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ORIGINAL PAPER

Land use and rent gradients with a monopoly vendor and two central business districts

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Abstract Dispersed consumer amenities such as shopping and cultural attractions greatly influence land use patterns and rent gradients. Lai and Tsai (J Urban Econ 63:536–543, 2008) generalize the traditional Alonso–Mills–Muth model by introducing a monopoly vendor and show that the vendor will choose a boundary location. We generalize their model by allowing for inter-city shopping, which is very common in the real world. The location choice of a monopoly vendor and the consequent changes of urban configuration in two adjacent cities are discussed. This paper shows that as the distance between two CBDs decreases, the vendor may choose to locate at an inner city boundary, at the mid-point between the two cities, and at one of the outer boundaries, respectively. In many cases, a core-periphery urban structure will result.

JEL Classification R30 · R14 · L12

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1 Introduction

There is a long and rich history of research about urban land use and rent gradients in urban economics, including papers by Alonso (1964), Mills (1967), Muth (1969), and Brueckner (1987). Much of the literature in this tradition assumes a single exogenously determined central business district (CBD) and derives land use and rent gradient as a function of distance to this central attraction. The model described by this traditional literature has only limited relevance to many modern metropolitan and even rural areas. In many areas, we see polycentric configurations with an older core downtown area, and job or residential agglomerations at various surrounding points. Rent gradients are influenced by exogenous geographical structures (such as lakes and mountains) and endogenous choices (such as the location of roads, airports, and major commercial centers). Our goal is to use a simple analytical model to elucidate some of the fundamental forces that drive land uses and rent gradients around employment clusters both within and between cities. Although our formal model treats the two cities as distinct, and perhaps politically independent, all of our analytical insights apply equally well to regions near adjacent job clusters within a single city.

Our model also relates to the discussions of edge cities. Fujita et al. (1997) consider a large firm choosing its location in a monocentric city to form a secondary employment center. They find that the equilibrium urban configuration depends on the competition in labor and land markets and the technological externalities. Henderson and Mitra (1996) discuss the decision of a developer when choosing the business district capacity and location of the edge city. Based on numerical analysis with different values of parameters, many types of urban configurations can be identified. Zhang and Sasaki (2005) also employ numerical analysis to analyze the "vacated business district" in the process of the formation of an edge city. They find that sometimes an existing business district becomes vacated when an edge city is formed, especially when the conversion cost (from business usage to residential usage) is high. However, all above three papers focus on the formation of the edge city from an existing monocentric city, while our model concerns the location choice of a monopoly vendor and the consequent changes of the urban configuration in two adjacent cities. Lai and Tsai (2008) conceptualize a very different model in which there is a single monopolist firm that endogenously influences the location of consumers and their land rents paid to the absentee landowners. They demonstrate that the monopoly firm will locate at one of the city boundaries to maximize profit.[1] We generalize their model by allowing for inter-city shopping, and the location choice of a monopoly vendor. Changes in urban configuration are also discussed.[2]

[1] Tsai and Lai (2012) generalize the Lai and Tsai (2008) model to include labor market competition between the firms in CBD and the vendor. The urban configuration in their model is very similar to the above edge city literature.

[2] Recently, Takahashi (2014) constructs a model with two firms strategically choosing a location game in an Alonso–Mills–Muth city. However, the product price is fixed in his model, while our monopoly vendor endogenously determines product price, which highly affects the land use and rent gradient in the cities.

Before the entry of a modern giant vendor, we assume that there are two identical adjacent cities along a highway with a distance d between these two CBDs.[3] All residents commute to their local CBDs for their jobs, and each one consumes one unit of land and one unit of a composite good purchased from ubiquitous mom-and-pop stores with zero shopping distance. Since a global retailer (say Wal-mart or Target) may have strong cost advantage, many local mom-and-pop stores will be forced out of business after entrance of this giant retailer.

We wish to study how vendors with market power may affect rent gradients and land use. Although it is rarely true that a retailer is literally a monopolist, by modeling vendors in this way, we can gain insight about basic forces driving rent gradients and land uses without the additional complexity of game-theoretic oligopolistic competition. We can think of this giant retailer as a monopoly vendor who can serve city 1, city 2, or both.[4] The vendor, perhaps for political reasons, cannot partially serve a city—if the vendor offers any service, it must serve all residents of the city.[5] Like residents in traditional Alsonso–Mills–Muth models, each resident in our model travels to the CBD of their city to work, and lives at, and pays rent on, land somewhere in the city. Residents also travel to and buy goods from the monopoly vendor on a regular basis. Since we assume that there is only one location for the vendor, residents living in another city must make inter-city shopping, which is common in reality but rarely noticed in the literature. Note that in equilibrium all residents have equal utility, because residents are free to choose their locations in these two cities.

We expand the work of Lai and Tsai (2008) by investigating how a monopoly chooses location and price for an economy with two adjacent cities and an endogenously distributed population of consumers. In our model, the cities may be distinct (i.e., connected by a space without population) or connected. We will show that, depending upon various parameter values, a monopoly firm may maximize profit by choosing locations either at the midpoint of the cities, or at one of the city boundaries. Specifically first, when d is extremely large (i.e., the two CBDs are far apart), the vendor will locate at one city's boundary and will not serve the other city. Second, when d is large enough, the vendor may locate at city 1's right boundary and serve both cities. Third, when d is smaller, the vendor may locate at the mid-point between the cities,

[3] Note that the CBD in our model corresponds to a job center in the real world. One could think of these job centers as being parts of a metropolitan area (e.g., edge cities) or as being smaller provincial cities (e.g., Eau Claire, Stevens Point, Racine or Kenosha, Wisconsin).

