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Technical Note

Reaching more states for control of FMS

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Uzam and Zhou (Uzam, M. and Zhou, M., 2006. An improved iterative synthesis approach for liveness enforcing supervisors of flexible manufacturing systems. *International Journal of Production Research*, 44 (10), 1987–2030) synthesised a near optimal controlled model for a well-known S^3PR with 21,562 good states, 19 states short of the optimal one (21,581). It is non-trivial and interesting to synthesise an optimal controlled model as we will present it in this paper.

Keywords: petri nets; siphons; deadlocks

Uzam and Zhou (2006) were able to obtain a near optimal controlled model for the net in Figure 1 with 21,562 good states, 19 states short of the optimal 21,581 good states. It is non-trivial and interesting to synthesise an optimal controlled model as we will present it in this paper.

We shall start with the model shown in Table 1 where we have added 14 monitors and 78 control arcs versus 19 monitors and 120 control arcs in Uzam and Zhou (2006) but with the same 21,562 good states.

By removing monitor *V11* (*V12*, *V13*, and *V14*) and its associated control elements, the number of good and dead states are 21,585 (21,563, 21,574, 21,564) and 4 (1, 2, 2) respectively. Thus, the loss of 19 states is due to the presence of monitor *V11* since $21{,}585 - 4 = 21{,}581$. Monitor *V11* is related to siphon $S = \{p21, V8, p22, p24, p10, p12, p18, p17\}$, synthesised from circuit [*p21 t8 V8 t16 p22 t5 p24 t4 p21*], with complementary siphon $[S] = \{p2, p3, p8, p9, p11, p18, p19\}$. When S is empty of tokens, $M([S]) = M(p2) + M(p3) + M(p8) + M(p9) + M(p11) + M(p18) + M(p1) = 7$ reaches its maximal, $M(p24) = 0$, $M(p9) + M(p3) = 2$ and $M(p9) > 0$ (if $M(p9) = 0 \Rightarrow M(p3) = 2$, contradicting the fact that $M(p8) + M(p2) = 1$ and $M(p3) + M(p2) + M(p8) \leq 2 = M_0(V3)$).

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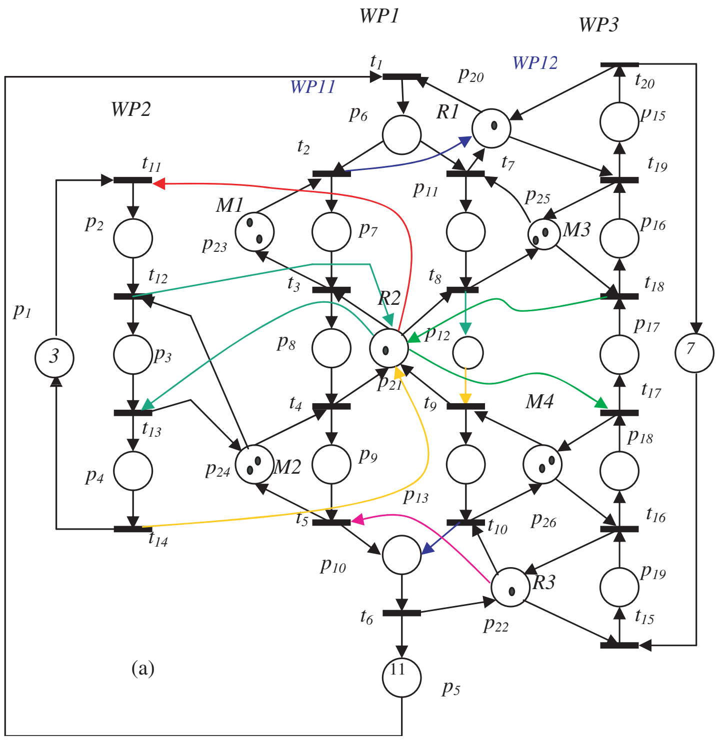

Figure 1. An S^3PR in Uzam *et al.* (2006).

Table 1. Control elements for the S^3PR in Figure 1.

V_i	$\cdot V_i$	$V_i\cdot$	$M_0(V_i)$
1	*t10 t16*	*t9 t15*	2
2	*t5 t10 t13 t17*	*t3 t9 t11 t15*	5
3	*t4 t13*	*t3 t11*	2
4	*t10 t17*	*t8 t15*	3
5	*t9 t17*	*t8 t16*	2
6	*t3 t8 t19*	*t1 t17*	5
7	*t8 t18*	*r7 t17*	2
8	*t8 t17*	*t7 t16*	3
9	*t9 t17*	*t7 t15*	4
10	*t8 t10 t17*	*t7 t9 t15*	4
11	*t5 t8 t13 t17*	*t7 t3 t11 t15*	6
12	*t5 t8 t10 t18*	*t1 t9 t15*	9
13	*t5 t8 t10 t17 t19*	*t1 t9 t15 t18*	9
14	*t5 t9 t17 t19*	*t1 t15 t18*	9

Table 2. Revised control elements to reach more states than optimal.

