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Endogenous fertility and human capital in a Schumpeterian growth model

Angus C. Chu · Guido Cozzi · Chih-Hsing Liao

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Abstract This study develops a scale-invariant Schumpeterian growth model with endogenous fertility and human capital accumulation. The model features two engines of long-run economic growth: R&D-based innovation and human capital accumulation. One novelty of this study is endogenous fertility, which negatively affects the growth rate of human capital. Given this growth-theoretic framework, we characterize the dynamics of the model and derive comparative statics of the equilibrium growth rates with respect to structural parameters. As for policy implications, we analyze how patent policy affects economic growth through technological progress, human capital accumulation, and endogenous fertility. In summary, we find that strengthening patent protection has (a) a positive effect on technological progress, (b) a negative

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A. C. Chu (✉) · G. Cozzi
Durham Business School, Durham University, Durham, UK
e-mail: anguscccc@gmail.com

G. Cozzi
e-mail: guido.cozzi@durham.ac.uk

C.-H. Liao
Institute of Economics, Academia Sinica, Taipei, Taiwan
e-mail: chihhsingliao@gmail.com

C.-H. Liao
Department of Economics, National Chengchi University, Taipei, Taiwan

A. C. Chu
School of Economics, Shanghai University of Finance and Economics, Shanghai, China

effect on human capital accumulation through a higher rate of fertility, and (c) an ambiguous overall effect on economic growth.

Keywords Economic growth · Endogenous fertility · Patent policy

JEL Classification O31 · O34 · O40

1 Introduction

In this study, we develop a scale-invariant Schumpeterian growth model with endogenous fertility and human capital accumulation. In the model, there are two engines of long-run economic growth. The first growth engine is R&D-based innovation, whereas the second growth engine is human capital accumulation. One novelty of this study is that we consider endogenous fertility, which negatively affects the growth rate of human capital per capita through two channels. First, a higher rate of fertility has a crowding-out effect on households' time endowment, which in turn decreases the accumulation of human capital. Second, a higher rate of fertility has a diluting effect on human capital per member of households. Given the growth-theoretic framework, we characterize the dynamics of the model and derive comparative statics of the equilibrium growth rates with respect to structural parameters. As for policy implications, we analyze how patent policy affects economic growth through technological progress, human capital accumulation, and endogenous fertility. In summary, we find that strengthening patent protection has (a) a positive effect on technological progress, (b) a negative effect on human capital accumulation through a higher rate of fertility, and (c) an ambiguous overall effect on economic growth.

In the model, optimizing households choose the fertility rate by trading off the marginal utility of higher fertility against its marginal costs arising from (a) foregone wages, (b) the dilution of financial assets per capita, and (c) the dilution of human capital per capita. We find that strengthening patent protection that increases the market power of firms weakens the *foregone-wage* effect and the *human-capital-diluting* effect but strengthens the *asset-diluting* effect of fertility. On the one hand, weakening the foregone-wage effect and the human-capital-diluting effect leads to a higher fertility rate. On the other hand, strengthening the asset-diluting effect leads to a lower fertility rate. We find that the effects of patent policy on the dilution of financial assets and the dilution of human capital cancel each other. As a result, strengthening patent protection unambiguously increases fertility through weakening the foregone-wage effect, and this higher rate of fertility reduces human capital accumulation, which in turn leads to a negative effect on economic growth. Together with the positive effect of patent protection on R&D and technological progress, the overall effect on economic growth is ambiguous. Furthermore, we find that a stronger preference for fertility (i.e., a larger value of the fertility-preference parameter) tends to strengthen the negative effect of patent policy on economic growth.

The intuition of the above results can be explained as follows. Strengthening patent protection that increases the market power of firms raises the share of income that goes to monopolistic profits giving rise to a conventional positive effect on R&D and technological progress. However, it also reduces the share of income that goes to other factor inputs including labor. As a result of lower wages, the opportunity cost of nonmarket activities decreases; consequently, households reallocate their time from labor supply to nonmarket activities including child rearing. This is the weakening foregone-wage effect discussed above. The higher rate of fertility in turn reduces the rate of human capital accumulation by crowding out parents' time and reducing the amount of resources per child. Because economic growth is driven by both technological progress and human capital accumulation, the overall effect of patent policy on economic growth is ambiguous. Finally, we also calibrate the model to aggregate data of the US economy to provide a quantitative analysis on the relative strength of these opposing effects of patent policy.

Our study relates to the literature on R&D-based growth models. In this literature, there has been a very important debate about the presence of counterfactual scale effects in the first-generation models, such as Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992). In response to this critique, subsequent generations of R&D-based growth models have been developed to remove the strong scale effect (i.e., a positive relationship between population size and long-run growth).[1] In these scale-invariant models, the long-run growth rate is either solely or partly determined by the population growth rate that is *assumed* to be exogenous. However, in a more realistic framework, the fertility rate should be treated as an endogenous variable chosen by optimizing households. In this study, we develop a scale-invariant quality-ladder growth model with endogenous fertility and human capital accumulation. In some recent vintages of R&D-based growth models, the long-run growth rate is increasing in the population growth rate (i.e., a *weak* scale effect); however, even this weak scale effect is not supported empirically.[2] Therefore, we follow Strulik (2005) to model human capital accumulation in order to generate a negative relationship between fertility and economic growth.

Our study also relates to the literature on endogenous fertility and R&D-driven growth for which Growiec (2006) provides an excellent review.[3] Jones (2001) develops a semi-endogenous growth model with endogenous fertility

[1] See Jones (1999) for an excellent review of these subsequent generations of R&D-based growth models.

[2] See, for example, Strulik (2005) for a discussion.

[3] See also Barro and Becker (1989) for a seminal study on endogenous fertility in an overlapping-generation model with exogenous growth.

to analyze the emergence of rapid growth and demographic transitions.[4] To simplify their analysis, Jones (2001, 2003) and Growiec (2006) consider a model in which the allocation of inputs to R&D is exogenously determined. The present study differs from Jones (2001, 2003) and Growiec (2006) by developing a quality-ladder model in which both fertility and the allocation of factor inputs are endogenously determined through the market equilibrium. Therefore, our model follows more closely the footsteps of Connolly and Peretto (2003), who develop an R&D-based growth model with vertical and horizontal innovations to analyze demographic shocks and industrial policies that affect the costs of R&D and/or entry. However, our model differs from Connolly and Peretto (2003) by featuring human capital accumulation as well as creative destruction that gives rise to the importance of patent breadth that protects an innovation against previous innovations. Therefore, the present study complements their interesting analysis by analyzing another important set of industrial policy: the effects of intellectual property rights on fertility, human capital accumulation, and economic growth.

