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核心邊陲模型的模擬分析[1]

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摘要

本研究以核心邊陲模型爲基礎，分別探討以調整的動態模式解釋工作者的移動，或加上非市場聚集效果的生產模式，模擬分析聚集、成長與區域差異的關係。模擬結果顯示區域所得的差異隨者聚集程度而增加，核心邊陲結構地區的平均所得比分散型態地區的平均所得高，區域成長的差異可能會導致邊陲地區所得降低。聚集與成長如預期互相正向影響，成長主要是在核心區域。加入非市場空間聚集效果的生產模式加強了區域的向心力，更強化了核心邊陲型態。

關鍵詞：核心邊陲模型，非市場性聚集效果

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Evolution in a core-periphery model

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Abstract

The author proposes an alternative dynamic pattern to explain workers migration, and a production behavior to include the non-market agglomeration effect into the model. The dynamic process of worker migration is derived from the model implicitly with microfoundation rather than explicitly apply a migration law from evolutionary game which is independent of the model and without microfoundation.

Simulation results suggest that the difference between income in core and periphery regions rises with agglomeration. The average income is higher in the core-periphery structure than in a dispersed pattern. The increase of regional disparities may cause impoverishment of the peripheral region. Agglomeration and growth reinforce each other; however, inter-regional integration may benefit only the core region. The periphery is better off in a more dispersed pattern. Economic growth in the core region does not necessarily benefit the whole region. Inclusion of the non-market spatial agglomeration effect enhances the centripetal forces, which further leads the system to a core-periphery pattern.

Keywords: Core-periphery model, Bifurcation, Replicator dynamics

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I. Introduction

Inter-regional integration increases economic efficiency in the spatial economy. Fujita and Thisse (2002) strongly supported the idea that agglomeration and growth reinforce each other. Cities are often considered engines of growth (Hohenberg and Lees, 1985; Feldman and Florida, 1994). However, it has long been argued that growth is localized (Hirschman, 1958; Myrdal, 1957). Krugman (1991) applied a Dixit-Stiglitz type monopolistic competition model (known as the core-periphery model) to explain how economic activity may be agglomerated. This model shows “how the interactions among increasing returns at the level of firm, transport costs, and factor mobility can cause spatial economic structure to emerge and change” (Fujita et al., 1999).

The dual role of individuals as workers and consumers adds both production and consumption capabilities to a region’s economy. Initial expansion of a market pushes nominal wages up (the home market effect), and, consequently, leads to a rise in real wages (the price index effect). Migration of workers is explained by a given ad hoc dynamics: “replicator dynamics” which is routinely used in evolutionary game theory found in the classical Wright–Haldane–Fisher theory (Akin, 1979). It assumes that workers’ migration decisions depend on the difference in real wages. This dynamics process is not generated from the core-periphery model itself. The properties of the core-periphery model with three regions are provided by Castro et al. (2012). The migration law in this study is the replicator dynamics in core-periphery model with utility level instead of real wage rate.

Ikeda, Akamatsu and Kono (2012) investigate the progress of agglomeration of the core-periphery model based on the same replicator dynamics with more than 2 cities by numerical simulation. They find the speed of bifurcations varied by the system of cities. Lange and Quaas (2010) proposed a novel approach to analyze migration incentives of the core-periphery models and show that migration is more driven by differences in the price index than by differences in the nominal wage rate. This result suggests that migration choice is implicit rather than explicit as replicator dynamics in core-periphery models. Accetturo (2010) allows external effect from agglomeration and use the explicit migration law from evolutionary game without micro foundation. It assumes migration occurs whenever the utility level in one region is higher than the utility level in the other region. This is the same replicator dynamics in core-periphery models.

Replicator dynamics from evolutionary game is a general alternative to define population migration without economic foundation. It is applied in all related studies. The population change in a region is assumed to be proportional to the difference between real wage (local utility level) and the average real wage (average utility level) (Weibull, 1996, Baldwin et al., 2003, Berliant and Kung, 2009). This migration rules is based on population game considering payoffs to all strategies. The mean dynamic is defined by the expected increments such as proportional imitation (Sandholm, 2011; Schlag, 1998). One

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of the aim of this paper is to contribute to filling this gap by implicitly deriving a migration law with microfoundation from the original core-periphery model rather than the originally exogenous replicator dynamics.

The externality assumed in the model relies only on market interactions involving economies of scale at the level of the individual firm. Due to supposition of the externality, transport costs are the key factor that determines distribution of industries. However, non-market interactions that yield increasing returns, external to firms, are viewed as crucial in related studies (Baldwin and Martin, 2004; Fujita and Thisse, 2002). This is not addressed in the core-periphery model. The existing analysis of the multi-region core-periphery model relies on numerical simulations exclusively (see Krugman 1993; Jujita et al. 1999; brakman et al. 2001; Ago et al. 2006). Accetturo (2010) obtain both a Krugman-type catastrophic agglomeration and a core-periphery reversal according to the interplay between knowledge spillovers and the congestion costs.

The purpose of this paper is to investigate the features of regional discrepancies and bifurcation of the core-periphery model (Fujita et al., 1999) and two proposed modifications: derived implicit dynamic process of migration law and firm's production function allowing external agglomeration effect. We first simulate the core-periphery model to investigate features of the limiting distribution of manufactures. We apply the location decision model and Polya process to derive a dynamic process of worker migration instead of the given replicator dynamics term in the core-periphery model. A production behavior incorporating the non-market agglomeration effect is proposed. The contribution of this paper is to endogenously derive a migration law which is essential in the evolution analysis to explain how workers move with microfoundation rather than explicitly apply a migration law which is independent of the model and not model specific.

