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A NOTE ON INDETERMINACY AND INVESTMENT ADJUSTMENT COSTS IN AN ENDOGENOUSLY GROWING SMALL OPEN ECONOMY

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This note analytically examines the interrelations between macroeconomic (in)stability and investment adjustment costs in a one-sector endogenously growing small–open economy representative-agent model. We show that under costly capital accumulation, the economy exhibits indeterminacy and sunspots if and only if the equilibrium wage–hours locus slopes upward and is steeper than the household's labor supply curve. By contrast, the economy without adjustment costs for capital investment always displays saddlepath stability and equilibrium uniqueness, regardless of the degree of increasing returns in aggregate production.

Keywords: Indeterminacy, Investment Adjustment Costs, Endogenous Growth, Small Open Economy

1. INTRODUCTION

It is now well known that dynamic general equilibrium macroeconomic models may possess an indeterminate steady state or balanced-growth path (BGP) that can be exploited to generate business cycle fluctuations driven by agents' self-fulfilling beliefs. In particular, Benhabib and Farmer (1994) showed that

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one-sector representative-agent models for a closed economy exhibit indeterminacy and sunspots under sufficiently strong increasing returns in aggregate production.[1] Subsequently, Kim (2003) found that (both analytically and quantitatively) in the no–sustained growth version of Benhabib and Farmer's model, the minimum level of returns to scale needed for equilibrium indeterminacy is *ceteris paribus* a monotonically increasing function of the size of investment adjustment costs.[2] It is straightforward to show that the same result also holds in the endogenous-growth specification of the Benhabib–Farmer closed economy. Intuitively, investment adjustment costs are observationally equivalent to a state-contingent tax that "leans against the wind" of intertemporal capital accumulation. As a result, when the representative agent becomes optimistic about the economy's future and decides to invest more, the presence of investment adjustment costs will raise the degree of aggregate increasing returns required to fulfill the household's initial optimism.[3]

In this note, we build upon Kim's (2003) analyses and explore the theoretical interrelations between equilibrium indeterminacy and investment adjustment costs in the endogenous-growth version of Benhabib and Farmer's (1994) one-sector representative-agent model for a small open economy. This specification provides a useful analytical benchmark, as well as facilitating comparison with recent studies [e.g., Lahiri (2001), Weder (2001), and Meng and Velasco (2003, 2004)] that examine the condition(s) for indeterminacy and sunspots in closed-economy versus small–open economy macroeconomic models with two sectors and no adjustment costs. Under the assumption of perfect international capital markets, the domestic households are able to lend to and borrow from abroad freely.[4] To guarantee positive equilibrium growth of consumption, the constant world real interest rate is postulated to be strictly higher than the representative agent's utility discount rate. Moreover, without loss of generality, we consider a specific nonlinear capital accumulation formulation that incorporates convex investment adjustment costs à la Lucas and Prescott (1971). The zero degree of joint homogeneity in capital stock and gross investment is imposed to ensure the economy's sustained growth.

We show that our model economy possesses a unique BGP and that its local dynamics depend on whether capital accumulation is costly or not. Specifically, in sharp contrast to Kim (2003) and other previous studies of closed economies, the level of increasing returns to scale needed for equilibrium indeterminacy is independent of the (positive) degree of investment adjustment costs in our small–open economy version of Benhabib and Farmer's (1994) one-sector endogenous growth model. It follows that with sustained economic growth and costly capital accumulation, indeterminacy and sunspots are easier to obtain, in the sense that lower aggregate increasing returns are needed, within a small open economy than in its closed-economy counterpart. This result turns out to be qualitatively consistent with those of Lahiri (2001), Weder (2001), and Meng and Velasco (2003, 2004) in various two-sector small–open economy macroeconomic models without adjustment costs for capital investment.[5] We also find that under costly capital accumulation, our endogenously growing small open economy exhibits the same local stability properties as those in the original Benhabib–Farmer closed-economy model without investment adjustment costs. That is, indeterminacy and belief-driven fluctuations arise *if and only if* the equilibrium wage–hours locus slopes upward and is steeper than the labor supply curve.

When the household's investment decision does not involve adjustment costs, we show that the BGP's shadow prices of physical capital and foreign debt are equal, and that the equilibrium level of labor hours remains constant over time. It follows that our model economy always exhibits saddlepath stability and equilibrium uniqueness in this setting. As a corollary, the same stability/uniqueness result will continue to hold if our analysis starts with a fixed labor supply in the household's utility function, no matter whether capital accumulation is subject to investment adjustment costs or not.