[4] For example, a large retailer (Walmart or K-Mart) could easily serve only Racine, only Kenosha, or both Racine and Kenosha. In fact, many smaller US cities are served by essentially a single shopping mall for durable goods. Moreover, if the assumption that the composite goods are provided ubiquitously is still kept, then the monopoly vendor in our model can be replaced by a large public facility such as a hospital, library, airport, or sports stadium to which residents travel on a regular basis, and all results will be similar to that in the current paper.

[5] In a much more general case, development and zoning authorities approve only those developments that will adequately serve the population. In these cases, political forces are certainly involved and developments that do not adequately serve the entire population are unlikely to garner approval. Even when there are two separate jurisdictions, regional planning agencies, or state (or provincial) or national governments may become involved in issuing permits to the vendor. Generalizations in which a vendor endogenously chooses to serve a smaller portion of the population are beyond the scope of this paper but may be fruitful areas for future research.

that is, $z = d/2$, where z is the vendor location. Fourth, as d becomes even smaller, the vendor may choose to locate at one of the outer boundaries to minimize its land rent.

An important implication of our model is that a big vendor or public facility may induce an endogenous economic behavior, say inter-city shopping or visiting, and reshape the land use patterns as well as rent gradients in a system of cities. Specifically, a (local) core-periphery urban structure—with a dominant and a smaller city—may emerge naturally from our model. Moreover, the vendor may exert monopoly power to set a high price for the composite good and consequently lower the land rent that he pays.

The rest of this paper is organized as follows: Sect. 2 presents the model, and the concluding remarks are offered in Sect. 3.

2 The model

2.1 The urban configuration before the entry of a monopoly vendor

Suppose that there are two cities (city 1 and 2) with the same city size along a straight line (i.e., a highway) with city 1 at the west of city 2. The urban configuration is depicted in Fig. 1. The distance from the CBD in city 1 (location "0") to the CBD of city 2 (location "d") is d. Each resident has a job in their local CBDs and earns an identical income y that is spent to buy one unit of the composite good (with a price $\overline{p}$) from ubiquitous mom-and-pop stores and to pay his commuting costs and land rent. Residents are purely mobile, and each attains the same level of utility in equilibrium. The per unit, per distance, cost of a commuting trip is denoted by t. All land is owned by absentee landowners, and each unit of land is rented to the highest bidder. Since the CBD points (0 and d) are the most convenient places for residents, the equilibrium land rents should be $y - \overline{p}$ at these two points. For a location with a distance x to the CBD point 0 (d), the equilibrium land rent should be $y - p - tx$. The urban land rent equals the agriculture land rent (assumed to be zero) at a city boundary.

2.2 The urban configuration after the entry of the vendor

Suppose that there is a giant retailer with a strong cost advantage in producing and selling a composite good, and it is considering where to locate a store to serve one or

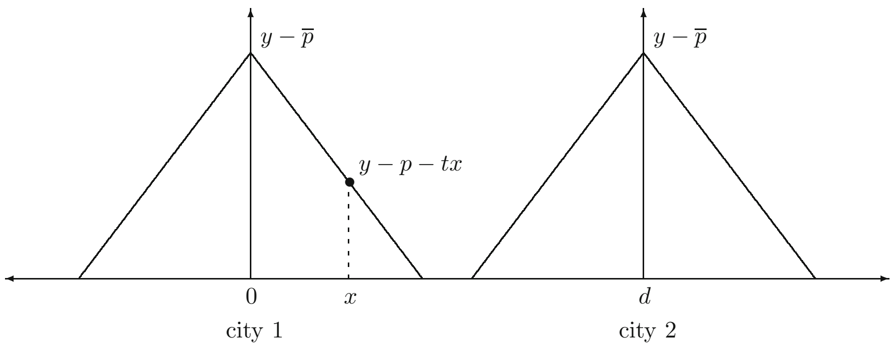

Fig. 1 The land rent pattern before the entry of a monopoly vendor

Table 1 Values of x for which $r(x)$ (rent) equals zero

Boundaries	Location range	$x \leq z$	$x > z$
City 1's West Boundary	$x \leq 0$	$x = -\left(\frac{y-p-kz}{k+t}\right)$	$x = \left(\frac{y-p+kz}{k-t}\right)$
City 1's East Boundary (if it exists)	$0 < x \leq d/2$	$x = \left(\frac{y-p-kz}{-k+t}\right)$	$x = \left(\frac{y-p+kz}{k+t}\right)$
City 2's West Boundary (if it exists)	$d/2 < x < d$	$x = -\left(\frac{y-p-kz-td}{k+t}\right)$	$x = \left(\frac{y-p+kz-td}{k-t}\right)$
City 2's East Boundary	$x \geq d$	$x = \left(\frac{y-p-kz+td}{-k+t}\right)$	$x = \left(\frac{y-p+kz+td}{k+t}\right)$

both of these cities. Assume this vendor can set a price much lower than $\overline{p}$, and thus, all mom-and-pop stores will be forced out of business after the entry of the vendor. Now residents must pay a shopping cost to travel from their homes to the location of the vendor to buy the composite good. The shopping transport rate is denoted by k. In general, people commute to the office five times per week, while they go shopping one or two times per week. Therefore, we assume $t > k$ in our model.[6]