II. The model

1. The proposed production behavior

In the core-periphery model (Fujita et al., 1999), every consumer shares the same Cobb-Douglas tastes for the two types of goods: manufactured goods and agricultural goods (Appendix). The quantity index is a subutility function defined over a continuum of varieties of manufactured goods. There are two sectors in the economy; monopolistically competitive manufacturing and perfectly competitive agriculture. The agricultural good is assumed to be produced using a constant-returns technology. Manufacturing involves economies of scale. Production of quantity q of manufacturing goods requires labor input l, as follows.

$$l_i = F + c_i q_i \cdots\cdots (1)$$

where F indicates fixed inputs and c_i is marginal input requirement.

The basic force that drives spatial agglomeration in the core-periphery model relies only on market

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interactions, which is different from most of the existing literature dealing with causes of agglomeration. In the model, marginal input requirement c_i is constant in all locations. This assumption leads to a constant equilibrium output q^* and a constant equilibrium labor input l^* for all firms in all locations. This result implies that all scale (or market-size) effects in the model do not work through a larger market or production at a larger scale, but only work through changes in variety. In this section, we modify the production behavior to relax the limitation of the market-size effect.

In the proposed production behavior, the marginal input requirement at location *I*, c_i, is assumed to be negatively related to the manufacture share X_r of location *i*. The higher the manufacture share of the location, the larger will be the agglomeration economies at the location. Consequently, equilibrium output q_r^M and equilibrium labor input l_r^M vary from location to location. The difference between original CP model and this proposed model is that agglomeration effect in the original CP model is triggered only from market-size effect. In our proposed model, the location agglomeration of firms give positive external effect to production.

2. The proposed dynamic process: Polya process and the probability of worker location choice Polya process:

In the core-periphery model (Fujita et al., 1999), the optimal solutions from both consumer and producer behaviors derive four endogenous variables of each location: income, price index of manufactures, the nominal wage rate of workers, and the real wage rate (in Appendix). Worker's migration decision mainly depends on the difference in real wages. The dynamic process used in the model is the "replicator dynamics" from the evolutionary game theory.[3]

$$\dot{X}_i = r\left(\omega_i - \varpi\right) X_i \quad \cdots\cdots(2)$$

where X_i describes worker share at location *i*,; w_i is the real wage at location *i;* $\overline{w}$ is the average real wage; and r denotes the adjustment speed.

This dynamic process in the original model (replicator dynamics) is exogenous from the evolutionary game theory. It is independent from the model in terms of economics meaning. In this section, we derive a dynamic pattern from the model to explain the migration process of workers. The Polya processes introduced in Arthur (2000) are applied in this study which is based on a class of path-dependent stochastic processes. Let s_{it} describe the size of the total population of location *i* at time *t*, and X_{it} describe the proportion of population in location *i* at time *t*. Assume the change of population at location *i* follows the dynamic process:

$$s_{i,t+1} = s_{it} + z_{it}, \quad \cdots\cdots(3)$$

where z_{it} equals one with probability P_{it}, zero with probability $(1 - P_{it})$. Probability P_{it} is the probability that worker will choice to reside in location *i*. Consequently, the evolution of worker share at location *i*,, X_{it}, is as follows:

[3] Oyama (2009) gives its application in economic geography.

$$X_{i,t+1} = X_{it} + \frac{1}{(a+t)}[z_{it} - X_{it}] \quad \text{(4)}$$

The expected change of worker share at location *i* depends on previous share and the probability of residential location choice of worker.

$$E[X_{it+1} | X_{it}] = X_{it} + \frac{1}{(a+t)}[P_{it} - X_{it}] \quad \text{(5)}$$

where a is the initial total population. We apply this derived change of worker share at location *i* to explain the evolution process instead of the exogenous dynamic process in original core-periphery model (equation (2)). The differences between this proposed model and original core-periphery model is that the evolution process of worker share for each location in the original core-periphery model is independent of the model itself. It is exogenous given from game theory. The proposed model in this paper derives the dynamic process of worker share direct from the model itself. It is endogenous. In the derived dynamic process (equation (5)), the worker share of location *i* depends on previous worker share at this location, X_{it}, and the probability of worker location choice, P_{it}.

The probability of worker location choice, P_{it}:

The utility of resident at location *i*, U_{it}, consists of two components: the observed part V_{it} and the unobserved part e_{it}.

$$U_{it} = V_{it} + e_{it} \quad \text{(6)}$$

The probability of residents preferring location *i* over all other locations is:

$$P_{it} = \Pr ob\{U_{it} > U_{it}, \quad for\ all \quad j \neq i\}. \quad \text{(7)}$$

Assume that each unobserved part of utility, e_{it} is distributed independently, and identically in accordance with the extreme value distribution. The probability of worker's residential choice is determined by the observed utility, V_{it}, as the following:

$$P_{it} = \frac{e^{V_{it}}}{\sum_j e^{V_{it}}} \quad \text{(8)}$$

We apply worker's indirect utility in core-periphery model (Fujita *et al*., 1999) as the observed utility, V_{it}, in equation (6). It is a function of income, Y_{it}, and price level, G_{it}, solved from worker's utility maximization and firm's profit maximization:

$$V_{it} = aY_{it}G_{it} \quad \text{(9)}$$

Consequently, the state of equilibrium of the manufacture distribution depends on the relation between the manufacture share X_{it} and the location choice probability P_{it} (equation (5)); the choice probability relies on the observed utility (equation (8)) which is solved as a function of corresponding income and price level (equation (9)); both endogenous income and price level (A.8~9) are related to local worker share. The derived location choice probability and local worker share interacted with each other.

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The derivatives of the choice probabilities are calculated. The change in the probability of choosing location *i* given a change in the manufacture share at location *i* is

$$\frac{\partial P_{it}}{\partial X_{it}} = \left(\frac{\partial V_{it}}{\partial X_{it}}\right) P_{it}\,(1 - P_{it}) = \beta_{it} P_{it}\,(1 - P_{it}) \quad \cdots\cdots(10)$$

$$\beta_{it} = \left(\frac{\partial V_{it}}{\partial X_{it}}\right) = \mu\left[\left(\frac{w_{it}}{Y_{it}}\right) - \left(\frac{1}{1-\sigma}\right)\left(\frac{w_{it}}{G_{it}}\right)^{1-\sigma}\right] \quad \cdots\cdots(11)$$

Where μ is a constant representing the expenditure share of manufactured goods; parameter σ represents the elasticity of substitution between any two varieties; Y is income and G is the price index (see Appendix).