The remainder of this note is organized as follows. Section 2 describes our model and analyzes the equilibrium conditions. Section 3 examines local dynamics of the economy's BGP with or without investment adjustment costs. Section 4 concludes.

2. THE ECONOMY

We incorporate a small open economy with perfect capital mobility and convex investment adjustment costs into the endogenous-growth version of Benhabib and Farmer's (1994) one-sector representative agent model. To facilitate comparison with existing studies, we adopt the same preference and technology formulations as in the Benhabib–Farmer economy.

There is a continuum of identical competitive firms, with the total number normalized to one. Each firm produces output y_t using the Cobb–Douglas production function as follows:

$$y_t = x_t k_t^{\alpha} h_t^{1-\alpha}, \quad 0 < \alpha < 1, \tag{1}$$

where k_t and h_t are capital and labor inputs, respectively, and x_t represents productive externalities that are taken as given by the individual firm. We postulate that externalities take the form

$$x_t = K_t^{1-\alpha} H_t^{(1-\alpha)\eta}, \quad \eta > 0, \tag{2}$$

where K_t and H_t denote the economywide levels of capital and labor services. In a symmetric equilibrium, all firms make the same decisions, such that $k_t = K_t$ and $h_t = H_t$, for all t. As a result, (2) can be substituted into (1) to obtain the following aggregate production function that displays increasing returns to scale:

$$y_t = k_t h_t^{(1-\alpha)(1+\eta)}. \tag{3}$$

Notice that the economy exhibits sustained endogenous growth, because the social technology (3) displays linearity in physical capital. Under the assumption that factor markets are perfectly competitive, the first-order conditions for the firm's profit-maximization problem are given by

$$u_t = \alpha \frac{y_t}{k_t}, \tag{4}$$

$$w_t = (1-\alpha)\frac{y_t}{h_t}, \tag{5}$$

where u_t is the capital rental rate and w_t is the real wage.

The economy is also populated by a unit measure of identical infinitely lived households. Each has one unit of time endowment and maximizes a discounted stream of utilities over its lifetime,

$$\int_0^\infty \left\{ \log c_t - A\frac{h_t^{1+\gamma}}{1+\gamma} \right\} e^{-\rho t} dt, \quad A > 0, \tag{6}$$

where c_t is the individual household's consumption, $\gamma \geq 0$ denotes the inverse of the intertemporal elasticity of substitution in labor supply, and $\rho > 0$ is the discount rate. The budget constraint faced by the representative household is given by

$$c_t + i_t + \dot{b}_t = u_t k_t + w_t h_t + r b_t, \tag{7}$$

where $k_0 > 0$ and b_0 are given, i_t is gross investment, and $r > 0$ is the exogenously given constant world real interest rate on risk-free foreign bonds b_t. Under the assumption of perfect international capital markets, the domestic household is able to lend to and borrow from abroad freely.

As in Kim (2003, p. 400), the law of motion for capital stock is specified as

$$\frac{\dot{k}_t}{k_t} = \Psi\left(\frac{i_t}{k_t}\right) = \begin{cases} \dfrac{\delta\left[\left(\dfrac{i_t}{\delta k_t}\right)^{1-\theta} - 1\right]}{1-\theta}, & \theta \geq 0,\ \theta \neq 1, \\ \delta \log\left(\dfrac{i_t}{\delta k_t}\right), & \theta = 1, \end{cases} \tag{8}$$

where $\delta \in (0, 1)$ is the capital depreciation rate. When $\theta > 0$, the resulting nonlinear formulation is an increasing and concave function of the investment-to-capital ratio i_t/k_t. This feature can be viewed as reflecting convex adjustment costs à la Lucas and Prescott (1971); and the elasticity parameter θ $\left(\equiv -\delta\Psi''(\delta)/\Psi'(\delta)\right)$ represents the degree (or size) of investment adjustment costs.[6] When $\theta = 0$, (8) becomes the standard linear capital accumulation equation without adjustment costs. Finally, the zero degree of joint homogeneity in i_t and k_t is needed to maintain the economy's balanced growth in output, consumption, and investment.[7]

