

On the competition between an online bookstore and a physical bookstore

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Abstract This study examines the relative competitiveness of online and physical bookstores. Online bookstores have the advantage of being able to provide a wide range of books while minimizing inventory costs, but customers must wait several days for their books. Physical bookstores allow consumers to immediately obtain their books, but consumers must pay a transportation cost to visit the store. We can find the condition such that online bookstores charge a lower price than physical bookstores and take a larger market share, and attract a higher proportion of consumers who prefer variety. The implication of the welfare analysis is also discussed.

Keywords Variety competition · Price competition · Online bookstores

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1 Introduction

In July 1995, Amazon, the first online bookstore, launched a new business model by selling books online. This event initiated competition between online bookstores and physical (brick-and-mortar) bookstores. Now, Amazon has grown to become the largest online bookstore in the world.¹ Although some physical bookstores have competed with Amazon by providing their own online sales systems, many consumers prefer Amazon.²

The basic premise on which the competitiveness of an online bookstore is based on that it can provide customers with an almost infinite variety of books without needing to keep those books in its own stock, thus saving on storage, maintenance, and other costs. Brynjolfsson et al. [2] show that Amazon provides 23 times more book titles than a typical Barnes and Noble superstore.³ Moreover, consumers can also search for and order a book within minutes, without the need to travel to a bookstore.

The only obvious disadvantage of an online store is that consumers must wait several days to receive their parcels, and readers cannot do browsing of the content of the books (or can only do so for select items which have partial scans of the contents). Furthermore, an online bookstore may not be able to match the atmosphere of a brick-and-mortar bookstore. Meanwhile, since physical bookstores have limited shelf space, they must consider the popularity of each book they stock. Profit considerations demand that books on the shelf be popular.

Conversely, an online bookstore can provide almost any book in its virtual inventory at near zero cost. Online bookstores thus can offer a considerably wider variety of books than a physical bookstore. That is, physical bookstores can only provide the most popular books (known within the industry as the “head”), whereas online bookstores can provide both popular and obscure books (i.e. “head” and “tail”).⁴ Anderson [1] terms this market for low

¹ Actually, Amazon sells not only books but also CDs, DVDs, electronics, jewelry, office products, shoes, software, toys and other items.

² In fact, Amazon may still have a “first-mover advantage” in e-commerce [10]. Many people experienced their first online purchase at Amazon. Once they set up their account in Amazon, it is easy to revisit it again. Actually, the website of Amazon is very user-friendly and thus this early experience largely affected people’s choice among many websites. Moreover, the online sector and non-online sectors in a physical bookstore may have some conflicts, such as business types, profit sharing, and professional skills, which may cause their online business to be less attractive than that of Amazon.

³ Another advantage is that online stores can stay open 24 hours a day, whereas this is very costly for physical stores. Moreover, most online bookstores provide personalized recommendation tools, whereas physical stores have difficulty providing service staff on continuous standby. Therefore, the searching cost for online customers is lower than that in physical bookstores. Finally, online stores can collect reader reviews for potential purchasers to browse. However, to simplify the model, this study does not consider these factors.

⁴ Actually, Brynjolfsson et al. [3, p. 67] found that “30–40 % of (Amazon) sales are of books that would not normally be found in a brick-and-mortar store.”

popularity products the “long tail,” and stresses that it can be endless if inventory costs are eliminated.

Although numerous book industry surveys and electronic business discussions exist, such as Brynjolfsson et al. [2, 3], Chevalier and Goolsbee [4], Clay et al. [6], Forman et al. [8], and Sorensen [13],⁵ very limited theoretical models describe the competition between online and physical bookstores in the areas of product variety and price. This article provides a theory describing the competition between a physical bookstore and an online bookstore in price and variety. Involving competition of an online firm forces the physical firm to stay near the market center, which is different from the case of two physical firms competing with both horizontal and vertical differentiation in Neven and Thisse [11], in which either vertical dominance or horizontal dominance appears, that is, there exist both vertical differentiation and horizontal differentiation. Our model also shows that online bookstores may provide more variety than physical bookstores, and thus attract a higher proportion of consumers who prefer variety. In general the online bookstore will not charge a price equal to that of the physical bookstore.

The remainder of this paper is organized as follows: Section 2 provides the model and equilibrium prices and varieties, and Section 3 discusses the first-best solution and provides a welfare analysis. Conclusions are in the last section.