Since the land market is purely competitive, rent must adjust, so that all residents at any location have equal utility in equilibrium. We assume that residents bear the cost of travel to their work locations (either 0 or d) and to the vendor's location. Rent will absorb all residual income after residents pay to purchase the monopoly vendor's good, costs to commute to the vendor, and costs to commute to work. Since urban land rent cannot be less than the agriculture land rent (normalized to zero), locations that require the sum of purchase and travel costs to exceed income will be unoccupied—they might be conceived of as agricultural land. Utility-maximizing residents will eliminate wasteful commuting (Hamilton and Roell 1982) and will work at the nearest CBD (point 0 or point d). Therefore, the rent at location x will be:

$$r(x) = \max\{(y - p - k|z - x| - t \cdot \min\{|0 - x|, |d - x|\}), 0\}, \quad (1)$$

where $|\alpha - \beta|$ is the distance between α and β, and z is the location of the vendor. We show the different values of x for which $r(x)$ (land rent) equals zero in Table 1.[7]

We model the vendor as simultaneously choosing location and product price to maximize profit. The vendor in our model takes into consideration the fact that the store location will influence the distribution of population in the two cities. The vendor must compare profitability along four line segments: locations to the left of city 1's

[6] If $t = k$, then land rent at the vendor's point may be higher than that of in two CBDs. For example, if the vendor chooses to locate at the mid-point of these two cities, then the distance to shopping is longer than the distance to the office for remote consumers. Therefore, the highest rent $(y - p)$ will emerge at the vendor's location, instead of the original two CBDs. This situation is less likely happen in the real world, because a traditional CBD has many functions other than a job center, such as a traffic center, culture center, or financial center.

[7] For example, when $x \leq z$ and $x \leq 0$, from (1), we have the zero rent condition: $r(x) = y - p - k(z - x) - t(0 - x) = 0$. Solving this equation yields $x = -(\frac{y-p-kz}{k+t})$. Similarly, when $x > z$ and $x \leq 0$, we have the zero rent condition: $r(x) = y - p - k(x - z) - t(0 - x) = 0$. Solving this equation yields $x = (\frac{y-p+kz}{k-t})$. Detailed calculations underlying Table 1 are available upon request to the authors.

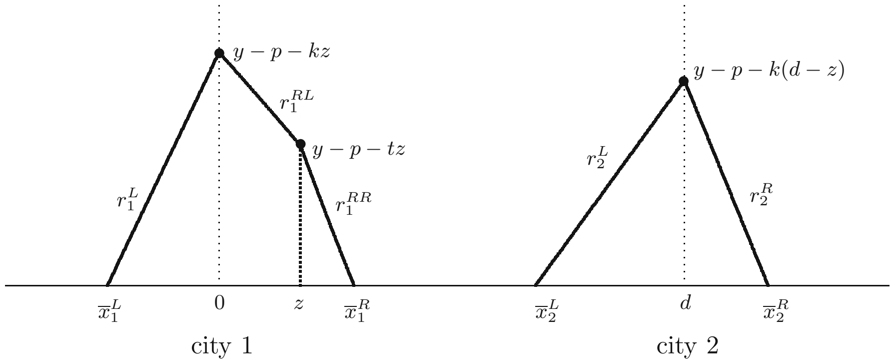

Fig. 2 A possible bid rent pattern (a large d and no overlapping in equilibrium)

CBD (point 0), locations between city 1's CBD and the right boundary of city 1 (e.g., the eastern-most location at which rent equals zero if it exists), locations between the left boundary of city 2 and city 2's CBD (point d), and locations to the right of city 2's CBD.

First, consider the case in which the vendor locates somewhere in city 1 (denoted by z), to the right of its CBD (point 0). Discussion of the case in which the vendor locates in city 2 is omitted due to symmetry.[8] We can use Eq. (1) to find the distance from each city's CBD to its left- and right-hand side boundaries—i.e., the distance from the CBD at which rent falls to zero–as a function of income, commuting and shopping costs, and the vendor's location.

Figure 2 depicts the boundaries and rent gradients of both cities when the vendor locates somewhere in city 1, to the right of its CBD, and because d is large, the two cities do not overlap. Note that r_1^{RL} (r_1^{RR}) represents the land rent to the right part of city 1, but to the left (right) side of z. Figure 3 depicts the case when the two cities have a mild overlap—x_{12} marks the intersection point.

Note that unlike conventional models, in our model, rent gradients will be asymmetric as we move away from the CBD since rents depend on the distance to both the (local) CBD and the vendor. Rents for locations nearer to the vendor will be higher.

The vendor's profitability at a particular location z is:[9]

$$\pi(z) = p(z)\,Q(z) - r(z)\,, \tag{2}$$

where $p(z)$ is the profit-maximizing monopoly price, $Q(z)$ is the quantity sold, and $r(z)$ is the rent paid by the vendor.

[8] Since both cities are identical in the absence of the vendor, the analysis of the case in which the vendor chooses the right part of city 1 (inner city area) is completely analogous to the case in which the vendor chooses the left part of city 2, the only difference being that the city in which the vendor locates will be bigger. Similarly, analysis of the case in which the vendor chooses the left part of city 1 (outer city area) is analogous to the case in which it selects the right part of city 2.

[9] If the vendor has two stores, one store serves one city. Then, this situation can be directly applied to the scenario in Lai and Tsai (2008).

