al.

The second term on the right-hand side of equation (8) can be derived by the following equation:

$$\begin{aligned} E_Q[e^{-rT_{\bar{B}}}[A_{T_{\bar{B}}}]1_{\{A_{T_{\bar{B}}^-}<\lambda L_{T_{\bar{B}}}\}}1_{\{T_{\bar{B}}\le T\}}] &= A_0 E_P[e^{-(r-g+(1/2)m^2)T_b^-}\ \exp((m+\sigma)Z_{T_b^-})1_{\{Z_{T_b^-}<1/\sigma \ln(\lambda\alpha)\}}1_{\{T_b^-\le T\}}] \\ &= A_0 E_P[\exp((m+\sigma)Z_{T_b^-})1_{\{Z_{T_b^-}<b^*\}}]E_P[e^{-(r-g+(1/2)m^2)T_b^-}1_{\{T_b^-\le T\}}]. \end{aligned}$$

We can calculate $E_P[e^{-(r-g+(1/2)m^2)T_b^-}1_{\{T_b^-\le T\}}]$ by its Laplace transform, which is denoted by $\hat{E}_P[e^{-(r-g+(1/2)m^2)T_b^-}1_{\{T_b^-\le T\}}]$, and we apply the numerical inversion technique developed by Labart and Lelong (2009) to invert $\hat{E}_P[e^{-(r-g+(1/2)m^2)T_b^-}1_{\{T_b^-\le T\}}]$. It needs the following propositions to find $E_P[\exp(mZ_{T_b^-})1_{\{Z_{T_b^-}<b^*\}}]$, $\hat{E}_P[e^{-(r-g+(1/2)m^2)T_b^-}1_{\{T_b^-\le T\}}]$, and $E_P[\exp((m+\sigma)Z_{T_b^-})1_{\{Z_{T_b^-}<b^*\}}]$.[9]

Proposition 3.1

$$L\{E_P[e^{-(r-g+(1/2)m^2)T_b^-}1_{\{T_b^-\le T\}}]\} = \hat{E}_P[e^{-(r-g+(1/2)m^2)T_b^-}1_{\{T_b^-\le T\}}] = \frac{1}{s}\frac{\exp\left(\sqrt{\left(s+r-g+\frac{1}{2}m^2\right)}b\right)}{\psi\left(\sqrt{\left(s+r-g+\frac{1}{2}m^2\right)}d\right)},$$

where L is the Laplace transform, $\psi(z) = 1 + z\sqrt{2\pi}e^{z^2/2}N(z)$, and $N(z)$ is the cumulative distribution function of the standard normal distribution.

Proposition 3.2

$$\begin{aligned} &E_P[\exp(mZ_{T_b^-})1_{\{Z_{T_b^-}<b^*\}}] \\ &\quad = e^{mb}\left[\exp\left(-\frac{(b^*-b)^2}{2d}+m(b^*-b)\right)-\sqrt{2\pi d}m\exp\left(\frac{dm^2}{2}\right)N\left(\frac{b^*-b-dm}{\sqrt{d}}\right)\right]. \end{aligned}$$

Proposition 3.3

$$\begin{aligned} E_P[\exp((m+\sigma)Z_{T_b^-})1_{\{Z_{T_b^-}<b^*\}}] &= e^{(m+\sigma)b}\left[\exp\left(-\frac{(b^*-b)^2}{2d}+(m+\sigma)(b^*-b)\right)\right. \\ &\quad \left. -\sqrt{2\pi d}(m+\sigma)\exp\left(\frac{d(m+\sigma)^2}{2}\right)N\left(\frac{b^*-b-d(m+\sigma)}{\sqrt{d}}\right)\right]. \end{aligned}$$

According to the above derivation and the numerical method of PDOP on Labart and Lelong (2009), we could simulate the premium of TIGF.

4. NUMERICAL ILLUSTRATIONS

We employ the inverse Laplace transform in numerical computations to investigate the fair premium and to compare the results by leverage ratio, asset volatility, grace period, and intervention criterion.

[9] The proofs of Propositions 3.1–3.3 are given in the Appendix.

Moreover, the trigger event is related with the value of assets and liabilities of the life insurer; therefore, the asset volatility is also a significant factor when we estimate $C(0)$.

Labart and Lelong (2009) use the Fourier series representation to find the inverse Laplace transform. Given the Laplace transform $\hat{f}$, function f can be inverted by the contour integral:

$$f(t) = \frac{1}{2\pi i}\int_{\beta-i\infty}^{\beta+i\infty} e^{st}\hat{f}(s)ds, \quad t > 0,$$

where β is a positive number and $i = \sqrt{-1}$. Labart and Lelong (2009) introduced the trapezoidal rule to approximate the above integration, and we find the numerical result in the same way.[10]

4.1 The Fair Premium

In Section 4.1 we illustrate the risk-based premium under different parameters settings that are based on the insurance industry experience in Taiwan. We summarize the parameters settings in Table 2.

