Appl Intell (2008) 28: 183–193
DOI 10.1007/s10489-007-0060-4

Evaluating students' answerscripts using vague values

Hui-Yu Wang · Shyi-Ming Chen

Received: 31 December 2006 / Accepted: 18 April 2007 / Published online: 20 June 2007

Abstract It is obvious that education institutions must provide students with the evaluation reports regarding their test/examination as sufficient as possible and with the unavoidable error as small as possible. In this paper, we present a new method for evaluating students' answerscripts using vague values, where the evaluating marks awarded to the questions in the students' answerscripts are represented by vague values. The vague mark awarded to each question of a student's answerscript can be regarded as a vague set, where each element in the universe of discourse belonging to the vague set is represented by a vague value. An index of optimism λ determined by the evaluator is used to indicate the degree of optimism of the evaluator, where $\lambda \in [0, 1]$. The larger the value of λ, the more optimistic the evaluator. The smaller the value of λ, the more pessimistic the evaluator. The proposed method can evaluate students' answerscripts in a more flexible and more intelligent manner.

Keywords Vague sets · Vague grade sheets · Generalized vague grade sheets · Letter grades · Index of optimism · Fuzzy sets

H.-Y. Wang
Department of Education, National Chengchi University, Taipei, Taiwan

S.-M. Chen (✉)
Department of Computer Science and Information Engineering, National Taiwan University of Science and Technology, 43, Section 4, Keelung Road, Taipei, Taiwan
e-mail: smchen@mail.ntust.edu.tw

1 Introduction

In recent years, some methods for students' evaluation have been presented [1–3, 10, 12–16, 18–27]. In [1], Altun and Cakan presented an investigation of cognitive styles, achievement scores and attitudes toward computers among university students. In [2], Biswas pointed out that the chief aim of education institutions is to provide students with the evaluation reports regarding their test/examination as sufficient as possible and with the unavoidable error as small as possible. Therefore, he presented a fuzzy evaluation method (*fem*) for applying fuzzy sets [28] in students' answerscripts evaluation. He also modified the fuzzy evaluation method to propose a generalized fuzzy evaluation method (*gfem*) for students' answerscripts evaluation. In [3], Chang and Sun presented a method for fuzzy assessment of learning performance of junior high school students. In [10], Chen and Lee pointed out that the methods presented in [2] have two drawbacks, i.e., (1) It will take a large amount of time to deal with the matching operations of the matching function; (2) Two different fuzzy marks may be translated into the same awarded letter grade which is unfair for students' evaluation. Thus, they presented two methods for evaluating students' answerscripts using fuzzy sets. In [12], Cheng and Yang presented a method for using fuzzy sets in education grading systems. In [13], Chiang and Lin presented a method for applying the fuzzy set theory to teaching assessment. In [15], Echauz and Vachtsevanos presented a fuzzy grading system to translate a set of scores into letter grades. In [16], Frair presented a method for student peer evaluations using the analytic hierarchy process method. In [18], Kaburlasos et al. presented a software tool, called PARES, for computer-based testing and evaluation, used in the Greek higher education system. In [19], Law presented a method for applying fuzzy numbers in education grading systems. In [20], Ma and Zhou presented a fuzzy set approach for assessing the performance of student-centered learning. In [21], McMartin et al. used scenario assignments as assessment tools for undergraduate engineering education. In [22], Pears et al. presented a method for student evaluation in an international collaborative project course. In [23], Rasmani and Shen presented a method for evaluating student academic performance based on data-driven fuzzy rule induction. In [25], Weon and Kim presented a leaning achievement evaluation strategy in students' learning procedure using fuzzy membership functions. In [26], Wilson et al. presented a flexible, adaptive and automatic fuzzy-based grade-assigning system. In [27], Wu presented a method for applying the fuzzy set theory and the item response theory to evaluate learning performance.

In [2], the fuzzy marks awarded to answers in the students' answerscripts are represented by fuzzy sets [28]. In a fuzzy set, the grade of membership of an element u_i in the universe of discourse U belonging to a fuzzy set is represented by a real value between zero and one. However, in [17], Gau and Buehrer pointed out that this single value between zero and one combines the evidence for $u_i \in U$ and the evidence against $u_i \in U$. They pointed out that it does not indicate the evidence for $u_i \in U$ and the evidence against $u_i \in U$, respectively, and it does not indicate how much there is of each. They also pointed out that the single value between zero and one tells us nothing about its accuracy. Thus, they proposed the theory of vague sets, where each element in the universe of discourse belonging to a vague set is represented by a vague value. Therefore, if we can allow the marks awarded to the questions of the students' answerscripts to be represented by vague sets, then there is room for more flexibility.

In this paper, we present a new method for students' answerscripts evaluation using vague values. The vague marks awarded to the answers in the students' answerscripts are represented by vague sets. An index of optimism λ [12] determined by the evaluator is used to indicate the degree of optimism of the evaluator, where $\lambda \in [0, 1]$. The larger the value of λ, the more optimistic the evaluator. The smaller the value of λ, the more pessimistic the evaluator. The proposed method can evaluate students' answerscripts in a more flexible and more intelligent manner.

2 Vague sets and vague values

In [17], Gau and Buehrer presented the vague set theory. In [4], Chen has presented the arithmetic operations for vague sets. In [7] and [8], Chen has presented similarity measures between vague sets. A vague set $\tilde{A}$ in the universe of discourse U is characterized by a truth-membership function $t_{\tilde{A}}$

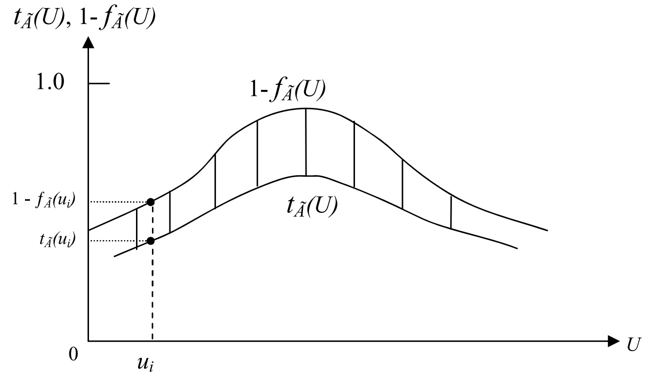

Fig. 1 A vague set [17]