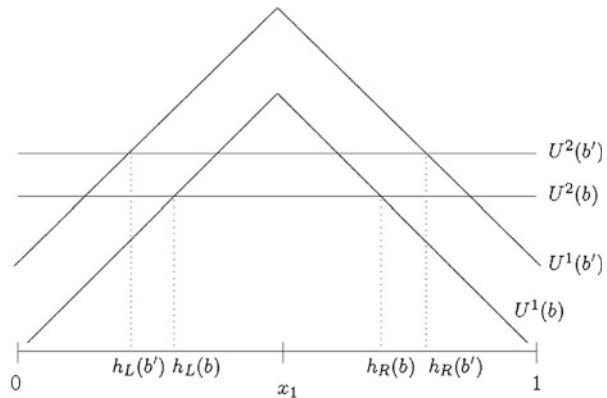
2 The model

Assume that two bookstores, a physical bookstore (firm 1) and an online bookstore (firm 2) compete on variety and price.⁶ Furthermore, suppose that individual consumers buy a book from one of these two bookstores, and the total

⁵Clay et al. [6] found that the average online prices are similar to those of the physical stores, but there is a significant price dispersion online. Sorensen [13] found that the effect of being a New York Times bestseller on consumers' purchase is insignificant for typical titles, but is substantial for new authors. Chevalier and Goolsbee [4] developed a method to directly measure the own- and cross-price elasticities of demand at the two leading online booksellers. They show that both Amazon.com and BarnesandNoble.com have significant price sensitivities for online book purchases.

⁶Several previous studies also address the term variety with different concepts. For instance, in Dixit and Stiglitz [7] and Chou and Shy [5], there are many firms, each of which provides one variety. For example, there are many automobile companies in the world, and each company may provide a differential style of automobile. Consumers enjoy the fact that they can select the most preferred style among so many varieties of automobiles. In particular, price and variety competition has been discussed in Peng and Tabuchi [12], where they employed the new economic geographic framework to analyze price and variety competition. But in the bookstore industry, most bookstores provide the same hot (popular) books, and the essential difference is how long their tails (obscure books) are. Therefore, the “variety” in those models is in fact different from ours. We are grateful to one of the anonymous referees for pointing out this difference.

Fig. 1 A possible utility configuration with $b' > b$



number of consumers is normalized to one ($N = 1$).⁷ Consider a two-stage game, and only pure strategies are discussed. In the first stage, the physical bookstore decides its location, and then in the second stage, both the physical firm and the online firm decide their prices and variety simultaneously.⁸

Consumers at location x are divided into continuous types of variety preference, indexed by b (*à la* Hotelling [9]). Consumers with a large b assign a high value to the variety of books (v) provided by a bookstore, and those with low b are less concerned with this.⁹ Assume b is uniformly distributed in $b \in [\underline{b}, \bar{b}]$. Suppose \underline{b} is not too low, such that the reservation price for each consumer is greater than the purchasing costs plus transportation costs to guarantee that all consumers will participate to purchase one book from either the physical or the online store. Suppose the physical bookstore is located at x_1 .

The number of customers with type b of firm 1 is $N_1(b) = (h_L(b) - h_R(b))$, $b \in [\underline{b}, \bar{b}]$. Thus, the total number of customers of firm 1 is then

$$N_1 = \int_{\underline{b}}^{\bar{b}} (h_R(b) - h_L(b)) \frac{1}{\bar{b} - \underline{b}}, b \in [\bar{b}, \underline{b}], \quad (1)$$

where $h_L(b)$ ($h_R(b)$) denotes the left (right) indifferent consumer for a variety type b (see Fig. 1). Notably, $h_L(b)$ and $h_R(b)$ are endogenous functions of b .

⁷In the real world, people may buy several books per store visit. The current model assumes that people buy the same number of books when visiting either firm. Thus, the number of books per sale is normalized to one.

⁸Alternatively, one may consider a three-stage game with location-variety-price decisions. Similar results are obtained with more mathematical complexity. We consider a two-stage game to simplify the model. Moreover, it seems reasonable for bookstores to decide the price and variety simultaneously.

⁹Higher v brings higher utility because it makes it easier for shoppers to find a suitable book. For example, if shoppers want a book on “Economics,” but numerous such books written by different authors exist, shoppers can easily choose a suitable book in a large v bookstore, but may have difficulty doing the same in a small v store.

Without loss of generality, this study focuses on the interior solutions of $h_L(b)$ and $h_R(b)$ with $h_L(b) > 0$, $h_R(b) < 1$, and $h_L(b) < h_R(b)$, for all $b \in [\underline{b}, \bar{b}]$. If equilibrium $h_L^*(b) < 0$ and $h_R^*(b) > 1$, this indicates that all consumers with a type b may buy books from firm 1. We skip this situation. Moreover, the market share of an online firm is $N_2 = 1 - N_1$.

