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# Insurance bargaining under ambiguity

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## HIGHLIGHTS

- We examine both cooperative and non-cooperative insurance bargaining games.
- In the presence of ambiguity, full coverage is optimal.
- The optimal premium is higher in the presence than in the absence of ambiguity.
- The optimal premium will increase with the degree of ambiguity aversion.
- The optimal premium will increase with an increase in ambiguity.

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## 1. Introduction

Both cooperative and non-cooperative bargaining between insurance companies and clients are commonly observed in reality. One case for cooperative bargaining is that the insurance companies and their clients are in the same conglomerate. These insurance companies have interlocking business relationships with the firms in the same group due to top-down management, centralized control, or equity ownership connections. The insurance companies and their clients negotiate over the terms of the insurance and seek to draw up contracts which can benefit both parties. Another case for non-cooperative bargaining is that the insurance company could settle the property and casualty insurance contract with a

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## ABSTRACT

This paper investigates the effects of an increase in ambiguity aversion and an increase in ambiguity in an insurance bargaining game with a risk-and-ambiguity-neutral insurer and a risk-and-ambiguity-averse client. Both a cooperative and a non-cooperative bargaining game are examined. We show that, in both games, full coverage is optimal in the presence of ambiguity, and that the optimal premium is higher in the presence of ambiguity than in the absence of it. Furthermore, the optimal premium will increase with both the degree of ambiguity aversion and an increase in ambiguity.

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large corporation, or the unemployment insurance with a union through bargaining. Therefore, analysis under a bargaining context is important and deserves attention in insurance.

Kihlstrom and Roth (1982) were the first to analyze the Nash equilibrium of a cooperative bargaining game between a riskneutral insurance company and a risk-averse client. They found that the optimal insurance contract is a full-coverage one. In addition, they found that the more risk-averse the client is, the higher the premium that he/she will pay.<sup>1</sup> Schlesinger (1984) further generalized Kihlstrom and Roth's (1982) model and obtained similar results. Recently, Viaene et al. (2002) proposed a sequential



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<sup>&</sup>lt;sup>1</sup> Some papers have obtained different results under different frameworks. For example, Safra and Zilcha (1993) found that this result does not necessarily hold under non-expected utility preferences such as the rank-dependent utility preference and the weighted utility preference. Volij and Winter (2002) arrived at an opposite result by using Yaari's dual theory.

bargaining game and found that the insurance company obtains a higher premium when the client has a lower discount factor.<sup>2</sup> Quiggin and Chambers (2009) studied the interaction between bargaining power and the efficiency of insurance contracts, and found that an increase in the bargaining power of the clients will increase social welfare.

In this paper, we extend this line of the literature by considering the impact of ambiguity and ambiguity aversion in insurance bargaining games. Ambiguity describes a case where a decision maker is uncertain about the payoff probability which affects his/her decisions, and ambiguity aversion is an aversion to such an uncertainty. The literature has demonstrated that ambiguity and ambiguity aversion could have significant effects on individuals' decisions under risk.<sup>3</sup> Regarding insurance, the demand for insurance and the design of insurance contracts will be different in the presence of ambiguity from in the absence of it. For example, Snow (2011) proved that the demands for both self-insurance and self-protection increase with ambiguity aversion.<sup>4</sup> Although the above literature has provided many fruitful findings, these papers all focus on a non-bargaining-based context. To the best of our knowledge, our paper is the first to examine insurance bargaining under ambiguity.

Specifically, we respectively investigate the effects on the negotiation outcomes of an increase in ambiguity aversion and an increase in ambiguity in two-player cooperative and noncooperative insurance bargaining games. For the cooperative bargaining game, we follow the framework of Kihlstrom and Roth (1982). For the non-cooperative bargaining game, we consider sequential bargaining games as modeled in Rubinstein (1982) and White (2008).<sup>5</sup> In both games, we analyze the case where the insurance company and the client negotiate on the insurance coverage and the premium.

As in Alary et al. (2013) and Gollier (2013), we assume that the insurance company is risk and ambiguity neutral. The client is assumed to be not only risk averse, but also ambiguity averse. To model ambiguity aversion, the literature has provided several approaches.<sup>6</sup> In this paper, we employ Klibanoff et al.'s (2005) smooth model of ambiguity aversion.<sup>7</sup> Their model can separate the ambiguity preferences and the ambiguous beliefs, and thus help us to

discuss the effects of an increase in ambiguity aversion and an increase in ambiguity on the bargaining outcome.

In cooperative and non-cooperative bargaining games, we find that full coverage is optimal. This result shows that the optimal full coverage first found by Kihlstrom and Roth (1982) is robust in the presence of ambiguity. Moreover, the optimal premium is found to be higher in the presence of ambiguity than in the absence of ambiguity. We further find that the impacts on the premium of an increase in ambiguity aversion and an increase in ambiguity are robust in both types of bargaining game: (1) an increase in the client's degree of ambiguity aversion increases the optimal premium; (2) the optimal premium becomes higher when an increase in ambiguity occurs.

The rest of the paper is organized as follows. Section 2 first studies a cooperative insurance bargaining game under ambiguity, and then examines the impact on the bargaining outcomes of an increase in ambiguity aversion and an increase in ambiguity. Section 3 examines identical questions, but uses a non-cooperative insurance bargaining model. Finally, Section 4 concludes the paper, and appendices provide proofs of lemmas.

#### 2. A cooperative insurance bargaining game

This section consists of two subsections. In the first subsection, a cooperative insurance bargaining model is introduced to investigate what the optimal insurance contract will be in the presence of ambiguity. In the subsequent subsection, we respectively examine the effects on the optimal insurance contract of an increase in ambiguity aversion and an increase in ambiguity.

#### 2.1. The presence of ambiguity

The model setting and the notation are as follows. Suppose that there are two agents in an economy. One is a risk-and-ambiguityneutral insurance company endowed with  $\omega_I$  and the other is a risk-and-ambiguity-averse client endowed with  $\omega_C$ . The client has a potential loss *L*, and its probability of occurrence is  $1-\pi \in (0, 1)$ . On  $\pi$ , the client has a subjective belief following an *F* distribution. This is common knowledge between the client and the insurance company. To isolatedly evaluate the effect of the presence of ambiguity, for simplicity, it is assumed that the insurance company prices the contract by the unbiased ambiguous belief, i.e.,

$$\alpha = \int_0^1 \pi \, dF(\pi). \tag{1}$$

To hedge the risk, the client negotiates with the insurance company. The negotiation could turn out to be successful or it could break down. If it is successful, the two agents will sign an insurance contract and simultaneously determine the terms of the insurance contract  $C = \{P, Q\}$ , where *P* is the insurance premium and  $Q \in [0, L]$  is the coverage. However, if the negotiation breaks down, no insurance contract will be agreed upon.