and a false-membership function $f_{\tilde{A}}$, where $t_{\tilde{A}} : U \to [0, 1]$, $f_{\tilde{A}} : U \to [0, 1]$, $t_{\tilde{A}}(u_i)$ is a lower bound of the grade of membership of u_i derived from the evidence for u_i, $f_{\tilde{A}}(u_i)$ is a lower bound of the negation of u_i derived from the evidence against u_i, $t_{\tilde{A}}(u_i) + f_{\tilde{A}}(u_i) \leq 1$, and $u_i \in U$. It should be noted that the truth-membership function and the false-membership function of a vague set are subjectively defined. The grade of membership of u_i in the vague set $\tilde{A}$ is bounded by a subinterval $[t_{\tilde{A}}(u_i), 1 - f_{\tilde{A}}(u_i)]$ of $[0, 1]$. The vague value $[t_{\tilde{A}}(u_i), 1 - f_{\tilde{A}}(u_i)]$ indicates that the exact grade of membership $\mu_{\tilde{A}}(u_i)$ of u_i is bounded by $t_{\tilde{A}}(u_i) \leq \mu_{\tilde{A}}(u_i) \leq 1 - f_{\tilde{A}}(u_i)$, where $t_{\tilde{A}}(u_i) + f_{\tilde{A}}(u_i) \leq 1$. Figure 1 shows an example of a vague set $\tilde{A}$ in the universe of discourse U.

A vague set $\tilde{A}$ of the universe of discourse U, $U = \{u_1, u_2, \ldots, u_n\}$, can be represented as follows:

$$\tilde{A} = \sum_{i=1}^{n} [t_{\tilde{A}}(u_i), 1 - f_{\tilde{A}}(u_i)]/u_i. \tag{1}$$

If the universe of discourse U is an infinite set, then a vague set $\tilde{A}$ of the universe of discourse U can be represented as follows:

$$\tilde{A} = \int_U [t_{\tilde{A}}(u_i), 1 - f_{\tilde{A}}(u_i)]/u_i, \quad u_i \in U, \tag{2}$$

where the symbol $\int$ denotes the union operator.

In this paper, we present the concept of the expected truth value $E(X)$ of a vague value X. Let X be a vague value, where $X = [t_x, 1 - f_x] = [t_x, f_x^*]$, t_x denotes the degree of truth, f_x denotes the degree of false, $t_x \in [0, 1]$, $f_x \in [0, 1]$, and $t_x + f_x \leq 1$. Let $f_x^* = 1 - f_x$, i.e., the vague value $X = [t_x, 1 - f_x] = [t_x, f_x^*]$. Then, the unknown part $N(X)$ of the vague value X is defined as follows:

$$N(X) = f_x^* - t_x = (1 - f_x) - t_x. \tag{3}$$

Because different evaluators may have different characteristics in students' evaluation, in [12], Cheng and Yang divided the evaluators into three types according to their characteristics in students' evaluation, i.e., the strict-type evaluators, the normal-type evaluators and the lenient-type evaluators. They also use an index of optimism λ to denote the degree of optimism of an evaluator. If $0 \le \lambda < 0.5$, then the evaluator is a pessimistic evaluator (i.e., a strict-type evaluator). If $\lambda = 0.5$, then the evaluator is a normal evaluator (i.e., a normal-type evaluator). If $0.5 < \lambda \le 1.0$, then the evaluator is an optimistic evaluator (i.e., a lenient-type evaluator). The larger the value of λ, the more optimistic the evaluator. The smaller the value of λ, the more pessimistic the evaluator.

Assume that the index of optimism determined by the evaluator is λ, where $\lambda \in [0, 1]$. Based on (3), the expected truth value $E(X)$ of the vague value X based on the index of optimism λ determined by the evaluator is defined as follows:

$$\begin{aligned} E(X) &= (1-\lambda) \times t_x + \lambda \times (t_x + N(X)), \\ &= (1-\lambda) \times t_x + \lambda \times (t_x + f_x^* - t_x), \\ &= (1-\lambda) \times t_x + \lambda \times (t_x + (1-f_x) - t_x), \\ &= (1-\lambda) \times t_x + \lambda \times (1-f_x), \end{aligned} \tag{4}$$

where $f_x^* = 1 - f_x$ and $0 \le t_x \le E(X) \le t_x + f_x^* \le 1$. For example, assume that X be a vague value, where $X = [0.6, 0.9]$, and assume that the index of optimism λ determined by the evaluator is 0.6 (i.e., $\lambda = 0.6$). In this situation, we can see that $t_x = 0.6$ and $f_x = 0.1$. Based on (4), we can see that the expected truth value $E(X)$ of the vague value X based on the index of optimism $\lambda = 0.6$ is calculated as follows:

$$\begin{aligned} E(X) &= (1-0.6) \times 0.6 + 0.6 \times (1-0.1) \\ &= 0.4 \times 0.6 + 0.6 \times 0.9 \\ &= 0.78. \end{aligned}$$

That is, the expected truth value $E(X)$ of the vague value X based on the index of optimism $\lambda = 0.6$ is 0.78.

3 A review of Biswas's methods for students' answerscripts evaluation using fuzzy sets

In [2], Biswas used the matching function S to measure the degree of similarity between two fuzzy sets [5, 6, 28]. Let A and B be two fuzzy sets of the universe of discourse X,

$$\begin{aligned} X &= \{x_1, x_2, \ldots, x_n\}, \\ A &= \{(x_1, \mu_A(x_1)), (x_2, \mu_A(x_2)), \ldots, (x_n, \mu_A(x_n))\}, \\ B &= \{(x_1, \mu_B(x_1)), (x_2, \mu_B(x_2)), \ldots, (x_n, \mu_B(x_n))\}, \end{aligned}$$

where μ_A is the membership function of the fuzzy set A, $\mu_A(x_i) \in [0, 1]$, μ_B is the membership function of the fuzzy set B, $\mu_B(x_i) \in [0, 1]$, and $1 \le i \le n$. The fuzzy sets A and B can be represented by the vectors $\overline{A}$ and $\overline{B}$, respectively [9, 11], where

$$\begin{aligned} \overline{A} &= \langle \mu_A(x_1), \mu_A(x_2), \ldots, \mu_A(x_n) \rangle, \\ \overline{B} &= \langle \mu_B(x_1), \mu_B(x_2), \ldots, \mu_B(x_n) \rangle. \end{aligned}$$

Then, the degree of similarity $S(\overline{A}, \overline{B})$ between the fuzzy sets A and B can be calculated as follows [5]:

$$S(\overline{A}, \overline{B}) = \frac{\overline{A} \cdot \overline{B}}{\mathrm{Max}(\overline{A} \cdot \overline{A}, \overline{B} \cdot \overline{B})}, \tag{5}$$

where $S(\overline{A}, \overline{B}) \in [0, 1]$. The larger the value of $S(\overline{A}, \overline{B})$, the higher the similarity between the fuzzy sets A and B.