A b type consumer located at x buying a book from firm 1 yields a utility

$$U_1(b) = \begin{cases} b \cdot \sqrt{v_1} - p_1 - k \cdot (x_1 - x), & \forall x \in [0, x_1], \\ b \cdot \sqrt{v_1} - p_1 - k \cdot (x - x_1), & \forall x \in [x_1, 1], \end{cases} \quad (2)$$

where v_1 is the book variety provided by firm 1, p_1 is the book price, and k is the per unit transportation cost. Similarly, the utility of the customers of firm 2 is

$$U_2(b) = b \cdot \sqrt{v_2} - p_2 - z, \quad (3)$$

where v_2 represents the book variety provided by firm 2, p_2 is the book price (shipping and handling costs are assumed to be zero),¹⁰ and z is the disutility of waiting for the arrival of the parcel.¹¹ Backward induction is employed to determine the equilibrium prices and variety. The first step is to find the indifferent consumers belonging to each type. Solving $U_1(b) = U_2(b)$ yields:

$$h_L(b) = \frac{1}{2} - \frac{b(\sqrt{v_1} - \sqrt{v_2})}{k} - \frac{p_2 - p_1}{k} - \frac{z}{k}, \quad (4)$$

$$h_R(b) = \frac{1}{2} + \frac{b(\sqrt{v_1} - \sqrt{v_2})}{k} + \frac{p_2 - p_1}{k} + \frac{z}{k}. \quad (5)$$

Let $\beta = (\bar{b} + \underline{b})/2$ to simplify the notations, which represents the average valuation of the bookstore variety. Using Eqs. 4, 5, and β , yields

$$N_1 = 2 \left(\frac{\beta(\sqrt{v_1} - \sqrt{v_2})}{k} + \frac{p_2 - p_1}{k} + \frac{z}{k} \right) \quad (6)$$

and

$$N_2 = 1 - 2 \left(\frac{\beta(\sqrt{v_1} - \sqrt{v_2})}{k} + \frac{p_2 - p_1}{k} + \frac{z}{k} \right). \quad (7)$$

¹⁰Our major results will be robust when a handling cost (h) is considered for online transactions. Under this situation, the profit function for firm 2 is $\pi_2 = (p_2 - h)N_2 - c_2V_2$. In order to simplify our presentation, we ignore the handling cost. We are grateful to an anonymous referee for providing this point.

¹¹The waiting cost for online book purchasing is not absolute. For example, a user to an online bookstore can buy a book as an electronic version with immediate access, while a user to a physical bookstore may get immediate access to the electronic version and look inside. If satisfied, they order the book and wait for its arrival. The zero waiting cost in this situation is included in our model. Broadly speaking, the “waiting cost” in our model can include the uncertainty about sellers’ credit and shipping safety. We thank one of the anonymous referees for this suggestion.

The profit functions can be represented as follows:

$$\pi_1 = (p_1 - c)N_1 - c_1 v_1, \quad (8)$$

$$\pi_2 = (p_2 - c)N_2 - c_2 v_2, \quad (9)$$

where the physical bookstore and the online bookstore incur constant marginal costs c_1 , and c_2 , to provide one book title, respectively, and assuming that an online bookstore incurs a much lower cost to provide v_2 variety, that is, $c_1 \gg c_2$.¹² Moreover, without loss of generality, assume that the marginal costs for providing each book are zero ($c = 0$).

Solving $\partial\pi_1/\partial p_1 = 0$, $\partial\pi_1/\partial v_1 = 0$, $\partial\pi_2/\partial p_2 = 0$, and $\partial\pi_2/\partial v_2 = 0$ yields

$$v_1^* = \frac{\beta^2 (\beta^2 - 2c_2(k + 2z))^2}{4(\beta^2(c_1 + c_2) - 3kc_1c_2)^2}, \quad (10)$$

$$v_2^* = \frac{\beta^2 (\beta^2 - 2c_1(k - z))^2}{4(\beta^2(c_1 + c_2) - 3kc_1c_2)^2}, \quad (11)$$

$$\begin{aligned} p_1^* &= \frac{k + 2z}{6} + \frac{\beta^2}{6} \left(\sqrt{\frac{(\beta^2 - c_2(k + 2z))^2}{(\beta^2(c_1 + c_2) - 3kc_1c_2)^2}} - \sqrt{\frac{(\beta^2 - 2c_1(k + z))^2}{(\beta^2(c_1 + c_2) - 3kc_1c_2)^2}} \right) \\ &= \frac{k + 2z}{6} + \frac{\beta^2}{6} \left(\frac{2c_1(k - z) - c_2(k + 2z)}{\beta^2(c_1 + c_2) - 3kc_1c_2} \right), \end{aligned} \quad (12)$$

$$\begin{aligned} p_2^* &= \frac{k - z}{3} + \frac{\beta^2}{6} \left(-\sqrt{\frac{(\beta^2 - c_2(k + 2z))^2}{(\beta^2(c_1 + c_2) - 3kc_1c_2)^2}} + \sqrt{\frac{(\beta^2 - 2c_1(k + z))^2}{(\beta^2(c_1 + c_2) - 3kc_1c_2)^2}} \right) \\ &= \frac{k - z}{3} + \frac{\beta^2}{6} \left(\frac{c_2(k + 2z) - 2c_1(z - k)}{\beta^2(c_1 + c_2) - 3kc_1c_2} \right). \end{aligned} \quad (13)$$