The first-order conditions for the representative household with respect to the indicated variables and the associated transversality conditions (TVC) are

$$c_t: \quad \frac{1}{c_t} = \lambda_{at}, \tag{9}$$

$$h_t: \quad Ah_t^{\gamma} = \lambda_{at} w_t, \tag{10}$$

$$i_t: \quad \lambda_{kt} \left(\frac{i_t}{\delta k_t}\right)^{-\theta} = \lambda_{at}, \tag{11}$$

$$k_t: \quad \dot{\lambda}_{kt} = \rho\lambda_{kt} - \frac{\delta\lambda_{kt}\left[\theta\left(\frac{i_t}{\delta k_t}\right)^{1-\theta} - 1\right]}{1-\theta} - \lambda_{at} u_t, \tag{12}$$

$$b_t: \quad \dot{\lambda}_{at} = (\rho - r)\lambda_{at}, \tag{13}$$

$$\mathrm{TVC}_1: \quad \lim_{t\to\infty} e^{-\rho t}\lambda_{at} b_t = 0, \tag{14}$$

$$\mathrm{TVC}_2: \quad \lim_{t\to\infty} e^{-\rho t}\lambda_{kt} k_t = 0, \tag{15}$$

where λ_{at} and λ_{kt} are the shadow prices of the asset (foreign bonds) and physical capital, respectively. Equation (9) states that the marginal benefit of consumption equals its marginal cost, which is the marginal utility of having an additional unit of internationally traded bonds. Moreover, equation (10) equates the slope of the representative household's indifference curve to the real wage, and equations (11) and (12) together govern the evolution of capital stock over time. Finally, equation (13) states that the marginal utility values of foreign debt holdings are equal to their respective marginal costs.

Combining equations (9) and (13) yields the following standard Keynes–Ramsey rule:

$$\frac{\dot{c}_t}{c_t} = r - \rho. \tag{16}$$

To guarantee positive equilibrium growth of consumption, we assume that the world interest rate is strictly higher than the household's utility discount rate, i.e., $r > \rho$.

3. ANALYSIS OF DYNAMICS

We focus on the economy's BGP, along which labor hours are stationary, and output, consumption, physical capital, and foreign bonds all exhibit a common, positive constant growth rate given by $r - \rho$. To facilitate the analysis of perfect-foresight dynamics under sustained economic growth, we make the following transformation of variables: $q_t \equiv \lambda_{kt}/\lambda_{at}$, $z_t \equiv c_t/k_t$, and $x_t \equiv b_t/c_t$. Notice that q_t corresponds to "Tobin's q," which defines the marginal utility value of physical capital measured in terms of the shadow price of foreign bonds.

With these transformations, the model's equilibrium conditions can be expressed as an autonomous system of differential equations:

$$\frac{\dot{q}_t}{q_t} = r - \frac{\delta\left(\theta q_t^{\frac{1-\theta}{\theta}} - 1\right)}{1-\theta} - \frac{\alpha}{q_t}\left(\frac{Az_t}{1-\alpha}\right)^{\frac{(1-\alpha)(1+\eta)}{(1-\alpha)(1+\eta)-(1+\gamma)}}, \quad \textbf{(17)}$$

$$\frac{\dot{z}_t}{z_t} = r - \rho - \frac{\delta\left(q_t^{\frac{1-\theta}{\theta}} - 1\right)}{1-\theta}, \quad \textbf{(18)}$$

$$\frac{\dot{x}_t}{x_t} = \rho + \frac{1}{z_t x_t}\left[\left(\frac{Az_t}{1-\alpha}\right)^{\frac{(1-\alpha)(1+\eta)}{(1-\alpha)(1+\eta)-(1+\gamma)}} - \delta q_t^{\frac{1}{\theta}}\right] - \frac{1}{x_t}. \quad \textbf{(19)}$$

