Pattern Recognition Letters 34 (2013) 453–455

Contents lists available at SciVerse ScienceDirect

Pattern Recognition Letters

journal homepage: www.elsevier.com/locate/patrec

# A comment on "On the Mitchell similarity measure and its application to pattern recognition"

Cheng-Tan Tung [a,*], Sophia Liu [b], Bonnie Shuan Wang [c]

[a] Department of Information Management, Central Police University, Taiwan
[b] Department of Chemistry, National Tsing Hua University, Taiwan
[c] Department of Land Economics, National Chengchi University, Taiwan

ARTICLE INFO

*Article history:*
Received 11 June 2012
Available online 8 December 2012

Communicated by G. Borgefors.

*Keywords:*
Pattern recognition
Intuitionistic fuzzy sets
Similarity measures

ABSTRACT

We provide a careful analysis of the similarity measures of Mitchell [Mitchell, H.B., 2003. On the Dengfend–Chuntian similarity measure and its application to pattern recognition. Pattern Recognition Lett. 24, 3101–3104] and Julian et al. [Julian, P., Hung, K.C., Lin, S.J., 2012. On the Mitchell similarity measure and its application to pattern recognition. Pattern Recognition Lett. 33, 1219–1223] for Julian's application to pattern recognition problem. In this paper we will first point out that the similarity measure of Julian et al. (2012) does not satisfy the system of axioms for similarity measures and then provide a counter example to support our assertion. Second, we will show that foundation that the differentiability property[1] proposed by Julian et al. (2012) is questionable, hindering the progress of the similarity measure. Our analysis should help researchers to realize new similarity measures under intuitionistic fuzzy sets environment.


## 1. Introduction

Pattern recognition under intuitionistic fuzzy sets (IFSs) environment had been applied to many areas as data analysis, artificial intelligence, and decision making problems. An IFS $\tilde{A}$ in $X$ is defined by Atanassov (1986) as $\tilde{A} = \{\langle u, \mu_A(u), \nu_A(u)\rangle | u \in X\}$ where $X$ is the disclose of universe, $\mu_{\tilde{A}}(x) : X \to [0,1]$ is the membership function, and $\nu_{\tilde{A}}(x) : X \to [0,1]$ is the non-membership function, with $\mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \leqslant 1$, for $x \in X$. How to construct similarity measures to assess resemblance between a sample and several patterns becomes the essential issue to performance of pattern recognition problems. Li and Cheng (2002) introduced a system for axioms of similarity measures with respect to IFSs and then defined some similarity measures. Mitchell (2003) pointed out that the similarity measures proposed by Li and Cheng (2002) may contain counter intuitive results and then offered an improved system for axioms for similarity measures with IFSs. Many authors have constructed many similarity measures and applied them to different fields, such as Hung and Yang (2004), Park et al. (2007), Vlachos and Sergiadis (2007), Xu and Yager (2009), and Yusoff et al. (2011). Recently, Julian et al. (2012) examined the similarity measures of Mitchell (2003) to point out that the calculation of the similarity measures by Mitchell (2003) contained questionable results, and so Julian et al. (2012) proposed a new similarity measures. Finally, Julian et al. (2012) discussed the advantage of their own new similarity measures. The main purpose of this paper is to show that Julian's new similarity measures violate the axioms of similarity measures proposed by Li and Cheng (2002), Mitchell (2003) and Xu (2007). We will also provide a detailed discussion of the self-proclaimed advantages of the similarity measures of Julian et al. (2012), revealing the problems that are presented in Julian's results.

## 2. Review similarity measures of Mitchell (2003) and Julian et al. (2012)

We recall the definition for IFSs as $\tilde{A} = \{\langle u, \mu_A(u), \nu_A(u)\rangle | u \in X\}$ where $X$ is the disclose of universe, $\mu_A$ is the membership function, and $\nu_A$ is the non-membership function. We recall Mitchell (2003), where she assumed that $S(\tilde{A}, \tilde{B})$ is a well defined similarity measure if the following properties are satisfied:

(P1) $0 \leqslant S(\tilde{A}, \tilde{B}) \leqslant 1$.

(P2) $S(\tilde{A}, \tilde{B}) = 1$ if and only if $\tilde{A} = \tilde{B}$.

(P3) $S(\tilde{A}, \tilde{B}) = S(\tilde{B}, \tilde{A})$.