According to the market data,[11] we set the insurer's initial asset value, A_0, at 100 and the insurer's initial liability at 95, which means that the leverage ratio is 95%. Moreover, the risk-free rate is 1.75% based on the statistics of the yield for the 30-year government bond in Taiwan.[12] On the other hand, the minimal guarantee rate g is assumed to be 2.00%,[13] and the maturity T is 20 years. The grace period d is 6 months because the insurer has to report the RBC ratio twice a year. Moreover, the RBC ratio has to exceed 200% under the regulations in Taiwan. However, the formula of RBC ratio could not directly transform to the minimum capital. Moreover, we also consider the capital forbearance. Thus, following Lee et al. (2005), we set the monitoring ratio η at 0.9, and the volatility of asset σ is assumed to be 5%. In this scenario the fair premium of TIGF is 198 basic points (b.p.). Comparing this with the current setting of TIGF (10 b.p. of the total annual written premium income), the empirical premium is underestimated.

4.2 Sensitivity Analysis

In order to illustrate the impact of the vital parameters on the fair premium, in this section we display a table to describe the sensitivity analysis.

Table 3 summarizes the fair premiums based on different leverage ratios, asset volatilities, grace periods, and monitoring ratios. Table 3 illustrates that TIGF should charge different premiums to the life insurers according to their riskiness. First, we fix the intervention criterion at 90%, and the volatility

Table 2

Parameter Definition and Base Values

	Parameter	Value
A_0	Insurer's initial asset value	100
σ	Volatility	5%
r	Risk-free rate	1.75%
L_0	Insurer's initial liabilities	95
g	Minimal guarantee rate	2.00%
η	Monitoring ratio	0.9
d	Grace period	0.5
T	Maturity	20

[10] Beside the trapezoidal rule, Labart and Lelong (2009) use Euler summation to reduce the computation.

[11] According to the statistics data of Taiwan Insurance Institute, the average leverage ratio of the whole life insurance industry is about 95% from 2001 to 2010.

[12] This information is from the database of GreTai Securities Market.

[13] The maximum valuation interest rate in determining minimum Taiwan statutory reserves and minimum life policy nonforfeiture values for lie insurance policy of more than 20 years in 2011 is 2%. Hence 2% is employed for our numerical example and sensitivity analysis.

Table 3
The Fair Premium with Parameters (b.p.)

		Leverage Ratio								
		0.85			0.9			0.95		
		Monitoring Ratio								
Grace Period	**Asset Volatility (%)**	**0.9**	**0.95**	**1**	**0.9**	**0.95**	**1**	**0.9**	**0.95**	**1**
0.25	1	0	0	0	0	0	0	1	0	0
	2	1	0	0	6	0	0	19	0	0
	3	14	0	0	31	0	0	57	0	0
	4	42	0	0	68	1	0	101	1	0
	5	80	4	0	113	5	0	149	6	0
	6	124	12	0	160	15	0	198	18	0
	7	171	26	1	210	32	1	249	37	1
	8	221	47	3	261	55	3	300	62	3
	9	272	74	7	313	84	8	352	93	8
	10	324	105	15	365	116	16	404	128	18
0.5	1	0	0	0	0	0	0	1	0	0
	2	2	0	0	8	0	0	25	0	0
	3	18	0	0	40	1	0	75	1	0
	4	54	5	0	90	8	0	134	11	0
	5	104	19	1	148	26	1	198	33	1
	6	162	44	4	212	56	4	265	68	5
	7	225	78	12	278	95	14	332	110	15
	8	291	120	26	347	140	30	401	158	34
	9	359	168	48	416	192	55	470	214	60
	10	429	220	78	486	247	86	540	272	94
1	1	0	0	0	0	0	0	1	0	0
	2	2	0	0	9	0	0	32	1	0
	3	22	4	0	50	7	0	96	12	0
	4	68	20	2	114	32	2	173	45	3
	5	132	53	9	190	74	12	257	96	15
	6	206	101	28	273	130	36	344	160	44
	7	288	160	60	359	196	71	433	231	84
	8	374	227	103	448	268	122	523	308	140
	9	462	300	155	538	345	176	613	388	196
	10	553	377	215	630	425	240	704	471	264

Note: $r = 1.75\%$ and $g = 2.00\%$.

of assets is 10% under a three-month grace period; then the risk-based premiums are 324 b.p., 365 b.p., and 404 b.p. under the leverage ratios 85%, 90%, and 95%, respectively. Note that the takeover cost increases 24.7% when the leverage ratio rises from 85% to 95%. This result shows that the financial distress can be more costly for a higher leverage ratio firm. Moreover, if we extend the grace period to one year, then the additional cost under the leverage ratio rises from 85% to 95% increases to be 27.3% (553 b.p. to 704 b.p.). This indicates that the grace period is significantly influenced when the insurer maintains a higher leverage ratio.

Subsequently Table 3 shows that the premium of TIGF increases when the life insurer's asset is more volatile. For example, comparing $\sigma = 5\%$ with $\sigma = 10\%$, the cost is over fourfold larger (325 b.p. increment) when the leverage ratio is 0.85, monitoring ratio is 0.9, and grace period is 6 months. In addition, as expected, the volatility of asset return quickly increases the premium with a longer grace period. For example, the premium increases 421 b.p. to 553 b.p. when the grace period is extended to be one year. It shows that the grace period has a significant influence when asset volatility increases.