In [2], Biswas presented a "fuzzy evaluation method" (*fem*) for evaluating students' answerscripts based on the matching function S. Let X be the universe of discourse, $X = \{0\%, 20\%, 40\%, 60\%, 80\%, 100\%\}$. He used five fuzzy linguistic hedges, called Standard Fuzzy Sets (SFS), for students' answerscripts evaluation, i.e., E (excellent), V (very good), G (good), S (satisfactory) and U (unsatisfactory), where

$$\begin{aligned} E &= \{(0\%, 0), (20\%, 0), (40\%, 0.8), (60\%, 0.9), \\ &\quad (80\%, 1), (100\%, 1)\}, \\ V &= \{(0\%, 0), (20\%, 0), (40\%, 0.8), (60\%, 0.9), \\ &\quad (80\%, 0.9), (100\%, 0.8)\}, \\ G &= \{(0\%, 0), (20\%, 0.1), (40\%, 0.8), (60\%, 0.9), \\ &\quad (80\%, 0.4), (100\%, 0.2)\}, \\ S &= \{(0\%, 0.4), (20\%, 0.4), (40\%, 0.9), (60\%, 0.6), \\ &\quad (80\%, 0.2), (100\%, 0)\}, \\ U &= \{(0\%, 1), (20\%, 1), (40\%, 0.4), (60\%, 0.2), \\ &\quad (80\%, 0), (100\%, 0)\}. \end{aligned}$$

Biswas pointed out that "$\boldsymbol{A}$", "$\boldsymbol{B}$", "$\boldsymbol{C}$", "$\boldsymbol{D}$" and "$\boldsymbol{E}$" are letter grades, where $0 \le \boldsymbol{E} < 30$, $30 \le \boldsymbol{D} < 50$, $50 \le \boldsymbol{C} < 70$, $70 \le \boldsymbol{B} < 90$ and $90 \le \boldsymbol{A} \le 100$. Furthermore, he presented the concept of "mid-grade-points", where the mid-grade-points of the letter grades $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{D}$ and $\boldsymbol{E}$ are $P(\boldsymbol{A})$, $P(\boldsymbol{B})$, $P(\boldsymbol{C})$, $P(\boldsymbol{D})$ and $P(\boldsymbol{E})$, respectively, $P(\boldsymbol{A}) = 95$, $P(\boldsymbol{B}) = 80$, $P(\boldsymbol{C}) = 60$, $P(\boldsymbol{D}) = 40$ and $P(\boldsymbol{E}) = 15$. Assume that an evaluator evaluates the first question (i.e., $Q.1$) of an answerscript of a student using a fuzzy grade sheet as shown in Table 1. In Table 1, the fuzzy mark awarded to the answer of question $Q.1$ indicates that the degrees of the evaluator's satisfaction for that answer in 0%, 20%, 40%, 60%, 80% and 100% are 0.1, 0.2, 0.3, 0.6, 0.8 and 0.9, respectively.

Table 1 A fuzzy grade sheet [2]

Question No.	Fuzzy mark						Grade
$Q.1$	0.1	0.2	0.3	0.6	0.8	0.9	
$Q.2$							
$Q.3$							
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$Q.n$							
							Total mark =

In the following, we briefly review Biswas's algorithm from [2] for students' answerscript evaluation as follows:

Step 1: For each question in the answerscript repeatedly perform the following tasks:

(1) The evaluator awards a fuzzy mark F_i to each question $Q.i$ and fills up each cell of the ith row for the first seven columns shown in Table 1, where $1 \le i \le n$. Let $\overline{F_i}$ be the vector representation of F_i, where $1 \le i \le n$.
(2) Calculate the values of $S(\overline{E}, \overline{F_i})$, $S(\overline{V}, \overline{F_i})$, $S(\overline{G}, \overline{F_i})$, $S(\overline{S}, \overline{F_i})$ and $S(\overline{U}, \overline{F_i})$, respectively, where $\overline{E}$, $\overline{V}$, $\overline{G}$, $\overline{S}$ and $\overline{U}$ are the vector representations of the standard fuzzy sets E (excellent), V (very good), G (good), S (satisfactory) and U (unsatisfactory), respectively.
(3) Find the maximum value among the values of $S(\overline{E}, \overline{F_i})$, $S(\overline{V}, \overline{F_i})$, $S(\overline{G}, \overline{F_i})$, $S(\overline{S}, \overline{F_i})$ and $S(\overline{U}, \overline{F_i})$. Assume that $S(\overline{V}, \overline{F_i})$ is the maximum value among the values of $S(\overline{E}, \overline{F_i})$, $S(\overline{V}, \overline{F_i})$, $S(\overline{G}, \overline{F_i})$, $S(\overline{S}, \overline{F_i})$ and $S(\overline{U}, \overline{F_i})$, then award grade "***B***" to the question $Q.i$ due to the fact that grade "***B***" corresponds to V (very good) of the standard fuzzy set.

Step 2: Calculate the total mark of the student as follows:

$$\mathit{Total\ Mark} = \frac{1}{100} \times \sum_{i=1}^{n} [T(Q.i) \times P(g_i)], \tag{6}$$

where $T(Q.i)$ denotes the mark allotted to $Q.i$ in the question paper, g_i denotes the grade awarded to $Q.i$ by Step 1 of the algorithm, and $P(g_i)$ denotes the mid-grade-point of g_i. Put this total score in the appropriate box at the bottom of the fuzzy grade sheet.

4 A new method for students' answerscripts evaluation using vague values

In this section, we present a new method for students' answerscripts evaluation, where the evaluating values are represented by vague values and an index of optimism λ [12] determined by the evaluator is used to indicate the degree of optimism of the evaluator for evaluating students' answerscripts, where $\lambda \in [0, 1]$. If $0 \le \lambda < 0.5$, then the evaluator is a pessimistic evaluator. If $\lambda = 0.5$, then the evaluator is a normal evaluator. If $0.5 < \lambda \le 1.0$, then the evaluator is an optimistic evaluator. The larger the value of λ, the more optimistic the evaluator. The smaller the value of λ, the more pessimistic the evaluator. Eleven satisfaction levels shown in Table 2 are used to evaluate the students' answerscripts regarding a question of a test/examination, where the corresponding vague satisfaction values of the eleven satisfaction levels are shown in Table 2.