Finally, our study also relates to the literature on patent policy and economic growth. The seminal study in the literature on optimal patent design is Nordhaus (1969).[5] While studies in this patent-design literature mostly analyze patent policy in partial-equilibrium models, the present study follows more closely a related macroeconomic literature by analyzing the effects of patent policy in a quantitative dynamic general equilibrium model. The seminal dynamic general equilibrium analysis on optimal patent length is Judd (1985), who finds that optimal patent length can be infinite. Subsequent studies by Iwaisako and Futagami (2003) and Futagami and Iwaisako (2007) show that optimal patent length is usually finite in the Romer model due to an additional distortionary effect on intermediate goods that is absent in Judd (1985).[6] While this branch of studies focuses on characterizing optimal patent length, another branch of studies in the literature analyzes the effects of other patent-policy levers on innovation and growth. See, for example, Li (2001) and Iwaisako and Futagami (2011) on patent breadth; O'Donoghue and Zweimuller (2004) on forward patent protection and patentability requirement; Cozzi (2001) and Cozzi and Spinesi (2006) on intellectual appropriability; Kwan and Lai (2003), Horii and Iwaisako (2007), Furukawa (2007, 2010), and Cysne and Turchick (2012) on patent protection against imitation; Dinopoulos and Syropoulos (2007) and Davis and Sener (2012) on rent protection activities; and Chu (2009) and Chu et al. (2012) on blocking patents. Some of these studies find that strengthening patent protection generates a negative effect on economic

[4]See also Jones (2003), who analyzes the effects of an exogenous increase in the R&D share of labor chosen by the government. He finds that this policy change increases growth in the short run but decreases growth in the long run through a *lower* rate of fertility due to a *crowding-out* effect on labor supply. In contrast, our result of a negative effect of patent breadth on economic growth is based on a *higher* rate of fertility through an *opportunity-cost* effect of lower foregone wages.

[5]See Scotchmer (2004) for a comprehensive review of this patent-design literature.

[6]See also Horowitz and Lai (1996).

growth, and this finding is consistent with the detailed case studies analyzed in Jaffe and Lerner (2004), Bessen and Meurer (2008), and Boldrin and Levine (2008). The present study contributes to this literature by analyzing a novel mechanism through endogenous fertility that patent policy reduces human capital accumulation causing a negative effect on economic growth. To our knowledge, this interaction between patent policy, endogenous fertility, human capital accumulation, and economic growth has never been explored in the literature.

The rest of this study is organized as follows. Section 2 describes the model. Section 3 derives the equilibrium allocation and characterizes the dynamics of the model. Section 4 analyzes the effects of patent policy on economic growth and social welfare. Section 5 considers an extension of the model. The final section concludes.

2 A quality-ladder model with endogenous fertility and human capital accumulation

In this section, we develop a scale-invariant version of the Grossman and Helpman (1991) quality-ladder model. The key changes in our model are as follows: First, we consider endogenous fertility instead of exogenous fertility following the setup in Razin and Ben-Zion (1975) and Yip and Zhang (1997). Second, we allow for variable patent breadth as in Li (2001) and Iwaisako and Futagami (2011) in order to analyze the effects of patent policy. Third, we remove the strong scale effect through diluting R&D inputs by the scale of the economy following Laincz and Peretto (2006). In the literature, there are two seminal approaches to remove the strong scale effect. The first approach is the semi-endogenous growth model in which long-run economic growth is solely determined by the population growth rate.[7] The second approach is the second-generation model in which long-run economic growth is determined by both the population growth rate and the R&D share of labor.[8] In our model, economic growth depends on both the population growth rate and the share of human capital allocated to R&D resembling a second-generation model.[9] Finally, we introduce human capital accumulation as in Strulik (2005) to generate a negative effect of fertility on economic growth. Given that the quality-ladder model has been well studied, we will describe the familiar features briefly to conserve space and discuss the new features in details.

[7] Early studies on the R&D-based semi-endogenous growth model include Jones (1995), Kortum (1997), and Segerstrom (1998).

[8] Early studies on the second-generation R&D-based *endogenous* growth model include Young (1998), Dinopoulos and Thompson (1998), and Peretto (1998).

[9] See Laincz and Peretto (2006) and Ha and Howitt (2007) for empirical evidence that supports the second-generation R&D-based growth model.

2.1 Households

There is a unit continuum of identical households. As is standard in the literature on endogenous fertility, households derive utility from fertility. Here, we consider a continuous-time setup similar to Yip and Zhang (1997), which in turn is based on the discrete-time setup in the seminal study by Razin and Ben-Zion (1975). The intergenerational utility of households is the discounted sum of per capita utility across time.[10] Specifically, the utility function of a household is given by

$$U = \int_0^\infty e^{-\rho t} u(c_t, n_t) dt, \tag{1}$$

where $u(c_t, n_t) = \ln c_t + \alpha \ln n_t$. c_t is the per capita consumption of final goods (numeraire), and n_t is the number of births per person at time t. Given N_t as the size of population, the total number of births is $\dot{N}_t = n_t N_t$. In this simple model with zero mortality, n_t is also the population growth rate. $\alpha > 0$ is a fertility-preference parameter. $\rho > 0$ is the discount rate.

Each household maximizes Eq. 1 subject to the following asset-accumulation equation.

$$\dot{a}_t = (r_t - n_t)a_t + w_t l_t - c_t. \tag{2}$$

a_t is the amount of financial assets per capita, and r_t is the rate of return on assets. An increase in n_t reduces the amount of assets per capita, and we refer to this effect as the asset-diluting effect of fertility. w_t is the wage rate, and l_t is human capital-embodied labor supply. Each person has one unit of time to allocate between fertility, work, and education. The time spent on fertility is given by $n_t/\theta < 1$, where $\theta > 0$ is a parameter that is inversely related to the time cost of fertility.[11] At time t, the stock of human capital per capita is h_t. Each person combines her remaining time endowment $1 - n_t/\theta$ with her human capital h_t for work l_t and education e_t subject to

$$h_t(1 - n_t/\theta) = l_t + e_t. \tag{3}$$

Increasing n_t reduces the amount of time available for work and education capturing the foregone-wage effect of fertility. The law of motion for human capital per capita is

$$\dot{h}_t = \xi e_t - (n_t + \delta)h_t, \tag{4}$$

[10] See Growiec (2006) for an interesting discussion on alternative ways of modeling endogenous fertility in the growth literature.

[11] We follow a common approach in the literature to assume that θ is independent of capital accumulation or technological progress; see also Yip and Zhang (1997) and Connolly and Peretto (2003). Otherwise, as technology or human capital accumulates, θ increases causing a lower time cost of fertility, which in turn leads to a rising fertility rate instead of a constant fertility rate (i.e., ruling out a balanced growth path). However, we think it is reasonable that parental human capital contributes to the health and education of children, and this positive effect is captured by the law of motion for human capital per capita in Eq. 4.

where $\xi > \rho$ is a productivity parameter for human capital accumulation. $n_t h_t$ captures the human-capital-diluting effect of fertility as in Strulik (2005). The parameter $\delta \geq 0$ is the depreciation rate of human capital.