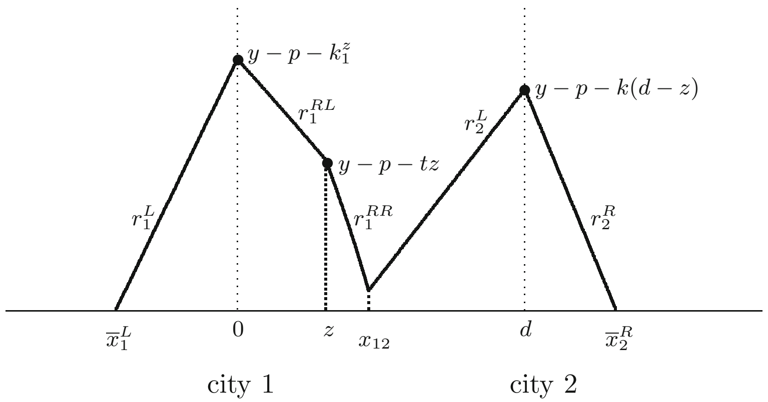

Fig. 3 A possible bid rent pattern (d is sufficiently small, so that the cities overlap in equilibrium)

There are four possible profit functions:

$$\pi_r^N = p\left(\left(\overline{x}_1^R - \overline{x}_1^L\right) + \left(\overline{x}_2^R - \overline{x}_2^L\right)\right) - (y - p - tz), \tag{3}$$

$$\pi_r^O = p\left(\left(x_{12} - \overline{x}_1^L\right) + \left(\overline{x}_2^R - x_{12}\right)\right) - (y - p - tz), \tag{4}$$

$$\pi_l^N = p\left(\left(\overline{x}_1^R - \overline{x}_1^L\right) + \left(\overline{x}_2^R - \overline{x}_2^L\right)\right) - (y - p + tz), \tag{5}$$

$$\pi_l^O = p\left(\left(x_{12} - \overline{x}_1^L\right) + \left(\overline{x}_2^R - x_{12}\right)\right) - (y - p + tz), \tag{6}$$

where '‘N” represents the nonoverlapping case, while “O” represents the overlapping case, “r” represents the case that z is in the right part of city 1, “l” represents the case that z is in the left part of city 1, and $\overline{x}_i^j$, $i = 1, 2$, $j = R, L$ is the value of x for the i's city at jth (R=right, L=left) boundary. Note that when cities just touch $z = d/2$, $\overline{x}_1^R = x_{12} = \overline{x}_2^L$, and thus $\pi_r^N = \pi_r^O$.

Case 1: When d is large

If the vendor locates in the right part of city 1, then the Lagrangian is:

$$\mathcal{L} = \pi_r^N + \lambda\left(y - p - t\left(\overline{x}_2^R - d\right) - k\left(\overline{x}_2^R - z\right)\right) + \mu\left(y - p - tz\right), \tag{7}$$

where the first constraint assures that the most remote consumer at the right boundary of city 2 ($\overline{x}_2^R$) is able to afford the inter-city shopping, and the last constraint assures that the land rent at the vendor's location is nonnegative. If the vendor chooses to locate in city 1, then the right part is clearly at least as good as the left part, because it is closer to the other city, and thus, the vendor can set a higher price for its product.[10] Solving the Kuhn–Tucker conditions ($p\partial\mathcal{L}/\partial p = 0, z\partial\mathcal{L}/\partial z = 0, \lambda\partial\mathcal{L}/\partial\lambda = 0,$ $\mu\partial\mathcal{L}/\partial\mu = 0$) yields:

[10] If we restrict $z \in [\overline{x}_1^L, 0]$, then one of the boundaries $z = \overline{x}_1^L$ or $z = 0$ will be chosen, but profits will be less than when $z = \overline{x}_1^R$. Detailed calculations are available upon request to the authors.

$$p^* = \frac{y}{2} - \frac{dk}{4},\ z^* = \frac{2y + dk}{4t}. \tag{8}$$

The maximum profit is then:

$$\pi_r^{N*} = \frac{(2y - dk)^2\, t}{4\,(t + k)\,(t - k)}\ , \tag{9}$$

If serving just one of the cities maximizes profits, then the analysis of Lai and Tsai (2008) can be applied to demonstrate that the vendor will locate at the boundary of whichever city is chosen. In this case, profit will be

$$\pi^0 = \frac{y^2}{2\,(t + k)}. \tag{10}$$

Comparing (9) and (10) yields:

$$\pi_r^{N*} - \pi^0 = \frac{2y^2t - 4tydk + td^2k^2 + 2y^2k}{4\,(t + k)\,(t - k)}. \tag{11}$$

The sign of Eq. (11) will depend on the value of the parameters. Intuitively, if d is large—the two CBDs are far apart—then the vendor may not be able to decrease price to serve both cities, and thus, it will only serve one city. The critical values for d are [11]

$$\hat{d} = \frac{\left(4\,t - 2\,\sqrt{2\,t^2 - 2\,tk}\right)y}{2tk} \quad \text{and} \quad \check{d} = \frac{\left(4t + 2\sqrt{2t^2 - 2tk}\right)y}{2tk}. \tag{12}$$

If $d > \widehat{d}$, the vendor will serve city 1 only and we can apply the analysis of Lai and Tsai (2008), so we do not discuss it here. Suppose $d < \widehat{d}$, hereafter, so that the vendor maximizes profit by serving both cities.[12] The profit-maximizing location is at the right boundary of city 1, because

$$z^* = \frac{2y + dk}{4t} = \overline{x}_1^R, \tag{13}$$

After some calculations, the analytical solutions of the four boundary points $(\overline{x}_1^L, \overline{x}_1^R, \overline{x}_2^L, \overline{x}_2^R)$ are, respectively,

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[11] Equation (12) means that $\pi_r^{N*} > \pi^0$ when $d < \hat{d}$ or $d > \check{d}$. However, if $d \geq \check{d}$, it will violate the condition that the composite good price must be positive. Because when $d > \underline{d} = 2y/k$, then $p < 0$. Checking $\check{d} - \underline{d} = \frac{y\sqrt{2\,t\,(t-k)}}{kt} > 0$, it is then $d \geq \check{d}$ is infeasible. Numerically, if $y = 100, t = 1$, and $k = 1/5$, then $\hat{d} = 367.54$.