The indirect utility V_{it} is as equation (A.1) solved from the original model. If the observed utility V_{it} is linear with parameters, the term β_{it} is constant. Consequently, the derivative is maximized when $P_{it} = 1/2$, and becomes smaller as P_{it} approaches zero or one. However, the term β_{it} is not a constant; rather, it is a function of endogenous variables: real wage, price index and income. It is varied by time and location. This shows that the probability of worker location choice is affected by worker shares endogenously.

The difference our proposed implicit migration law and the explicit migration law in the original core-periphery model is that worker migrates depends mainly on real wage rate (or utility level) in the explicit migration law which is independent of model. The migration law derived in this study is solved as a function of income and price level. These two endogenous variables includes wage and utility. The proposed migration law is changed by the change of model.

3. Simulation

We simulate the proposed model (according to section 2) based on the original core-periphery model (Fujita *et al.*, 1999) in this section. The indirect utility and endogenous variables solved from model are income level, price level; nominal wage and real wage at each location (see Appendix). The dynamic process in the original core-periphery model is as equation (2), and the proposed dynamic process in this study is as equation (5). There are three experiments: (1) Simulations based on the original core-periphery model. (2) Simulations based on the core-periphery model with the proposed production behavior as in section 2.1. (3) Simulations based on the core-periphery model with proposed dynamic process as in equation (5) in section 2.2. There are *n* locations in study region.

(1) The original core-periphery model

The original core-periphery model is simulated by computing all the endogenous variables (see Appendix) for each location through time. The variables are: income, price index, nominal wage and real wage (equation A.8~A.11). At the first period, given the initial condition and parameter value, we solve these endogenous variables. And we apply the dynamic process of equation (A.7) to derive the

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workers share of next period. This process is repeated to the end period we set.

We first examine the features of core-periphery model in a two regions experiment. The simulation result of the limiting manufacture share for σ = 5 and μ =0.4 is in Figure 1, where parameter σ represents the elasticity of substitution between any two varieties, and μ is a constant representing the expenditure share of manufactured goods. The value we use for parameters (σ = 5 and μ =0.4) to simulate the original model is the same as in Fujita's work. This is a starting simulation condition for comparison. This result shows how workers share among two regions varied by transport cost. As in Krugman (1991), when the manufacturing share of each region starts from either a high or a low initial value, the economy converges to a core-periphery pattern, with all manufacture locating in a single core region, when transport costs are sufficiently low. If transport costs are sufficiently high, then inter-regional shipments of goods are discouraged. The economy converges to a symmetric regional pattern of production. The transport cost is the key determining factor of the state of equilibria. The home market effect gets magnified through the mobility of workers. The core-periphery structure emerges as the equilibrium balance in a system of opposing forces.

The corresponding simulated distribution of income (Y) and weighted average income (AY) at time t=100 is depicted in Figure 2 When manufacture gets concentrated in a core region, the income level in that core region is much higher than that in its periphery. On the other hand, if transport costs are sufficiently high, manufacture is more dispersed. The difference between incomes in core and periphery declines as transport cost increases. The average income of residents of the core-periphery structure is higher than in a symmetric regional pattern. The limiting distribution of income (t=1000) (see Figure 3) implies that regional disparities of income eventually diminish when transport costs are sufficiently high. The income level of the peripheral region is less in a core-periphery structure than in a symmetrical pattern.

Figures 4 and 5 depict the limiting share of manufacture in regions against μ , the expenditure share of manufactured goods. These two figures are calculated for σ = 5 and T =1.1, and 2.1, the remaining parameters being the same as in Figure 1 The value of transport cost T=1.1 and 2.1 are chosen by considering other values within range and pick these two relatively high and low values which show different simulation results. The types of equilibria vary with μ . The larger the value of μ , the more concentrated the limiting distribution becomes. The agglomeration effect in core-periphery model arises only from the manufacturing sector, due to the assumptions of the model. The larger the share of the manufactures, the stronger the agglomeration forces are.

In the case of more than two regions, evolution of manufacturing shares of ten locations is simulated. In Figures 6 and 7, transport costs are 2.4 and 7, respectively. When transport cost is relatively high, limiting distribution becomes uniform. On the contrary, a small transport cost may cause regional concentration because of the market effect. In addition, regressions of log of rank versus log of size of the limiting distributions of manufacture shares are in Table 1. As T increases, distributions of

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log of rank versus log of size come close to a linear relationship. It shows that the limiting distributions of manufacture shares support the power law.

(2) The core-periphery model with the proposed production behavior

The model we use in this case is the same as in case 1 except the production equation (1) (the same as (A.5)). We simulate the model with the proposed production behavior as in Section 2.1. The marginal input requirement is not a constant; it varies by the location's manufacture share: $l_i = F + c(x_i)q_i$. The major difference in this case is to change the marginal input c from exogenous to endogenous $c(x_i)$. All other solved endogenous variables are changed accordingly: income, price index, nominal wage, and real wage and worker shares at each location.

This assumption allows spatial agglomeration effect not only from the market but also from the production perspective. The corresponding endogenous variables for each location (income, price index, wage rate and real wage rate) are respectively derived according to the change of the production behavior.

The limiting distribution of manufacture share is shown in Figure 8, given the same value of parameters as in Figure 1 of Case 1. The economy converges to a symmetric equilibrium when T is more than 6, which is much larger than in Figure 1 of Case 1. Inclusion of external spatial agglomeration enhances the centripetal forces, and, therefore, it is more likely to lead the system to a core-periphery pattern. The symmetrical state is not stable until the transport cost is sufficiently high.