The last two equations are derived because it is assumed that β is not too low, in particular, to guarantee $\beta^2 > \max \{c_2(k + 2z), 2c_1(k + z), \frac{3kc_1c_2}{c_1 + c_2}\}$.¹³

¹²In reality, an online bookstore should prepare a storage facility for packing and handling and should apply a fixed cost F for this service. However, F will not affect decisions regarding variety and will complicate the calculation of profit. Therefore, $F = 0$ is assumed hereafter.

¹³The second-order conditions are assumed satisfied as follows. For firm 1, $\partial^2\pi_1/\partial p_1^2 = -4/k < 0$, $\partial^2\pi_1/\partial v_1^2 = -(p_1\beta)/(kv_1^{3/2}) < 0$, and $\frac{\partial^2\pi_1}{\partial p_1^2} \frac{\partial^2\pi_1}{\partial v_1^2} - (\frac{\partial^2\pi_1}{\partial p_1 \partial v_1})^2 = \frac{\beta(2p_1 - \sqrt{v_1}\beta)}{k^2 v_1^{3/2}} > 0$. Similarly, for firm 2, $\partial^2\pi_2/\partial p_2^2 = -4/k < 0$, $\partial^2\pi_2/\partial v_2^2 = -(p_2\beta)/(2kv_2^{3/2}) < 0$, and $\frac{\partial^2\pi_2}{\partial p_2^2} \frac{\partial^2\pi_2}{\partial v_2^2} - (\frac{\partial^2\pi_2}{\partial p_2 \partial v_2})^2 = \frac{\beta(2p_2 - \sqrt{v_2}\beta)}{k^2 v_2^{3/2}} > 0$.

Replacing Eqs. 10, 11, 12 and 13 into Eqs. 4 and 5 yields

$$h_L(b) = x_1 - \frac{1}{6k} \left(k + 2z + \frac{\beta(2\beta - 3b)(c_2(k + 2z) - 2c_1(k - z))}{(c_1 + c_2)\beta^2 - 3c_1c_2k} \right), \quad (14)$$

$$h_R(b) = x_1 + \frac{1}{6k} \left(k + 2z + \frac{\beta(2\beta - 3b)(c_2(k + 2z) - 2c_1(k - z))}{(c_1 + c_2)\beta^2 - 3c_1c_2k} \right). \quad (15)$$

Differentiating Eq. 15 with b yields:

$$\frac{\partial h_L(b)}{\partial b} = \left(\frac{\beta(c_2(k + 2z) - c_1(k - z))}{2(c_1 + c_2)\beta^2 - 6c_1c_2k} \right) > 0, \text{ if } z > \frac{k(2c_1 - c_2)}{2(c_1 + c_2)}, \quad (16)$$

$$\begin{aligned} \frac{\partial N_1(b)}{\partial b} &= \frac{\partial(h_R(b) - h_L(b))}{\partial b} \\ &= - \left(\frac{\beta(c_2(k + 2z) - c_1(k - z))}{(c_1 + c_2)\beta^2 - 3c_1c_2k} \right) < 0, \text{ if } z > \frac{k(2c_1 - c_2)}{2(c_1 + c_2)}. \end{aligned} \quad (17)$$

This result shows that $N_1(b)$ may be either increasing or decreasing in b . Therefore, the physical bookstore may have either smaller or larger market share for high variety type consumers. The necessary conditions for the existence of equilibrium are $0 < h_L(b) < h_R(b) < 1$, for $b \in (\underline{b}, \bar{b})$. To ensure $h_L(b) > 0$ for all $b \in (\underline{b}, \bar{b})$, it is required that $h_L(b) > 0$, therefore, plugging \underline{b} and \bar{b} into Eq. 14 yields

$$x_1 > x_1^{\min} \equiv \max(A(\underline{b}), A(\bar{b})), \text{ where} \quad (18)$$

$$A(b) = x_1 - h_L(b) = \frac{1}{6} + \frac{z}{3k} + \frac{\beta(2\beta - 3b)(c_2(k + 2z) - 2c_1(k - z))}{3k((c_1 + c_2)\beta^2 - 3c_1c_2k)}. \quad (19)$$