Table 2 Satisfaction levels and their corresponding vague satisfaction values

Satisfaction levels	Vague satisfaction values
extremely good (*EG*)	[1, 1]
very very good (*VVG*)	[0.90, 99]
very good (*VG*)	[0.80, 0.89]
good (*G*)	[0.70, 0.79]
more or less good (*MG*)	[0.60, 0.69]
fair (*F*)	[0.50, 0.59]
more or less bad (*MB*)	[0.40, 0.49]
bad (*B*)	[0.25, 0.39]
very bad (*VB*)	[0.10, 0.24]
very very bad (*VVB*)	[0.01, 0.09]
extremely bad (*EB*)	[0, 0]

Assume that an evaluator evaluates the students' answerscripts by using a vague grade sheet as shown in Table 3, where X_i denotes a vague truth value defined in [0, 1] and $1 \le i \le 11$. In any row of Table 3, the columns from the second to the twelfth indicate the vague mark awarded to the answer to the corresponding question shown in the first column, where the vague mark is represented as a vague set. The last (i.e., the thirteenth) column of the vague grade sheet shown in Table 3 indicates the degree of satisfaction evaluated by the proposed method awarded to each question. The box at the bottom of the vague grade sheet shown in Table 3 indicates the total mark awarded to the student. For example, assume that an evaluator uses a vague grade sheet as shown in Table 3 to evaluate the vague mark for the first question (i.e., $Q.1$) of a test/examination of a student, shown as follows:

$$\begin{aligned} FN_{Q.1} = \{&(EG, [0, 0]), (VVG, [0.8, 0.9]), (VG, [0.7, 0.8]), \\ &(G, [0.5, 0.6]), (MG, [0, 0]), (V, [0, 0]), \\ &(MB, [0, 0]), (B, [0, 0]), (VB, [0, 0]), \\ &(VVB, [0, 0]), (EB, [0, 0])\}. \end{aligned}$$

For convenience, the fuzzy set $FN_{Q.1}$ can also be abbreviated into

$$\begin{aligned} FN_{Q.1} = \{&(VVG, [0.8, 0.9]), (VG, [0.7, 0.8]), \\ &(G, [0.5, 0.6])\}. \end{aligned}$$

Table 3 Vague mark represented by vague values of the question $Q.i$ in a vague grade sheet

Question No.	Satisfaction levels											Degree of satisfaction
	EG	*VVG*	*VG*	*G*	*MG*	*F*	*MB*	*B*	*VB*	*VVB*	*EB*	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$Q.i$	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
												Total mark =

It indicates that the vague satisfaction values of the student's answerscript with respect to the first question is: [0.8, 0.9] very very good, [0.7, 0.8] very good and [0.5, 0.6] good, where [0.8, 0.9], [0.7, 0.8] and [0.5, 0.6] are vague values.

Assume that the vague mark of the question $Q.i$ of a student's answerscript evaluated by an evaluator is as shown in Table 3, where X_i is a vague value in the universe of discourse [0, 1] and $1 \le i \le 11$. Assume that the degree of optimism of the evaluator determined by the evaluator for evaluating students' answerscript is λ, where $\lambda \in [0, 1]$. The proposed method for students' answerscripts evaluation based on vague values is now presented as follows:

Step 1: Based on (4) and the index of optimism λ determined by the evaluator, where $\lambda \in [0, 1]$, calculate the expected truth value $E(X_i)$ of each vague truth value X_i in the vague grade sheet shown in Table 3, where $E(X_i) \in [0, 1]$ and $1 \le i \le 11$, as shown in Table 4.

Step 2: From Table 2, we can see that the corresponding vague satisfaction truth values of the satisfaction levels *EG*, *VVG*, *VG*, *G*, *MG*, *F*, *MB*, *B*, *VB*, *VVB* and *EB* are as follows: $T(EG) = [1, 1]$, $T(VVG) = [0.90, 0.99]$, $T(VG) = [0.80, 0.89]$, $T(G) = [0.70, 0.79]$, $T(MG) = [0.60, 0.69]$, $T(F) = [0.50, 0.59]$, $T(MB) = [0.40, 0.49]$, $T(B) = [0.25, 0.39]$, $T(VB) = [0.10, 0.24]$, $T(VVB) = [0.01, 0.09]$ and $T(EB) = [0, 0]$, where [1, 1], [0.90, 0.99], [0.80, 0.89], [0.70, 0.79], [0.60, 0.69], [0.50, 0.59], [0.40, 0.49], [0.25, 0.39], [0.10, 0.24], [0.01, 0.09] and [0, 0] are vague truth values. Based on (4) and the index of optimism λ determined by the evaluator, where $\lambda \in [0, 1]$, calculate the corresponding expected truth value $E(Y)$ of each satisfaction level Y in the vague grade sheet shown in Table 4, where $Y \in \{EG, VVG, VG, G, MG, F, MB, B, VB, VVB, EB\}$ and $E(Y) \in [0, 1]$. For example, from Table 2, we can see that $T(VG) = [0.80, 0.89]$, where $t_{VG} = 0.80$ and $1 - f_{VG} = 0.89$. Assume that the index of optimism λ determined by the evaluator is 0.60 (i.e., $\lambda = 0.60$), then based on (4), we can see that the expected truth value $E(VG)$ of the satisfaction level *VG* is calculated as follows:

$$\begin{aligned} E(VG) &= (1-\lambda) \times t_{VG} + \lambda \times (1 - f_{VG}) \\ &= (1 - 0.60) \times 0.80 + 0.60 \times 0.89 \\ &= 0.32 + 0.534 \\ &= 0.854. \end{aligned}$$

It indicates that the expected truth value of the satisfaction level *VG* is 0.854 when the index of optimism λ determined by the evaluator is 0.60 (i.e., $\lambda = 0.60$). The degree of satisfaction $D(Q.i)$ of the question $Q.i$ of the student's answerscript can be evaluated by the function D,

$$\begin{aligned} D(Q.i) = {} & [E(X_{i1}) \times E(EG) + E(X_{i2}) \times E(VVG) \\ & + \cdots + E(X_{i11}) \times E(EB)] \\ & /[E(X_{i1}) + E(X_{i2}) + \cdots + E(X_{i11})], \end{aligned} \tag{7}$$

where $E(X_i)$ denotes the expected satisfaction value of the vague satisfaction value X_i, $1 \le i \le 11$, and $0 \le D(Q.i) \le 1$. The larger the value of $D(Q.i)$, the higher the degree of satisfaction that the question $Q.i$ of the student's answerscript satisfies the evaluator's opinion.