(3) The core-periphery model with proposed dynamic process

The model we use in this case is the same as in case 1 except the dynamic process of workers shares (equation (2)) (the same as (A.7)). We simulate the model with the proposed dynamic process (equation (5)) derived in Section 2.2. We use equation (5) instead of equation (2) for each time period.

The Polya process is applied in Section 2.2 as the dynamic term, instead of the replicator dynamics in the core-periphery model to examine the bifurcation features. Numerical examples show that the types of equilibria vary with transport costs in a very different pattern. The resulting paths of the equilibrium state are shown in Figures 9 and 10 They are calculated for σ=5 and different μ. The patterns are rather different from those of the original core-periphery model (Case 1). Simulation results show that in most cases, the limiting distribution of manufacture shares are converging. When the expenditure share of manufactured goods μ goes up, bifurcation occurs because of higher transport cost. It supports that limiting distributions is highly sensitive to the proposed dynamic process.

Finally, we assume the workers migrate when there is a positive difference in resident's utility, instead of the real wage in the original core-periphery model. The real wage in the replicator dynamics is replaced with the level of utility. The simulation result is in Figure 11, where all other parameters are the same as in Figure 1. The states of equilibria of two kinds of dynamic terms are quite different. In core-periphery model, the real wage rate is distributed more symmetrically than the utility level. This shows that the state of equilibria is sensitive to formulation of the dynamic pattern. The choice probability versus manufacture share can display the feature of the state of equilibria.

Figure 12 shows implies that the limiting manufacture distributions converge to a stable disperse structure. On the contrary, in Figure 13, the choice probability curve indicates that manufacture share converge to a stable core-periphery structure. The disperse pattern in Figure 13 is unstable. Change in the value of parameters will lead to change of the endogenous variables in the system; further, the slope of the choice probability (equation (9)) which is highly related to the property of the state of equilibria will be affected by the change of endogenous variables.

4. Concluding remarks

The contribution of this paper is to endogenously derive a migration law to explain how workers move with microfoundation rather than explicitly apply a migration law independent of the model.

As manufacturers get concentrated in a region, the income level is much higher in the core location than in the periphery. The difference between income in core and periphery regions increases with the degree of agglomeration. The average income is higher in the core-periphery structure than in a dispersed pattern. However, increase of regional disparities may cause impoverishment of the peripheral region.

The simulated result supports the idea that agglomeration and growth reinforce each other. Nevertheless, inter-regional integration may benefit only the core region, i.e. increased income of the core region may come at the expense of the peripheral region. In general, the core region benefits from agglomeration. On the contrary, the periphery is better off in a more dispersed pattern. Economic development of the core region does not necessarily benefit the whole region. This result is valuable for policy consideration. It would be interested to analyze further the influences of economic development of the core region conditional on different situation in future work.

Inclusion of non-market spatial agglomeration enhances the centripetal forces, which further leads the system to a core-periphery pattern. The functional relation between the choice probability and the manufacture share is affected by the value of parameters. Different value of parameters leads to different relation between choice probability and manufacture share. A robust and systematic simulation criteria and evidence from the real world survey will be future direction to extend this work.

Appendix: The original core-periphery model (Fujita et al., 1999)

Utility of consumer is assumed in Cobb-Douglas form of two kinds of goods:

$$U = M^{u} A^{1-u}, \quad \cdots\cdots (A.1)$$

Where *M* is composite manufactured goods, and *A* is the agricultural goods. μ is a constant representing the expenditure share of manufactured goods.

$$M = \left[\int_0^n m(i)^{\rho} di \right]^{1/\rho}, \qquad 0 < \rho < 1, \quad \cdots\cdots (A.2)$$

The parameter ρ represents the intensity of the preference for variety in manufactured goods. A smaller value of ρ implies more desire for variety of manufactured goods. The budget constraint for consumer is the follows:

$$p^A A + \int_0^n p(i)m(i)di = Y, \quad \text{(A.3)}$$

where Y is income and p^A is the price of agricultural good.

Worker utility maximization solved the indirect utility function:

$$V_i = u^u (1-u)^{1-u} Y_i G_i^{-u} (p^A) - (1-u) \quad \text{(A.4)}$$

where Y_{it} is income and p^A is the price of agricultural good, G_{it} is the price index.

Firm's production function is assumed that to quantity q of manufacturing goods requires labor input l, as follows.

$$l = F + cq \quad \text{(A.5)}$$

where F indicates fixed inputs and c_i is marginal input requirement. The profit is given by

$$\pi = pq - w(F + cq), \quad \text{(A.6)}$$

The dynamic process used in the model is the "replicator dynamics" from the evolutionary game theory.

$$\dot{X}_i = r(\omega_i - \varpi) X_i \quad \text{(A.7)}$$

Parameter r denotes the adjustment speed, and does not vary by location. In the most general form, this parameter equals one.

The solutions from consumer's utility maximization and producer's profit maximization derive the following four endogenous variables.

Income is:

$$Y_i = uX_i W_i + (1-u)\phi_i \quad \text{(A.8)}$$

where W is the nominal wage rate, and ϕ is the exogenous region share.

Price index is:

$$G_i = [\sum_j X_j (WT_{ji})^{1-\sigma}]^{1/1-\sigma} \quad \text{(A.9)}$$

where T_{ji} represents the transport cost for moving goods between location *i* and *j*. Parameter $\sigma = 1/(1-\rho)$, represents the elasticity of substitution between any two varieties; a larger value of σ implies higher degree of substitution.