[12] For $d < \hat{d}$, $\partial\, p/\partial\, d = -\frac{k}{4} < 0$, and $\partial\, \pi_r^{N*}/\partial\, d = \frac{kt\,(dk-2y)}{2(t+k)(t-k)} < 0$, if $d < \frac{2y}{k}$. Since $\hat{d} - \frac{2y}{k} = -\frac{y\sqrt{2t\,(t-k)}}{kt} < 0$, therefore, in the feasible region of d, the larger d, the lower equilibrium price and the lower equilibrium profit. It is straightforward that d is a negative factor to the profit of the vendor.

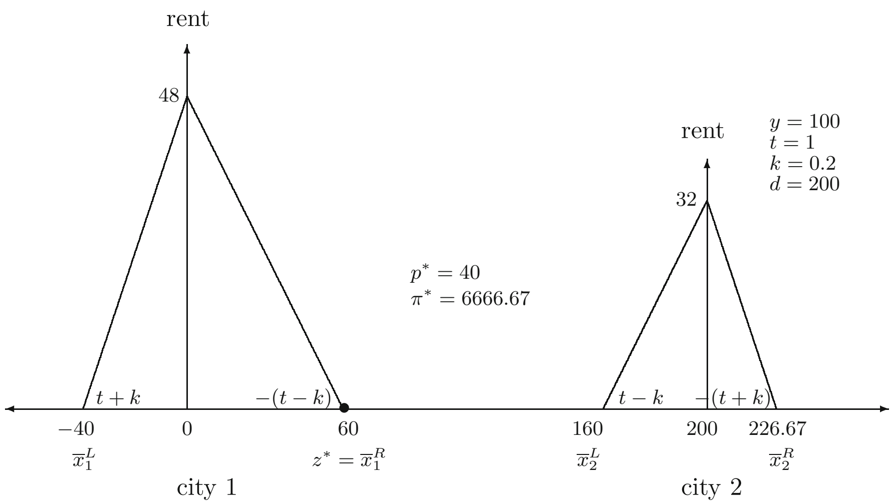

Fig. 4 An example of equilibrium land rent pattern in two cities without overlapping

$$\left[-\frac{(t-k)\,(2\,y+dk)}{4t\,(t+k)},\ \frac{2\,y+dk}{4t},\ \frac{-2\,y(t+k)-tdk+4\,t^2d-dk^2}{4t\,(t-k)},\ \frac{2\,yt+tdk+4\,t^2d+2\,yk+dk^2}{4t\,(t+k)}\right]. \tag{14}$$

As d gets smaller the right boundary of city 1 will remain the location of the vendor until the boundaries of the two cities just touch. Solving $\overline{x}_1^R = \overline{x}_2^L$ in (14) yields

$$d = \frac{2y}{2t-k} \equiv d_t, \tag{15}$$

where the subscript "t" represents "touching" and the vendor will locate at the midpoint of the two cities. We can summarize our results as the following proposition.

Proposition 1 *(i) When $d > \widehat{d} = (4t - 2\sqrt{2t^2 - 2tk}y/(2tk)$, the vendor will only serve city 1 and locate at one of the boundaries. (ii) When $d_t < d < \widehat{d}$ as $k < 2t(\sqrt{2}-1) \cong 0.828t$, the vendor will locate at the right boundary of city 1 ($z^* = \overline{x}_1^R$), and all residents in city 2 must do inter-city shopping in each time period.*[13] *(iii) If $k > 2t(\sqrt{2}-1)$, then $d_t > \hat{d}$, then vendor will serve one city. (iv) When $k = 2t(\sqrt{2}-1)$, the vendor is indifferent in serving one city or two cities, because the profits are identical.*

Using (8) and (14), Fig. 4 depicts the rent gradients in this urban configuration when $y = 100, d = 200, t = 1$, and $k = 0.2$. Note that city 1 is larger than city 2, because residence in city 1 is more attractive in that it reduces distance to the vendor.

Case 2: when the cities just touch in equilibrium ($d = \frac{2y}{2t-k} \equiv d_t$)

From (15), we have the following proposition.

[13] We are grateful to one of the referees for pointing out the possibility of $d_t > (=, <)\hat{d}$.

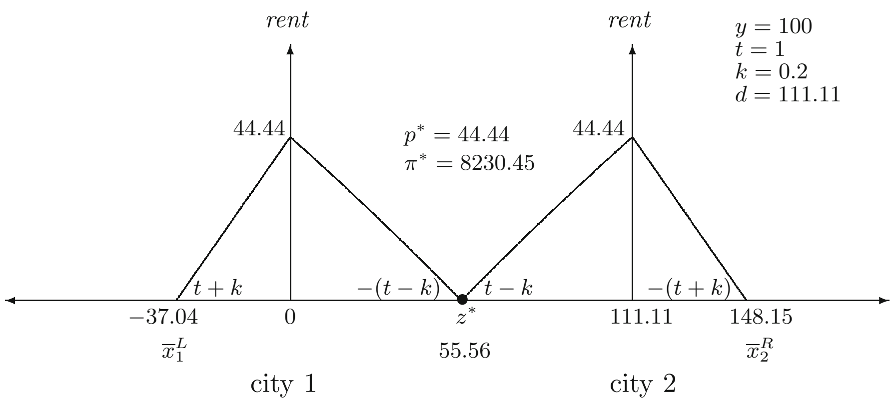

Fig. 5 An equilibrium land rent pattern of a twin city with $z^* = \frac{d}{2}$

Proposition 2 *When $d = d_t$, then $z^* = \frac{d}{2}$ and these two cities just touch in equilibrium.*

Using (8), (14), and (15), Fig. 5 is a numerical example of such an equilibrium obtained by assuming $y = 100$, $t = 1$, $k = 0.2$ and solving for $d_t = \frac{2y}{2t-k} = 111.11$. This yields $z^* = \frac{d}{2} = 55.56$.