Solving $h_R(b) - h_L(b) \geq 0$ for all $b \in [\underline{b}, \bar{b}]$ requires that $h_R(\bar{b}) - h_L(\bar{b}) > 0$ and $h_R(\underline{b}) - h_L(\underline{b}) > 0$. Plugging \underline{b} and \bar{b} into Eqs. 14 and 15 yields

$$\min(A(\underline{b}), A(\bar{b})) > 0. \quad (20)$$

Similarly, solving $1 - h_R(b) \geq 0$ by plugging \underline{b} and \bar{b} into Eq. 15 yields

$$x_1 < x_1^{\max} \equiv 1 - x_1^{\min}. \quad (21)$$

From $x_1^{\max} > x_1^{\min}$ it follows that $1 - x_1^{\min} > x_1^{\min}$ to yield $x_1^{\min} < \frac{1}{2}$, that is,

$$\max(A(\underline{b}), A(\bar{b})) < \frac{1}{2}, \quad (22)$$

From Eq. 19, if $z < \frac{k(2c_1 - c_2)}{2(c_1 + c_2)}$, then $A(\bar{b}) < A(\underline{b})$. Therefore, Eq. 22 becomes $A(\underline{b}) < 1/2$ and thus we can solve Eqs. 19 and 20 to have

$$z < \frac{k(2\beta^2 c_1 + \underline{b}\beta(c_1 - 2c_1) - 2kc_1 c_2)}{2\beta^2(c_1 + c_2) - 2kc_1 c_2 - 2\underline{b}\beta(c_1 + c_2)} = z_1^m.$$

Similarly, if $z < \frac{k(2c_1 - c_2)}{2(c_1 + c_2)}$, then $A(\bar{b}) < A(\underline{b})$. Therefore, Eq. 22 becomes $A(\bar{b}) < 1/2$ and thus we can solve Eqs. 19 and 22 to have

$$z < \frac{k(2\beta^2 c_1 + \bar{b}\beta(c_1 - 2c_1) - 2kc_1 c_2)}{2\beta^2(c_1 + c_2) - 2kc_1 c_2 - 2\bar{b}\beta(c_1 + c_2)} = z_2^m.$$

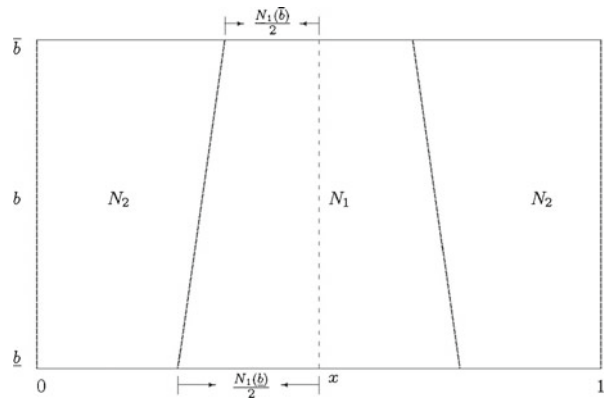
Therefore, if $z < z^m \equiv \min\{z_1^m, z_2^m\}$, then $x_1^{\min} < x_1 < 1 - x_1^{\min}$. Therefore, our model ensures that an equilibrium exists whenever the physical bookstore is located near the center of the market.¹⁴

Proposition 1 *When a physical bookstore competes with an online bookstore, there exists a possible equilibrium in which the physical bookstore is located near the market center and occupies a range of the market for each type of consumer, and the online firm takes all other markets when $z < z^m$. The equilibrium prices and variety $(p_1^*, p_2^*, v_1^*, v_2^*)$ satisfy Eqs. 10, 11, 12 and 13, and the equilibrium locations (x_1^*) satisfy $x_1^{\min} < x_1^* < (1 - x_1^{\min})$, where x_1^{\min} satisfies Eq. 18.*

Proposition 1 describes the equilibrium location, prices, and variety. It shows that involving competition of an online firm forces the physical firm to stay near the market center to attract more consumers. A similar framework is found in the literature with both horizontal and vertical differentiation in Neven and Thisse [11]. They find that there exist both vertical differentiation and horizontal differentiation. In contrast, in our framework the market share of the physical bookstore is the area near the market center, regardless of the preference type to variety type. The following example demonstrates the situation of equilibrium.

Example 1 Using $\beta = 0.398$, $c_1 = 0.5$, $c_2 = 0.4$, $k = 0.2$, $z = 0.067$, $\bar{b} = 0.7$, and $\underline{b} = 0.095$ it is possible to obtain $(v_1, v_2) = (0.477, 0.501)$, $(p_1, p_2) = (0.550, 0.450)$, $(N_1, N_2) = (0.550, 0.450)$, $N_1(b) = 0.571 - 0.0537b$, and the equilibrium location is $0.2829 < x_1^* < 0.7171$. In this case $z > \frac{k(2c_1 - c_2)}{2(c_1 + c_2)} = 0.0667$, therefore, $N_1(b)$ is decreasing in b . The equilibrium market shares are depicted in Fig. 2.