To model the decision making under ambiguity, we adopt the smooth model of ambiguity aversion proposed by Klibanoff et al. (2005). Under their model, the expected utility of a decision maker facing ambiguity in the insurance bargaining game can be obtained in two steps. The first step is to compute all the expected utilities under a specific belief of the loss probability. The second step is to obtain the expected utility under ambiguity by

<sup>&</sup>lt;sup>2</sup> Viaene et al. (2002) described the effect of a lower discount factor as the effect of more risk aversion or more impatience.

<sup>&</sup>lt;sup>3</sup> For example, Epstein and Schneider (2008) found that, when the reliability of information quality is uncertain, ambiguity-averse investors require more excess returns for poor signals, especially when fundamentals are volatile. Gollier (2011) showed that a more ambiguity-averse agent will demand fewer ambiguous assets when the distribution of a risky asset's return is uncertain. He further demonstrated that an increase in ambiguity aversion raises equity premiums when the distribution of states is uncertain.

<sup>&</sup>lt;sup>4</sup> Alary et al. (2013), who considered more than two states of Nature, investigated the effect of ambiguity aversion on self-insurance and self-protection. They showed that, under certain conditions, ambiguity aversion increases the demand for self-insurance but decreases the demand for self-protection. Gollier (2013) studied the effect of ambiguity aversion on the optimal insurance contract and found that, under different ambiguity structures, ambiguity aversion results in different optimal insurance contracts. Huang (2012) examined the impact of ambiguity aversion on effort when either the target wealth distribution or the initial wealth distribution is ambiguous. She found that a decision maker with greater ambiguity aversion will make more effort when the target distribution is ambiguous, but may make less effort when the starting distribution is ambiguous.

<sup>&</sup>lt;sup>5</sup> Our model is similar to that in Viaene et al. (2002), but it differs in two ways. First, they did not consider the effect of ambiguity. Second, they assumed that both parties only bargain on the premium rate, whereas we assume that both parties can bargain on the premium and the coverage.

<sup>&</sup>lt;sup>6</sup> For example, the maxmin expected utility model (Gilboa and Schmeidler, 1989), the Choquet expected utility (Schmeidler, 1989), the α-maxmin (Ghirardato et al., 2004), and the smooth model of ambiguity aversion (Klibanoff et al., 2005).

 $<sup>^7</sup>$  Klibanoff et al. (2005) set up a two-stage model in which they decomposed the decision process into risk and ambiguity: the "expected utility" of an

ambiguity-averse agent is the expected ambiguity function over the ambiguous beliefs, and the ambiguity function is a concave function of the traditional expected utility over risk. The ambiguity function captures the attitude related to ambiguity aversion and the distribution of ambiguous beliefs captures ambiguity.

transforming each expected utility obtained in the previous step with an increasing function and then computing the expectation of the transformed expected utility given the subjective distribution of the loss probability. Thus, the decision maker's expected utility under ambiguity can be expressed as

$$\Phi = \int \phi \left( \mathsf{E} U \left( \pi \right) \right) dF \left( \pi \right),$$

where  $\Phi$  is the expected utility under ambiguity, EU is the expected utility given the value of  $\pi$ , and the function  $\phi$  captures the decision maker's attitude toward ambiguity with  $\phi' > 0$ . When  $\phi$  is linear, the decision maker is ambiguity neutral, and when  $\phi'' < 0$ , the decision maker is ambiguity averse. It is noted that an important characteristic of their model is that the effect of ambiguity aversion (represented by the shape of the ambiguity function  $\phi$ ) and the effect of ambiguity (represented by the distribution of ambiguous beliefs *F*) can be separated, which helps us to respectively explore the effects of an increase in ambiguity aversion and an increase in ambiguity in the later subsection.

The utility of an ambiguity-averse client should be reduced to the expected utility when there is no ambiguity. To make this hold, we take an inverse function of the ambiguity function based on the setting in Klibanoff et al. (2005) as in Treich (2009), Gollier (2011), and Alary et al. (2013). As a result, the client's utility function could be expressed as

$$\Phi_{\mathcal{C}}(P,Q;F) = \phi^{-1} \left[ \int \phi \left( \pi u \left( \omega_{\mathcal{C}} - P \right) + (1-\pi) u \left( \omega_{\mathcal{C}} - P - L + Q \right) \right) dF(\pi) \right], \quad (2)$$

when there is an insurance agreement, and

$$\Phi_{C}(0, 0; F) = \phi^{-1} \left[ \int \phi \left( \pi u \left( \omega_{C} \right) + (1 - \pi) u \left( \omega_{C} - L \right) \right) dF(\pi) \right], \quad (3)$$

when there is a disagreement. In the above equations, u is the client's utility function with u' > 0 and u'' < 0, and  $\phi$  is the client's ambiguity function with  $\phi' > 0$  and  $\phi'' < 0$ . For simplicity, let us assume that  $\Phi_C(0, 0; F) = 0$ . Thus, the client's utility gain from reaching an agreement as opposed to a disagreement with the insurance company is  $\Phi_C(P, Q; F)$ .

Without losing generality, we assume that  $\omega_l = 0$ , so the gain for the insurance company from reaching an agreement as opposed to a disagreement with the client is

$$\alpha P + (1 - \alpha) (P - Q) = P - (1 - \alpha) Q.$$
(4)

Now, let us introduce the cooperative insurance bargaining model. Kihlstrom and Roth (1982), who adopted Nash's solution (1950), have proposed that, in a cooperative insurance bargaining game, the insurance company and the client will jointly set up an insurance contract to maximize the social welfare function *SW*, which is the product of the utility gains from the insurance of both agents.<sup>8</sup> Therefore, the objective function is as follows:

$$\max_{P,Q} SW = [P - (1 - \alpha) Q]^{\beta} \times [\Phi_{C} (P, Q; F)]^{1 - \beta},$$
(5)

where  $\beta$  and  $1 - \beta$  denote the bargaining power of the insurer and the client respectively, and  $\beta \in (0, 1)$ . The corresponding first-order conditions (FOCs) are

$$\frac{\partial SW}{\partial P} = \beta \left[ P - (1 - \alpha) Q \right]^{\beta - 1} \left[ \Phi_{C} \left( P, Q; F \right) \right]^{1 - \beta} + (1 - \beta) \left[ P - (1 - \alpha) Q \right]^{\beta} \left[ \Phi_{C} \left( P, Q; F \right) \right]^{-\beta} \times \frac{\partial \Phi_{C} \left( P, Q; F \right)}{\partial P} = 0,$$
(6)

and

$$\frac{SW}{\partial Q} = -(1-\alpha)\beta \left[P - (1-\alpha)Q\right]^{\beta-1} \left[\Phi_{C}(P,Q;F)\right]^{1-\beta} + (1-\beta)\left[P - (1-\alpha)Q\right]^{\beta} \left[\Phi_{C}(P,Q;F)\right]^{-\beta} \times \frac{\partial\Phi_{C}(P,Q;F)}{\partial Q} = 0.$$
(7)

Assume that the second-order conditions (SOCs) of the objective function (5) hold, and that both agents could obtain positive utility gains from reaching an agreement to sign an insurance contract, i.e.,

$$\Phi_{C}(P,Q;F) > 0 \text{ and } P - (1-\alpha)Q > 0.$$
 (8)

Thus, there exists an optimal allocation ( $P^*$ ,  $Q^*$ ) which satisfies FOCs (6) and (7) and maximizes the social welfare. From these FOCs, we find that  $Q^* = L$ , as shown in Lemma 1. Note that, in the absence of ambiguity, full coverage is optimal,<sup>9</sup> which is the result found by Kihlstrom and Roth (1982).