(P4) *If* $\tilde{A} \subseteq \tilde{B} \subseteq \tilde{C}$, then $S(\tilde{A}, \tilde{C}) \leqslant S(\tilde{A}, \tilde{B})$ and $S(\tilde{A}, \tilde{C}) \leqslant S(\tilde{B}, \tilde{C})$.

* Corresponding author. Tel.: +886 3 3282312 4491; fax: +886 3 3282312 4329.
*E-mail addresses:* tung@mail.cpu.edu.tw (C.-T. Tung), sophia0liu@hotmail.com (S. Liu), BonnieWang@hotmail.com (B.S. Wang).

[1] The differentiability between the best pattern and the second one for a given sample.

0167-8655/$ - see front matter © 2012 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.patrec.2012.11.010

For two functions, $f$ and $g$, under the p-norm, Li and Cheng (2002) assumed that

$$S_p(f,g) = 1 - \left[\int_{-\infty}^{\infty} w(x)|f(x) - g(x)|^p dx\right]^{1/p}, \tag{1}$$

where $f$ and $g$ are functions from the set of real numbers to interval [0,1] and $w(x)$ is the weight function under the conditions of $w(x) \geqslant 0$ and $\int_{-\infty}^{\infty} w(x)dx = 1$ with $p \geqslant 1$.

In Mitchell (2003) an example with three patterns $\tilde{A}$, $\tilde{B}$ and $\tilde{C}$, was constructed with membership functions, $\mu_A$, $\mu_B$ and $\mu_C$, and non-membership functions $\nu_A$, $\nu_B$ and $\nu_C$, defined for domain $0 \leqslant x \leqslant 1$ such that

$$\mu_A(x) = \nu_A(x) = 0.4, \quad \text{for} \quad 0.2 \leqslant x \leqslant 0.8; \\ \mu_A(x) = \nu_A(x) = 0, \quad \text{otherwise}, \tag{2}$$

$$\mu_B(x) = \nu_B(x) = 0.3, \quad \text{for} \quad 0.2 \leqslant x \leqslant 0.8; \\ \mu_B(x) = \nu_B(x) = 0, \quad \text{otherwise} \tag{3}$$

and

$$\mu_C(x) = \nu_C(x) = 0.1, \quad \text{for} \quad 0.2 \leqslant x \leqslant 0.8; \\ \mu_C(x) = \nu_C(x) = 0, \quad \text{otherwise}. \tag{4}$$

For two intuitionistic fuzzy sets, $\tilde{A}$ and $\tilde{B}$, based on the formula of Eq. (1) proposed by Li and Cheng (2002), with $w(x) = 1$ for $0 \leqslant x \leqslant 1$, and $p = 2$, the similarity measure between two IFSs, $\tilde{A}$ and $\tilde{B}$, was proposed to be

$$S_{\text{mod},2}(\tilde{A},\tilde{B}) = \frac{1}{2}\left(\rho_\mu(\tilde{A},\tilde{B}) + \rho_\phi(\tilde{A},\tilde{B})\right), \tag{5}$$

with

$$\rho_\mu(\tilde{A},\tilde{B}) = S_2(\mu_A,\mu_B) \quad \text{and} \quad \rho_\phi(\tilde{A},\tilde{B}) = S_2(\phi_A,\phi_B), \tag{6}$$

where $\phi_A(x) = 1 - \nu_A(x)$. We must point out that $S_2(\phi_A,\phi_B) = S_2(\nu_A,\nu_B)$. In Mitchell (2003), it derived to be

$$S_{\text{mod},2}(\tilde{A},\tilde{B}) = 0.85 \quad \text{and} \quad S_{\text{mod},2}(\tilde{A},\tilde{C}) = 0.54. \tag{7}$$

For future comparison, we have written down the detailed expression for the similarity measure proposed by Mitchell (2003) below.

$$S_{\text{mod},2}(\tilde{A},\tilde{B}) = 1 - \frac{1}{2}\left(\int_{-\infty}^{\infty} w(x)|\mu_A(x) - \mu_B(x)|^2 dx\right)^{1/2} \\ - \frac{1}{2}\left(\int_{-\infty}^{\infty} w(x)|\phi_A(x) - \phi_B(x)|^2 dx\right)^{1/2}. \tag{8}$$

Julian et al. (2012) defined a new similarity measure as

$$S_{new,2}(\tilde{A},\tilde{B}) = 1 - \left(\int_{-\infty}^{\infty} w(x)|\mu_A(x) - \mu_B(x)|^2 dx\right)^{1/2} \\ - \left(\int_{-\infty}^{\infty} w(x)|\phi_A(x) - \phi_B(x)|^2 dx\right)^{1/2} \tag{9}$$

and then showed that

$$S_{new,2}(\tilde{A},\tilde{B}) = 0.8451 \quad \text{and} \quad S_{new,2}(\tilde{A},\tilde{C}) = 0.5352. \tag{10}$$

Julian et al. (2012) claimed that the results of Eq. (10) are the same as those derived in Eq. (7) of Mitchell (2003) after round off to the second decimal place. Hence, Julian et al. (2012) declared that their own new similarity measure is a valid new approach to estimate the degree of similarity between two IFSs.