The nominal wage is:

$$W_i = [\sum_j Y_j T_{ij}^{1-\sigma} G_j^{\sigma-1}]^{1/\sigma} \quad \text{(A.10)}$$

Real wage is:

$$w_i = W_i G_i^{-u} \quad \text{(A.11)}$$

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Table 1 The regression result of the log (Rank) versus the log (Share)

TC	Estimated Slope	R-square
1.1	-0.21	0.89
1.5	-0.08*	0.97
1.6	-0.11	0.95
1.7	-0.11*	0.96
1.8	-0.11	0.96
1.9	-0.15*	0.97
2.0	-0.31	0.84
2.1	-0.33*	0.93
2.2	-0.52	0.92
2.3	-0.81*	0.98
2.4	-1.16	0.98
2.5	-1.82	0.90
2.6	-2.34*	0.92

Source: Calculations by author.

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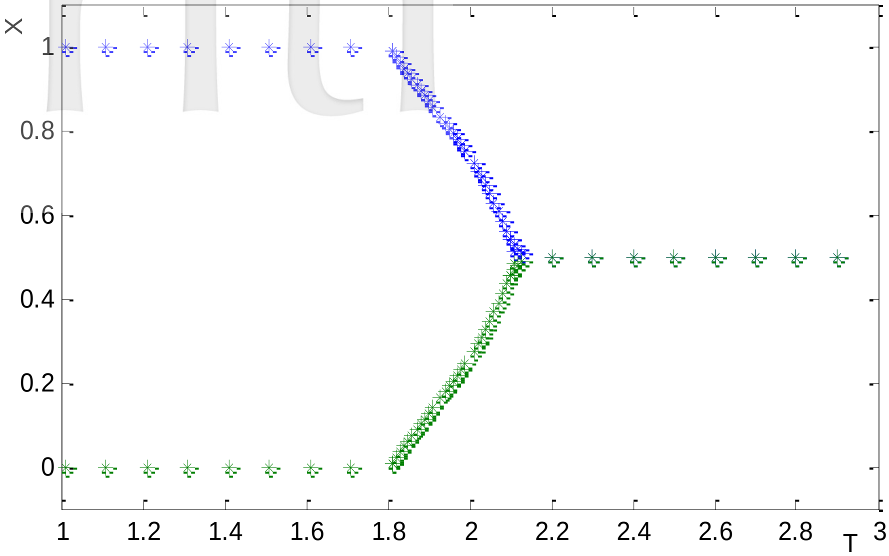

Figure 1 The limiting manufacture share of Case 1 for σ = 5 and μ =0.4

Sourse: Simulation by author.

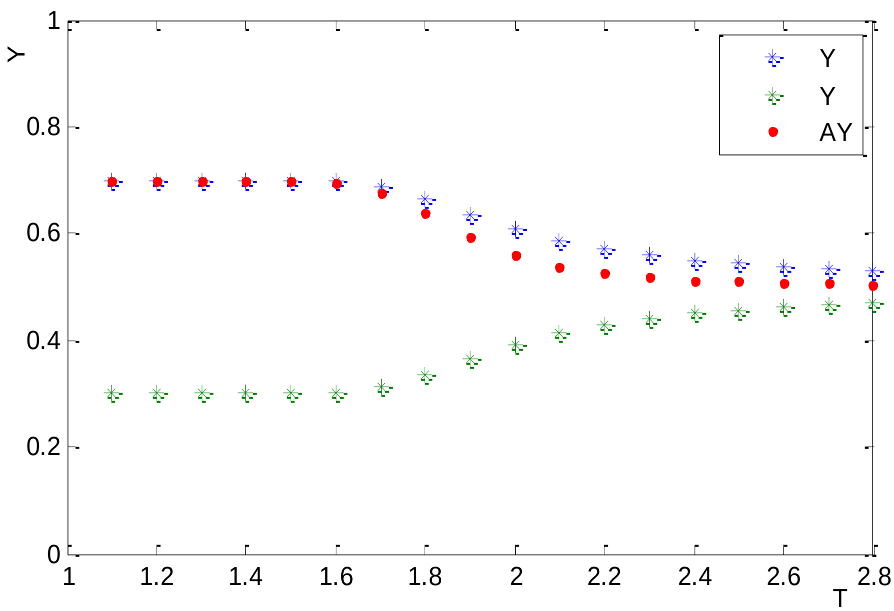

Figure 2 The limiting income and average income at t=100

Sourse: Simulation by author.

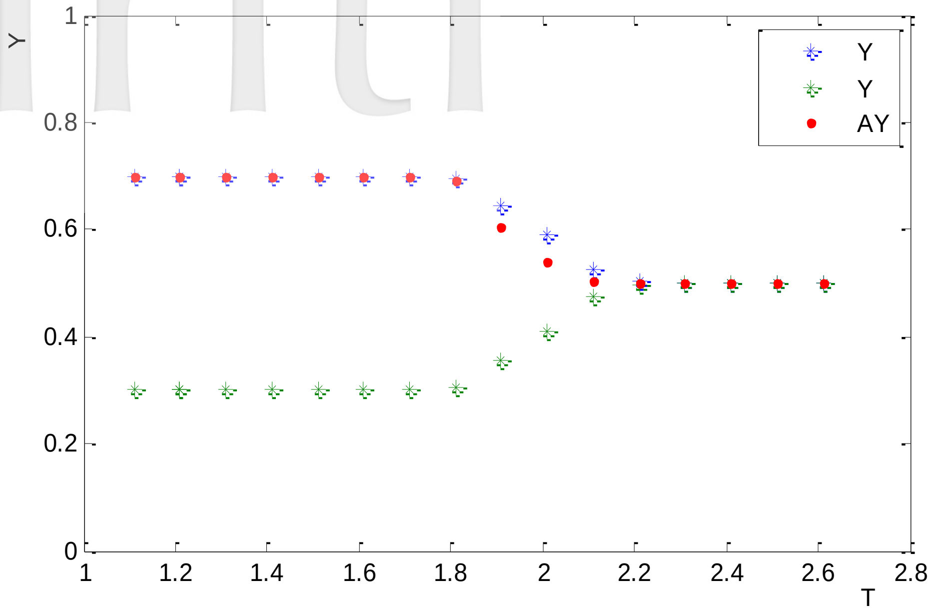

Figure 3 The limiting income and average income at t=1000

Sourse: Simulation by author.