With these parameters, the two cities have symmetric rent gradients around the vendor's location (z^*), but rent gradients to the left of city 1's CBD ($|t+k| = 1.2$) and to the right of city 2's CBD ($|t+k| = 1.2$) are much steeper than the rent gradients around $z^*(|t-k| = 0.8)$. Each city is asymmetric, with the bulk of the population closer to the vendor's location.

Case 3: d is small $\left(d < \frac{2y}{2t-k} \equiv d_t\right)$

In this case, we analyze three subcases in which d is smaller than in case 2. In subcase 3.1, the cities overlap and the vendor locates on the right side of city 1; in subcase 3.2, the vendor locates on the left side of city 1 and the two cities overlap in equilibrium; and in subcase 3.3, the vendor locates on the left side of city 1 and the two cities do not overlap in equilibrium.

Subcase 3.1: Cities overlap in equilibrium and $z \in [0, x_{12}]$

When $d < d_t$, a candidate for the equilibrium location is $z = d/2$ and both cities are connected. The vendor can adjust the composite good price to minimize the land rent at $d/2$. We set up a Lagrangian function:

$$\mathcal{L} = \pi_r^O + \lambda\left(y - p - t(\overline{x}_2^R - d) - k(\overline{x}_2^R - z)\right) + \mu(y - p - tz) + \theta(x_{12} - z),$$

the bracket after λ ensures that the most remote consumer (at $\overline{x}_2^R$) can afford to do inter-city shopping, the bracket after μ ensures nonnegative rent at z, and the last constraint ensures that z can never be larger than x_{12}. Solving $p\frac{\partial\mathcal{L}}{\partial p} = 0$, $z\frac{\partial\mathcal{L}}{\partial z} = 0$, $\lambda\frac{\partial\mathcal{L}}{\partial\lambda} = 0$,

$\mu \frac{\partial \mathcal{L}}{\partial \mu} = 0$, and $\theta \frac{\partial \mathcal{L}}{\partial \theta} = 0$ simultaneously yields

$$p^* = y - \frac{td}{2}, \quad z^* = \frac{d}{2}.$$

Then, equilibrium profit is

$$\pi_m^* = \frac{(2y - td)td}{t + k},$$

where "m" represents the case in which the vendor locates at the mid-point. Since π_m^* is quadratic in d, $d = y/t$ maximizes profit. In short, when $d < d_t$, the mid-point between these two cities could be a profit-maximizing location for the vendor. Since

$$y - p - tz = y - \left(y - \frac{td}{2}\right) - t\left(\frac{d}{2}\right) = 0, \tag{16}$$

we know that

$$\frac{\partial p}{\partial d} = -\frac{t}{2}. \tag{17}$$

From (16) and (17), we find an important result that the vendor will choose the mid-point location and raise the product price as d decreases in order to minimize the land rent and thus also maximize profit. Note that both Case 2 and Subcase 3.1 are shown as "just touching," but the parameter d in these two cases is different ($d = d_t$ in Case 2, $d < d_t$ in Subcase 3.1).

Subcase 3.2: When cities are overlapping in equilibrium and $z \in [\overline{x}_1^L, 0]$

If the vendor chooses to locate at the left part of city 1, solutions in which the cities overlap (unequal twin cities) and do not overlap (Subcase 3.3) are possible. For the first situation (unequal twin cities),[14] the Lagrangian is

$$\mathcal{L} = \pi_l^O + \lambda(y - p - t(\overline{x}_2^R - d) - k(\overline{x}_2^R - z)) + \mu(y - p + tz), \tag{18}$$

[14] In case of $z \in [x_1^L, 0]$, then the bid rent function for the left side of city 1 is

$$r_1^{LR} = y - p + tx - k(x - z),$$
$$r_1^{LL} = y - p + tx - k(z - x).$$

and the bid rent for the right side of city 1 is

$$r_1^R = y - p - tx - k(x - z).$$

Solving the Kuhn–Tucker conditions simultaneously yields

$$p^* = \frac{y}{2} + \frac{td}{4}, z^* = \frac{td - 2y}{4t} = \overline{x}_1^L. \tag{19}$$

and

$$\pi_l^{O*} = \frac{(2y + td)^2}{8(t + k)}, \tag{20}$$

where π_l^{O*} is profit when the vendor locates at the left boundary of city 1 with both cities overlapping in equilibrium.[15] The analytical solutions of boundary points $(\overline{x}_1^L, \overline{x}_1^R, x_{12}, \overline{x}_2^L, \overline{x}_2^R)$ are, respectively:

$$\left[\frac{-2y+td}{4t}, \frac{(k-t)(td-2y)}{4t(t+k)}, \frac{d}{2}, \frac{2yt-5t^2d-2yk+tdk}{4t(k-t)}, \frac{2yt+3t^2d-2yk+tdk}{4t(t+k)}\right]. \tag{21}$$

Comparing the above two strategies: locate at the mid-point of the cities (Subcase 3.1) or locate at the left-most boundary of city 1 (Subcase 3.2), the vendor calculates

$$\pi_l^{O*} - \pi_m^* = \frac{(3td - 2y)^2}{8(t + k)} > 0. \tag{22}$$

However, π_l^{O*} is valid only when $\overline{x}_1^R \geq \overline{x}_2^L$. From (21), we find

$$d \leq \frac{2y(t - k)}{t(3t + k)} \equiv d_l^O. \tag{23}$$

Therefore, when $d < d_l^O$, Subcase 3.2 is valid and the vendor will choose $z = \overline{x}_1^L$.