It is straightforward to show that our model economy possesses a unique balanced-growth equilibrium, characterized by $\{q^*, z^*, x^*\}$, that satisfies $\dot{q}_t = \dot{z}_t = \dot{x}_t = 0$. In terms of the BGP's local stability properties, we note that the above dynamical system (17)–(19) is block recursive, in that the evolutions of q_t and z_t do not depend on the foreign debt–to–consumption ratio x_t. As a result, whether the model exhibits equilibrium (in)determinacy will be completely determined by the two-by-two subsystem in q_t and z_t. We linearize equations (17) and (18) around (q^*, z^*), and then compute the resulting Jacobian matrix J. The trace and determinant of the Jacobian are

$$\text{Tr} = \rho > 0, \quad \textbf{(20)}$$

$$\text{Det} = \frac{\alpha\delta\,(1-\alpha)\,(1+\eta)\,(q^*)^{\frac{1-2\theta}{\theta}}}{\theta\,[1+\gamma-(1-\alpha)\,(1+\eta)]}\left(\frac{Az^*}{1-\alpha}\right)^{\frac{(1-\alpha)(1+\eta)}{(1-\alpha)(1+\eta)-(1+\gamma)}}. \quad \textbf{(21)}$$

Because q_t and z_t are both jump variables, the dynamical subsystem (17)–(18) has no initial condition. Therefore, the BGP displays saddle-path stability and equilibrium uniqueness when both eigenvalues have positive real parts. Given that the trace of the Jacobian J is positive,[8] the BGP equilibrium is locally indeterminate (a sink) *if and only if* the two eigenvalues are of opposite sign, i.e., the Jacobian's determinant (21) is negative. In this case, the economy exhibits endogenous growth fluctuations driven by agents' self-fulfilling expectations or sunspots.

3.1. When $\theta > 0$

In this specification, the nonlinear capital accumulation equation (8) exhibits convex investment adjustment costs. It is straightforward to show that the BGP equilibrium is characterized by a pair of positive real numbers (q^*, z^*) given by[9]

$$q^* = \begin{cases} \left[\dfrac{(1-\theta)(r-\rho)+\delta}{\delta}\right]^{\frac{\theta}{1-\theta}}, & \theta \geq 0,\ \theta \neq 1, \\ \exp\left[\frac{r-\rho}{\delta}\right], & \theta = 1, \end{cases} \tag{22}$$

$$z^* = \begin{cases} \left(\dfrac{1-\alpha}{A}\right)\left[\dfrac{(1-\theta)r+\delta+\theta\rho}{\alpha}\right]^{\frac{(1-\alpha)(1+\eta)-(1+\gamma)}{(1-\alpha)(1+\eta)}} \\ \quad \left[\dfrac{(1-\theta)(r-\rho)+\delta}{\delta}\right]^{\frac{\theta[(1-\alpha)(1+\eta)-(1+\gamma)]}{(1-\theta)(1-\alpha)(1+\eta)}}, & \theta \geq 0,\ \theta \neq 1, \\ \left(\dfrac{1-\alpha}{A}\right)\left\{\left(\dfrac{\rho+\delta}{\alpha}\right)\exp\left[\dfrac{r-\rho}{\delta}\right]\right\}^{\frac{(1-\alpha)(1+\eta)-(1+\gamma)}{(1-\alpha)(1+\eta)}}, & \theta = 1. \end{cases} \tag{23}$$

because $q^*, z^*, A, \eta, \theta > 0$, $0 < \alpha, \delta < 1$, and $\gamma \geq 0$, the Jacobian's determinant (21) is negative if and only if

$$(1-\alpha)(1+\eta) - 1 > \gamma, \tag{24}$$

which is also the *necessary and sufficient* condition for our model economy to display equilibrium indeterminacy and belief-driven aggregate fluctuations.

To understand this indeterminacy condition, we first substitute (9) into (10) and find that agents' equilibrium decision on hours worked is governed by

$$Ac_t h_t^{\gamma} = w_t. \tag{25}$$

Next, plugging the aggregate production function (3) into the logarithmic version of firms' labor-demand condition (5) shows that the slope of the equilibrium wage–hours locus is equal to $(1-\alpha)(1+\eta)-1$. In addition, taking logarithms on both sides of (25) indicates that the slope of the household's labor-supply curve is γ (≥ 0), and its position or intercept is affected by the level of consumption. It follows that the condition that is needed to generate indeterminacy and sunspots, as in (24), states that the equilibrium wage–hours locus is positively sloped and steeper than the labor supply curve. Interestingly, (24) turns out to be identical to the necessary and sufficient condition that leads to an indeterminate BGP in Benhabib and Farmer's (1994) closed economy. It follows that given identical preference and technology specifications, our one-sector endogenous-growth model under costly capital accumulation for a small open economy exhibits exactly the same local stability properties as the original Benhabib–Farmer closed-economy counterpart without investment adjustment costs.