V_i	$\cdot V_i$	$V_i\cdot$	$M_0(V_i)$
1	*t10 t16*	*t9 t15*	2
2	*t5 t10 t13 t17 t21*	*t3 t9 t9 t11*	5
3	*t4 t13*	*t3 t11*	2
4	*t10 t17*	*t8 t15*	3
5	*t9 t17*	*t8 t16*	2
6	*t3 t8 t19*	*t1 t17*	5
7	*t8 t18*	*r7 t17*	2
8	*t8 t17*	*t7 t16*	3
9	*t9 t17*	*t7 t15*	4
10	*t8 t10 t17*	*t7 t9 t15*	4
12	*t5 t8 t10 t18 t21*	*t1 t9 t15 t18*	9
13	*t5 t8 t10 t17 t19 t21*	*t1 t9 t15 t18*	9
14	*t5 t9 t17 t19 t21*	*t1 t15 t18*	9
11′	*t3 t7 t11 t15 t21:6*	*t5 t8 t13 t17 t21:7*	0

t21· = {p5, p24, V2, V12, V13, V14, V′11:6}
·t21 = {p9, V′11:7}

It is impossible to refine or expand monitor *V11* to just prevent the occurrence of the four deadlock states to reach 21,581 good states. Hence, we employ a deadlock recovery approach. When the net reaches a deadlock state, we add control elements to return the net to a previous state.

We create monitor *V′11* to record the number of tokens in [*S*]. When $M(V'11) = M([S]) = 7$, *S* becomes empty of tokens to reach a deadlock state. The output transition *t21* of *V′11* then becomes enabled to fire once (weight of (*V′11 t21*) is 7 indicated by t21:7 in Table 2) to rid a token from *p9* and put a token in *p5*, *p24*, *V2*, *V12*, *V13*, and *V14* so that the sum of tokens in the support of minimal P-invariants containing them remain constant. Also it returns six tokens to *V′11* ($M(V'11) = 6$) since the weight of (*t21 V′11*) is 6 (indicated by t21:6 in Table 2); otherwise, $M(V'11) = 0$ – not a previous state of *V′11*.

Reachability analysis shows that the resulting controlled model (Table 2) is live and reaches 21,585 good states more than the optimal in Uzam and Zhou (2006). In general, given a near optimal design, we remove some monitor with more control arcs than most other monitors and see if more than optimal good states can be reached. If yes, then add a new monitor place and transition to return to a previous state.

To help the reader verify the results, Table 3 lists the text input file for INA analysis.

Note that we have introduced a new transition (none in previous approaches) in addition to an initially unmarked (marked in previous approaches) monitor place within the Petri net model of Figure 1. In general, this would entail adding one more monitor place. As a result, there would be two monitor places, which naturally obtained some more reachable states.

However, we have not added this extra place; the resulting four extra reachable good states are exactly the four deadlock states mentioned earlier. If we project the state space *R* in the final control model to that (*R′*) without *V′11*, *t21* and the associated control arcs, we find the projected space has 21,585 states also. This is because there is a one-one mapping between states in *R′* and *R*.

Table 3. Text input file for INA analysis of the model in Table 2.

```
  P  M     PRE,                POST  NETZ 1:a
  1  3     14,                 11
  2  0     11,                 12
  3  0     12,                 13
  4  0     13,                 14
  5 11      6 21,               1
  6  0      1,                  2 7
  7  0      2,                  3
  8  0      3,                  4
  9  0      4,                  5 21
 10  0      5 10,               6
 11  0      7,                  8
 12  0      8,                  9
 13  0      9,                 10
 14  7     20,                 15
 15  0     19,                 20
 16  0     18,                 19
 17  0     17,                 18
 18  0     16,                 17
 19  0     15,                 16
 20  1      2 7 20,             1 19
 21  1      4 9 12 14 18,       3 8 11 13 17
 22  1      6 16,               5 10 15
 23  2      3, 2
 24  2      5 13 21,            4 12
 25  2      8 19,               7 18
 26  2     10 17,               9 16
 27  2     10 16,               9 15
 28  5      5 10 13 17 21,      3 9 11 15
 29  2      4 13, 3 11
 30  3     10 17, 8 15
 31  2      9 17, 8 16
 32  5      3 8 19, 1 17
 33  2      8 18, 7 17
 34  3      8 17, 7 16
 35  4      9 17, 7 15
 36  4      8 10 17,            7 9 15
 37  9      5 8 10 18 21,       1 9 15
 38  9      5 8 10 17 19 21,    1 9 15 18
 39  9      5 9 17 19 21,       1 15 18
 40  0      3 7 11 15 21:6,     5 8 13 17 21:7
```

Conclusion

We have illustrated a way to reach more good states than the optimal one by adding a monitor place and transition to return to previous states from deadlock states. Thus, the original deadlock states are allowed, rather than avoided as in previous approaches.

Reference

Uzam, M and Zhou, M., 2006. An improved iterative synthesis approach for liveness enforcing supervisors of flexible manufacturing systems. *International Journal of Production Research*, 44 (10), 1987–2030.