PATTERNS OF CHILDREN'S EMOTIONAL RESPONSES TO MATHEMATICAL PROBLEM-SOLVING

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This study was based upon the responses of 116 Taiwanese primary school children aged nine-ten years to a questionnaire concerning their emotional and motivational responses to mathematical problems. A cluster analysis revealed four distinct patterns of response, which were differentially related to attainment. These patterns of emotional response were subsequently investigated further with a smaller sample of children using a repertory grid technique and an associated interview. The four patterns were found to have differential characteristics and development processes in terms of emotional variables and preferred problem types.

With the trend toward constructivist mathematics, involving the increasing introduction of non-routine, open-ended and project-based mathematical problems, there is growing concern about students' emotional responses to mathematical problem-solving. In order to explore students' emotional responses to problem-solving in mathematics, in-depth interviews and questionnaires were conducted over the course of a school year with primary school students and teachers experiencing the constructivist mathematics in Taiwan for the first time. The topic of 'fractions' in the participants' textbook was chosen as the focused topic, as this included significant ill- and well-structured problems, which are the distinct characteristics of constructivist and traditional mathematics respectively.

According to Nitko (1996) well-structured problems are tasks that are clearly laid out, give students all the information they need, and usually have one correct answer that students can obtain by applying a procedure taught in class. In contrast most authentic problems are ill-structured. In order to solve an ill-structured problem, students have to organise, clarify and obtain information not readily available to enable understanding of the problem. In addition, there are likely to be a number of correct answers for an ill-structured problem (ibid, p.185). Seven emotional and motivational variables were regarded as closely related to children's authentic experiences of mathematical problem-solving. The seven variables were disposition toward teaching, the liberal thinking style, the conservative thinking style, the deep approach, the surface approach, self-efficacy of effort and mistake anxiety.

The study aimed to answer the following research questions:

Research question 1: Are there distinct patterns of children's emotional and motivational responses to mathematical problem-solving?

Research question 2: What are the characteristics defining these patterns?

Research question 3: Are there differences in attainments between these patterns?

METHOD

Participants

The participants in the study for the fractions topic were four mathematics teachers and their respective Year 5 pupils, aged 9-10, in a public primary school in Taiwan. Each class had 29 children, altogether 116 children. There were 51 children interviewed.

Focused problems

There were two ill-structured problems (Problems 1 and 2) and two well-structured problems (Problems 3 and 4) chosen as the focused problems for the fractions topic. These problems were taken from the textbook used in classes and had all been attempted by the children during the teaching of the topic.

Problem 1: Please use calculation procedure, $7 \div 5 = 1^2/_5$, to make a mathematical problem.

Problem 2: Mother made several pizzas and Betty got $^3/_4$ pizza. By which ways could the pizzas be divided?

Problem 3: Thirty-six scenery postcards are packed in a box. Equally divide ten boxes of postcards between nine persons. How much of a box of scenery postcards will one person get?

Problem 4: Two ribbons (of equal length) are equally divided between six persons. How much ribbon will one person get?

Measure 1: Learning Experience and Emotion Questionnaire (LEEQ)

After the teaching of the fractions topic, children in the four classes completed the questionnaire four times, once in relation to each of the four focused problems. The questionnaire consisted of 28 items and two practice items. All items were rated on a five-point Likert scale, 1= strongly disagree to 5= strongly agree. The variable of disposition toward teaching was developed based on positive aspects of teaching methods in traditional and constructivist mathematics in Taiwan. The three items regarding the liberal thinking style and three items concerning the conservative thinking style were adapted from the Sternberg-Wagner Self-Assessment Inventory on liberal and conservative styles (Sternberg, 1997, pp.71-73). The items on deep approaches (six items) and surface approaches (five items) were adapted from the Revised two-Factor Study Process Questionnaire (R-SPQ-2F, Biggs, Kember and Leung, 2001). The three items in the self-efficacy of effort section were adapted from the part of the Indiana Mathematics Belief Scales that deals with the belief that 'effort can increase mathematical ability' (Kloosterman and Stage, 1992). The three items on mistake anxiety were adapted from the scale of 'affect' in the School Failure Tolerance Scale (Clifford, 1988).

Measure 2. Child-Interview Questions with the Repertory Grid Technique

Children were interviewed using a 33-step process, including the repertory grid technique, a procedure first designed by Kelly (Kelly, 1955, p.219).

Measure 3. Mathematical Attainments

The attainment scores used were the mean standardised scores of the children's mathematical attainments from the last semester and the first school test for this semester (including the fractions topic).

RESULT 1: PATTERNS/CLUSTERS AND THEIR CHARACTERISTICS

A cluster analysis was conducted to identify the patterns of children's responses to the questionnaire. Two analyses were carried out to establish the validity of the clusters. Firstly, the data from the children's interviews were explored to interpret the clusters. Secondly, the relationships were investigated between the clusters and attainments in mathematics (Whitebread, 1996, p.4) as criterion or predictive validity. The analysis dealt with 14 variables, the seven emotional and motivational variables for the ill-structured problems and well-structured problems respectively. The mean scores of the 14 variables were firstly transformed into standardised Z scores. As distance measures are 'sensitive to outliers' (Turner, 1998, p.762), 25 cases identified as outliers were deleted. Using Ward linkage and complete linkage, a four-cluster solution was derived which was found to be reliable and interpretable. K-means cluster analyses with cluster centres from the Ward linkage and complete linkage were performed respectively. The cross-tabulation of case numbers of clusters from both K-means analyses further identified nine outliers and these were deleted. This resulted in 82 cases left for the four-cluster solution. Cluster means (in Z scores) were calculated for the 14 variables, as shown in Table 1.