In terms of economic intuition, suppose that households become optimistic about the future of the economy and anticipate a higher return on investment.

Acting upon this belief, agents will invest more and consume less today, thus substituting out of foreign bonds and into domestic physical capital (a *portfolio-substitution* effect). This in turn raises the relative shadow price of capital because of higher demand. Using (25), lower spending an current consumption also shifts out the household's labor-supply curve, which causes labor hours to rise and the real wage to fall. If the degree of external effects in firms' production function η is strong enough to yield a more-than-unity equilibrium labor elasticity of output in the social technology, namely $(1-\alpha)(1+\eta) > 1$ in (3), increases in hours worked lead to higher labor productivity. It follows that the labor demand curve will shift outward, reinforcing the initial employment effect generated by the reduction of consumption expenditures. Subsequently, higher labor hours raise the household's projected income stream, thereby increasing its ability to consume and shifting the labor supply curve to the left. When such a leftward shift makes the equilibrium wage–hours locus intersect the labor-supply curve from below, i.e., $(1-\alpha)(1+\eta)-1 > \gamma$, a positive sunspot shock generates simultaneous expansions in output, consumption, investment, hours worked, and labor productivity. Moreover, because of higher levels of employment and labor productivity, the marginal product of capital and its relative shadow price both will rise, validating agents' initial optimistic expectations. In contrast, if the strength of productive externalities η is not sufficiently high so that the equilibrium wage–hours locus is flatter than the labor-supply curve, consumption will become countercyclical. As a result, agents' optimism cannot become self-fulfilling in equilibrium.

In sharp contrast to previous studies on closed economies [e.g., Georges (1995), Wen (1998), Guo and Lansing (2002), and Kim (2003)], we have shown that in the small–open economy version of one-sector representative-agent models that allow ongoing economic growth, the level of labor externalities needed for equilibrium indeterminacy, $\eta_{\min}$, is independent of the degree of investment adjustment costs; i.e., $\partial\eta_{\min}/\partial\theta = 0$. As long as $\theta > 0$ and condition (24) is satisfied, the shadow-price wedge between physical capital and foreign bonds will produce the above-mentioned effects of portfolio substitution and labor adjustments in response to agents' optimistic expectations. In contrast, such portfolio-substitution opportunities do not exist within a closed economy because the only asset available to households is domestic capital stock. Consequently, $\eta_{\min}$ is monotonically increasing with respect to the size of adjustment costs for capital investment in order to fulfill households' optimism ($\partial\eta_{\min}/\partial\theta > 0$).

This "independence" finding also implies that under sustained economic growth and costly capital accumulation, indeterminacy and sunspots are more likely to arise, in the sense that lower increasing returns are required, within a small–open economy than a closed-economy setting.[10] Notice that Lahiri (2001), Weder (2001), and Meng and Velasco (2003, 2004) obtain the same result in various two-sector small–open economy macroeconomic models without investment adjustment costs. Because of perfect capital mobility, the models that these authors examine behave like a closed economy with linear preferences in consumption. It follows that there are no utility costs associated with constructing alternative equilibrium paths when agents become optimistic. In comparison with the corresponding closed economy, this feature in turn enlarges the range of parameter values under which indeterminacy and sunspots may occur in a small–open economy formulation.

Finally, in the no–sustained growth version of our one-sector small–open economy model with investment adjustment costs, we find that international lending and borrowing lead to complete consumption smoothing; i.e., the level of equilibrium consumption c_t is a fixed constant over time [see equation (16) under the knife-edge condition $r = \rho$, and Turnovsky (2002) for more general discussions]. Therefore, in response to the agent's optimism about the economy's future, the outward shift of the household's labor-supply curve described earlier will not take place. On the other hand, the portfolio substitution (from foreign bonds to domestic physical capital) effect alone is not sufficiently strong to raise the rate of return on investment because the firm's production technology now displays a diminishing marginal product of capital. It follows that the steady state is a saddlepoint within this configuration. The same intuition for saddlepath stability can be applied to our model with exogenous growth, based on labor-augmenting technological progress s_t, in that the detrended consumption c_t/s_t will remain time-invariant along the economy's equilibrium growth path.[11] Overall, the above analysis illustrates that endogenous growth is necessary for the possibility of indeterminacy and sunspots in a small open economy.