## 3. Our comments for the similarity measure of Julian et al. (2012)

The four main points in Julian et al. (2012) are: (a) to revise the computation of similarity measures of Mitchell (2003), (b) to define a new similarity measure, (c) to point out the advantages of the new similarity measure, and (d) to provide a theoretical proof to support Julian's computation.

We accept that the calculations and the statements in Julian et al. (2012) regarding Mitchell's similarity measure are still valid. In this section, we will point out that the second point (b), regarding the newly proposed similarity measure of Julian et al. (2012) contains questionable results. In the axiom system for a reasonable similarity measure, Li and Cheng (2002), Mitchell (2003) and Xu (2007) all defined that the similarity measure of two IFSs $\tilde{A}$ and $\tilde{B}$ must satisfy the property (P1)

$$0 \leqslant S(\tilde{A},\tilde{B}) \leqslant 1. \tag{11}$$

Now we analyze the similarity measure proposed by Julian et al. (2012) to see if it indeed satisfies Eq. (11). Thus, the following inequality must always be present:

$$0 \leqslant \left(\int_{-\infty}^{\infty} w(x)|\mu_D(x) - \mu_E(x)|^2 dx\right)^{1/2} \\ + \left(\int_{-\infty}^{\infty} w(x)|\phi_D(x) - \phi_E(x)|^2 dx\right)^{1/2} \leqslant 1, \tag{12}$$

for any two IFSs, $\tilde{D}$ and $\tilde{E}$, with weight function $w(x)$. Intuitively, the inequality in Eq. (12) means that the sum of two distances between (a) two membership functions $\mu_D$ and $\mu_E$, and (b) two non-membership functions $\upsilon_D$ and $\upsilon_E$ (owing to $(\phi_D - \phi_E)^2 = ((1 - \upsilon_D) - (1 - \upsilon_E))^2 = (\upsilon_D - \upsilon_E)^2$) will be less than one. We know that each of distances being less than one does not imply that the sum of two distances will also be less than one.

We changed the weight function from $w(x) = 1$ for $0 \leqslant x \leqslant 1$ to $w(x) = \frac{5}{3}$ for $0.2 \leqslant x \leqslant 0.8$ and two new patterns, $\tilde{D}$ and $\tilde{E}$ with

$$\mu_D(x) = 0.8, \quad \text{for} \quad 0.2 \leqslant x \leqslant 0.8; \quad \mu_D(x) = 0, \quad \text{otherwise}, \tag{13}$$

$$\nu_D(x) = 0.1, \quad \text{for} \quad 0.2 \leqslant x \leqslant 0.8; \quad \nu_D(x) = 0, \quad \text{otherwise}, \tag{14}$$

$$\mu_E(x) = 0.2, \quad \text{for} \quad 0.2 \leqslant x \leqslant 0.8; \quad \mu_E(x) = 0, \quad \text{otherwise} \tag{15}$$

and

$$\nu_E(x) = 0.6, \quad \text{for} \quad 0.2 \leqslant x \leqslant 0.8; \quad \nu_E(x) = 0, \quad \text{otherwise}. \tag{16}$$

We found that

$$\left(\int_0^{0.8} w(x)|\mu_D(x) - \mu_E(x)|^2 dx\right)^{1/2} = \frac{6}{10} \tag{17}$$

and

$$\left(\int_0^{0.8} w(x)|\phi_D(x) - \phi_E(x)|^2 dx\right)^{1/2} = \frac{5}{10}, \tag{18}$$

such that inequality of Eq. (12) is invalid and

$$S_{new,2}(\tilde{D},\tilde{E}) = -0.1, \tag{19}$$

clearly indicating that the similarity measure proposed by Julian et al. (2012) contains questionable results. We can now say that the similarity measure proposed by Julian et al. (2012) violates the system of axioms for similarity measures.