**Lemma 1.** In a cooperative bargaining game with an ambiguous loss probability, a risk-and-ambiguity-neutral insurance company and a risk-and-ambiguity-averse client will settle on full coverage, i.e.,  $Q^* = L$ .

## **Proof.** Please see Appendix A. ■

Kihlstrom and Roth (1982) have pointed out that full coverage is optimal in a cooperative insurance bargaining game when the client is risk averse and the insurer is risk neutral. Lemma 1 indicates that introducing ambiguity and ambiguity preferences for the client does not change their findings. The intuition for Lemma 1 is similar to the intuition for Kihlstrom and Roth (1982). From the client's side, since Klibanoff et al.'s (2005) smooth model sets the ambiguity function as an "expected-utility-like functional form" (Baillon et al., 2011), the ambiguity-averse decision maker would prefer a mean-preserving contraction in terms of the expected utility value. This characteristic is similar to the characteristic of a risk-averse decision maker who would prefer a mean-preserving contraction in terms of the payoff. Hence, a fullcoverage contract which can equalize the payoffs and the expected utility values for different states is preferred by the client. From the insurer's view, since the insurer is risk and ambiguity neutral, he/she is willing to take all the risk and the ambiguity in order to obtain the premium. Therefore, under the unbiased ambiguous beliefs assumption, we will find full coverage to be optimal, as in Kihlstrom and Roth (1982). In addition, from the proof of Lemma 1, we also find that the result has no relation to the bargaining power β.

Although introducing ambiguity does not affect the optimal coverage, it does affect the optimal premium. We find that an ambiguity-averse client will pay a higher premium in the presence of ambiguity than in the absence of it, which is shown in the following lemma.

<sup>&</sup>lt;sup>8</sup> Nash (1950) proposed that this methodology can be applied to find the solution to a bargaining game when the model satisfies the following four properties: Pareto optimality, symmetry, the independence of irrelevant alternatives, and the independence of equivalent utility representatives. Since our model possesses these four properties as in Kihlstrom and Roth (1982)'s model, we adopt the same approach.

 $<sup>^9\,</sup>$  The result can be obtained by assuming that  $\phi$  is linear.

**Lemma 2.** In a cooperative bargaining game, a risk-and-ambiguityneutral insurance company and a risk-and-ambiguity-averse client will settle on a higher premium in the presence of ambiguity than in the absence of ambiguity.

#### **Proof.** Please see Appendix B.

Snow (2010) has shown that introducing a mean-preserving spread of the beliefs will reduce the utility of an ambiguity-averse individual. In other words, for the ambiguity-averse client, the utility in the presence of ambiguity is lower than that in the absence of ambiguity. A full-coverage contract eliminates the ambiguity, thereby avoiding the reduction in the client's utility. As a result, the client is willing to pay a higher premium for a full-coverage contract in the presence of ambiguity than in the absence of ambiguity, which leads to the result in Lemma 2.

#### 2.2. An increase in ambiguity aversion and an increase in ambiguity

Let us first analyze the effect of an increase in ambiguity aversion and then analyze the effect of an increase in ambiguity. Let  $\psi = h(\phi)$ , where h' > 0 and h'' < 0. Since  $\psi$  is a concave transformation of  $\phi$ , as defined in Klibanoff et al. (2005), a client with ambiguity function  $\psi$  will be more ambiguity averse than a client with ambiguity function  $\phi$ . Although we have taken an inverse function of the ambiguity function based on the setting in Klibanoff et al. (2005), the way in which we compare the ambiguity attitudes between individuals does not change.

Let  $P_{\psi}^*$  and  $P_{\phi}^*$  denote the optimal premiums under ambiguity function  $\psi$  and ambiguity function  $\phi$ , respectively. The following proposition indicates that an increase in ambiguity aversion will increase the optimal premium.

**Proposition 1.** The optimal premium will be higher if a risk-andambiguity-averse client becomes more ambiguity averse.

**Proof.** Let  $SW_{\psi}$  denote the social welfare function when the client's ambiguity function is  $\psi$ . Because the SOCs hold,  $P_{\psi}^* \ge P_{\phi}^*$  if and only if

$$\frac{\partial SW_{\psi}}{\partial P}\Big|_{P_{\phi}^{*}} = \beta \left[P_{\phi}^{*} - (1-\alpha)L\right]^{\beta-1} \\
\times \left[u\left(\omega_{C} - P_{\phi}^{*}\right) - \Psi_{C}\left(0,0;F\right)\right]^{1-\beta} \\
- (1-\beta)\left[P_{\phi}^{*} - (1-\alpha)L\right]^{\beta} \\
\times \left[u\left(\omega_{C} - P_{\phi}^{*}\right) - \Psi_{C}\left(0,0;F\right)\right]^{-\beta}u'\left(\omega_{C} - P_{\phi}^{*}\right) \\
\geq 0,$$
(9)

where  $\Psi_C(0, 0; F)$  denotes the utility under ambiguity function  $\psi$  when there is no insurance.

From the FOC (Eq. (6)) evaluated at full coverage, Eq. (9) can be written as

$$\beta \left[ P_{\phi}^{*} - (1 - \alpha)L \right]^{\beta - 1} \\ \times \left[ \left( u \left( \omega_{C} - P_{\phi}^{*} \right) - \Psi_{C} \left( 0, 0; F \right) \right)^{1 - \beta} - u^{1 - \beta} \left( \omega_{C} - P_{\phi}^{*} \right) \right] \\ \geq (1 - \beta) \left[ P_{\phi}^{*} - (1 - \alpha)L \right]^{\beta} u' \left( \omega_{C} - P_{\phi}^{*} \right) \\ \times \left[ \left( u \left( \omega_{C} - P_{\phi}^{*} \right) - \Psi_{C} \left( 0, 0; F \right) \right)^{-\beta} - u^{-\beta} \left( \omega_{C} - P_{\phi}^{*} \right) \right].$$

As  $P_{\phi}^* - (1 - \alpha)L$  is positive,  $\beta \in (0, 1)$ , and u' > 0, the above condition holds if  $\Psi_{\mathcal{C}}(0, 0; F) \leq 0$ .