From standard dynamic optimization, the Euler equation is

$$\frac{\dot{c}_t}{c_t} = r_t - n_t - \rho, \tag{5}$$

and the consumption-fertility optimality condition is

$$\frac{\alpha}{n_t} = \frac{1}{c_t}\left[a_t + \left(\frac{1}{\theta} + \frac{1}{\xi}\right) w_t h_t\right]. \tag{6}$$

This condition equates the marginal utility of fertility given by α/n_t to the marginal utility of consumption (in response to a change in fertility) given by $\left[a_t + w_t h_t \left(1/\theta + 1/\xi\right)\right]/c_t$. The first term a_t/c_t captures the asset-diluting effect of fertility, and this effect is positively related to the value of assets per capita. The second term $\theta^{-1} w_t h_t / c_t$ captures the foregone-wage effect of fertility, and the third term $\xi^{-1} w_t h_t / c_t$ captures the human-capital-diluting effect of fertility. Both of these effects are positively related to the wage rate. From dynamic optimization, we can also derive an equilibrium condition that equates the returns on assets and human capital.

$$r_t = \frac{\dot{w}_t}{w_t} - \delta + \xi(1 - n_t/\theta). \tag{7}$$

We will show that this condition determines the equilibrium growth rate of human capital.

2.2 Final goods

Final goods are produced by competitive firms that aggregate intermediate goods using a standard Cobb–Douglas aggregator given by

$$Y_t = \exp\left(\int_0^1 \ln X_t(i) di\right). \tag{8}$$

$X_t(i)$ denotes intermediate goods $i \in [0, 1]$. From profit maximization, the conditional demand function for $X_t(i)$ is

$$X_t(i) = Y_t / p_t(i), \tag{9}$$

where $p_t(i)$ is the price of $X_t(i)$.

2.3 Intermediate goods

There is a unit continuum of industries producing differentiated intermediate goods. Each industry is temporarily dominated by an industry leader until the arrival of the next innovation, and the owner of the new innovation becomes the next industry leader.[12] The production function for the leader in industry i is

$$X_t(i) = z^{q_t(i)} L_{x,t}(i). \tag{10}$$

The parameter $z > 1$ is the step size of productivity improvement, and $q_t(i)$ is the number of productivity improvements that have occurred in industry i as of time t. $L_{x,t}(i)$ is the production labor in industry i . Given $z^{q_t(i)}$, the marginal cost of production for the industry leader in industry i is $mc_t(i) = w_t/z^{q_t(i)}$. It is useful to note that we here adopt a cost-reducing view of vertical innovation as in Peretto (1998).

Standard Bertrand price competition leads to a profit-maximizing price given by

$$p_t(i) = \mu(z, b) mc_t(i), \tag{11}$$

where $\mu = z^b > 1$ and $b \in (0, 1)$ denote patent breadth. In the original Grossman and Helpman (1991) model, the patent holder is assumed to have complete protection against imitation such that $b = 1$. Li (2001) considers a more general policy environment with incomplete patent protection against potential imitation such that $b \in (0, 1)$. Here, we follow the formulation in Li (2001), and this simple setup captures Gilbert and Shapiro's (1990) seminal insight on "breadth as the ability of the patentee to raise price." From Eq. 9, the amount of monopolistic profit is

$$\pi_t(i) = \left(\frac{\mu - 1}{\mu}\right) p_t(i) X_t(i) = \left(\frac{\mu - 1}{\mu}\right) Y_t. \tag{12}$$

Therefore, a larger patent breadth b increases the markup μ and the amount of monopolistic profit improving the incentives for R&D. For the rest of this study, we simply use μ to measure the strength of patent breadth. Finally, production-labor income is

$$w_t L_{x,t}(i) = \left(\frac{1}{\mu}\right) p_t(i) X_t(i) = \left(\frac{1}{\mu}\right) Y_t. \tag{13}$$

Equations 12 and 13 show that strengthening patent protection increases the share of profit income (i.e., π_t/Y_t) and decreases the share of wage income (i.e., $w_t L_{x,t}/Y_t$). Through these effects, patent policy affects the equilibrium rate of fertility.

2.4 R&D

Denote $v_t(i)$ as the share value of the monopolistic firm in industry i . Because $\pi_t(i) = \pi_t$ for $i \in [0, 1]$ from Eq. 12, $v_t(i) = v_t$ in a symmetric equilibrium that

[12] This is known as the Arrow replacement effect in the literature. See Cozzi (2007) for a discussion on the Arrow effect.

features an equal arrival rate of innovation across industries.[13] In this case, the familiar no-arbitrage condition for v_t is

$$r_t v_t = \pi_t + \dot{v}_t - \lambda_t v_t. \tag{14}$$

This condition equates the interest rate to the asset return per unit of asset. The asset return is the sum of (a) monopolistic profit π_t, (b) potential capital gain $\dot{v}_t$, and (c) expected capital loss $\lambda_t v_t$ from creative destruction for which λ_t is the arrival rate of the next innovation.

There is a unit continuum of R&D firms indexed by $j \in [0, 1]$. They hire R&D labor $L_{r,t}(j)$ for innovation. The zero-expected-profit condition of firm j is

$$v_t \lambda_t(j) = w_t L_{r,t}(j), \tag{15}$$

where the *firm-level* arrival rate of innovation is

$$\lambda_t(j) = \overline{\varphi}_t L_{r,t}(j). \tag{16}$$

To remove the strong scale effect, we follow Laincz and Peretto (2006) to specify that $\overline{\varphi}_t$ is decreasing in the scale of the economy. Specifically, we assume that $\overline{\varphi}_t = \varphi L_{r,t}^{\phi-1}/(h_t N_t)^{\phi}$,[14] where $h_t N_t$ captures the scale of the economy and the parameter $\phi \in (0, 1)$ inversely measures the negative duplication externality commonly discussed in the literature, see, for example, Jones (1995) and Jones and Williams (2000). Given $L_{r,t} = \int_0^1 L_{r,t}(j)dj$, the *aggregate* arrival rate λ_t of innovation features decreasing returns to scale in $L_{r,t}$.[15]

3 Decentralized equilibrium

The equilibrium is a time path of allocations $\{c_t, n_t, h_t, l_t, N_t, Y_t, X_t(i), L_{x,t}(i), L_{r,t}(j)\}$ and a time path of prices $\{p_t(i), w_t, r_t, v_t\}$. Also, at each instance of time, the following holds:

- Households maximize utility taking $\{r_t, w_t\}$ as given;
- Competitive final-goods firms produce $\{Y_t\}$ to maximize profit taking $\{p_t(i)\}$ as given;

[13] We follow the standard approach in the literature to focus on the symmetric equilibrium. See Cozzi et al. (2007) for a theoretical justification for the symmetric equilibrium to be the unique rational-expectation equilibrium in the quality-ladder growth model.

