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Corrigendum

Corrigendum to “The evolution of traveling waves in a simple isothermal chemical system modeling quadratic autocatalysis with strong decay” [J. Differential Equations 256 (10) (2014) 3335–3364]

Sheng-Chen Fu ^{a,*}, Je-Chiang Tsai ^b

^a Department of Mathematical Sciences, National Chengchi University, 64, S-2 Zhi-nan Road, Taipei 116, Taiwan

^b Department of Mathematics, National Chung Cheng University, 168, University Road, Min-Hsiung, Chia-Yi 621, Taiwan

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In our recent work [1], we construct a pair of generalized super/sub-solutions for the approximate system of the autocatalytic reaction system to show the existence of traveling wave solution of (1.2) of [1] and investigate the long time behavior of the solution of the initial value problem of (1.2)–(1.3) of [1]. However, there is a gap in our proofs of [1, Lemma 3.10] and [1, Lemma 5.2]. To fix it, we first need to replace system (3.3a)–(3.3c) by

$$\begin{aligned} \delta U'' + cU' - UV_0 &= 0 \quad \text{in } (-l, l), \\ V'' + cV' + U_0V_0 - K(V_0)^{q-1}V &= 0 \quad \text{in } (-l, l), \\ (U, V)(-l) &= (U^-, V^-)(-l), \quad (U, V)(l) = (U^-, V^-)(l). \end{aligned}$$

Second, (3.8) of [1] should be replaced by

$$L = L(x_1, x_0) > \max \left\{ \frac{M}{u^*} \cdot e^{\lambda x_1}, - \frac{(Me^{\lambda x_1} + Ke^{\lambda(q-1)x_0+\lambda x_1})}{p(\lambda + \eta)} \right\}.$$

With this change, [1, Lemma 3.4] and its proof can be replaced by the following.

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* Corresponding author.

E-mail addresses: fu@nccu.edu.tw (S.-C. Fu), tsajc.math@gmail.com (J.-C. Tsai).

Lemma 0.1. *The function $V_\lambda^-(z; x_1, x_0) := \max\{0, V_\lambda^+(z; x_1) - Le^{-(\lambda+\eta)z}\}$ satisfies the inequality*

$$\begin{aligned} & (V_\lambda^-(z; x_1, x_0))'' + c(V_\lambda^-(z; x_1, x_0))' + U_\lambda^-(z; x_0)V_\lambda^-(z; x_1, x_0) \\ & - K(V_\lambda^+(z; x_0))^{q-1}V_\lambda^-(z; x_1, x_0) \geq 0 \end{aligned} \quad (0.1)$$

for all $z \neq z_1$, where the prime denotes the differentiation with respect to z .

Proof. For $z < z_1$, the inequality (0.1) holds immediately since $V_\lambda^- \equiv 0$ in $(-\infty, z_1)$. For $z > z_1$, $V_\lambda^-(z; x_1, x_0) = V_\lambda^+(z; x_1) - Le^{-(\lambda+\eta)z}$ and $U_\lambda^-(z; x_0) = u^* - Me^{-\gamma z}$. A simple computation gives that

$$\begin{aligned} (V_\lambda^-(z; x_1, x_0))' &= (V_\lambda^+(z; x_1))' + (\lambda + \eta)L e^{-(\lambda+\eta)z}, \\ (V_\lambda^-(z; x_1, x_0))'' &= (V_\lambda^+(z; x_1))'' - (\lambda + \eta)^2 L e^{-(\lambda+\eta)z}, \\ U_\lambda^-(z; x_0)V_\lambda^-(z; x_1, x_0) &= (u^* - M e^{-\gamma z})(V_\lambda^+(z; x_1) - L e^{-(\lambda+\eta)z}) \\ &\geq u^* V_\lambda^+(z; x_1) - u^* L e^{-(\lambda+\eta)z} - M e^{\lambda x_1 - (\lambda+\gamma)z}, \end{aligned}$$

and

$$(V_\lambda^+(z; x_0))^{q-1}V_\lambda^-(z; x_1, x_0) \leq (V_\lambda^+(z; x_0))^{q-1}V_\lambda^+(z; x_1) = e^{\lambda(q-1)x_0 + \lambda x_1 - \lambda q z}.$$

Together with (3.5) of [1] and definition of p , we get

$$\begin{aligned} & (V_\lambda^-(z; x_1, x_0))'' + c(V_\lambda^-(z; x_1, x_0))' + U_\lambda^-(z; x_0)V_\lambda^-(z; x_1, x_0) \\ & - K(V_\lambda^+(z; x_0))^{q-1}V_\lambda^-(z; x_1, x_0) \\ & \geq e^{-(\lambda+\eta)z}[-p(\lambda + \eta)L - M e^{\lambda x_1 + (\eta - \gamma)z} - K e^{\lambda(q-1)x_0 + \lambda x_1 + (\lambda + \eta - \lambda q)z}] \\ & \geq e^{-(\lambda+\eta)z}[e^{\lambda x_1}M(1 - e^{(\eta - \gamma)z}) + e^{\lambda(q-1)x_0 + \lambda x_1}K(1 - e^{(\lambda + \eta - \lambda q)z})] \geq 0 \\ & \quad (\text{since } L > -(M e^{\lambda x_1} + K e^{\lambda(q-1)x_0 + \lambda x_1})/p(\lambda + \eta), \text{ and} \\ & \quad \eta - \gamma < 0 \text{ and } \lambda + \eta - \lambda q < 0). \end{aligned}$$

The proof of this lemma is thus completed. \square

With the aid of Lemma 0.1, one can show [1, Lemma 3.10]. Indeed, following the proofs of [1, Lemma 3.1], [1, Lemma 3.7], [1, Lemma 3.8], [1, Lemma 3.9] with a slight modifications, one can easily show that the mapping \mathcal{F} satisfies the conditions of the Schauder fixed point theorem. So we have [1, Lemma 3.10].

To prove [1, Lemma 5.2], we should replace (5.6) of [1] by

$$v_t = v_{xx} + U_\lambda^-(x - c_\lambda t; x_0)v - K(V_\lambda^+(x - c_\lambda t; x_0))^{q-1}v.$$

Then following the identical arguments in [1, Lemma 5.2], we finish the proof of [1, Lemma 5.2].

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References

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