## 4. Further discussion for the advantage of the similarity measure of Julian et al. (2012)

In Section 3, we already showed that Julian's similarity measure did not satisfy the axiom for similarity measure. In this section, we will demonstrate that the advantage of differentiability of Julian et al. (2012), point (c), is an illusion.

Julian et al. (2012) mentioned that their own similarity measure has an advantage to differentiate the best alternative from the

**Table 1**
For $p = 2$, comparison between $S_{new,2}(\tilde{P}_k, \tilde{Q})$ and $S_{\mathrm{mod},2}(\tilde{P}_k, \tilde{Q})$.

| | $k = 1$ | $k = 2$ | $k = 3$ |
|---|---|---|---|
| $S_{\mathrm{mod},2}(\tilde{P}_k, \tilde{Q})$ | 0.7697 | 0.7556 | 0.8346 |
| $S_{new,2}(\tilde{P}_k, \tilde{Q})$ | 0.5393 | 0.5111 | 0.6692 |

**Table 2**
For $p = 2$, computation of $f(\tilde{P}_k, \tilde{Q})$.

| | $k = 1$ | $k = 2$ | $k = 3$ |
|---|---|---|---|
| $f(\tilde{P}_k, \tilde{Q})$ | 0.0787 | 0.0222 | 0.3384 |

**Table 3**
Comparison among $1 - S_{new,2}(\tilde{P}_k, \tilde{Q}),\ 1 - S_{\mathrm{mod},2}(\tilde{P}_k, \tilde{Q})$ and $1 - f(\tilde{P}_k, \tilde{Q})$.

| | $k = 1$ | $k = 2$ | $k = 3$ |
|---|---|---|---|
| $1 - S_{\mathrm{mod},2}(\tilde{P}_k, \tilde{Q})$ | 0.2303 | 0.2444 | 0.1654 |
| $1 - S_{new,2}(\tilde{P}_k, \tilde{Q})$ | 0.4607 | 0.4889 | 0.3308 |
| $1 - f(\tilde{P}_k, \tilde{Q})$ | 0.9213 | 0.9778 | 0.6616 |

second best. In the following, we will show that Julian's assertion was developed based on false reasoning. First, we will restate Julian's findings for a sample $Q$ and three patterns $P_k$ with $k = 1, 2, 3$. For the detailed data for $Q$ and $P_k$ with $k = 1, 2, 3$, please refer to Section 5 of Julian et al. (2012). We will only focus on Julian's results related to two-norm, which has been reproduced Tables 1 and 2 of Julian et al. (2012).

From Table 1, owing to

$$S_{\mathrm{mod},2}(\tilde{P}_2, \tilde{Q}) = 0.76 < S_{\mathrm{mod},2}(\tilde{P}_1, \tilde{Q}) = 0.77 < S_{\mathrm{mod},2}(\tilde{P}_3, \tilde{Q}) = 0.83 \tag{20}$$

for similarity measure $S_{\mathrm{mod},2}$, $\tilde{P}_3$ is the best alternative and $\tilde{P}_1$ is the second best one.

Julian et al. (2012) computed that

$$\frac{S_{\mathrm{mod},2}(\tilde{P}_1, \tilde{Q})}{S_{\mathrm{mod},2}(\tilde{P}_3, \tilde{Q})} = \frac{0.7697}{0.8346} = 0.9222 \tag{21}$$

and

$$\frac{S_{new,2}(\tilde{P}_1, \tilde{Q})}{S_{new,2}(\tilde{P}_3, \tilde{Q})} = \frac{0.5397}{0.6692} = 0.8065, \tag{22}$$

implying that the similarity between (a) the sample $\tilde{Q}$ and the best pattern $\tilde{P}_3$, and (b) the sample $\tilde{Q}$ and the second best pattern $\tilde{P}_1$, is more noticeable by Julian's approach $S_{new,2}(\tilde{P}_k, \tilde{Q})$ than that of $S_{\mathrm{mod},2}(\tilde{P}_k, \tilde{Q})$, proposed by Mitchell (2003).

We will construct a function to reveal the above discussion is based on a wrong foundation. We assume that

$$f(\tilde{A}, \tilde{B}) = 1 - 2\left(\int_{-\infty}^{\infty} w(x)|\mu_A(x) - \mu_B(x)|^2 dx\right)^{1/2} - 2\left(\int_{-\infty}^{\infty} w(x)|\phi_A(x) - \phi_B(x)|^2 dx\right)^{1/2}. \tag{23}$$

Applying Eq. (23), we find $f(\tilde{P}_k, \tilde{Q})$ for $k = 1, 2, 3$ to list them in our second table.