Step 3: Consider the situation that the total mark of a student's answerscript to an examination is 100 marks. Assume that there are n questions to be answered, i.e.,

TOTAL MARKS $= 100$,

$Q.1$ carries s_l marks,

$Q.2$ carries s_2 marks,

⋮

$Q.n$ carries s_n marks,

where $\sum_{i=1}^{n} s_i = 100$, $0 < s_i \le 100$, and $1 \le i \le n$. Assume that the evaluated degrees of satisfaction of the questions $Q.1, Q.2, \ldots,$ and $Q.n$ are $D(Q.1), D(Q.2), \ldots,$ and $D(Q.n)$, respectively, then the total mark of the student is evaluated as follows:

$$s_1 \times D(Q.1) + s_2 \times D(Q.2) + \cdots + s_n \times D(Q.n). \tag{8}$$

Put this total mark in the appropriate box at the bottom of the vague grade sheet.

Table 4 Expected truth values of the vague truth values of the question $Q.i$ of Table 3

Question No.	Satisfaction levels											Degree of satisfaction
	EG	*VVG*	*VG*	*G*	*MG*	*F*	*MB*	*B*	*VB*	*VVB*	*EB*	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$Q.i$	$E(X_1)$	$E(X_2)$	$E(X_3)$	$E(X_4)$	$E(X_5)$	$E(X_6)$	$E(X_7)$	$E(X_8)$	$E(X_9)$	$E(X_{10})$	$E(X_{11})$	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
												Total mark =

Table 5 Vague grade sheet of Example 4.1

Question No.	Satisfaction levels											Degree of satisfaction
	EG	*VVG*	*VG*	*G*	*MG*	*F*	*MB*	*B*	*VB*	*VVB*	*EB*	
$Q.1$	[0.8, 0.9]	[0.9, 0.95]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	
$Q.2$	[0, 0]	[0, 0]	[0, 0]	[0.6, 0.7]	[0.9, 0.95]	[0.55, 0.6]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	
$Q.3$	[0, 0]	[0, 0]	[0.85, 0.9]	[0.75, 0.8]	[0.5, 0.6]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	
$Q.4$	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.5, 0.6]	[0.9, 0.95]	[0.2, 0.4]	[0, 0]	
												Total mark =

Example 4.1 Consider a student's answerscript to an examination of 100 marks. Assume that in total there are four questions to be answered:

TOTAL MARKS $= 100$,

$Q.1$ carries 20 marks,

$Q.2$ carries 30 marks,

$Q.3$ carries 25 marks,

$Q.4$ carries 25 marks.

Assume that an evaluator awards the students' answerscript by a vague grade sheet as shown in Table 5 and assume that the optimism index λ of the evaluator is 0.60 (i.e., $\lambda = 0.60$).

[*Step 1*] Based on (4) and the index of optimism λ determined by the evaluator, where $\lambda = 0.60$, we can calculate the expected satisfaction value of each vague satisfaction value in the vague grade sheet shown in Table 5, as shown in Table 6.

[*Step 2*] From Table 2, we can see that the corresponding vague truth values of the satisfaction levels *EG*, *VVG*, *VG*, *G*, *MG*, *F*, *MB*, *B*, *VB*, *VVB* and *EB* are [1, 1], [0.90, 0.99], [0.80, 0.89], [0.70, 0.79], [0.60, 0.69], [0.50, 0.59], [0.40, 0.49], [0.25, 0.39], [0.10, 0.24], [0.01, 0.09] and [0, 0], respectively, i.e., $T(EG) = [1, 1]$, $T(VVG) = [0.90, 0.99]$, $T(VG) = [0.80, 0.89]$, $T(G) = [0.70, 0.79]$, $T(MG) = [0.60, 0.69]$, $T(F) = [0.50, 0.59]$, $T(MB) = [0.40, 0.49]$, $T(B) = [0.25, 0.39]$, $T(VB) = [0.10, 0.24]$, $T(VVB) = [0.01, 0.09]$ and $T(EB) = [0, 0]$, where [1, 1], [0.90, 0.99], [0.80, 0.89], [0.70, 0.79], [0.60, 0.69], [0.50, 0.59], [0.40, 0.49], [0.25, 0.39], [0.10, 0.24], [0.01, 0.09] and [0, 0] are vague truth values. Because the index of optimism λ determined by the evaluator is 0.60 (i.e., $\lambda = 0.60$), based on (4), we can get the following results:

$$E(EG) = (1 - 0.60) \times 1 + 0.60 \times 1 = 1,$$

$$E(VVG) = (1 - 0.60) \times 0.90 + 0.60 \times 0.99 = 0.954,$$

$$E(VG) = (1 - 0.60) \times 0.80 + 0.60 \times 0.89 = 0.854,$$

$$E(G) = (1 - 0.60) \times 0.70 + 0.60 \times 0.79 = 0.754,$$

$$E(MG) = (1 - 0.60) \times 0.60 + 0.60 \times 0.69 = 0.654,$$

$$E(F) = (1 - 0.60) \times 0.50 + 0.60 \times 0.59 = 0.554,$$

$$E(MB) = (1 - 0.60) \times 0.40 + 0.60 \times 0.49 = 0.454,$$

$$E(B) = (1 - 0.60) \times 0.25 + 0.60 \times 0.39 = 0.334,$$

$$E(VB) = (1 - 0.60) \times 0.10 + 0.60 \times 0.24 = 0.184,$$

$$E(VVB) = (1 - 0.60) \times 0.01 + 0.60 \times 0.09 = 0.058,$$

$$E(EB) = (1 - 0.60) \times 0 + 0.60 \times 0 = 0.$$

Based on (7), we can get the following results:

$$\begin{aligned} D(Q.1) = (&0.86 \times 1 + 0.93 \times 0.954 + 0 \times 0.854 + 0 \\ &\times 0.754 + 0 \times 0.654 + 0 \times 0.554 + 0 \times 0.454 \\ &+ 0 \times 0.334 + 0 \times 0.184 + 0 \times 0.058 + 0 \times 0) \\ &/(0.86 + 0.93 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 \\ &+ 0) = 0.976, \end{aligned}$$