Let  $y(\phi)$  denote the willingness to pay of a client with ambiguity function  $\phi$  to eliminate ambiguity  $F(\pi)$ , i.e.,

$$\alpha u \left(\omega_{\mathsf{C}} - y(\phi)\right) + (1 - \alpha) u \left(\omega_{\mathsf{C}} - L - y(\phi)\right)$$
  
=  $\phi^{-1} \left( \int \phi \left(\pi u \left(\omega_{\mathsf{C}}\right) + (1 - \pi) u \left(\omega_{\mathsf{C}} - L\right)\right) dF(\pi) \right).$ 

Thus, we have

 $\Psi_{C}(0,0;F)$ 

$$= \psi^{-1} \left[ \int \psi (\pi u (\omega_{C}) + (1 - \pi) u (\omega_{C} - L)) dF (\pi) \right] \\ = \psi^{-1} \left[ \int h (\phi (\pi u (\omega_{C}) + (1 - \pi) u (\omega_{C} - L))) dF (\pi) \right] \\ \leq \psi^{-1} \left[ h \left( \int \phi (\pi u (\omega_{C}) + (1 - \pi) u (\omega_{C} - L)) dF (\pi) \right) \right] \\ = \psi^{-1} [h (\phi (\alpha u (\omega_{C} - y (\phi)) \\ + (1 - \alpha) u (\omega_{C} - L - y (\phi))))] \\ = \alpha u (\omega_{C} - y (\phi)) + (1 - \alpha) u (\omega_{C} - L - y (\phi)) \\ = \phi^{-1} \left[ \int \phi (\pi u (\omega_{C}) + (1 - \pi) u (\omega_{C} - L)) dF (\pi) \right] = 0,$$

where the second line follows from the definition of  $\psi$ , the third line follows from Jensen's inequality, the fourth line follows from the definition of  $y(\phi)$ , the fifth line follows from the property of the inverse function, and the last line follows from the definition of  $y(\phi)$  and the assumption that  $\Phi_C(0, 0; F) = 0$ .

The intuition underlying Proposition 1 is as follows. When the client becomes more ambiguity averse, he/she is willing to pay more for the elimination of uncertainty regarding ambiguous beliefs (Snow, 2010). An increase in the premium will increase the insurer's gain from bargaining. As a result, both parties will negotiate for a higher premium to make them better off.

Now suppose that an increase in ambiguity occurs. The distribution of the client's ambiguous beliefs shifts from F to G. As noted by Snow (2010, 2011), due to the unbiased assumption (Eq. (1)), an increase in ambiguity is a mean-preserving spread on the distribution of the no-loss probability, i.e., G is a mean-preserving spread of F, which is defined as follows.

**Definition 1.** A distribution *G* is a mean-preserving spread of the distribution *F* (written as "*G* MPS *F*") if

$$F^{(2)}(\pi) \le G^{(2)}(\pi), \quad \forall \pi, \text{ and } \int \pi \, dF(\pi) = \int \pi \, dG(\pi),$$

where  $F^{(2)}(\pi) = \int_0^{\pi} F(t) dt$  and  $G^{(2)}(\pi) = \int_0^{\pi} G(t) dt$ .

Since these two distributions have the same mean, full coverage is still optimal. The following proposition demonstrates the effect of an increase in ambiguity on the optimal premium.

**Proposition 2.** If G MPS F, then the optimal premium under F will be lower than the optimal premium under G for all risk-and-ambiguity-averse individuals.

**Proof.** Since the SOCs hold, the optimal premium under  $F(P_F^*)$  will be lower than the optimal premium under  $G(P_G^*)$  if and only if

$$\frac{\partial SW}{\partial P}\Big|_{G} = \beta \left[P_{F}^{*} - (1 - \alpha)L\right]^{\beta - 1} \left[u\left(\omega_{C} - P_{F}^{*}\right) - \Phi_{C}\left(0, 0; G\right)\right]^{1 - \beta} - (1 - \beta)\left[P_{F}^{*} - (1 - \alpha)L\right]^{\beta} \times \left[u\left(\omega_{C} - P_{F}^{*}\right) - \Phi_{C}\left(0, 0; G\right)\right]^{-\beta}u'\left(\omega_{C} - P_{F}^{*}\right) \ge 0. \quad (10)$$

From the FOC (Eq. (6)) evaluated at full coverage, Condition (10) can be rewritten as

$$\beta \left[ P_F^* - (1 - \alpha) L \right]^{\beta - 1} \\ \times \left[ \left( u \left( \omega_C - P_F^* \right) - \Phi_C \left( 0, 0; G \right) \right)^{1 - \beta} - u^{1 - \beta} \left( \omega_C - P_F^* \right) \right]$$

$$\geq (1 - \beta) \left[ P_F^* - (1 - \alpha) L \right]^{\beta} u' \left( \omega_C - P_F^* \right) \\ \times \left[ \left( u \left( \omega_C - P_F^* \right) - \Phi_C \left( 0, 0; G \right) \right)^{-\beta} - u^{-\beta} \left( \omega_C - P_F^* \right) \right]$$

Since  $P_F^* - (1 - \alpha)L$  is positive,  $\beta \in (0, 1)$ , and u' > 0, the above condition is satisfied as long as  $\Phi_C(0, 0; G) \le 0$ . Since  $\Phi_C(0, 0; F) = 0$ ,  $P_F^* \le P_G^*$  if

$$\int \phi \left(\pi u \left(\omega_{C}\right) + (1 - \pi) u \left(\omega_{C} - L\right)\right) \left[dF(\pi) - dG(\pi)\right] \ge 0. (11)$$

Integrating the above equation by parts yields

$$\int \phi (\pi u (\omega_{C}) + (1 - \pi) u (\omega_{C} - L)) [dF (\pi) - dG (\pi)]$$
  
= - [u (\omega\_{C}) - u (\omega\_{C} - L)]  $\int \phi' (\pi u (\omega_{C})$   
+ (1 - \pi) u (\omega\_{C} - L)) [F (\pi) - G (\pi)] d\pi  
= [u (\omega\_{C}) - u (\omega\_{C} - L)]^{2}  $\int \phi'' (\pi u (\omega_{C})$   
+ (1 - \pi) u (\omega\_{C} - L)) [F^{(2)} (\pi) - G^{(2)} (\pi)] d\pi.