[14] In an earlier version of this study, see Chu and Cozzi (2011), we consider a semi-endogenous-growth version of the model by specifying $\overline{\varphi}_t$ to be decreasing in aggregate technology. In that model, we find that patent breadth has the same effects on fertility as in the current framework. However, the current framework is more general because long-run growth depends also on the R&D share of human capital whereas this R&D share only plays a role on short-run growth, but not on long-run growth in the semi-endogenous growth model.

[15] We assume constant returns to scale at the firm level in order to be consistent with free entry and zero expected profit.

- Monopolistic intermediate-goods firms produce $\{X_t(i)\}$ and choose $\{L_{x,t}(i), p_t(i)\}$ to maximize profit taking $\{w_t\}$ as given;
- R&D firms choose $\{L_{r,t}(j)\}$ to maximize expected profit taking $\{w_t, v_t\}$ as given;
- The market-clearing condition for human capital-embodied labor supply holds such that $l_t N_t = L_{x,t} + L_{r,t}$;
- The market-clearing condition for final goods holds such that $Y_t = c_t N_t$; and
- The share value of monopolistic firms adds up to the total value of household assets such that $v_t = a_t N_t$.

The aggregate production function is given by

$$Y_t = Z_t L_{x,t}, \tag{17}$$

where aggregate technology Z_t is defined as

$$Z_t = \exp\left(\int_0^1 q_t(i) di \ln z\right) = \exp\left(\int_0^t \lambda_\tau d\tau \ln z\right). \tag{18}$$

The second equality of Eq. 18 applies the law of large numbers. Differentiating the log of Eq. 18 with respect to t yields the growth rate of aggregate technology given by

$$g_{z,t} \equiv \frac{\dot{Z}_t}{Z_t} = \lambda_t \ln z = (\varphi \ln z)\left(\frac{L_{r,t}}{h_t N_t}\right)^{\phi}. \tag{19}$$

As for the dynamics of the model, Proposition 1 shows that the economy is always on a unique and saddle-point stable balanced growth path.

Proposition 1 *Given a constant level of patent breadth μ, the economy immediately jumps to a unique and saddle-point stable balanced growth path along which each variable grows at a constant (possibly zero) rate.*

Proof See Appendix A (available online). □

3.1 Balanced growth path

Given Proposition 1, we analyze the equilibrium allocation on the balanced growth path in this section. On the balanced growth path, the arrival rate of innovation is constant so that $L_{r,t}$ and $h_t N_t$ must grow at the same rate. The steady-state growth rate of technology is

$$g_z = (\varphi \ln z) s_r^{\phi}, \tag{20}$$

where we define $s_r \equiv L_{r,t}/(h_t N_t)$ and $s_x \equiv L_{x,t}/(h_t N_t)$ as the shares of human capital devoted to R&D and production, respectively.

Combining Eqs. 13 and 17 yields $w_t = Z_t/\mu$, which implies

$$\frac{\dot{Z}_t}{Z_t} = \frac{\dot{w}_t}{w_t} = \frac{\dot{c}_t}{c_t} + n_t + \rho + \delta - \xi(1 - n_t/\theta), \tag{21}$$

where the second equality of Eq. 21 is derived by substituting Eq. 5 into Eq. 7. The steady-state growth rate of consumption per capita is

$$g_c = g_y - n = g_z + g_h, \tag{22}$$

where g_y is the steady-state growth rate of Y_t. In other words, our model features two engines of growth (i.e., technological progress g_z and human capital accumulation g_h). Substituting Eq. 22 into Eq. 21 yields

$$g_h = \xi(1 - n/\theta) - n - \rho - \delta. \tag{23}$$

Therefore, the growth rate of human capital per capita is decreasing in n. The first negative effect (i.e., $-\xi n/\theta$) arises from the crowding out of fertility on time endowment. The second negative effect (i.e., $-n$) is the human-capital-diluting effect of fertility. Finally, the growth rate of consumption c_t is

$$g_c = g_z + g_h = (\varphi \ln z)s_r^{\phi} - (1 + \xi/\theta)n + \xi - \rho - \delta. \tag{24}$$

Equation 24 shows that economic growth g_c is increasing in s_r and decreasing in n. In other words, by introducing human-capital accumulation into the R&D-based growth model, we are able to generate a negative relationship between fertility and economic growth as in Strulik (2005). Furthermore, in our model, endogenous fertility generates an additional negative effect on human capital accumulation through the crowding out of time endowment that is absent in the exogenous-fertility model in Strulik (2005).

Using Eqs. 13 and 15, we derive the first equation for solving the model as follows:

$$\frac{v_t \lambda_t}{L_{r,t}} = w_t = \frac{Y_t}{\mu L_{x,t}} \Leftrightarrow \frac{s_r}{s_x} = (\mu - 1)\frac{\lambda}{\rho + \lambda}, \tag{25}$$

where $\lambda = \varphi s_r^{\phi}$. The second equation for solving the model can be obtained by combining the time-endowment constraint and the labor-market clearing condition:

$$1 - \frac{n}{\theta} = \frac{l_t}{h_t} + \frac{e_t}{h_t} = s_r + s_x + \frac{e_t}{h_t}. \tag{26}$$

From Eq. 4, the steady-state growth rate of h_t is

$$g_h \equiv \frac{\dot{h}_t}{h_t} = \xi \frac{e_t}{h_t} - n - \delta. \tag{27}$$

Equating Eqs. 27 and 23 yields

$$\frac{e_t}{h_t} = 1 - \frac{n}{\theta} - \frac{\rho}{\xi}, \tag{28}$$

which describes a negative relationship between n and e_t/h_t. Using Eq. 28, we can simplify Eq. 26 to

$$\frac{\rho}{\xi} = s_r + s_x. \tag{29}$$

Combining Eqs. 25 and 29 yields the following polynomial function that solves the equilibrium s_r^* as an implicit function in structural parameters.

$$\varphi\mu s_r^* + \rho(s_r^*)^{1-\phi} = \varphi(\mu - 1)\rho/\xi. \tag{30}$$

Finally, to solve for the equilibrium fertility rate n^*, we make use of the consumption-fertility optimality condition in Eq. 6:[16]

$$\begin{aligned}\frac{\alpha}{n^*} &= \frac{a_t}{c_t} + \frac{1}{\theta}\left(\frac{w_t h_t}{c_t}\right) + \frac{1}{\xi}\left(\frac{w_t h_t}{c_t}\right)\\ &= \left(\frac{\mu-1}{\mu}\right)\frac{1}{\rho + \varphi(s_r^*)^{\phi}} + \frac{1}{\theta}\left(\frac{1}{\mu s_x^*}\right) + \frac{1}{\xi}\left(\frac{1}{\mu s_x^*}\right),\end{aligned} \tag{31}$$

where s_r^* and s_x^* are implicit functions in structural parameters from Eqs. 29 and 30.