Table 6 Expected satisfaction values of the vague grade sheet shown in Table 5

Question No.	Satisfaction levels											Degree of satisfaction
	EG	*VVG*	*VG*	*G*	*MG*	*F*	*MB*	*B*	*VB*	*VVB*	*EB*	
$Q.1$	0.86	0.93	0	0	0	0	0	0	0	0	0	
$Q.2$	0	0	0	0.66	0.93	0.58	0	0	0	0	0	
$Q.3$	0	0	0.88	0.78	0.56	0	0	0	0	0	0	
$Q.4$	0	0	0	0	0	0	0	0.56	0.93	0.32	0	
												Total mark =

$$
\begin{aligned}
D(Q.2) = {} & (0 \times 1 + 0 \times 0.954 + 0 \times 0.854 + 0.66 \times 0.754 \\
& + 0.93 \times 0.654 + 0.58 \times 0.554 + 0 \times 0.454 \\
& + 0 \times 0.334 + 0 \times 0.184 + 0 \times 0.058 + 0 \times 0) \\
& /(0 + 0 + 0 + 0.66 + 0.93 + 0.58 + 0 + 0 + 0 \\
& + 0 + 0) = 0.658, \\
D(Q.3) = {} & (0 \times 1 + 0 \times 0.954 + 0.88 \times 0.854 \\
& + 0.78 \times 0.754 + 0.56 \times 0.654 + 0 \times 0.554 \\
& + 0 \times 0.454 + 0 \times 0.334 + 0 \times 0.184 \\
& + 0 \times 0.058 + 0 \times 0) \\
& /(0 + 0 + 0.88 + 0.78 + 0.56 + 0 + 0 + 0 \\
& + 0 + 0 + 0) \\
= {} & 0.768, \\
D(Q.4) = {} & (0 \times 1 + 0 \times 0.954 + 0 \times 0.854 + 0 \times 0.754 \\
& + 0 \times 0.654 + 0 \times 0.554 + 0 \times 0.454 \\
& + 0.56 \times 0.334 + 0.93 \times 0.184 \\
& + 0.32 \times 0.058 + 0 \times 0) \\
& /(0 + 0 + 0 + 0 + 0 + 0 + 0 + 0.56 + 0.93 \\
& + 0.32 + 0) \\
= {} & 0.208.
\end{aligned}
$$

[*Step 3*] Based on (8), the total mark of the student is evaluated as follows:

$$
\begin{aligned}
& 20 \times D(Q.1) + 30 \times D(Q.2) + 25 \times D(Q.3) \\
& \quad + 25 \times D(Q.4) \\
& = 20 \times 0.976 + 30 \times 0.658 + 25 \times 0.768 + 25 \times 0.208 \\
& = 19.52 + 19.74 + 19.2 + 5.2 \\
& = 63.66 \\
& \cong 64
\end{aligned}
$$

(assuming that no half mark is given in the total mark).

5 A generalized students' answerscripts evaluation method using vague values

In this section, we present a generalized students' answerscripts evaluation method using vague values. Assume that there are n questions to be answered:

TOTAL MARKS $= 100$,

$Q.1$ carries s_l marks,

$Q.2$ carries s_2 marks,

$\vdots$

$Q.n$ carries s_n marks,

where $\sum_{i=1}^{n} s_i = 100$, $0 < s_i \leq 100$, and $1 \leq i \leq n$. Assume that the degree of optimism of the evaluator is λ, where $\lambda \in [0, 1]$. If $0 \leq \lambda < 0.5$, then the evaluator is a pessimistic evaluator. If $\lambda = 0.5$, then the evaluator is a normal evaluator. If $0.5 < \lambda \leq 1.0$, then the evaluator is an optimistic evaluator. The larger the value of λ, the more optimistic the evaluator. The smaller the value of λ, the more pessimistic the evaluator. Assume that an evaluator evaluates the answers of students' answerscripts using the following four criteria [2]:

C_1: Accuracy of information,

C_2: Adequate coverage,

C_3: Conciseness,

C_4: Clear expression,

and assume that the weights of the criteria C_1, C_2, C_3 and C_4 are w_l, w_2, w_3 and w_4, respectively, where $0 \leq w_i \leq 1$ and $1 \leq i \leq 4$. Furthermore, assume that the evaluator can evaluate each question of the students' answerscripts using the above four criteria based on the method described previously. In this case, an evaluator can evaluate students' answerscripts using a generalized vague grade sheet as shown in Table 7, where the evaluating values in Table 7 are represented by vague values and the degrees of satisfaction of the question $Q.i$ of a student's answerscript

Table 7 A generalized vague grade sheet

Question No.	Criteria	Satisfaction levels											Degree of satisfaction for criteria	Degree of satisfaction for questions
		EG	*VVG*	*VG*	*G*	*MG*	*F*	*MB*	*B*	*VB*	*VVB*	*EB*		
$Q.1$	C_1												$D(C_{11})$	$P(Q.1)$
	C_2												$D(C_{12})$	
	C_3												$D(C_{13})$	
	C_4												$D(C_{14})$	
$Q.2$	C_1												$D(C_{21})$	$P(Q.2)$
	C_2												$D(C_{22})$	
	C_3												$D(C_{23})$	
	C_4												$D(C_{24})$	
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$Q.n$	C_1												$D(C_{n1})$	$P(Q.n)$
	C_2												$D(C_{n2})$	
	C_3												$D(C_{n3})$	
	C4												$D(C_{n4})$	
Total mark $= s_1 \times P(Q.1) + s_2 \times P(Q.2) + \cdots + s_n \times P(Q.n)$														

regarding the criteria C_1, C_2, C_3 and C_4 evaluated by the proposed method presented in Sect. 4 are $D(C_{il})$, $D(C_{i2})$, $D(C_{i3})$, and $D(C_{i4})$, respectively, where $0 \le D(C_{il}) \le 1$, $0 \le D(C_{i2}) \le 1$, $0 \le D(C_{i3}) \le 1$, $0 \le D(C_{i4}) \le 1$, and $1 \le i \le n$. The degree of satisfaction $P(Q.i)$ of the question $Q.i$ of the student's answerscript can be evaluated as follows:

$$P(Q.i) = [w_1 \times D(C_{i1}) + w_2 \times D(C_{i2}) + w_3 \times D(C_{i3}) + w_4 \times D(C_{i4})] / [w_1 + w_2 + w_3 + w_4], \tag{9}$$

where $0 \le P(Q.i) \le 1$ and $1 \le i \le n$. The total mark of the student can be evaluated and is equal to

$$s_1 \times P(Q.1) + s_2 \times P(Q.2) + \cdots + s_n \times P(Q.n). \tag{10}$$

Put this total score in the appropriate box at the bottom of the generalized vague grade sheet.

6 Experimental results

We have made an experiment to compare the evaluating results of the proposed method with Biswas's method [2] for different days. In our experiment, there are four questions to be answered in a student's answerscript, where

TOTAL MARKS $= 100$,

$Q.1$ carries 20 marks,

$Q.2$ carries 25 marks,

$Q.3$ carries 25 marks,

$Q.4$ carries 30 marks.