Table 1 Cluster means and test results for the 14 variables

Variables	Problem type	F	P<	Cluster 1 (N=21)	Cluster 2 (N=33)	Cluster 3 (N=17)	Cluster 4 (N=11)
1. Disposition toward teaching	Ill-structured	35.5	.0005	.34*	-1.00**	.37*	.34
	Well-structured	25.0	.0005	.20	-.97**7	.40**	.26
2. Liberal thinking style	Ill-structured	24.2	.0005	.69**	-.09	.51**	-1.02**
	Well-structured	21.6	.0005	.64**	-.06	.59**	-.93**
3. Conservative thinking style	Ill-structured	6.7	.0005	-1.9	-.50**	.28	-.003
	Well-structured	8.9	.0005	-.25	-.55**	.35*	.01
4. Deep approach	Ill-structured	31.4	.0005	.64**	-.20*	.59**	-1.15**
	Well-structured	32.7	.0005	.65**	-.40**	.71**	-1.00**
5. Surface approach	Ill-structured	27.9	.0005	-.83**	-.12	.76**	.69**
	Well-structured	20.3	.0005	-.83**	-.07	.74**	.66**
6. Self-efficacy of effort	Ill-structured	23.2	.0005	.79**	-.63**	.16	-.79**
	Well-structured	26.6	.0005	.71**	-.69**	.43**	-.71*
7. Mistake anxiety	Ill-structured	14.7	.0005	-.30	-.17	.88**	.70*
	Well-structured	7.9	.0005	-.11	-.23*	.71**	.61

* significantly different from 0 at the .05 level

** significantly different from 0 at the .01 level

Cluster 1: involvement. The profile of this cluster shows that these children were active and engaged learners in mathematics classrooms. They had high self-efficacy of effort, took a deep approach rather than a surface approach to learning mathematics, solved problems in a liberal way, professed an insignificant mistake anxiety, and valued and relied on teachers' teaching for ill-structured problems, but not for well-structured problems. They seemed to be able to use adaptive learning strategies to adjust to different problem types. In accordance with the Cluster 1 identified in Turner et al.'s (1998) study, children from this cluster displayed an image of active learners, with the positive characteristics required in the learning of mathematics. Interview data revealed that children in this cluster enjoyed difficult and time-consuming mathematical problems. They seemed to have a special ability to accurately describe and sincerely appreciated teachers' teaching and intentions in a positive way.

Cluster 2: rebellion or negative ambivalence. This group of children scored negatively on all 14 variables, most significantly in relation to the variables of disposition toward teaching, self-efficacy of effort, conservative style and deep approach. They also had significantly low mistake anxiety about solving well-structured problems. The children in this cluster had overwhelmingly the most negatively perceived disposition toward teaching. They engaged in learning mathematics but in an entirely instrumental way, not perceiving teachers as worth following, with little self-efficacy, not obeying existing rules to solve problems, not investing much time in learning mathematics, not perceiving mathematics as interesting, and not even worrying much about solving well-structured problems. Children in this cluster seemed to be an 'anti-mathematics' group. A possible reason for this 'anti-mathematics' is that their learning anticipation and styles were not rewarded by or not a good match with mathematics or the current teaching, as revealed by their significant low disposition toward teaching. Interview data showed that children in this cluster viewed mathematics as a tool only, not an aim in itself. They were not keen on listening to teachers and followed the principles of efficiency, economy and practicality in mathematics learning.

Cluster 3: conformity or positive ambivalence. This cluster, in contrast to Cluster 2, presented a positive value for each variable and mostly in a significant way. However, the most significant characteristic of this cluster was their high mistake anxiety about both types of problem. A paradoxical phenomenon for this cluster was that they manifested significant deep and surface approaches at the same time, and displayed liberal thinking styles toward both problem types and conservative thinking styles to well-structured problems at the same time. They displayed self-efficacy of effort and conservative thinking styles toward well-structured problems, but not toward ill-structured problems, perhaps because of the awareness that their effort and following teachers' teaching were successful learning strategies for making progress in solving well-structured problems, but not in ill-structured problems. They also had high disposition toward teaching for both types of problem. Cluster 3 children appeared to be resilient, accepting 'everything', such as different approaches to learning, teachers' teaching, mathematics and different thinking styles. From interview data, children in this cluster had identified themselves as possessing an 'inability to do mathematics'. They had a special ability to describe adversities as 'funny experiences', in a humourous way. On the other hand, they tended to lay the blame on themselves for bad results.