3.2 Comparative statics

In this subsection, we analyze the comparative statics of the equilibrium growth rates with respect to structural parameters. For simplicity, we present the results for the limiting case of $\phi \to 1$ under which we obtain closed-form solutions; however, the results for the general case of $\phi \in (0, 1)$ are qualitatively the same.[17] For patent breadth μ , the results for $\phi \to 1$ and $\phi \in (0, 1)$ are drastically different,[18] and we will present the analysis of patent breadth for the general case of $\phi \in (0, 1)$ in the next section.

Taking the limit of $\phi \to 1$, Eq. 30 simplifies to

$$s_r^*(\underset{+}{\varphi}, \underset{-}{\xi}, \underset{+}{\rho}) = \rho\left[\left(\frac{\mu-1}{\mu}\right)\frac{1}{\xi} - \frac{1}{\varphi\mu}\right]. \tag{32}$$

Then, substituting Eq. 32 into Eq. 20 yields

$$g_z^*(\underset{+}{\varphi}, \underset{-}{\xi}, \underset{+}{\rho}, \underset{+}{z}) = (\varphi \ln z)s_r^* = (\rho \ln z)\left[\left(\frac{\mu-1}{\mu}\right)\frac{\varphi}{\xi} - \frac{1}{\mu}\right]. \tag{33}$$

An increase in R&D productivity φ raises s_r^*, which in turn increases g_z^*. An increase in the productivity ξ of human capital accumulation reduces the share of human capital allocated to production s_x^* and R&D s_r^* as shown in Eq 29; as a result, g_z^* decreases. A larger step size z of innovation has a positive externality effect on g_z^*. Finally, unlike the usual R&D-based growth model, a higher discount rate ρ has a positive effect on s_r^* and g_z^* here. Intuitively, a larger ρ reduces the incentives to invest in human capital, which in turn increases the share of human capital allocated to production s_x^* and R&D s_r^* as shown in Eq. 29.

[16] It is useful to recall that $a_t = v_t/N_t$.

[17] See Appendix B (available online) for derivations.

[18] See footnote 20 for a discussion.

Substituting Eqs. 32 and 29 into Eq. 31 yields

$$n^*(\underset{-}{\varphi}, \underset{-}{\xi}, \underset{+}{\rho}, \underset{+}{\theta}, \underset{+}{\alpha}) = \frac{\rho\alpha\theta}{\theta + \xi\varphi/(\xi + \varphi)}. \tag{34}$$

An increase in R&D productivity φ raises $w_t h_t / c_t$ and leads to a higher opportunity cost of fertility; as a result, n^* decreases. Similarly, an increase in the productivity ξ of human capital accumulation reduces the share of human capital allocated to production, which in turn increases $w_t h_t / c_t$ and leads to a higher opportunity cost of fertility. A higher discount rate ρ reduces the incentives to invest in human capital, which in turn increases the time spent on fertility. A larger θ implies a lower time cost of fertility and naturally increases n^*. Finally, as households value fertility more (i.e., a larger α), they choose a higher rate of fertility n^*.

Substituting Eq. 34 into Eq. 23 yields

$$g_h^*(\underset{+}{\varphi}, \underset{+}{\xi}, \underset{-}{\rho}, \underset{+}{\theta}, \underset{-}{\alpha}, \underset{-}{\delta}) = \xi - \delta - \rho\left[1 + \frac{\alpha(\theta + \xi)}{\theta + \xi\varphi/(\xi + \varphi)}\right]. \tag{35}$$

The negative effect of φ on n^* translates into a positive effect on g_h^* because of the inverse relationship between n^* and g_h^*. As for ξ, it has a direct positive effect on g_h^* as well as an indirect positive effect on g_h^* through a smaller n^*.[19] Similarly, a larger ρ has a direct negative effect on g_h^* as well as an indirect negative effect on g_h^* through a larger n^*. As for θ, although it has an indirect negative effect on g_h^* through a larger n^*, this indirect negative effect is dominated by the direct positive effect of θ on g_h^*. A larger depreciation rate δ of human capital has a negative effect on g_h^*. Finally, a larger α leads to a higher rate of fertility n^*; as a result, the economy exhibits a lower growth rate of human capital per capita. Therefore, a stronger preference for fertility has a negative effect on economic growth.

4 Growth and welfare effects of patent breadth

In addition to the comparative statics with respect to other parameters analyzed in the previous section, we devote this section to explore the effects of a policy variable, namely, patent breadth μ. Taking the total differentials of Eq. 30, we obtain

$$\frac{ds_r^*}{d\mu} = \frac{\varphi s_x^*}{\varphi\mu + (1 - \phi)\rho(s_r^*)^{-\phi}} > 0. \tag{36}$$

Therefore, the R&D share s_r^* of human capital is increasing in μ, and this is the standard positive effect of patent breadth on R&D through a larger share of monopolistic profits. Equations 29 and 36 together imply that the production share s_x^* of human capital is decreasing in μ.

[19] To see this, differentiating Eq. 23 with respect to ξ yields $\partial g_h/\partial\xi = (1 - n/\theta) - (1 + \xi/\theta)\partial n/\partial\xi > 0$. Recall that $1 - n/\theta > 0$ and $\partial n/\partial\xi < 0$.

Equation 31 determines the equilibrium n^* as a function in μ . As for the comparative statics of n^* with respect to μ, we need to consider all the general equilibrium effects of μ on n^* . The first term on the right-hand side of Eq. 31 captures the asset-diluting effect of fertility. For a given s_r^*, a larger patent breadth strengthens this effect by increasing a_t/c_t (i.e., the ratio of asset value to consumption) and leads to a lower rate of fertility. The second term on the right-hand side of Eq. 31 captures the foregone-wage effect of fertility. For a given s_x^*, a larger patent breadth weakens this effect by decreasing $w_t h_t/c_t$ (i.e., the ratio of wage income to consumption) and leads to a higher rate of fertility. The third term on the right-hand side of Eq. 31 captures the human-capital-diluting effect of fertility. For a given s_x^*, a larger patent breadth also weakens this effect by decreasing $w_t h_t/c_t$ and leads to a higher rate of fertility.