¹⁴Notably, $p_1 > 0$ and $p_2 > 0$ imply $N_1 > 0$ and $N_2 > 0$.

Fig. 2 The market shares in Example 1

From Eq. 17, we have the following proposition.

Proposition 2 When $z > (<) \frac{(2c_1 - c_2)k}{2(c_1 + c_2)}$, the market share of the physical bookstore is decreasing (increasing) as b increases. That is, $\partial N_1(b)/\partial b < (>) 0$, iff $z > (<) k(2c_1 - c_2)/(2(c_1 + c_2))$.

The intuition of Proposition 2 is that whether the online bookstore attracts more consumers preferring high variety depends on the relative scales of z , k , c_1 and c_2 . Moreover, the difference in variety between these two bookstores is:

$$\sqrt{\frac{v_2^*}{v_1^*}} - 1 = \frac{(c_2(k + 2z) - 2c_1(k - z))}{\beta^2 - c_2(k + 2z)} > 0, \quad \text{iff } z > \frac{k(2c_1 - c_2)}{2(c_1 + c_2)}. \quad (23)$$

It can be summarized as follows.

Proposition 3 If $z > \frac{k(2c_1 - c_2)}{2(c_1 + c_2)}$, then the online bookstore must provide a larger variety than the physical bookstore.

The intuition of Proposition 3 is straightforward. If the waiting cost is large, then the online bookstore must provide a larger variety in order to compensate for the disutility of waiting cost.¹⁵ Moreover, the price difference is

$$p_2^* - p_1^* = \frac{[4c_1c_2z + \beta^2(c_2 - c_1) - kc_1c_2] \cdot k}{2[(c_1 + c_2)\beta^2 - 3kc_1c_2]} < 0, \\ \text{iff } z < \frac{kc_1c_2 + \beta^2(c_1 - c_2)}{3c_1c_2}. \quad (24)$$

¹⁵If z is small, then $p_2 < p_1$ from Proposition 4. Since the online bookstore has provided a lower price, it has no need to provide more variety to consumers. Therefore, a physical bookstore may provide a larger variety when z is small.

Proposition 4 *Book prices in the online bookstore are lower than those in the physical bookstore ($p_2^* < p_1^*$) if and only if $z < \frac{kc_1c_2 + \beta^2(c_1 - c_2)}{4c_1c_2}$.*

The intuition of Proposition 4 is that given a sufficiently low waiting cost, the online bookstore can charge a lower price than the physical bookstore. Replacing equilibrium 10, 11, 12, and 13 into Eqs. 6 and 7 yields

$$N_1^* = \frac{c_1(\beta^2 - c_2(k + 2z))}{\beta^2(c_1 + c_2) - 3kc_1c_2},$$

$$N_2^* = \frac{c_2(\beta^2 - 2c_1(k - z))}{\beta^2(c_1 + c_2) - 3kc_1c_2}.$$

and

$$\frac{N_1^*}{N_2^*} - 1 = \frac{\beta^2(c_2 - c_1) + c_1c_2(4z - k)}{c_1(\beta^2 - c_2(k + 2z))} < 0, \quad \text{iff } z < \frac{\beta^2(c_1 - c_2) + kc_1c_2}{4c_1c_2}.$$

Therefore, this result can be summarized as the following proposition:

Proposition 5 *The market share of the online bookstore is larger than that of the physical bookstore if and only if $z < \frac{\beta^2(c_1 - c_2) + kc_1c_2}{4c_1c_2}$.*

Again, the relative scale of waiting cost (with respect to β , c_1 , and k) significantly influences the distribution of market shares. Clearly, the market share of the online bookstore exceeds that of the physical bookstore when the waiting cost is low enough.

Corollary 1 *When $\frac{k(2c_1 - c_2)}{2(c_1 + c_2)} < z < \frac{\beta^2(c_1 - c_2) + kc_1c_2}{4c_1c_2}$ (that is $\beta > \frac{\sqrt{3kc_1c_2(c_1 - c_2)}}{c_1 + c_2}$), the online bookstore provides more variety of books, charges a lower price, and has a larger market share.*

From Propositions 3, 4, and 5,

$$\frac{kc_1c_2 + \beta^2(c_1 - c_2)}{4c_1c_2} - \frac{k(2c_1 - c_2)}{2(c_1 + c_2)} = \frac{(c_1 - c_2)(\beta^2(c_1 + c_2) - 3kc_1c_2)}{4c_1c_2(c_1 + c_2)} > 0,$$

$$\text{iff } \beta > \frac{\sqrt{3kc_1c_2(c_1 - c_2)}}{c_1 + c_2}.$$

Therefore, we can draw an equilibrium result under $\beta > \sqrt{3kc_1c_2(c_1 - c_2)}/(c_1 + c_2)$ (Fig. 3). There are two critical points z_1 and z_2 such that when $z < z_1$