If we compute the ratio between the best and the second best to find that

$$\frac{f(\tilde{P}_1, \tilde{Q})}{f(\tilde{P}_3, \tilde{Q})} = \frac{0.0787}{0.3384} = 0.2326. \tag{24}$$

If we compare the findings of Eqs. (21), (22), and (24), can we announce that our new function of Eq. (23) preserve a better differentiability between the best and the second best?

If we merge the results from Tables 1 and 2 and then rewrite them in a distance expression to list them in the next Table 3.

From Table 3, if we compute the ratio between the best and the second best then

$$\frac{0.1654}{0.2303} = \frac{0.3308}{0.4607} = \frac{0.6616}{0.9213}. \tag{25}$$

The above computation of distance reveals that three approaches, $1 - S_{\mathrm{mod},2}(\tilde{P}_k, \tilde{Q})$, $1 - S_{new,2}(\tilde{P}_k, \tilde{Q})$ and $1 - f(\tilde{P}_k, \tilde{Q})$ will imply the same ratio. Hence, the differentiability between the best and the second best of the similarity measure proposed by Julian et al. (2012) is a bias. After we changed its expression in distance, then the differentiability should be the same for $1 - S_{\mathrm{mod},2}(\tilde{P}_k, \tilde{Q})$, $1 - S_{new,2}(\tilde{P}_k, \tilde{Q})$ and $1 - f(\tilde{P}_k, \tilde{Q})$. Our (false) similarity measure $f$ has the same feature as the similarity measure of Julian et al. (2012) that both do not satisfy the axiom of similarity measure. Based on the comparison of Eqs. (22) and (24), our similarity measure $f$ has better differentiability than that of Julian et al. (2012). Equation (9) of Julian et al. (2012) should be divided by 2 before taking subtraction and our formula of Eq. (23) for $f$ should be divided by 4 before taking subtraction that more bias formula has better differentiability. It reveals that the logic foundation for the comparison used by Julian et al. (2012) is questionable. Therefore, the corner stone of Julian's similarity measure, differentiability between the best and the second best, no longer exists.

## 5. Conclusions

Li and Cheng (2002), Mitchell (2003), and Xu (2007) have constructed a set of axioms for which all similarity measures must abide by. In this paper we have constructed a similarity measure that has identical features to that of the one in Julian et al. (2012), implicating that the similarity measure proposed by Julian et al. (2012) has violated the fundamental axioms of similarity measure. Furthermore, after comparing the similarity measures, it was revealed that the methodology utilized in Julian et al. (2012) to compare differentiability is a bias.

## References


Atanassov, K., 1986. Intuitionistic fuzzy sets. Fuzzy Sets Systems 20, 87–96.

Hung, W.L., Yang, M.S., 2004. Similarity measures of intuitionistic fuzzy sets based on Hausdorff distance. Pattern Recognition Lett. 25 (14), 16031611.

Julian, P., Hung, K.C., Lin, S.J., 2012. On the Mitchell similarity measure and its application to pattern recognition. Pattern Recognition Lett. 33, 1219–1223.

Li, D.F., Cheng, C.T., 2002. New similarity measures of intuitionistic fuzzy sets and application to pattern recognitions. Pattern Recognition Lett. 23, 221–225.

Mitchell, H.B., 2003. On the Dengfend–Chuntian similarity measure and its application to pattern recognition. Pattern Recognition Lett. 24, 31013104.

Park, J.H., Park, J.S., Kwun, Y.C., Lim, K.M., 2007. New similarity measures on intuitionistic fuzzy sets. Adv. Soft Comput. 40, 2230.

Vlachos, I.K., Sergiadis, G.D., 2007. Intuitionistic fuzzy information – applications to pattern recognition. Pattern Recognition Lett. 28 (2), 197206.

Xu, Z.S., 2007. Some similarity measures of intuitionistic fuzzy sets and their applications to multiple attribute decision making. Fuzzy Optimiz. Decision Making 6 (2). Fuzzy Optim. Decis. Making 6, 109–121.

Xu, Z., Yager, R.R., 2009. Intuitionistic and interval-valued intuitionistic fuzzy preference relations and their measures of similarity for the evaluation of agreement within a group. Fuzzy Optimiz. Decis. Making 8 (2), 123139.

Yusoff, B., Taib, I., Abdullah, L., Wahab, A.F., 2011. A new similarity measure on intuitionistic fuzzy sets. Internat. J. Comput. Math. Sci. 5 (2), 7074.