Although there are two positive effects and one negative effect on fertility, we nonetheless derive an unambiguously positive effect because the human-capital-diluting effect and the asset-diluting effect cancel each other. To see this result, we first differentiate α/n^* with respect to μ and then substitute Eqs. 25 and 36 into the resulting expression to obtain

$$\frac{\partial \alpha/n^*}{\partial \mu} = \frac{1}{\mu}\left(\frac{1}{\rho+\varphi(s_r^*)^{\phi}}\right)\left(\frac{1}{\mu} - \frac{\phi\varphi}{\varphi\mu+(1-\phi)\rho(s_r^*)^{-\phi}}\right) - \frac{1}{\mu s_x^*}\left(\frac{1}{\theta}+\frac{1}{\xi}\right)\left(\frac{1}{\mu} - \frac{\varphi}{\varphi\mu+(1-\phi)\rho(s_r^*)^{-\phi}}\right). \tag{37}$$

It can be shown that

$$\frac{\partial \alpha/n^*}{\partial \mu} < 0 \Leftrightarrow \frac{s_x^*}{\rho+\varphi(s_r^*)^{\phi}} < \frac{\rho}{\varphi\mu(s_r^*)^{\phi}+\rho}\left(\frac{1}{\theta}+\frac{1}{\xi}\right). \tag{38}$$

Applying Eqs. 25 and 30, this inequality further simplifies to $1/\theta > 0$. Therefore, unless the foregone-wage effect is absent (i.e., $\theta \to \infty$), n^* is increasing in μ for $\phi \in (0, 1)$.[20] Differentiating Eq. 24 with respect to μ yields

$$\frac{\partial g_c^*}{\partial \mu} = \underbrace{\frac{\partial g_z^*}{\partial \mu}}_{>0} + \underbrace{\frac{\partial g_h^*}{\partial \mu}}_{<0} = (\varphi \ln z)\underbrace{\frac{\partial (s_r^*)^{\phi}}{\partial \mu}}_{>0} - (1+\xi/\theta)\underbrace{\frac{\partial n^*}{\partial \mu}}_{>0}. \tag{39}$$

Also, Eqs. 31 and 37 imply that the value of $\partial n^*/\partial \mu$ is increasing in α, whereas Eq. 30 implies that $\partial (s_r^*)^{\phi}/\partial \mu$ is independent of α. We summarize our main results in Proposition 2.

Proposition 2 *An increase in the strength of patent protection μ increases the equilibrium fertility rate n^* and decreases the growth rate g_h^* of human capital. However, it also increases the R&D share s_r^* of human capital and the growth rate g_z^* of technology. Therefore, the overall effect of μ on the growth rate g_c^* of*

[20] In the special case of $\phi = 1$, it can be shown that patent breadth μ has no effect on the fertility rate n^* leaving only the positive effect on s_r^*.

consumption is ambiguous. If the fertility-preference parameter α *is sufficiently large, then the negative effect of* μ *on* g_c^* *through* g_h^* *dominates the positive effect through* g_z^*.

Proof Proven in the text. □

The positive effect of patent breadth on R&D and the growth rate of technology is consistent with previous studies such as Li (2001) and Iwaisako and Futagami (2011). The novel finding here is the negative effect of patent breadth on the accumulation of human capital. This result is complementary to the negative effect of patent breadth on the accumulation of *physical* capital in Iwaisako and Futagami (2011). The main difference is that our result is driven by an endogenous fertility rate n^*, whereas Iwaisako and Futagami (2011) consider neither human capital nor fertility in their analysis.[21]

4.1 Welfare analysis

In this section, we analyze the welfare effects of strengthening patent protection. First, we derive the welfare of households in the market equilibrium. Then, we also derive the first-best optimal allocation. On the balanced growth path, Eq. 1 simplifies to the following welfare expression that applies to both the market equilibrium and the first-best allocation:

$$U = \frac{1}{\rho}\left(\ln c_0 + \frac{g_c}{\rho} + \alpha \ln n\right), \tag{40}$$

where c_0 is initial consumption per capita. Using $c_t = Y_t/N_t$ and Eq. 17, we express c_0 as

$$c_0 = Z_0 h_0 s_x, \tag{41}$$

where Z_0 and h_0 are the initial *exogenous* levels of technology and per capita human capital, respectively. The steady-state growth rate of consumption is $g_c = g_z + g_h$, where g_z is given by Eq. 20 and g_h is given by Eq. 27. The resource constraint in Eq. 3 can be reexpressed as

$$1 = \frac{n}{\theta} + s_r + s_x + \frac{e_0}{h_0}, \tag{42}$$

where we will normalize $h_0 = 1$ and simply use $e \equiv e_0/h_0$ for convenience. Substituting the above conditions into Eq. 40 and dropping the exogenous terms yield

$$U = \frac{1}{\rho}\left(\ln s_x + \frac{\varphi \ln z}{\rho} s_r^{\phi} + \frac{\xi e - n}{\rho} + \alpha \ln n\right). \tag{43}$$

[21] In an extension of their model with human capital, Futagami and Iwaisako (2007) find that increasing *patent length* has a negative effect on the wage rate and human capital accumulation via an alternative mechanism other than endogenous fertility.

Under the market equilibrium denoted by a superscript *, differentiating Eq. 43 with respect to patent breadth μ yields

$$\rho\frac{\partial U^*}{\partial\mu}=\underbrace{\frac{\partial\ln s_x^*}{\partial\mu}}_{-}+\frac{\varphi\ln z}{\rho}\underbrace{\frac{\partial(s_r^*)^{\phi}}{\partial\mu}}_{+}+\frac{\xi}{\rho}\underbrace{\frac{\partial e^*}{\partial\mu}}_{-}-\frac{1}{\rho}\underbrace{\frac{\partial n^*}{\partial\mu}}_{+}+\alpha\underbrace{\frac{\partial\ln n^*}{\partial\mu}}_{+}. \tag{44}$$

Strengthening patent protection has the following effects on welfare. First, it decreases the production share s_x^* of human capital, which has a negative effect on welfare by reducing the initial level of consumption. Second, it increases the R&D share s_r^* of human capital, which has a positive effect on welfare by increasing the growth rate of technology. Third, it decreases human-capital investment e^* as implied by Eq. 28 giving rise to a negative effect on welfare through a lower growth rate of human capital. Finally, it increases the fertility rate n^*, which has a direct positive effect on welfare as well as a negative welfare effect through a lower growth rate of human capital. Whether the overall effect of μ on U^* is positive or negative is an empirical question that we will explore in the subsequent quantitative analysis.