Assume that the optimism index λ of the evaluator is 0.60 (i.e., $\lambda = 0.60$). The evaluator uses Biswas's method presented in [2] and the proposed method to evaluate the student's answerscript on different days, respectively. The results are shown in Fig. 2 and Fig. 3, respectively. A comparison of the evaluating results of the student's answerscript is shown in Table 8. From Table 8, we can see that the total marks of the student evaluated by the evaluator using Biswas's method [2] for July 1, 2006, July 2, 2006, July 3, 2006 and July 4, 2006 are 69, 72, 55 and 55, respectively. We also can see that the total marks of the student evaluated by the evaluator using the proposed method for July 1, 2006, July 2, 2006, July 3, 2006 and July 4, 2006 are 65, 65, 65 and 65, respectively. It is obvious that the proposed method is more stable to evaluate the student's answerscript than Biswas's method [2]. It can evaluate students' answerscripts in a more flexible and more intelligent manner.

7 Conclusions

In this paper, we have presented a new method for evaluating students' answerscripts using vague values, where the evaluating marks awarded to the questions in the students' answerscripts are represented by vague values. The vague mark awarded to each question of a student's answer-

Fig. 2 Evaluating the student's answerscript on different days using Biswas's method [2]

July 1, 2006

Question No.	Satisfaction levels						Grade
$Q.1$	0	0	0	0.6	0.9	0.8	
$Q.2$	0	0	0.6	0.9	0.8	0	
$Q.3$	0	0	0	0.6	0.8	0.9	
$Q.4$	0	0.6	0.9	0.8	0.2	0	
							Total mark =

July 2, 2006

Question No.	Satisfaction levels						Grade
$Q.1$	0	0	0	0.8	0.9	1	
$Q.2$	0	0	0.7	0.8	0.9	0	
$Q.3$	0	0	0	0.7	0.9	0.8	
$Q.4$	0	0.5	0.8	0.7	0	0	
							Total mark =

July 3, 2006

Question No.	Satisfaction levels						Grade
$Q.1$	0	0	0	0.6	0.9	0.7	
$Q.2$	0	0	0.6	0.8	0.7	0	
$Q.3$	0	0	0	0.5	0.7	0.9	
$Q.4$	0	0.5	0.8	0.6	0	0	
							Total mark =

July 4, 2006

Question No.	Satisfaction levels						Grade
$Q.1$	0	0	0	0.6	0.8	0.7	
$Q.2$	0	0	0.5	0.9	0.7	0	
$Q.3$	0	0	0	0.7	0.9	0.8	
$Q.4$	0	0.6	0.9	0.7	0	0	
							Total mark =

Table 8 A comparison of the evaluating results for different methods

Days	Total mark	
	Methods	
	Biswas's method [2]	The proposed method
July 1, 2006	69	65
July 2, 2006	72	65
July 3, 2006	55	65
July 4, 2006	55	65

script can be regarded as a vague set, where each element in the universe of discourse belonging to a vague set is represented by a vague value in [0, 1]. An index of optimism λ determined by the evaluator is used to indicate the degree of optimism of the evaluator, where $\lambda \in [0, 1]$. From the experimental results shown in Table 8, we can see that the proposed method is more stable to evaluate students' answerscripts than the Biswas's method presented in [2]. The proposed method still can get good properties of Table 8 by making a small variation of the evaluating values. It can evaluate students' answerscripts in a more flexible and more intelligent manner.

July 1, 2006

Question No.	Satisfaction levels											Degree of satisfaction
	EG	*VVG*	*VG*	*G*	*MG*	*F*	*MB*	*B*	*VB*	*VVB*	*EB*	
Q.1	[1, 1]	[0.8, 0.9]	[0.7, 0.8]	[0.6, 0.7]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	
Q.2	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.7, 0.8]	[0.8, 0.9]	[0.5, 0.6]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	
Q.3	[1, 1]	[0.8, 0.9]	[0.7, 0.8]	[0.6, 0.7]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	
Q.4	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.2, 0.3]	[0.7, 0.8]	[0.8, 0.9]	[0.5, 0.6]	[0, 0]	[0, 0]	
												Total mark =

July 2, 2006

Question No.	Satisfaction levels											Degree of satisfaction
	EG	*VVG*	*VG*	*G*	*MG*	*F*	*MB*	*B*	*VB*	*VVB*	*EB*	
Q.1	[0.8, 0.9]	[0.7, 0.8]	[0.6, 0.7]	[0.5, 0.6]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	
Q.2	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.7, 0.8]	[0.8, 0.9]	[0.6, 0.7]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	
Q.3	[0.8, 0.9]	[0.7, 0.8]	[0.6, 0.7]	[0.5, 0.6]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	
Q.4	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.1, 0.2]	[0.7, 0.8]	[0.8, 0.9]	[0.6, 0.7]	[0, 0]	[0, 0]	
												Total mark =

July 3, 2006

Question No.	Satisfaction levels											Degree of satisfaction
	EG	*VVG*	*VG*	*G*	*MG*	*F*	*MB*	*B*	*VB*	*VVB*	*EB*	
Q.1	[1, 1]	[0.8, 0.9]	[0.7, 0.8]	[0.6, 0.7]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	
Q.2	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.7, 0.8]	[0.8, 0.9]	[0.5, 0.6]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	
Q.3	[1, 1]	[0.8, 0.9]	[0.7, 0.8]	[0.6, 0.7]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	
Q.4	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.2, 0.3]	[0.8, 0.9]	[1.0, 1.0]	[0.7, 0.8]	[0, 0]	[0, 0]	
												Total mark =

July 4, 2006

Question No.	Satisfaction levels											Degree of satisfaction
	EG	*VVG*	*VG*	*G*	*MG*	*F*	*MB*	*B*	*VB*	*VVB*	*EB*	
Q.1	[0.8, 0.9]	[0.7, 0.8]	[0.6, 0.7]	[0.5, 0.6]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	
Q.2	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.7, 0.8]	[0.8, 0.9]	[0.6, 0.7]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	
Q.3	[1, 1]	[0.8, 0.9]	[0.8, 0.9]	[0.7, 0.8]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	
Q.4	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.2, 0.3]	[0.8, 0.9]	[1.0, 1.0]	[0.7, 0.8]	[0, 0]	[0, 0]	
												Total mark =

Fig. 3 Evaluating the student's answerscript on different days using the proposed method

Acknowledgements The authors would like to thank Professor Jason Chihyu Chan, Department of Education, National Chengchi University, Taipei, Taiwan, Republic of China, for providing very helpful comments and suggestions. This work was supported in part be the National Science Council, Republic of China, under Grant NSC 95-2221-E-011-117-MY2.