3.2. When $\theta = 0$

In this specification, the capital-accumulation equation (8) is linear and does not exhibit investment adjustment costs. Substituting $\theta = 0$ into (11) shows that the price (in utility terms) of physical capital relative to internationally traded bonds $q_t \equiv \lambda_{kt}/\lambda_{at} = 1$ for all t. As a result, the dynamical subsystem (17)–(18) now becomes degenerate. Resolving our model with $\theta = 0$ and $q_t = 1$ leads to the following single differential equation in $x_t \equiv b_t/c_t$, which describes the equilibrium dynamics:

$$\frac{\dot{x}_t}{x_t} = \rho + \frac{1}{x_t}\left\{\frac{A\left[(1-\alpha)\,r + \alpha\rho\right]}{\alpha\,(1-\alpha)}\left(\frac{r+\delta}{\alpha}\right)^{\frac{(1+\gamma)-(1-\alpha)(1+\eta)}{(1-\alpha)(1+\eta)}} - 1\right\}, \qquad \textbf{(26)}$$

which has a unique interior solution x^* that satisfies $\dot{x}_t = 0$ along the balanced-growth equilibrium path. We then linearize (26) around the BGP and find that its local stability properties are governed by $\rho > 0$. Consequently, the economy always displays saddlepath stability and equilibrium uniqueness because there is no initial condition associated with (26).

The intuition for the above determinacy result is straightforward. Combining equations (3), (4), (12), and (13), together with $\lambda_{kt} = \lambda_{at}$, yields a constant level of equilibrium labor hours over time,

$$h_t = \left(\frac{r+\delta}{\alpha}\right)^{\frac{1}{(1-\alpha)(1+\eta)}}, \quad \text{for all } t. \tag{27}$$

Therefore, when households become optimistic and decide to increase their investment spending today, the mechanism described in the previous section that makes for indeterminate equilibria, i.e., movements of hours worked induced by the representative agent's portfolio substitution, is completely shut down, regardless of the degree of productive externalities η. This implies that given the initial holding of foreign bonds b_0, the household's period-0 consumption c_0 will be uniquely determined, so that the economy immediately jumps onto its balanced-growth equilibrium characterized by x^*, and always stays there without any possibility of deviating transitional dynamics. It follows that equilibrium indeterminacy and endogenous-growth fluctuations can never occur in this setting. Notice that the same stability/uniqueness result will be obtained if our analysis starts with a fixed labor supply in the household utility function (6), no matter whether capital accumulation is subject to investment adjustment costs or not.

4. CONCLUSION

We have analytically explored the theoretical relationship between macroeconomic (in)stability and investment adjustment costs within one-sector endogenous-growth models for a small open economy. Under costly capital accumulation, the necessary and sufficient condition for our model economy to exhibit indeterminacy and sunspots requires that the equilibrium wage-hours locus be positively sloped and steeper than the household's labor-supply curve. Moreover, our model economy without investment adjustment costs always displays saddlepath stability and equilibrium uniqueness, regardless of the degree of increasing returns to scale in aggregate production. It would be worthwhile to examine the robustness of our results by introducing investment adjustment costs into a full-fledged open-economy model with distinct tradable and nontradable goods, or into the endogenous-growth version of Weder's (2001) two-sector small–open economy model, in which the minimum degree of increasing returns needed for equilibrium indeterminacy is much less stringent. We plan to pursue these research projects in the near future.

NOTES

1. See Benhabib and Farmer (1999) for an excellent survey of this strand of the indeterminacy literature.
2. Wen (1998) numerically verifies this positive relationship in a calibrated one-sector real–business cycle model where capital adjustment costs are postulated to be a quadratic function of the difference in gross investments between two consecutive time periods.
3. A corollary of this finding is that adjustment costs for capital investment can be used to eliminate equilibrium multiplicity and select a locally unique equilibrium in the Benhabib–Farmer economy [Guo and Lansing (2002, pp. 655–657)]. The same result is obtained in different one-sector macroeconomic

models for a closed economy [Georges (1995)] and in a two-sector representative-agent model with sector-specific externalities for a small open economy [Herrendorf and Valentinyi (2003)].

4. Aguiar-Conraria and Wen (2008) and Chen and Zhang (in press) examine the interrelations between equilibrium indeterminacy and production externalities in one-sector no–sustained growth models for an open economy without access to a perfect world capital market and investment adjustment costs.