Since *G* MPS *F*, by Definition 1,  $F^{(2)}(\pi) - G^{(2)}(\pi) \le 0$ ,  $\forall \pi$ . Moreover,  $[u(\omega_C) - u(\omega_C - L)]^2$  is nonnegative, and  $\phi''$  is negative because  $\phi$  is a concave function. Consequently, Eq. (11) holds.

Snow (2010) indicated that there is an increase in ambiguity if the distribution of the ambiguity beliefs has a mean-preserving spread. Proposition 2 shows that an increase in ambiguity will increase the optimal premium in our cooperative insurance bargaining game. The intuition is as follows. An ambiguity-averse individual is averse to mean-preserving spreads in the space of probabilities. Thus, the client has an incentive to pay a higher premium to eliminate such spreads, thereby settling with the insurance company on a higher premium for a full-coverage insurance contract.

#### 3. A non-cooperative bargaining game

In this section, the notation and the assumptions are the same as those in the previous section, except that both parties are performing a non-cooperative bargain.

#### 3.1. The presence of ambiguity

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The non-cooperative bargaining game is structured as the games in Rubinstein (1982) and White (2008), and is depicted in Fig. 1.

In the first period, the insurance company makes an offer that involves charging the client  $P_1$  for coverage  $Q_1$ . Assume that, as long as an offer makes an agent feel indifferent between accepting and rejecting it, he/she will accept it. After the client makes a response, Nature realizes whether a loss occurs or not. In the case where the insurance contract is agreed upon, the game ends after Nature makes a realization. The risk-and-ambiguity-neutral insurer's expected utility will be  $P_1 - (1 - \pi) Q_1$  once the client accepts the offer. The risk-and-ambiguity-averse client will receive an expected utility

$$\phi^{-1} \left[ \int \phi \Big( \pi u \, (\omega_C - P_1) + (1 - \pi) \, u \, (\omega_C - P_1 - L + Q_1) \Big) dF(\pi) \right].$$

In the case where the offer is turned down, the game will end provided that a loss occurs. The client, then, will obtain u ( $\omega_C - L$ ) due to suffering the loss L, and the insurer will maintain a zero endowment. If a loss does not occur, the game will proceed to the second period: the client's turn to make an offer.

In the second period, the client moves first to make an offer that involves paying the premium  $P_2$  to the insurer for coverage  $Q_2$ . Assume for simplicity that all the utilities across periods are determined without taking any discount into consideration, which guarantees that  $P_2$  must not be more than  $P_1$ . In the case where the offer is accepted, the game will be over after Nature moves. The client will obtain an expected utility

$$\phi^{-1} \left[ \int \phi \Big( \pi u \, (\omega_{\rm C} - P_2) + (1 - \pi) \, u \, (\omega_{\rm C} - P_2 - L + Q_2) \Big) dF(\pi) \right],$$

and the insurer will obtain  $P_2 - (1 - \pi) Q_2$ . If the offer is rejected, the procedure will be repeated as in the first period. The game will come to an end after Nature moves in the case where the offer is accepted or after a loss occurs in the case where the offer is rejected; otherwise it will not end until the two parties reach an agreement.

According to the literature (e.g., see Rubinstein, 1982; Osborne and Rubinstein, 1990; White, 2008), it is well known that the subgame perfect equilibrium of the Rubinstein bargaining game is settled such that, in the first period, the insurer will offer a contract  $(P_1, Q_1)$  which makes the client indifferent between agreeing and waiting until the next period for his/her own turn to offer a contract. When the client is offering, the client will also always offer a contract such that the insurer is indifferent between agreeing now and waiting for the next period.

The presence of ambiguity will not change the way to find the equilibrium. Accordingly, by backward induction, in period 2, the objective function for the client who offers a contract  $(P_2, Q_2)$  to the insurer is

$$\max_{P_2, Q_2} AEU_2^C = \phi^{-1} \left[ \int \phi \Big( \pi u \, (\omega_C - P_2) + (1 - \pi) \, u \, (\omega_C - P_2 - L + Q_2) \Big) dF \, (\pi) \right]$$
  
s.t.  $P_2 = \alpha P_1 - \alpha \, (1 - \alpha) \, Q_1 + (1 - \alpha) \, Q_2.$  (12)

In period 1, knowing that a contract will be offered according to the above problem, the insurer will offer a contract  $(P_1, Q_1)$  to the client such that the client will accept immediately. Therefore, the objective function for the insurer is

AFUI D (1

$$\max_{P_{1}, Q_{1}} AEU_{1}^{} = P_{1} - (1 - \alpha) Q_{1}$$
s.t.  $\phi^{-1} \left[ \int \phi \left( \pi u (\omega_{C} - P_{1}) + (1 - \pi) u (\omega_{C} - P_{1} - L + Q_{1}) \right) dF(\pi) \right]$ 

$$= \phi^{-1} \left[ \int \phi \left( \pi \left( \pi u (\omega_{C} - P_{2}) + (1 - \pi) u (\omega_{C} - P_{2} - L + Q_{2}) \right) + (1 - \pi) u (\omega_{C} - L) \right) dF(\pi) \right].$$
(13)

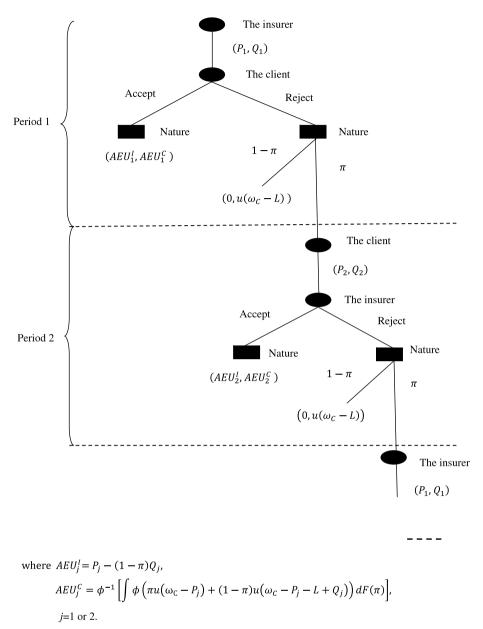


Fig. 1. The non-cooperative insurance bargaining game.

Assume that the SOCs of the above two objective functions hold. We prove that, in a non-cooperative bargaining game under ambiguity, the equilibrium will still be full coverage, which is stated in Lemma  $3.^{10}$ 

**Lemma 3.** In a non-cooperative bargaining game with an ambiguous loss probability, a risk-and-ambiguity-neutral insurance company and a risk-and-ambiguity-averse client will settle on full coverage, i.e.,  $Q_1^* = L$ .

**Proof.** Please see Appendix C.

From Lemma 3, we can find that the equilibria in both the absence of ambiguity and the presence of ambiguity are full coverage. Viaene et al. (2002) employed a full-coverage assumption in the non-cooperative bargaining game without ambiguity. Lemma 3 justifies their assumption.