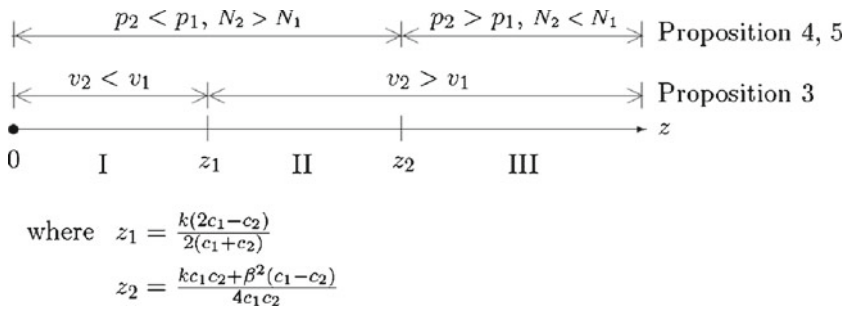


Fig. 3 The equilibrium results when $\beta > \frac{\sqrt{3kc_1c_2(c_1 - c_2)}}{c_1 + c_2}$

($z > z_1$), then $v_2 < v_1$ ($v_2 > v_1$) from Proposition 3, when $z < z_2$ ($z > z_2$), then $p_2 < p_1$ ($p_2 > p_1$) and $N_2 > N_1$ ($N_2 < N_1$) from Propositions 4 and 5. With z_1 and z_2 , there are three regions (I, II, and III) in Fig. 3. We emphasize region II, because it may be suitable to the reality such that the online bookstore provides more variety of books, charges a lower price, and has a larger market share than that of the physical bookstore.

For purposes of comparison from Eqs. 10 and 11, a change in the waiting cost can change the equilibrium variety:

$$\frac{\partial v_1}{\partial z} < 0, \quad \frac{\partial v_2}{\partial z} > 0,$$

$$\frac{\partial(\sqrt{v_2^*/v_1})}{\partial z} = \frac{4(\beta^2 - 2c_1(z - k))(\beta^2(c_1 + c_2) - 3kc_1c_2)}{(\beta^2 - c_2(k + 2z))^3} > 0.$$

Therefore, the influence of waiting costs on the dispersion of variety is shown as the following proposition.

Proposition 6 *Increased waiting cost reduces the variety that the online bookstore can offer, and thus reduces the variety ratio (v_2^*/v_1^*) between the two stores.*

Finally, from Eqs. 12 and 13 the equilibrium prices may also change because of changes in the waiting cost:

$$\frac{\partial p_1^*}{\partial z} = \frac{-kc_1c_2}{(c_1 + c_2)\beta^2 - 3kc_1c_2} < 0,$$

$$\frac{\partial p_2^*}{\partial z} = \frac{kc_1c_2}{(c_1 + c_2)\beta^2 - 3kc_1c_2} > 0.$$

Thus,

$$\frac{\partial(p_2^* - p_1^*)}{\partial z} > 0.$$

Therefore, these results can be summarized as the following proposition:

Proposition 7 *Increased waiting cost will decrease the price of the physical bookstore and increase the price of the online firm. Therefore, the price difference ($p_2 - p_1$) between these two bookstores increases with increasing z .*

3 Welfare analysis

Define a social welfare function

$$W = \int_{\underline{b}}^{\bar{b}} b \sqrt{v_1} N_1(b) \frac{db}{\bar{b} - \underline{b}} + \int_{b_l}^{\bar{b}} b \sqrt{v_2} (1 - N_1(b)) \frac{db}{\bar{b} - \underline{b}} - c_1 v_1 - c_2 v_2 \\ - \int_{b_l}^{\bar{b}} \frac{1}{4} k N_1(b)^2 \frac{db}{\bar{b} - \underline{b}} - \int_{\underline{b}}^{\bar{b}} z (1 - N_1(b)) \frac{db}{\bar{b} - \underline{b}},$$

where the first term in the right-hand side is the reservation value from purchasing from the physical bookstore, the second term is the reservation value from the online purchase, the third and fourth terms are the costs of variety, the fifth term is total transportation costs, and the sixth term is the total waiting cost. Differentiate W with respect to $N_1(b)$ to obtain

$$(\sqrt{v_1} - \sqrt{v_2})b - \frac{k N_1(b)}{2} + z = 0,$$

and thus the optimal market share of the physical bookstore is

$$N_1^O(b) = \frac{2b(\sqrt{v_1} - \sqrt{v_2}) + 2z}{k},$$

where the superscript “ O ” means “optimal”.