Subcase 3.3: When cities are nonoverlapping in equilibrium

If the cities do not overlap in equilibrium, the Lagrangian function is

$$\mathcal{L} = \pi_l^N + \lambda\left(p - t(\overline{x}_2^R - d)\right) - k(\overline{x}_2^R - z) + \mu(y - p + tz), \tag{24}$$

Solving the Kuhn–Tucker conditions yields

$$p^* = \frac{2tdk - 4yt + 2yk - t^2 + k^2}{4(-2t + k)}, \tag{25}$$

$$z^* = 0. \tag{26}$$

Note that (25) and (26) is a pair of local solutions. From a global view, we have shown that in the overlapping case (subcase 3.1), $z = 0$ is dominated by $z = d/2$. Also, in the

[15] When d is inside the unequal twin cities range, $\partial p/\partial d = \frac{t}{4} > 0$ and $\partial \pi_l^{O*}/\partial d > 0$. The larger d result in a larger equilibrium price and equilibrium profit. This is because the larger d represent a larger market size in this subcase.

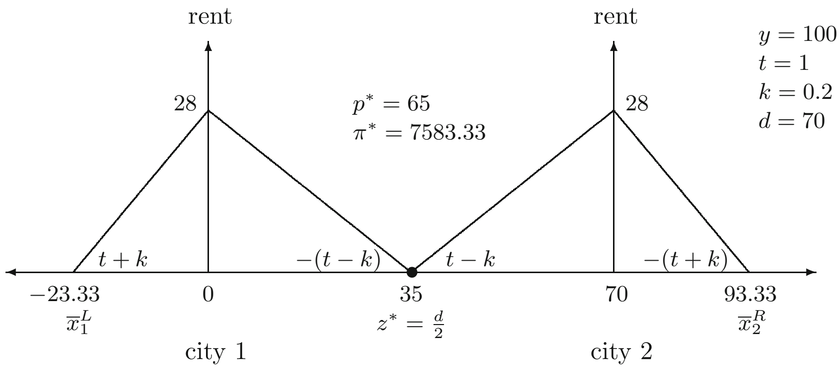

Fig. 6 An equilibrium land rent pattern in a mild $d = 70$

nonoverlapping case (Case 1), $z = 0$ is dominated by $z = \overline{x}_1^R$ [see (8)]. Therefore, this subcase will never be valid. From subcases 3.1, 3.2, and 3.3, we have the following proposition.

Proposition 3 *(i) When $d_l^O < d \leq d_t$, the vendor will locate at $z = d/2$ and pay a zero land rent by adjusting its product price. (ii) When $d \leq d_l^O$, the vendor will locate at $\overline{x}_1^L$ and pay a zero land rent.*

Subcases 3.1, 3.2, and 3.3 show that when d is small ($d_l^O < d \leq d_t$), the vendor will locate at the mid-point of the two cities. The vendor will adjust product price to keep a zero land rent at its site. Moreover, when d is very small ($d \leq d_l^O$) and two cities have a great deal of overlap, the problem is similar to that examined by Lai and Tsai (2008) and, for the reasons explained there, the vendor chooses an outer boundary.

Figure 6 simulates the case where ($d_l^O < d \leq d_t$), so that the two cities are just connected in equilibrium. Figure 7 is a numerical example when $d = d_l^O$ and $z = \overline{x}_1^L$. In this case, the two cities just touch in equilibrium. Figure 8 simulates a case where the cities overlap and the vendor locates at the left boundary of city 1, and Fig. 9 summarizes the overall results from the paper with various values of d.

2.3 Discussions

Our model can be extended to the case where the vendor needs more than one unit of land for its store. Suppose the vendor rents $h \geq 1$ unit of land, which is large enough to run his business. Then, Eq. (12) must be revised as

$$\pi(z) = p(z) \cdot Q(z) - r(z) \cdot h.$$

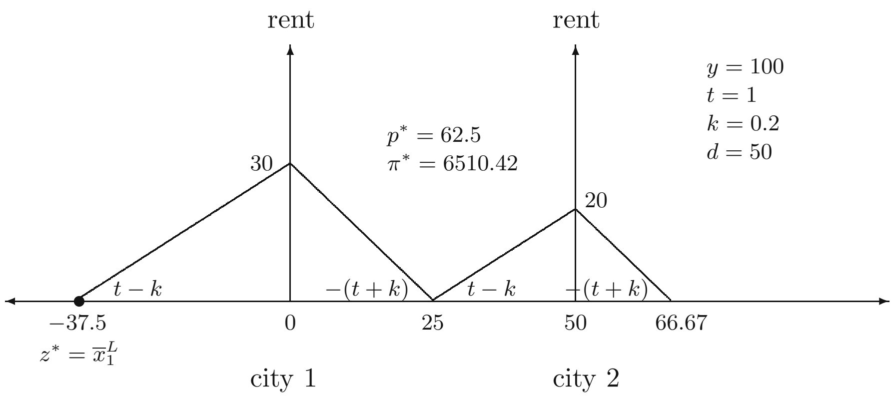

Fig. 7 When $d = 50$, $z^* = \overline{x}_1^L = -37.5$ and two cities just touch at 25