The next issue we are interested in is how ambiguity will affect the premium in equilibrium under full coverage. Consistent with the result in the cooperative bargaining game, we find that a client will pay a higher premium for full coverage in the presence of ambiguity than in the absence of it in a non-cooperative bargaining game. The result is summarized in the following lemma.

**Lemma 4.** In a non-cooperative bargaining game, a risk-and-ambiguity-neutral insurance company and a risk-and-ambiguity-averse client will settle on a higher premium in the presence of ambiguity than in the absence of ambiguity.

#### **Proof.** Please see Appendix D.

The intuition for Lemmas 3 and 4 is similar to that for Lemmas 1 and 2.

<sup>&</sup>lt;sup>10</sup> This solution is similar to giving the insurer a weight of  $\beta = 1$  in the cooperative bargaining model and fixing the client's utility at the reservation utility for accepting the offer in period 1. We thank the referee for mentioning this point.

### 3.2. An increase in ambiguity aversion and an increase in ambiguity

In this subsection, we respectively ask if an increase in ambiguity aversion and an increase in ambiguity increase the premium in equilibrium in the presence of ambiguity. The effect on the premium in equilibrium of an increase in the client's ambiguity aversion is first analyzed. Suppose that the client becomes more ambiguity averse, i.e., his/her ambiguity function changes from  $\phi$ to  $\psi$ , where  $\psi = h(\phi)$ , h' > 0 and h'' < 0. The result is shown in the following proposition.

**Proposition 3.** The premium in equilibrium will be higher if a riskand-ambiguity-averse client becomes more ambiguity averse.

**Proof.** Denote  $P_1^{\phi}$  as the premium in equilibrium when the client's ambiguity function is  $\phi$  and  $P_1^{\psi}$  as the premium in equilibrium under the ambiguity function  $\psi$ . Since the SOC holds,  $P_1^{\psi} \ge P_1^{\phi}$  if and only if

$$u\left(\omega_{C}-P_{1}^{\phi}\right)-\psi^{-1}\left[\int\psi\left(\pi u\left(\omega_{C}-\alpha P_{1}^{\phi}-(1-\alpha)^{2}L\right)\right.\right.\right.\right.$$
$$\left.+\left(1-\pi\right)u\left(\omega_{C}-L\right)\right)dF\left(\pi\right)\right]\geq0.$$
(14)

The above condition can be rewritten as

$$\phi^{-1} \left[ \int \phi \left( \pi u \left( \omega_{\mathsf{C}} - \alpha P_{1}^{\phi} - (1 - \alpha)^{2} L \right) \right. \\ \left. + (1 - \pi) u \left( \omega_{\mathsf{C}} - L \right) \right) dF \left( \pi \right) \right] \\ \left. - \psi^{-1} \left[ \int \psi \left( \pi u \left( \omega_{\mathsf{C}} - \alpha P_{1}^{\phi} - (1 - \alpha)^{2} L \right) \right. \\ \left. + (1 - \pi) u \left( \omega_{\mathsf{C}} - L \right) \right) dF \left( \pi \right) \right] \ge 0.$$

Let  $s(\phi)$  be the willingness to pay of a client with ambiguity function  $\phi$  to eliminate the ambiguity such that

$$\alpha u \left( \omega_{\rm C} - \alpha P_1^{\phi} - (1-\alpha)^2 L - s(\phi) \right) + (1-\alpha) u \left( \omega_{\rm C} - L - s(\phi) \right) = \phi^{-1} \Biggl[ \int \phi \Bigl( \pi u \Bigl( \omega_{\rm C} - \alpha P_1^{\phi} - (1-\alpha)^2 L \Bigr) + (1-\pi) u \left( \omega_{\rm C} - L \right) \Bigr) dF(\pi) \Biggr].$$

Therefore, we know that

y

$$\psi^{-1} \left[ \int \psi \left( \pi u \left( \omega_{C} - \alpha P_{1}^{\phi} - (1 - \alpha)^{2} L \right) + (1 - \pi) u \left( \omega_{C} - L \right) \right) dF(\pi) \right]$$
$$= \psi^{-1} \left[ \int h \left( \phi \left( \pi u \left( \omega_{C} - \alpha P_{1}^{\phi} - (1 - \alpha)^{2} L \right) + (1 - \pi) u \left( \omega_{C} - L \right) \right) \right) dF(\pi) \right]$$

$$\leq \psi^{-1} \left[ h \left( \int \phi \left( \pi u \left( \omega_{C} - \alpha P_{1}^{\phi} - (1 - \alpha)^{2} L \right) \right) \right) \right]$$
  
+  $(1 - \pi) u \left( \omega_{C} - L \right) dF (\pi) \right) \right]$   
$$= \psi^{-1} \left[ h \left( \phi \left( \alpha u \left( \omega_{C} - \alpha P_{1}^{\phi} - (1 - \alpha)^{2} L - s (\phi) \right) \right) \right) \right]$$
  
+  $(1 - \alpha) u \left( \omega_{C} - L - s (\phi) \right) \right) \right]$   
$$= \alpha u \left( \omega_{C} - \alpha P_{1}^{\phi} - (1 - \alpha)^{2} L - s (\phi) \right)$$
  
+  $(1 - \alpha) u \left( \omega_{C} - L - s (\phi) \right)$   
$$= \phi^{-1} \left[ \int \phi \left( \pi u \left( \omega_{C} - \alpha P_{1}^{\phi} - (1 - \alpha)^{2} L \right) \right) \right]$$
  
+  $(1 - \pi) u \left( \omega_{C} - L \right) dF (\pi) \right].$ 

Accordingly,  $P_1^{\psi} \ge P_1^{\phi}$ .

When a client becomes more averse to ambiguity, we find a similar result to Proposition 1. The intuition is as follows. An increase in ambiguity aversion makes the client more averse to the uncertainty about the loss probability, and he/she is therefore willing to get rid of her uncertainty at the expense of more premiums for the full-coverage insurance contract. The insurance company, however, is unaffected, but will take advantage of the fact that the client is more ambiguity averse to propose a higher premium. Consequently, the premium in equilibrium becomes higher.

Now, let us assume that other things are equal except that the no-loss probability becomes more ambiguous for the client. For the definition of an increase in ambiguity, because of the assumption about the unbiased beliefs, we still focus on a meanpreserving spread on the distribution of the no-loss probability. In other words, the distribution shifts from F to G, where G MPS F. The following proposition demonstrates the result of an increase in ambiguity.

**Proposition 4.** If G MPS F, then the premium in equilibrium under F will be lower than the premium in equilibrium under G for all riskand-ambiguity-averse individuals.