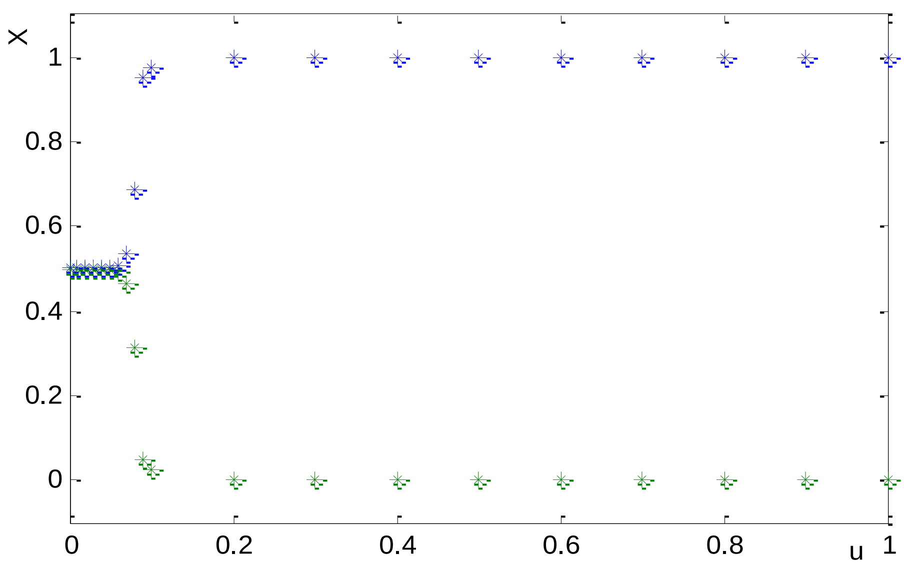

Figure 4 The limiting manufacture share of Case 1 for σ = 5 and T =1.1

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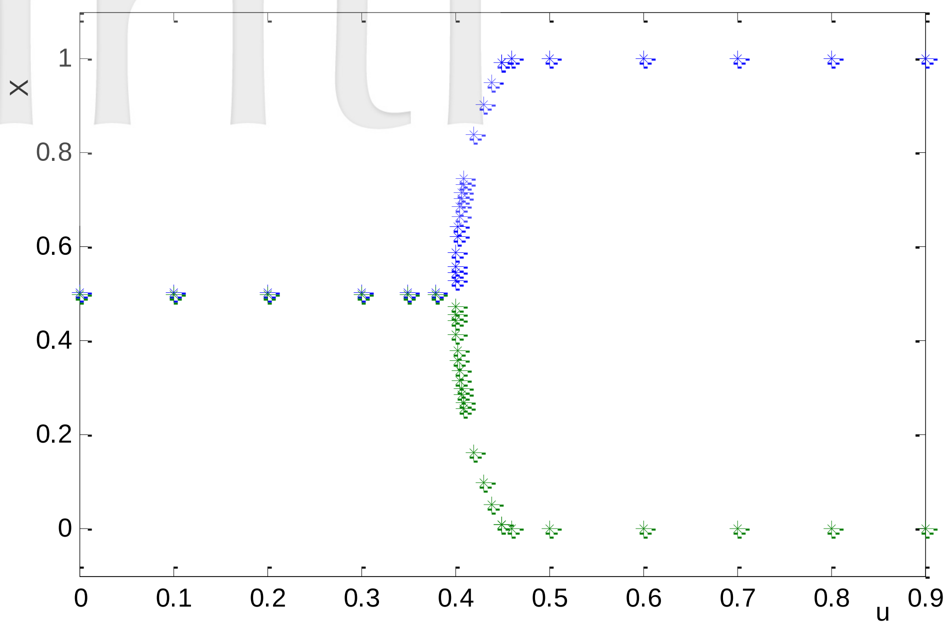

Figure 5 The limiting manufacture share of Case 1 for σ = 5 and T =2.1

Sourse: Simulation by author.

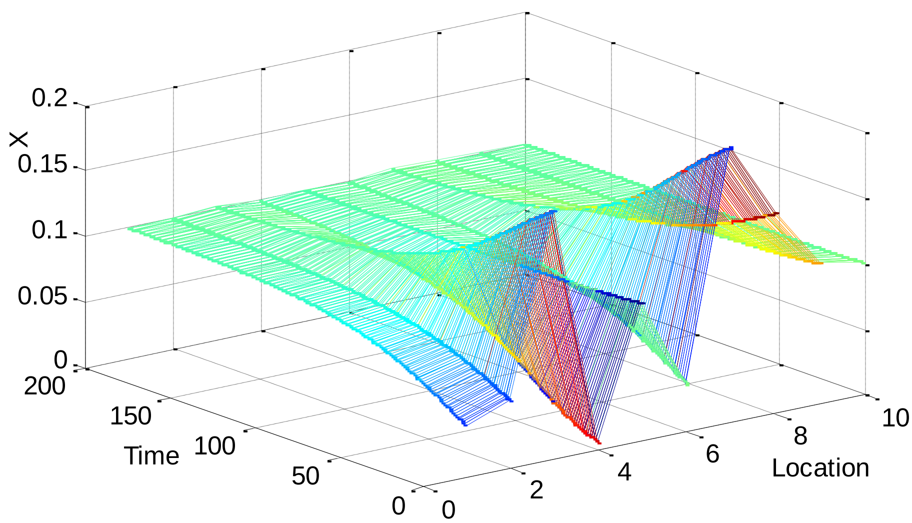

Figure 6 The limiting manufacture share of Case 1 with ten regions for σ = 5 and T =2.4

Sourse: Simulation by author.