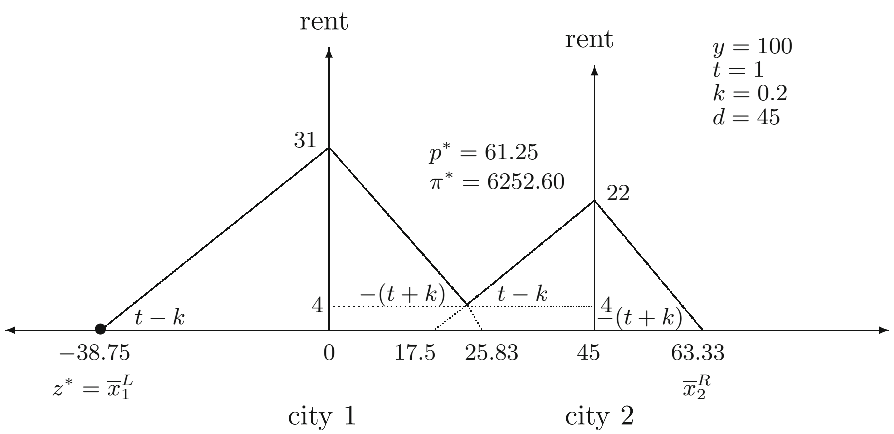

Fig. 8 An equilibrium land rent pattern in a twin city with $z^* = \overline{x}_1^L$

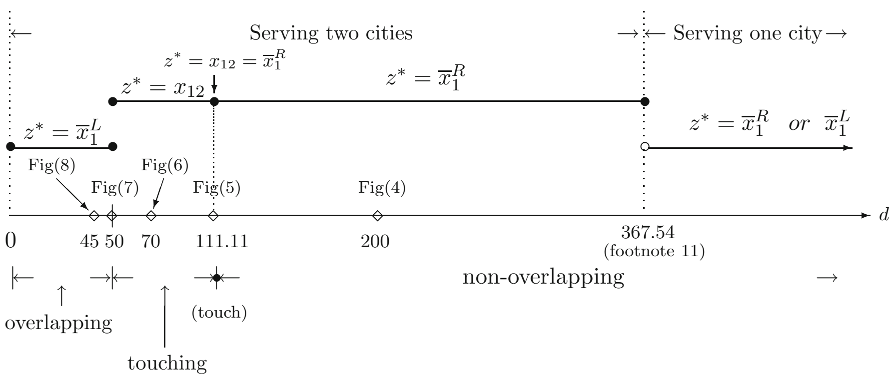

Fig. 9 An example of urban configuration and vendor locations ($y = 100$, $t = 1$, $k = 1/5$)

Then, for example, the Lagrangian of Eq. (7) (see page 7) must be revised as

$$\mathcal{L} = \pi_r^N + \lambda\left(y - p - t(\bar{x}_2^R - d) - k(\bar{x}_2^R - z)\right) + \mu \cdot h \cdot (y - p - tz)$$
$$= \pi_r^N + \lambda\left(y - p - t(\bar{x}_2^R - d) - k(\bar{x}_2^R - z)\right) + \mu' \cdot (y - p - tz)$$

where $\mu' = \mu \cdot h$. Therefore, all results will be the same as our previous analysis. This is because the vendor can exert his monopoly power via p and choose the location with the lowest rent payments. Therefore, the floor size for the vendor eventually plays no role in our model.

An important implication of our analysis is that the city which attracts the vendor will be larger than the other city except in the trivial case where the vendor locates at the exact midpoint of the two cities. In other words, a city with a commercial center that serves the entire urban area will induce the immigration of some people from other cities. Although there are just two cities in our model, the basic logic could be extended to an urban area with many cities.

One policy implication of our model is that a large private investment (such as a huge shopping center) or public investment (such as a large hospital, stadium, or museum) may significantly affect the original urban configurations. More importantly, this type investment may reinforce the advantage of the larger city and make it much bigger than the adjacent cities. In the real world, the largest city in many countries has grown faster than other domestic cities, partially because the largest city may invest in more facilities (such as an opera theater or football stadium) to attract people living in other cities.

3 Conclusions

This paper considers the possibility of inter-city shopping to generalize the work of Lai and Tsai (2008) to an urban area with two CBDs (or job centers). We show that when the two CBDs are far apart, a monopoly vendor will maximize profit by serving only one of the cities, and the results from Lai and Tsai (2008) can be applied without modification. If the two cities are closer, however, the vendor will serve both cities and locate at an inner city boundary. If the two CBDs are even closer, the vendor may locate at the point where the two cities just touch. In the extreme case, if the two CBDs are so close that the cities overlap, as they do, for example, in the Minneapolis–Saint Paul area, the vendor will locate at the outer boundary of one of the cities. An important implication of the paper is that some investments (such as a shopping mall, a library, hospital, museum, or a sports stadium) in one city may not only increase its population and rents but may also drain population from nearby cities. Unlike the new economic geography models such as Fujita and Thisse (1996), Fujita et al. (1999), and Forslid and Ottaviano (2003), this paper shows that a (local) core-periphery urban structure may emerge without relying on an assumption of increasing returns to scale. Although our model is based on the traditional urban configuration (Alonso–Mills–Muth model), strikingly, it may also be able to explain the core-periphery phenomena in the real world. Possible extensions of our work include allowing inter-city commuting for residents and modeling an endogenous labor market as in Tsai and Lai (2012). It is expected that the middle point between these two cities is more likely to be chosen by the vendor.

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