**Proof.** Suppose that  $P_1^F$  is the premium in equilibrium when the probability of no-loss  $\pi$  follows the *F* distribution, and  $P_1^G$  is the premium in equilibrium under the *G* distribution. Because the SOC holds,  $P_1^G \ge P_1^F$  if and only if

$$u\left(\omega_{\mathsf{C}} - P_{1}^{\mathsf{F}}\right) - \phi^{-1} \left[ \int \phi\left(\pi u\left(\omega_{\mathsf{C}} - \alpha P_{1}^{\mathsf{F}} - (1 - \alpha)^{2} L\right) + (1 - \pi) u\left(\omega_{\mathsf{C}} - L\right) \right) dG(\pi) \right] \geq 0.$$
(15)

The above condition can be rewritten as

$$\int \phi \left( \pi u \left( \omega_{\mathsf{C}} - \alpha P_{1}^{\mathsf{F}} - (1 - \alpha)^{2} L \right) + (1 - \pi) u \left( \omega_{\mathsf{C}} - L \right) \right) \left[ dF \left( \pi \right) - dG \left( \pi \right) \right] \ge 0.$$
(16)

Integrating by parts yields

$$\begin{split} &\int \phi \left( \pi u \left( \omega_{\rm C} - \alpha P_1^F - (1 - \alpha)^2 L \right) + (1 - \pi) u \left( \omega_{\rm C} - L \right) \right) \\ &\times \left[ dF \left( \pi \right) - dG \left( \pi \right) \right] \\ &= - \left[ u \left( \omega_{\rm C} - \alpha P_1^F - (1 - \alpha)^2 L \right) - u \left( \omega_{\rm C} - L \right) \right] \\ &\times \int \phi' \left( \pi u \left( \omega_{\rm C} - \alpha P_1^F - (1 - \alpha)^2 L \right) \right. \\ &+ \left( 1 - \pi \right) u \left( \omega_{\rm C} - L \right) \right) \left[ F \left( \pi \right) - G \left( \pi \right) \right] d\pi \\ &= \left[ u \left( \omega_{\rm C} - \alpha P_1^F - (1 - \alpha)^2 L \right) - u \left( \omega_{\rm C} - L \right) \right]^2 \\ &\times \int \phi'' \left( \pi u \left( \omega_{\rm C} - \alpha P_1^F - (1 - \alpha)^2 L \right) \right. \\ &+ \left( 1 - \pi \right) u \left( \omega_{\rm C} - L \right) \right) \left[ F^{(2)} \left( \pi \right) - G^{(2)} \left( \pi \right) \right] d\pi \,, \end{split}$$

where  $F^{(2)}(\pi) = \int_0^{\pi} F(t) dt$  and  $G^{(2)}(\pi) = \int_0^{\pi} G(t) dt$ . Because *G* MPS *F*,  $F^{(2)}(\pi) - G^{(2)}(\pi) \leq 0$ ,  $\forall \pi$ . In addition,  $\left[u\left(\omega_C - \alpha P_1^F - (1 - \alpha)^2 L\right) - u\left(\omega_C - L\right)\right]^2$  is nonnegative and  $\phi'' < 0$ . Therefore, Eq. (16) holds.

The intuition underlying the above proposition is as follows. When G MPS F, because the client is ambiguity averse, i.e., he/she dislikes any mean-preserving spread on the probability space, he/she is willing to pay a higher premium to eliminate it. However, such an increase in ambiguity does not have any impact on the insurer. Instead, he/she will take advantage of the client's ambiguity aversion to charge him/her more premiums. Finally, they settle on more premiums for the full-coverage insurance. This result is similar to that for Proposition 2.

## 4. Conclusions

In this paper, we have analyzed the effects of an increase in ambiguity aversion and an increase in ambiguity in a cooperative and a non-cooperative bargaining model. We first find that in the two models full coverage is always optimal, regardless of an increase in ambiguity aversion or an increase in ambiguity. Furthermore, we find that the premium increases with both the degree of ambiguity aversion of the client and an increase in the ambiguity of the loss probability.

It is worth noting that we assume that the insurer is ambiguity neutral. As documented by Cabantous (2007) and Cabantous et al. (2011), the insurer could be ambiguity averse. Thus, a future study considering an ambiguity-averse insurer would be fruitful.

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#### Appendix A. Proof of Lemma 1

Rearranging the two FOCs (6) and (7) yields

$$\beta \left[ \Phi_{C} \left( P, Q; F \right) \right]^{1-\beta} \left[ P - (1-\alpha) Q \right]^{\beta-1} \\ = -(1-\beta) \left[ P - (1-\alpha) Q \right]^{\beta} \left[ \Phi_{C} \left( P, Q; F \right) \right]^{-\beta} \\ \times \frac{\partial \Phi_{C} \left( P, Q; F \right)}{\partial P},$$

and

$$(1 - \alpha) \beta \left[ \Phi_{C} (P, Q; F) \right]^{1-\beta} \left[ P - (1 - \alpha) Q \right]^{\beta-1}$$
  
=  $(1 - \beta) \left[ P - (1 - \alpha) Q \right]^{\beta} \left[ \Phi_{C} (P, Q; F) \right]^{-\beta}$   
 $\times \frac{\partial \Phi_{C} (P, Q; F)}{\partial Q}.$ 

From Condition (8), the internal solution Q\* should satisfy the following equation:

$$\frac{1}{1-\alpha} = -\frac{\frac{\partial \Phi_{\mathcal{C}}(P,Q;F)}{\partial P}}{\frac{\partial \Phi_{\mathcal{C}}(P,Q;F)}{\partial Q}}.$$

If  $Q^* = L$ , the right-hand side of the above equation can be written as

$$-\frac{\frac{\partial \Phi_{C}(P,Q;F)}{\partial P}}{\frac{\partial \Phi_{C}(P,Q;F)}{\partial P}} = \frac{1}{\int (1-\pi) \, dF(\pi)} = \frac{1}{1-\alpha}$$

which is equal to the left-hand side. Since the SOCs hold, we have  $Q^* = L$ .  $\Box$ 

## Appendix B. Proof of Lemma 2

With full coverage, FOC (6) will become

$$\frac{\partial SW}{\partial P} = \beta \left[ P - (1 - \alpha) L \right]^{\beta - 1} \left[ u \left( \omega_{C} - P \right) \right]^{1 - \beta} - (1 - \beta) \left[ P - (1 - \alpha) L \right]^{\beta} \left[ u \left( \omega_{C} - P \right) \right]^{-\beta} u' (\omega_{C} - P) = 0.$$

From Condition (8), rearranging the above equation yields

$$sign\left\{\frac{\partial SW}{\partial P}\right\}$$
$$= sign\left\{\beta\left[\frac{u\left(\omega_{C}-P\right)}{P-(1-\alpha)L}\right] - (1-\beta)u'(\omega_{C}-P)\right\}.$$
(B.1)