As for the first-best allocation denoted by superscript **, we maximize Eq. 43 subject to Eq. 42 and obtain

$$s_x^{**}=\frac{\rho}{\xi}, \tag{45}$$

$$(s_r^{**})^{1-\phi}=\frac{\phi\varphi\ln z}{\xi}, \tag{46}$$

$$n^{**}=\frac{\alpha\rho}{1+\xi/\theta}, \tag{47}$$

$$e^{**}=1-\left(s_x^{**}+s_r^{**}+\frac{n^{**}}{\theta}\right). \tag{48}$$

The comparative statics with respect to the parameters are quite intuitive. Comparing Eqs. 45 and 29, we find that $s_x^{**}>s_x^*$ because $s_r^*>0$; in other words, the decentralized market allocates *an insufficient share of human capital to production*. Comparing Eqs. 46 and 30, we find that if $\phi\ln z\geq\mu-1$, then $s_r^{**}>s_r^*$ because $(s_r^*)^{1-\phi}<\varphi(\mu-1)/\xi$ from Eq. 30. In other words, *R&D underinvestment occurs if either ϕ or z is sufficiently large*. Intuitively, a larger ϕ implies a smaller degree of the negative duplication externality and a larger z implies a larger degree of the positive externality from z to technological progress g_z^* as shown in Eq. 20. Given R&D underinvestment, patent breadth μ may help to mitigate this market failure.

As for the comparison between n^{**} and n^*, we first note that from Eq. 31, $\lim_{\mu\to1}n^*=\alpha\rho/(1+\xi/\theta)$ because $\lim_{\mu\to1}s_r^*=0$ from Eq. 30 and $\lim_{\mu\to1}s_x^*=\rho/\xi$ from Eq. 29. Therefore, as μ approaches one, n^* approaches n^{**}. Given that n^* is increasing in μ from Proposition 2, we have $n^{**}<n^*$ for $\mu>1$; in other words, households choose *a suboptimally high rate of fertility* under the decentralized equilibrium. Finally, as μ approaches one, we have (a) $n^{**} = n^*$, (b) $s_x^{**} = s_x^*$, (c) $s_r^{**} > s_r^* = 0$, and (d) $e^{**} < e^*$. As μ increases above one, e^* decreases towards e^{**}, whereas s_r^* increases towards s_r^{**}; however, s_x^* and n^* deviate from their optimal values. As μ becomes sufficiently large, e^* may fall below e^{**}, and s_r^* may rise above s_r^{**}. In the quantitative analysis, we will compute the welfare changes from increasing patent breadth μ.

4.2 Quantitative analysis

In this section, we calibrate the model to examine quantitatively the effects of patent breadth on technological progress, fertility, human capital accumulation, economic growth, and social welfare. In the previous section, we show that strengthening patent protection has both positive and negative effects on economic growth. In this section, we calibrate the model to examine which effect is likely to dominate.

There are nine structural parameters $\{\rho, \delta, \phi, \alpha, \theta, \varphi, \mu, z, \xi\}$ that are relevant for this numerical exercise. First, we set the discount rate ρ to a standard value of 0.04. As for the depreciation rate of human capital, Stokey and Rebelo (1995) consider a range between 3 and 8 % to be reasonable for the US economy, so we set δ to an intermediate value of 0.055. As for the returns to scale in the R&D process, Kortum (1992) estimates a parameter similar to ϕ and finds that its value is 0.2; therefore, we set ϕ to 0.2.[22] We consider a range of values for the fertility-preference parameter $\alpha \in \{1, 2, 4, 8\}$. Finally, we use the following five empirical moments to pin down the values of the remaining five parameters. We consider a long-run population growth rate of 1 % for the US economy, and the equilibrium condition for n^* is given by Eq. 31. As for the arrival rate of innovation, we use the estimate in Laitner and Stolyarov (2011) to set $\lambda^* = \varphi(s_r^*)^\phi$ to 0.17. We set the equilibrium R&D share of GDP to 0.03 for the US economy, and this share is given by $S_r^* \equiv wL_r/Y$ in the model:

$$S_r^* = \left(\frac{\mu - 1}{\mu}\right) \frac{\lambda^*}{\rho + \lambda^*}. \tag{49}$$

We set the growth rate $g_z^* = \lambda^* \ln z$ of total factor productivity to 1 % and the growth rate $g_c^* = g_z^* + g_h^*$ of consumption per capita to 2 %. In other words, we consider a useful benchmark in which technological progress and human capital accumulation contribute equally to economic growth. Given a chosen value for each of $\{\rho, \delta, \phi, \alpha\}$, these five empirical moments determine

[22] Jones and Williams (2000) consider a lower bound for ϕ to be about 0.5 based on empirical estimates for the social rate of return to R&D. In this study, we intentionally choose a small value for ϕ in order for TFP growth g_z not to be overly responsive to the R&D share of GDP. In our calibration, the elasticity of TFP growth with respect to the R&D share of GDP is about 0.2. If we set ϕ to a higher value of 0.5, the elasticity increases to about 0.5. However, while R&D share of GDP in the USA has been steadily rising, TFP growth shows no significant upward trend.

Table 1 Calibration

α	θ	φ	μ	z	ξ
1.0	0.048	0.443	1.038	1.061	0.145
2.0	0.026	0.465	1.038	1.061	0.185
4.0	0.018	0.500	1.038	1.061	0.266
8.0	0.014	0.550	1.038	1.061	0.427

the values of $\{\theta, \varphi, \mu, z, \xi\}$, respectively. The calibrated parameter values are reported in Table 1.

Given these calibrated parameter values, we consider a counterfactual policy experiment by increasing patent breadth such that μ increases from 1.038 to 1.061 (i.e., patent breadth $b = \ln \mu / \ln z$ increases from 0.64 to 1.00). The numerical results are reported in Table 2. We see that S_r^* (i.e., the R&D share of GDP) increases by over one half. On the one hand, strengthening patent protection has a positive effect on technological progress. For all values of α, the arrival rate of innovation increases from 0.170 to 0.186 whereas the growth rate of technology increases from 1 to 1.095 %. On the other hand, strengthening patent protection raises the fertility rate from 1 % to roughly 1.003 % and decreases the growth rate of human capital. The magnitude of the decrease in g_h^* depends on α and is increasing in its parameter value. For a small value of α, the positive effect of μ on technological progress dominates the negative effect on human capital accumulation giving rise to a positive overall effect on economic growth g_c^*. For a sufficiently large value of α, the negative effect of μ on g_h^* becomes quantitatively significant and may completely offset or even dominate the positive effect on g_z^* giving rise to a slightly negative overall effect on g_c^*. As for social welfare, we find that it increases and the welfare gain ΔU (expressed in terms of equivalent variation in consumption flow) is slightly over 0.5 % of the consumption per year. If we decompose the welfare effects according to Eq. 40, the welfare gain mostly comes from (a) a higher consumption growth rate g_c^* when α is small and (b) a higher fertility rate n^* when α is large; in all cases, the welfare gain is partially offset by a reduction in initial consumption c_0.