5. Notice that the one-sector version of these authors' models exhibit degenerate macroeconomic dynamics, thus equilibrium indeterminacy cannot arise. It is the presence of an additional sector that leads to nondegenerate equilibrium dynamics and the possibility of multiple equilibria [Meng and Velasco (2004, p. 509)]. Section 3 shows that incorporating investment adjustment costs into our one-sector framework exerts the same qualitative effect on the model's local stability properties.

6. Notice that when $\theta = 1$, (8) corresponds to the log-linear law of motion for capital stock in discrete time whereby $k_{t+1} = k_t^{1-\delta} (i_t/\delta)^{\delta}, \quad 0 < \delta < 1$. In this case, Guo and Lansing (2003) find that Benhabib and Farmer's (1994) closed-economy model always exhibits saddlepath stability and equilibrium uniqueness. However, Section 3.1 shows that this result does not hold in our endogenously growing small–open economy model.

7. All the (in)determinacy results reported here remain unaffected under Hayashi's (1982) adjustment-cost formulation. In particular, the Appendix examines the following expenditure function for total investment, which includes capital installation costs and is consistent with sustained economic growth: $i_t((1 + \phi/2)\, i_t/k_t)$, where $\phi > 0$.

8. This implies that the case in which both eigenvalues have negative real parts is not possible.

9. The remaining endogenous variables on the economy's BGP can then be derived accordingly. In particular, the BGP's investment-to-capital ratio and hours worked are $(i/k)^* = \delta(q^*)^{\frac{1}{\theta}}$ and $h^* = (\frac{Az^*}{1-\alpha})^{\frac{1}{(1-\alpha)(1+\eta)-(1+\gamma)}}$, respectively. Moreover, the requirement of a positive q^* or $(\frac{i}{k})^*$ yields the following upper bound on the size of investment adjustment costs: $\theta < 1 + \delta/(r - \rho)$.

10. For example, under Kim's (2003, p. 399) benchmark parameterization with $\alpha = 0.3$, $\gamma = 0$, $\rho = 0.05$, $\delta = 0.1$, and $\theta = 0.05$, we find that $\eta_{\min} = 0.74$ for a closed-economy model, whereas $\eta_{\min} = 0.43$ in its small–open economy counterpart. Although the latter value of $\eta_{\min}$ is smaller, the resulting aggregate returns to scale are nevertheless empirically implausible. Therefore, the preceding calibrated example should be viewed more from a methodological perspective, as it quantitatively illustrates the stability effects of costly capital accumulation in a one-sector endogenously growing closed-economy versus small–open economy model.

11. The mathematical derivations of local dynamics for the no-growth and exogenous-growth cases of our model economy are straightforward and available upon request from the authors.

REFERENCES

Aguiar-Conraria, Luís and Yi Wen (2008) A note on oil dependence and economic instability. *Macroeconomic Dynamics* 12, 717–723.

Benhabib, Jess and Roger E.A. Farmer (1994) Indeterminacy and increasing returns. *Journal of Economic Theory* 63, 19–41.

Benhabib, Jess and Roger E.A. Farmer (1999) Indeterminacy and sunspots in macroeconomics. In John Taylor and Michael Woodford (eds.), *Handbook of Macroeconomics*, pp. 387–448. Amsterdam: North-Holland.

Chen, Yan and Yan Zhang (in press) A note on tariff policy, increasing returns, and endogenous fluctuations. *Macroeconomic Dynamics*.

Georges, Christophre (1995) Adjustment costs and indeterminacy in perfect foresight models. *Journal of Economic Dynamics and Control* 19, 39–50.

Guo, Jang-Ting and Kevin J. Lansing (2002) Fiscal policy, increasing returns, and endogenous fluctuations. *Macroeconomic Dynamics* 6, 633–664.