From Eqs. 4 and 5, we have the market share of the physical bookstore under market competition:

$$N_1(b) = h_R(b) - h_L(b) = 2 \left(\frac{b(\sqrt{v_1} - \sqrt{v_2})}{k} + \frac{p_2 - p_1}{k} + \frac{z}{k} \right).$$

Comparing $N_1^O(b)$ and $N_1(b)$ yields the following proposition.

Proposition 8 *In the first best solution, $p_1 = p_2$ in order to make $N_1(b) = N_1^O(b)$. In other words, the equilibrium prices must be identical to reach the social optimal solution.*

The intuition of Proposition 8 is that since both bookstores sell identical books, equal pricing ensures no distortion in the market.

Plugging $N_1^O(b)$ into W yields

$$W(v_1, v_2) = (\sqrt{v_1} - \sqrt{v_2})^2 \cdot \frac{(\bar{b}^3 - \underline{b}^3)}{3k(\bar{b} - \underline{b})} + (\sqrt{v_1} - 2\sqrt{v_2}) \frac{z}{k} \cdot (\bar{b} + \underline{b}) - c_1 v_1 - c_2 v_2 + \frac{z^2}{k} - z.$$

Solving $\partial W(v_1, v_2)/\partial v_1 = 0$ and $\partial W(v_1, v_2)/\partial v_2 = 0$ yields

$$v_1^O = \frac{(-6c_2 z \bar{b} - 6c_2 z \underline{b} + 2\bar{b}^2 \underline{b} + 2\bar{b} \underline{b}^2 + \bar{b}^3 + \underline{b}^3)^2}{16(c_1 \bar{b}^2 + \bar{b}^2 c_1 - 3c_2 c_1 k + \bar{b}^2 c_1 + c_2 \underline{b}^2 + \bar{b} \underline{b} c_1 + c_2 \bar{b} \underline{b})^2}$$

$$v_2^O = \frac{(\bar{b}^3 + 2\bar{b}^2 \underline{b} + 2\bar{b} \underline{b}^2 - 3c_1 k \bar{b} + 6\bar{b} z c_1 + 6z \underline{b} c_1 - 3c_1 k \underline{b} + \underline{b}^3)^2}{16(c_1 \bar{b}^2 + \underline{b}^2 c_1 - 3c_2 c_1 k + \bar{b}^2 c_1 + c_2 \underline{b}^2 + \bar{b} \underline{b} c_1 + c_2 \bar{b} \underline{b})^2}$$

Solving $v_1^O - v_2^O$ yields

$$v_1^O - v_2^O = \frac{3(-2zc_1 - 2c_2 z + c_1 k)(\bar{b} + \underline{b})^2(2\bar{b}^2 + 2\underline{b}\bar{b} - 3c_1 k + 2\bar{b}^2 \underline{b}^2 - 6c_2 z + 6zc_1)}{16(c_1 \bar{b}^2 + \underline{b}^2 c_1 - 3c_2 c_1 k + \bar{b}^2 c_1 + c_2 \underline{b}^2 + \bar{b} \underline{b} c_1 + c_2 \bar{b} \underline{b})^2}$$

Then, we have

Proposition 9 *If \bar{b} and \underline{b} are large enough, such that $2\bar{b}^2 + 2\underline{b}\bar{b} + 2\underline{b}^2 - 3c_1 k - 6c_2 z + 6zc_1 > 0$, then when $z > \frac{c_1 k}{2(c_1 + c_2)}$, we have $v_1^O < v_2^O$ in the first-best solution.*

The intuition of Proposition 9 is that z must be large enough to ensure that $v_1^O < v_2^O$, even though we know that the marginal cost of the physical bookstore is higher than that of the online bookstore ($c_1 > c_2$). On the contrary, when $z < \frac{c_1 k}{2(c_1 + c_2)}$, then the first-best solution does not mean that the online bookstore must provide a greater variety than the physical bookstore.

4 Conclusions

This article explores variety and price competition between an online and a physical bookstore. The online bookstore has the advantage of providing books with very low inventory costs, but customers must wait for several days to receive their packages. In contrast, physical stores allow consumers to obtain their books immediately, but consumers must pay a transportation cost to visit the bookstore. This study found that in some ranges of waiting cost the online bookstore provides more product variety, charges a lower price, has a larger market share than the physical bookstore and also attracts a higher

proportion of consumers who have high preference for variety. Finally, the first best solution should be no distortion in prices.

Up to now, the academic discussion of the interplay between physical and online bookstores has not been fully understood, while practical competitions are setting the pace at immense speed. In this paper we strive to extend the understanding by defining specific models under specific assumptions and by discussing related analytical results. We are fully aware that additional research is necessary to gain a fuller understanding and that new selling models in practice may necessitate new models, and it is not yet clear to which extent our models may help to support those cases.¹⁶

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