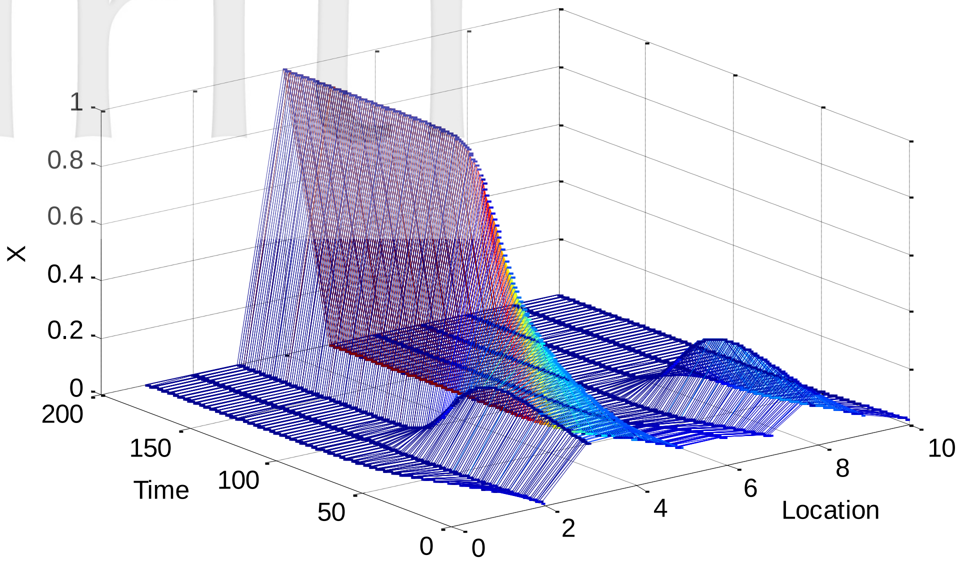

Figure 7 The limiting manufacture share of Case 1 with ten regions for σ = 5 and T =1.7

Sourse: Simulation by author.

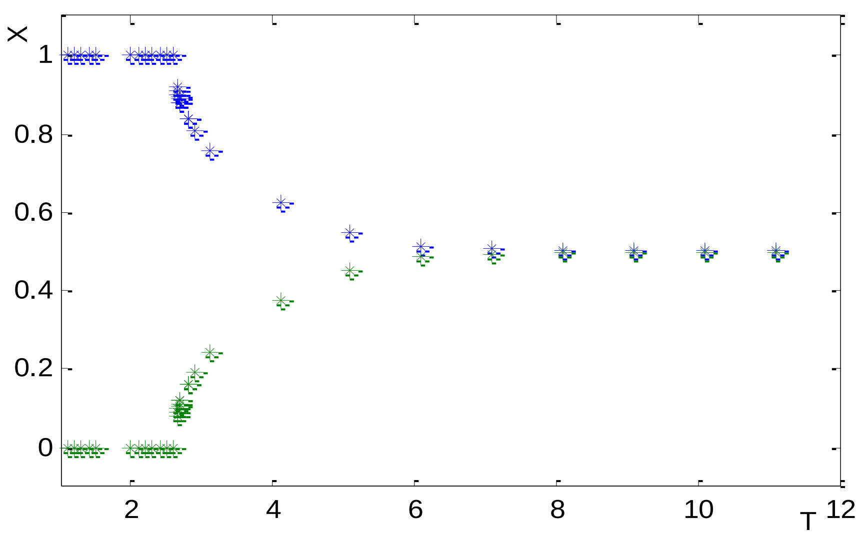

Figure 8 The limiting manufacture share of Case 2 for σ = 5 and μ =0.4

Sourse: Simulation by author.

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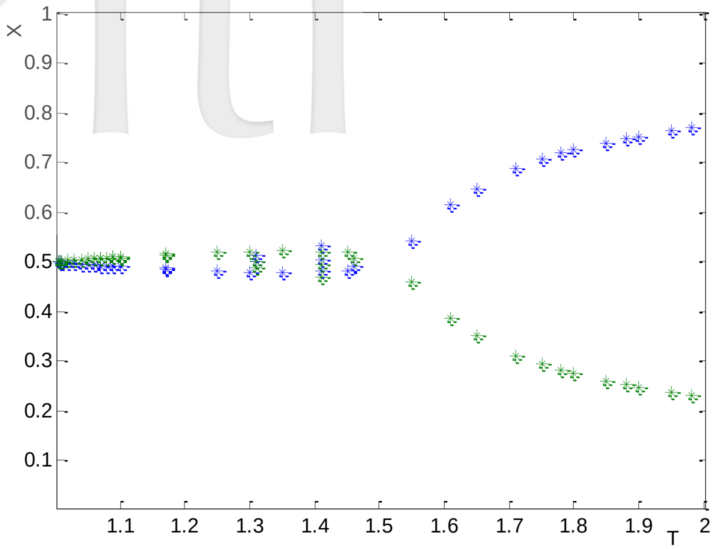

Figure 9 The limiting manufacture share of Case 3 (polya process) for σ = 5 and μ =0.85

Sourse: Simulation by author.

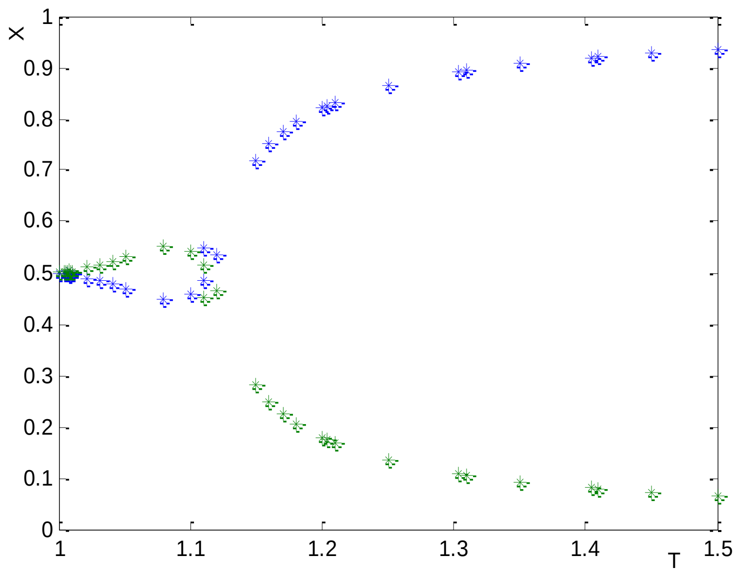

Figure 10 The limiting manufacture share of Case 3 (polya process) for σ = 5 and μ =0.95

Sourse: Simulation by author.