Cluster 4: avoidance. The profile of this cluster indicated a group of children who invested little time or motivation in learning mathematics (low in deep approaches and high in surface approach), refused to solve problems in a liberal way and had low self-efficacy of effort, completely contrary to the children in Cluster 1. They exhibited mistake anxiety about ill-structured problems, but not about well-structured problems, perhaps because ill-structured problems bear a high degree of difficulty and complexity, which seemed to be a poor match with their characteristics, such as negative liberal thinking styles and low self-efficacy. They revealed insignificant disposition toward teaching. Children in this cluster tended completely to withdraw their effort toward learning mathematics, perhaps related to their negative emotions such as low self-efficacy of effort, negative liberal thinking and high mistake anxiety. In contrast to Cluster 1 children in Cluster 4 presented a negative image, with 'self-handicapping' behaviours in order to avoid mathematics learning. This is consistent with the Cluster 4 identified in Turner et al.'s (1998) study. Interview data showed that Cluster 4 children were experiencing a very difficult period in learning mathematics. They were desperate to grasp at the most effective and direct 'stick', such as 'formula' and 'direct teaching' to get rid of their self-image of being losers. At the same time they did not perceive themselves as hard workers in order to keep their positive self-esteem in mathematics.

RESULT 2: ATTAINMENTS

A one-way ANOVA revealed that there were differences in the attainments between the four clusters. One-sample t tests were carried out to examine whether the mean scores were significantly different from the value of 0. The attainments were significant differently from the value of 0 for Cluster 3, but not for the other three clusters. In addition, the results of LSD post hoc tests of the ANOVA showed that the attainment of Cluster 3 was significantly lower than that of Cluster 1.

DISCUSSION

There has been a pervasive perception that mathematics learning is a destiny, as one girl in the present study stated, 'I feel mathematics belongs to people who are really smart'. One of the four teacher also indicated that 'mathematics is a very special school subject. Only in mathematics, not in other subjects, if you can do, then you can do it. If you cannot do, then you cannot.' The identification of the clusters in this study should not, however, be interpreted as an endorsement of this view of mathematics as a destiny.

Rather, emotional responses to mathematical problem-solving are viewed more as a transient state than an unchangeable trait. A mathematics classroom at any time is a semi-shared environment. Given the wide variation in characteristics of children, teachers are likely to face two choices. Firstly, in order to fit children's diverse characteristics, teachers can provide diverse teaching methods and materials, such as constructivist teaching, hands-on experimenting, direct teaching, and providing challenging tasks, ill-structured problems and well-structured problems with diverse degrees of difficulty. Children as independent learners are expected, with teachers' support, to be the right people to understand their own needs most and to be able to choose the most suitable teaching methods and materials for themselves. Project-based teaching (Burton, 1994; Boaler 1998) is likely to be a way to facilitate independent learning. Secondly, teachers can provide 'the ideal teaching', which attempts to cultivate 'the ideal characteristics of a mathematics learner', such as the characteristics of Cluster 1 children, namely preference for challenging or non-routine tasks without mistake anxiety, a positive deep approach to learning, high self-efficacy, and learning styles matching the current mathematics curriculum of constructivist mathematics in Taiwan. However, as we have seen, while this is an ideal to strive towards, it may not meet the needs of all children in real classrooms, particularly at a time of change in the approach to mathematics teaching. Any 'effective' teaching methods are potentially beneficial or not beneficial for some children in every classroom. If the effective or ideal teaching methods are potentially beneficial in the long term for children, then it is clearly necessary for teachers to have regard to the emotional and motivational responses of the children to particular teaching approaches, and to develop pedagogies which recognise the inter-relatedness of emotional and cognitive factors in mathematics learning.

REFERENCES

Biggs, J., Kember, D. and Leung, D.Y.P.: 2001, 'The revised two-factor Study Process Questionnaire: R-SPQ-2F,' *British Journal of Educational Psychology* 71, 133-149.

Boaler, J.: 1998, 'Open and closed mathematics: student experiences and understanding,' *Journal for Research in Mathematics Educations* 29(1), 41-62.

Burton, L.: 1994, Children Learning Mathematics: Patterns and Relationships, Simon and Schuster Education, Herts.

Clifford, M.M.: 1988, 'Failure tolerance and academic risk-taking in ten- to twelve-year-old students,' *British Journal of Educational Psychology* 58, 15-27.

Kelly, G. A.: 1995, The Psychology of Personal Constructs: Volume One, a Theory of Personality, W. W. Norton & Company, New York.

Kloosterman, P. and Stage, F.K.: 1992, 'Measuring beliefs about mathematical problem solving,' *School Science and Mathematics* 92(3), 109-115.

Nitko, A. J.: 1996, *Educational Assessment of Students*, Englewood cliff, NJ.

Sternberg, R.J: 1997, *Thinking Styles*, Cambridge University Press, Cambridge

Turner, J.C., Thorpe, P.K and Meyer, D.K.: 1998, 'Students' reports of motivation and negative affect: A theoretical and empirical analysis,' *Journal of Educational Psychology* 90(4), 758-771.

Whitebread, D.: 1996, 'The development of children's strategies on an inductive reasoning task,' *British journal of Educational Psychology* 66, 1-21.