Moving on to the effect of regulatory forbearance, which contains the intervention threshold and the grace period, Table 3 illustrates that as the ratio of monitoring decreases or the grace period increases, the fair premium increases. If we fix the asset volatility at 5% and the leverage ratio at 95%, the fair premium increases from 6 b.p. to 96 b.p. when the grace period increases from 3 to 12 months under a fixed 100% monitoring ratio. When the monitoring ratio increases from 90% to 100%, the fair premium decreases significantly from 257 b.p. to 15 b.p. under a 6-month grace period. These results show that regulatory forbearance is costly.

5. Conclusion

Sound insurance supervision is important for financial stability because the financial crisis might cause life insurers to face severe financial distress problems. Ex ante assessment schemes such as TIGF are efficient to protect the obligations of policyholders. In this paper we used the embedded Parisian option to price the fair premium of TIGF with regulatory forbearance schemes and investigated the effects under several important parameters.

In order to gain numerical accuracy and computational efficiency, we applied the Laplace transformation method and the numerical approximation proposed in Labart and Lelong (2009) to simulate the risk-based premium of TIGF under different parameter settings. In the basic scenario in Section 4.1, the fair premium of TIGF is 198 b.p., which is significantly higher than the current setting of TIGF (10 b.p. of the total amount of the annual written premium income).

In this study we also analyze the influence of the vital parameters, which include the leverage ratio, monitoring ratio, grace period, and asset volatility. The results show that the volatility of a life insurer's assets has a greater effect on premiums than do other factors. The premium increases with the higher asset volatility, and the higher financial leverage would deteriorate the negative influence of the asset volatility. Thus, we suggest that TIGF should charge a different premium according to the life insurer's financial structure. This conclusion is consistent with Cummins (1998) and Duan and Yu (2005). Furthermore, the longer grace period and looser intervention threshold cause expensive premiums. This demonstrates that regulatory forbearance is costly and suggests that the regulatory authorities have to set a reasonable forbearance mechanism.

Appendix

Proof of Proposition 3.1

Due to the definition of Laplace transform, we have

$$\begin{aligned}
\hat{E}_P[e^{-(r-g+1/2m^2)T_b^-}1_{\{T_b^-\le T\}}] &= \int_0^\infty e^{-st}E_P[e^{-(r-g+1/2m^2)T_b^-}1_{\{T_b^-\le t\}}]dt \\
&= E_P\left[\int_{T_b^-}^\infty e^{-st}e^{-(r-g+1/2m^2)T_b^-}dt\right].
\end{aligned}$$

Let $u = t - T_b^-$,

$$\begin{aligned}
\hat{E}_P[e^{-(r-g+1/2m^2)T_b^-}1_{\{T_b^-\le T\}}] &= E_P\left[\int_0^\infty e^{-s(u+T_b^-)}e^{-(r-g+1/2m^2)T_b^-}du\right] \\
&= E_P[e^{-(s+r-g+1/2m^2)T_b^-}]\int_0^\infty e^{-su}du \\
&= \frac{1}{s}\,\frac{\exp\left(\sqrt{\left(s+r-g+\frac{1}{2}m^2\right)}\,b\right)}{\psi\left(\sqrt{\left(s+r-g+\frac{1}{2}m^2\right)}\,d\right)},
\end{aligned}$$

where

$$E_P[e^{-(s+r-g+1/2m^2)T_b^-}] = \frac{\exp\left(\sqrt{\left(s + r - g + \frac{1}{2}m^2\right)} b\right)}{\psi\left(\sqrt{\left(s + r - g + \frac{1}{2}m^2\right)} d\right)}.$$

This can be found in Chesney et al. (1997, Appendix).

Q.E.D.

PROOF OF PROPOSITION 3.2

According to Chesney et al. (1997, Appendix), the probability density function of $Z_{T_b^-}$ is as shown in the following equation:

$$f(Z_{T_b^-}) = \frac{b - Z_{T_b^-}}{d} \exp\left(-\frac{(Z_{T_b^-} - b)^2}{2d}\right);$$

therefore,

$$E_P[\exp(mZ_{T_b^-})1_{\{Z_{T_b^-}<b^*\}}] = \int_{-\infty}^{b^*} e^{mx} \frac{b - x}{d} \exp\left(-\frac{(x - b)^2}{2d}\right) dx.$$

It is easy to find

$$\int_{-\infty}^{b^*} e^{mx} \frac{b - x}{d} \exp\left(-\frac{(x - b)^2}{2d}\right) dx$$
$$= e^{mb}\left[\exp\left(-\frac{(b^* - b)^2}{2d} + m(b^* - b)\right) - \sqrt{2\pi d}m \exp\left(\frac{dm^2}{2}\right) N\left(\frac{b^* - b - dm}{\sqrt{d}}\right)\right].$$

Q.E.D.

PROOF OF PROPOSITION 3.3

We can prove Proposition 3.3 using the same method as in Proposition 3.2.

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