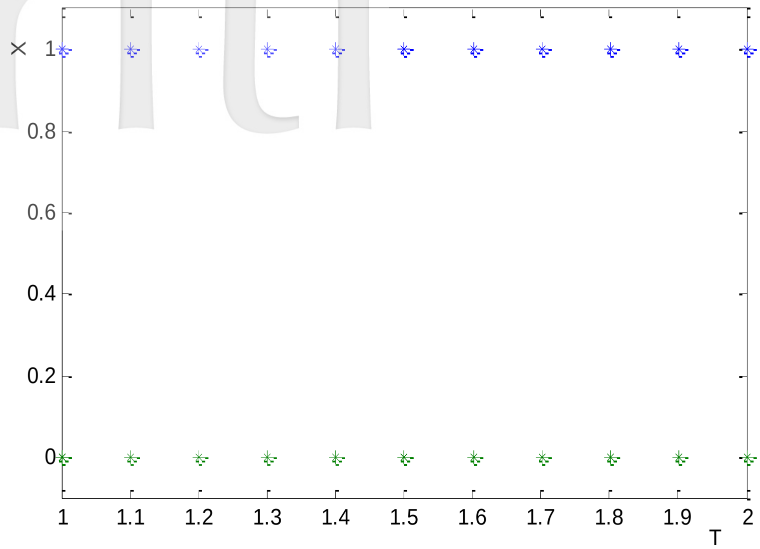

Figure 11 The limiting manufacture share of Case 3 (utility) for σ = 5 and μ =0.4

Sourse: Simulation by author.

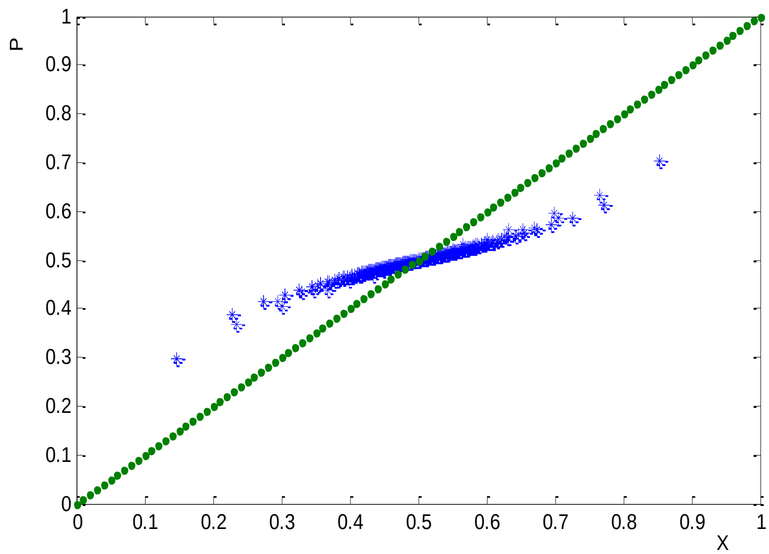

Figure 12 The choice probability versus manufacture share of Case 3 (polya process) for σ = 5, T =4.2 and μ =0.25

Sourse: Simulation by author.

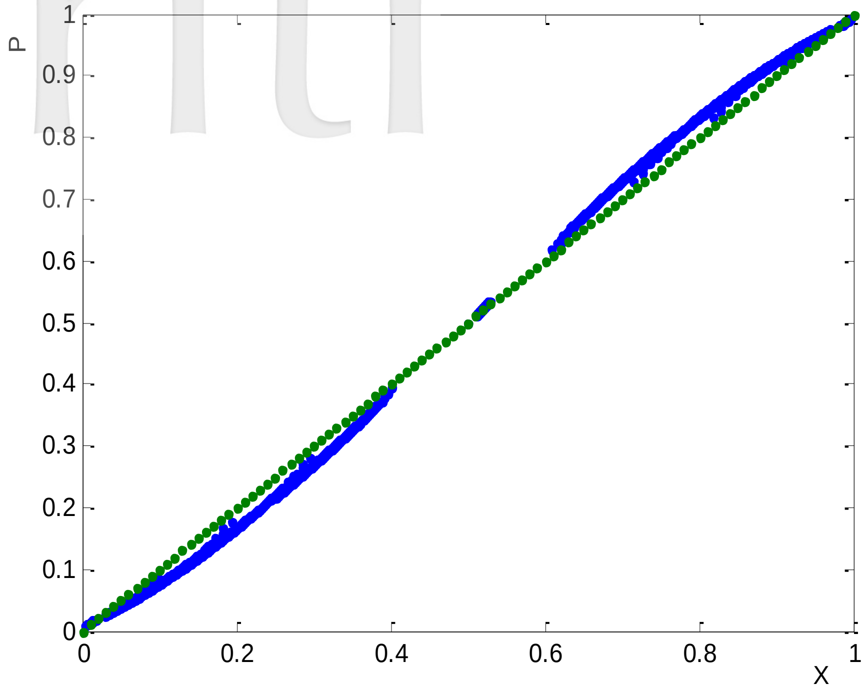

Figure 13 The choice probability versus manufacture share of Case 3 (polya process) for σ = 5, T =3.1 and μ =0.95

Sourse: Simulation by author.