Since the SOC holds, the optimal premium in the presence of ambiguity is greater than that in the absence of ambiguity, which is denoted by  $\widehat{P}$  if and only if  $\frac{\partial SW}{\partial P}|_{\widehat{P}} \geq 0$ . Note that the optimal premium  $\widehat{P}$  in the absence of ambiguity satisfies the following equation:

$$\beta \left\{ \frac{u\left(\omega_{\mathcal{C}} - \widehat{P}\right) - \left[\alpha u\left(\omega_{\mathcal{C}}\right) + (1 - \alpha)u\left(\omega_{\mathcal{C}} - L\right)\right]}{\widehat{P} - (1 - \alpha)L} \right\}$$
$$= (1 - \beta)u'(\omega_{\mathcal{C}} - \widehat{P}).$$

Thus, the sign of Eq. (B.1) evaluated at  $\widehat{P}$  is

$$\operatorname{sign}\left\{\left.\frac{\partial SW}{\partial P}\right|_{\widehat{P}}\right\} = \operatorname{sign}\left\{\alpha u\left(\omega_{C}\right) + (1-\alpha)u\left(\omega_{C}-L\right)\right\},$$

which is positive, as shown by Snow (2010).  $\Box$ 

#### Appendix C. Proof of Lemma 3

Substituting the constraint into the objective function (12) yields the FOC:

$$\frac{\partial AEU_2^C}{\partial Q_2} = \frac{1}{\phi'\left(AEU_2^C\right)} \left[ \int \phi'\left(\pi u\left(\omega_C - P_2\right) + (1 - \pi\right)\right. \\ \left. \times u\left(\omega_C - P_2 - L + Q_2\right)\right) \left[ -\pi \left(1 - \alpha\right) u'\left(\omega_C - P_2\right) \right. \\ \left. + \left(1 - \pi\right) \alpha u'\left(\omega_C - P_2 - L + Q_2\right) \right] dF\left(\pi\right) \right] \\ = 0.$$

If  $Q_2 = L$ , then  $\frac{\partial AEU_2^C}{\partial Q_2}\Big|_{Q_2 = L} = 0$ ,

because  $\alpha = \int \pi \, dF(\pi)$ . Since the SOC holds, we have  $Q_2^* = L$ . Thus,

$$P_2^* = \alpha P_1 - \alpha (1 - \alpha) Q_1 + (1 - \alpha) L.$$

Replacing  $P_2$  in the objective function (13) with the above condition and assuming a Lagrange function  $\Lambda$  yields the following FOCs:

$$\frac{\partial \Lambda}{\partial P_1} = 1 - \lambda \frac{1}{\phi'(y_1)} \left[ \frac{\partial \phi(y_1)}{\partial P_1} \right] + \lambda \frac{1}{\phi'(z_1)} \left[ \frac{\partial \phi(z_1)}{\partial P_1} \right]$$
  
= 0, (C.1)

$$\frac{\partial \Lambda}{\partial Q_1} = -(1-\alpha) - \lambda \frac{1}{\phi'(y_1)} \left[ \frac{\partial \phi(y_1)}{\partial Q_1} \right] \\ + \lambda \frac{1}{\phi'(z_1)} \left[ \frac{\partial \phi(z_1)}{\partial Q_1} \right]$$

$$= 0.$$
(C2)

$$\frac{\partial \Lambda}{\partial \lambda} = -y_1 + z_1 = 0, \tag{C.3}$$

where

$$y_{1} = \phi^{-1} \left[ \int \phi \left( \pi u (\omega_{C} - P_{1}) + (1 - \pi) u (\omega_{C} - P_{1} - L + Q_{1}) \right) dF(\pi) \right],$$
  

$$z_{1} = \phi^{-1} \left[ \int \phi \left( \pi u (\omega_{C} - \alpha P_{1} + \alpha (1 - \alpha) Q_{1} - (1 - \alpha) L) + (1 - \pi) u (\omega_{C} - L) \right) dF(\pi) \right],$$

and

 $\Lambda = P_1 - (1 - \alpha) Q_1 - \lambda (y_1 - z_1).$ Rearranging Eqs. (C.1) and (C.2) yields

$$\frac{-\frac{1}{\phi'(y_1)} \left[\frac{\partial\phi(y_1)}{\partial P_1}\right] + \frac{1}{\phi'(z_1)} \left[\frac{\partial\phi(z_1)}{\partial P_1}\right]}{-\frac{1}{\phi'(y_1)} \left[\frac{\partial\phi(y_1)}{\partial Q_1}\right] + \frac{1}{\phi'(z_1)} \left[\frac{\partial\phi(z_1)}{\partial Q_1}\right]} = \frac{-1}{(1-\alpha)}.$$
(C.4)

If  $Q_1 = L$ , then the left-hand side of Eq. (C.4) is  $\frac{-1}{(1-\alpha)}$ . Because the SOC holds, we have  $Q_1^* = L$ . We also check that the client (the insurer) is indeed better off when the insurer (the client) accepts the offer in period 2 (in period 1).  $\Box$ 

### Appendix D. Proof of Lemma 4

Substitute  $Q_1^* = L$  into Eq. (C.3), and denote  $P_1^*$  and  $\widehat{P}_1^*$  as the equilibrium premiums in the presence of ambiguity and in the absence of ambiguity, respectively. Hence, we know that

$$(\omega_{\rm C} - P_1^*) = \phi^{-1} \left[ \int \phi \left( \pi u \left( \omega_{\rm C} - \alpha P_1^* - (1 - \alpha)^2 L \right) + (1 - \pi) u \left( \omega_{\rm C} - L \right) \right) dF(\pi) \right]$$

$$\leq \phi^{-1} \left[ \phi \left( \alpha u \left( \omega_{C} - \alpha P_{1}^{*} - (1 - \alpha)^{2} L \right) \right. \\ \left. + (1 - \alpha) u \left( \omega_{C} - L \right) \right] \right] \\ = \alpha u \left( \omega_{C} - \alpha P_{1}^{*} - (1 - \alpha)^{2} L \right) \\ \left. + (1 - \alpha) u \left( \omega_{C} - L \right) \right] \\ \leq \alpha u \left( \omega_{C} - \alpha \widehat{P}_{1}^{*} - (1 - \alpha)^{2} L \right) \\ \left. + (1 - \alpha) u \left( \omega_{C} - L \right) \right] \\ = u \left( \omega_{C} - \widehat{P}_{1}^{*} \right).$$

The second line holds due to Jensen's inequality, the third line follows from the property of the inverse function, the fourth line holds because  $\widehat{P}_1^*$  is the equilibrium premium in the absence of ambiguity, and the last line is the condition that  $\widehat{P}_1^*$  satisfies. Since u' > 0, we have  $P_1^* \ge \widehat{P}_1^*$ .  $\Box$ 

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