Guo, Jang-Ting and Kevin J. Lansing (2003) Globally-stabilizing fiscal policy rules. *Studies in Nonlinear Dynamics and Econometrics* 7(2), 1–13.
Hayashi, Fumio (1982) Tobin's marginal q, average q: A neoclassical interpretation. *Econometrica* 50, 213–224.
Herrendorf, Bethold and Ákos Valentinyi (2003) Determinacy through intertemporal capital adjustment costs. *Review of Economic Dynamics* 6, 483–497.
Kim, Jinill (2003) Indeterminacy and investment adjustment costs: An analytical result. *Macroeconomic Dynamics* 7, 394–406.
Lahiri, Amartya (2001) Growth and equilibrium indeterminacy: The role of capital mobility. *Economic Theory* 17, 197–208.
Lucas, Robert E., Jr., and Edward C. Prescott (1971) Investment under uncertainty. *Econometrica,* 39, 659–681.
Meng, Qinglai and Andrés Velasco (2003) Indeterminacy in a small open economy with endogenous labor supply. *Economic Theory* 22, 661–669.
Meng, Qinglai and Andrés Velasco (2004) Market imperfections and the instability of open economies. *Journal of International Economics* 64, 503–519.
Turnovsky, Stephen J. (2002) Knife-edge conditions and the macroeconomics of small open economies. *Macroeconomic Dynamics* 6, 307–335.
Weder, Mark (2001) Indeterminacy in a small open economy Ramsey growth model. *Journal of Economic Theory* 98, 339–356.
Wen, Yi (1998) Indeterminacy, dynamic adjustment costs, and cycles. *Economics Letters* 59, 213–216.

APPENDIX

This Appendix reexamines our model under Hayashi's (1982) adjustment-cost formulation specified in Note 7. In this case, the representative household's budget constraint becomes

$$c_t + i_t\left(1 + \frac{\phi}{2}\frac{i_t}{k_t}\right) + \delta k_t + \dot{b}_t = u_t k_t + w_t h_t + r b_t, \quad k_0 > 0 \text{ and } b_0 \text{ are given,} \qquad \textbf{(A.1)}$$

where i_t is net investment, $\phi > 0$ captures the size (or degree) of investment adjustment costs, and $\delta \in (0, 1)$ is the capital depreciation rate. Using the same transformed variables $\{q_t, z_t, x_t\}$ as in the text, the autonomous system of differential equations that characterize the model's equilibrium conditions are now given by

$$\frac{\dot{q}_t}{q_t} = r - \frac{1}{q_t}\left[\alpha\left(\frac{Az_t}{1-\alpha}\right)^{\frac{(1-\alpha)(1+\eta)}{(1-\alpha)(1+\eta)-(1+\gamma)}} + \frac{(q_t-1)^2}{2\phi} - \delta\right], \qquad \textbf{(A.2)}$$

$$\frac{\dot{z}_t}{z_t} = r - \rho - \frac{q_t - 1}{\phi}, \qquad \textbf{(A.3)}$$

$$\frac{\dot{x}_t}{x_t} = \rho + \frac{1}{z_t x_t}\left[\left(\frac{Az_t}{1-\alpha}\right)^{\frac{(1-\alpha)(1+\eta)}{(1-\alpha)(1+\eta)-(1+\gamma)}} - \frac{q_t-1}{\phi} - \frac{(q_t-1)^2}{2\phi} - \delta\right] - \frac{1}{x_t}. \qquad \textbf{(A.4)}$$

It is straightforward to show that on the economy's (unique) BGP,

$$q^* = \phi(r - \rho) + 1, \quad z^* = (1 - \alpha)\left[\frac{2(r + \delta) + \phi(r^2 - \rho^2)}{2\alpha}\right]^{\frac{(1-\alpha)(1+\eta)-(1+\gamma)}{(1-\alpha)(1+\eta)}},$$

and $(i/k)^* = r - \rho$, which is invariant to the size of investment adjustment costs.

As with (17)–(19), the dynamical system (A.2)–(A.4) is block recursive in that x_t does not enter (A.2) and (A.3). Hence, we derive the following trace and determinant of the model's Jacobian matrix for the subsystem (A.2)–(A.3) evaluated at (q^*, z^*):

$$\text{Tr} = \rho > 0, \tag{A.5}$$

$$\text{Det} = \frac{\alpha(1-\alpha)(1+\eta)}{\phi[1+\gamma-(1-\alpha)(1+\eta)]}\left(\frac{Az^*}{1-\alpha}\right)^{\frac{(1-\alpha)(1+\eta)}{(1-\alpha)(1+\eta)-(1+\gamma)}}. \tag{A.6}$$

Following the same arguments as in Section 3.1, our model with Hayashi's (1982) adjustment-cost specification exhibits equilibrium indeterminacy and endogenous growth fluctuations if and only if condition (24) is satisfied.