From this quantitative analysis, we conclude that whether the positive or negative effect of patent policy on economic growth dominates depends on the empirical value of the fertility-preference parameter α. Here, we consider the calibrated values of n/θ (i.e., the fraction of time spent on fertility) to narrow down the empirical range of α. Using the calibrated values of θ in Table 1, one

Table 2 Policy experiment ($\mu = 1.061$)

α	S_r^*	λ^*	g_z^* (%)	n^* (%)	g_h^* (%)	g_c^* (%)	ΔU (%)
1.0	0.047	0.186	1.095	1.002	0.991	2.085	0.569
2.0	0.047	0.186	1.095	1.003	0.978	2.073	0.567
4.0	0.047	0.186	1.095	1.003	0.954	2.048	0.561
8.0	0.047	0.186	1.095	1.003	0.904	1.999	0.551
$\mu = 1.038$	0.030	0.170	1.000	1.000	1.000	2.000	n/a

can show that $\alpha \in \{1, 2, 4, 8\}$ corresponds to the following calibrated values of $n/\theta \in \{0.21, 0.38, 0.57, 0.73\}$. According to the American Time Use Survey from 2005 to 2009, an average person in households with youngest child under 6 years old spends less than 3 h per day for child caring as a primary activity.[23] Assuming an average of 16 h of nonsleeping time per day, the fraction of time spent on child caring in the US data is close to the lower bound of the calibrated values of n/θ implying that the empirical value of α should be reasonably small in the USA. Therefore, for the US economy, the positive effect of patent policy on technological progress is likely to dominate the negative effect on human capital accumulation. However, if the strength of fertility preference increases, the negative effect of patent policy on human capital accumulation would become quantitatively significant and offset the positive effect on technological progress.

5 Quality–quantity trade-off

In this section, we consider an extension of the model by incorporating human capital per capita into the utility function of households in order to capture a more explicit quality–quantity trade-off of fertility.[24] In particular, we explore the robustness of the positive effect of patent breadth on fertility under this extended model. The modified utility function is

$$U = \int_0^\infty e^{-\rho t} \left(\ln c_t + \alpha \ln n_t + \beta \ln h_t\right) dt, \tag{50}$$

which nests our baseline model as a special case with $\beta = 0$. In Appendix C (available online), we provide detailed derivations. Here, we sketch out the key equations of this extended model. With the additional term $\beta \ln h_t$ in the utility function, Eq. 7 derived from dynamic optimization becomes

$$r_t = \frac{\dot{w}_t}{w_t} - \delta + \xi\left(1 - \frac{n_t}{\theta}\right) + \beta \frac{\xi c_t}{w_t h_t}, \tag{51}$$

where $\beta\xi c_t/(w_t h_t)$ captures the additional benefit of accumulating human capital. The steady-state growth rate of human capital per capita in Eq. 23 becomes

$$g_h = \xi\left(1 - \frac{n}{\theta}\right) + \beta\frac{\xi c_t}{w_t h_t} - n - \rho - \delta, \tag{52}$$

where $c_t/(w_t h_t) = \mu s_x$ from Eq. 13 and $s_x \equiv L_{x,t}/(h_t N_t)$. Equating Eqs. 52 and 27 yields

$$\frac{e_t}{h_t} = 1 - \frac{n}{\theta} - \frac{\rho}{\xi} + \beta\xi\mu s_x, \tag{53}$$

[23] Persons in households with older children spend even less time for child caring.

[24] We would like to thank a referee for suggesting this interesting extension.

which replaces Eq. 28. Substituting Eq. 53 into Eq. 26 yields

$$\frac{\rho}{\xi} = s_r + (1 + \beta\xi\mu)s_x, \tag{54}$$

which replaces Eq. 29. Combining Eqs. 54 and 25 yields a polynomial function that implicitly solves the equilibrium s_r^* for $\phi \in (0.1)$. Finally, the equilibrium n^* is still given by Eq. 31.

We find that s_r^* is increasing in patent breadth μ as before. As a result, $g_z^* = (\varphi \ln z)(s_r^*)^{\phi}$ is also increasing in μ. As for n^*, we also find that n^* continues to be increasing in μ. In other words, the comparative statics of $\{s_r^*, g_z^*, n^*\}$ with respect to μ are the same as in the baseline model. This finding may not be surprising because the baseline model also contains an implicit quality–quantity trade-off of fertility as follows. On the one hand, fertility has a direct positive effect on utility. On the other hand, fertility has the negative crowding-out and dilution effects on the accumulation of human capital, which in turn affects utility through consumption. Therefore, allowing for an explicit quality–quantity trade-off should not alter our results. Interestingly, in the extended model, the effect of μ on g_h^* becomes ambiguous because the additional benefit of accumulating human capital is increasing in patent breadth (i.e., $\beta\xi c_t/(w_t h_t) = \beta\xi\mu s_x$ is increasing in μ). Rewriting Eq. 52 yields

$$g_h^* = \beta\xi\mu s_x^* - (1 + \xi/\theta)\, n^* + \xi - \rho - \delta, \tag{55}$$

where μs_x^* is increasing in μ. We find that there exists a threshold value of β below (above) which the negative effect of μ on g_h^* through n^* dominates (is dominated by) the positive effect of μ through μs_x^*. These results are summarized in the following proposition.

Proposition 3 *In the extended model, an increase in the strength of patent protection μ increases* (a) *the fertility rate n^*,* (b) *the R&D share s_r^* of human capital, and* (c) *the growth rate g_z^* of technology. However, it has opposing effects on the growth rate g_h^* of human capital. There exists a threshold value of β below (above) which the negative (positive) effect of μ on g_h^* dominates. If the negative effect dominates, then the overall effect of μ on the growth rate g_c^* of consumption would be ambiguous. In this case, if the fertility-preference parameter α is sufficiently large, then g_c^* would be decreasing in μ.*

Proof See Appendix C (available online). □

6 Conclusion

In this study, we have first developed a simple scale-invariant quality-ladder model with endogenous fertility and human capital accumulation and then apply the model to analyze the theoretical effects of patent policy on economic growth. We find that although strengthening patent protection has a positive effect on technological progress, it also has a negative effect on human capital accumulation. As a result, the overall effect on economic growth is ambiguous. In the quantitative analysis, we find that the relative magnitude of these two effects depends on the empirical value of a preference parameter on fertility. Calibrating this parameter to a reasonable value for the US economy, we find that the positive effect on technological progress is likely to dominate the negative effect on human capital accumulation rendering a positive overall effect on economic growth. However, if the country experiences a strengthening of fertility preference, the negative effect of patent policy on economic growth would become quantitatively significant and may even dominate the positive effect.

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