References

1. Altun A, Cakan M (2006) Undergraduate students' academic achievement, field dependent/independent cognitive styles and attitude toward computers. Educ Technol Soc 9(1):289–297
2. Biswas R (1995) An application of fuzzy sets in students' evaluation. Fuzzy Sets Syst 74(2):187–194
3. Chang DF, Sun CM (1993) Fuzzy assessment of learning performance of junior high school students. In: Proceedings of the 1993 first national symposium on fuzzy theory and applications, Hsinchu, Taiwan, Republic of China, pp 10-15
4. Chen SM (1995) Arithmetic operations between vague sets. In: Proceedings of the international joint conference of CFSA/IFIS/SOFT'95 on fuzzy theory and applications, Taipei, Taiwan, Republic of China, pp 206–211
5. Chen SM (1988) A new approach to handling fuzzy decision making problems. IEEE Trans Syst Man Cybern 18(6):1012–1016
6. Chen SM, Yeh MS, Hsiao PY (1995) A comparison of similarity measures of fuzzy values. Fuzzy Sets Syst 72(1):79–89
7. Chen SM (1995) Measures of similarity between vague sets. Fuzzy Sets Syst 74(2):217–223
8. Chen SM (1997) Similarity measures between vague sets and between elements. IEEE Trans Syst Man Cybern Part B: Cybern 27(1):153–158

9. Chen SM (1999) Evaluating the rate of aggregative risk in software development using fuzzy set theory. Cybern Syst 30(1):57–75
10. Chen SM, Lee CH (1999) New methods for students' evaluation using fuzzy sets. Fuzzy Sets Syst 104(2):209–218
11. Chen SM, Wang JY (1995) Document retrieval using knowledge-based fuzzy information retrieval techniques. IEEE Trans Syst Man Cybern 25(5):793–803
12. Cheng CH, Yang KL (1998) Using fuzzy sets in education grading system. J Chin Fuzzy Syst Assoc 4(2):81–89
13. Chiang TT, Lin CM (1994) Application of fuzzy theory to teaching assessment. In: Proceedings of the 1994 second national conference on fuzzy theory and applications, Taipei, Taiwan, Republic of China, pp 92–97
14. Devedzic V (2004) Web intelligence and artificial intelligence in education. Educ Technol Soc 7(4):29–39
15. Echauz JR, Vachtsevanos GJ (1995) Fuzzy grading system. IEEE Trans Educ 38(2):158–165
16. Frair L (1995) Student peer evaluations using the analytic hierarchy process method. In: Proceedings of 1995 frontiers in education conference, vol 2, pp 4c3.1–4c3.5
17. Gau WL, Buehrer DJ (1993) Vague sets. IEEE Trans Syst Man Cybern 23(2):610–614
18. Kaburlasos VG, Marinagi CC, Tsoukalas VT (2004) PARES: a software tool for computer-based testing and evaluation used in the Greek higher education system. In: Proceedings of the 2004 IEEE international conference on advanced learning technologies, pp 771–773
19. Law CK (1996) Using fuzzy numbers in education grading system. Fuzzy Sets Syst 83(3):311–323
20. Ma J, Zhou D (2000) Fuzzy set approach to the assessment of student-centered learning. IEEE Trans Educ 43(2):237–241
21. McMartin F, Mckenna A, Youssefi K (2000) Scenario assignments as assessment tools for undergraduate engineering education. IEEE Trans Educ 43(2):111–119
22. Pears A, Daniels M, Berglund A, Erickson C (2001) Student evaluation in an international collaborative project course. In: Proceedings of the 2001 symposium on applications and the Internet workshops, pp 74–79
23. Rasmani KA, Shen Q (2006) Data-driven fuzzy rule generation and its application for student academic performance evaluation. Appl Intell 25(3):305–319
24. Wang HY, Chen SM (2006) New methods for evaluating students' answerscripts using vague values. In: Proceedings of the 9th joint conference on information sciences, Kaohsiung, Taiwan, Republic of China, pp 1184–1187
25. Weon S, Kim J (2001) Learning achievement evaluation strategy using fuzzy membership function. In: Proceedings of the 31st ASEE/IEEE frontier in education conference, Reno, NV, pp T3A-19–T3A-24
26. Wilson E, Karr CL, Freeman LM (1998) Flexible, adaptive, automatic fuzzy-based grade assigning system. In: Proceedings of the 1998 North American fuzzy information processing society (NAFIPS) conference, pp 334–338
27. Wu MH (2003) A research on applying fuzzy set theory and item response theory to evaluate learning performance. Master thesis, Department of Information Management, Chaoyang University of Technology, Wufeng, Taichung County, Republic of China
28. Zadeh LA (1965) Fuzzy sets. Inf Control 8:338–353

Hui-Yu Wang received the B.S. degree and the Master degree from the Department of Education, National Chengchi University, Taipei, Taiwan, R.O.C., in 1993 and 1995, respectively. She is currently working toward her Ph.D. degree in the Department of Education, National Chengchi University, Taipei, Taiwan. His current research interests include the applications of the fuzzy set theory and the vague set theory in education.

Shyi-Ming Chen is a Professor of the Department of Computer Science and Information Engineering, National Taiwan University of Science and Technology, Taipei, Taiwan, R.O.C. He received the Ph.D. degree in Electrical Engineering from National Taiwan University, Taipei, Taiwan, in June 1991. He has published more than 250 papers in referred journals, conference proceedings and book chapters. His research interests include fuzzy systems, information retrieval, knowledge-based systems, artificial intelligence, neural networks, data mining, and genetic algorithms. He has received several honors and awards, including the 1994 Outstanding Paper Award of the Journal of Information and Education, the 1995 Outstanding Paper Award of the Computer Society of the Republic of China, the Best Paper Award of the 1999 National Computer Symposium, Republic of China, the 1999 Outstanding Paper Award of the Computer Society of the Republic of China, the 2001 Outstanding Talented Person Award, Republic of China, for the contributions in Information Technology, the Outstanding Electrical Engineering Professor Award granted by the Chinese Institute of Electrical Engineering (CIEE), Republic of China, the 2003 Outstanding Paper Award of the Technological and Vocational Education Society, Republic of China, and the 2006 Outstanding Paper Award of the 11th Conference on Artificial Intelligence and Applications. Dr. Chen is currently the President of the Taiwanese Association for Artificial Intelligence (TAAI). He is an Associate Editor of the IEEE Transactions on Systems, Man, and Cybernetics - Part C, an Associate Editor of the IEEE Computational Intelligence Magazine, an Associate Editor of the International Journal of Applied Intelligence, an Associate Editor of the Journal of Intelligent & Fuzzy Systems, an Associate Editor of the International Journal of Artificial Intelligence Tools, an Editor of the New Mathematics and Natural Computation Journal, an Associate Editor of the International Journal of Fuzzy Systems, an Editorial Board Member of the International Journal of Information and Communication Technology, an Editorial Board Member of the WSEAS Transactions on Systems, an Editor of the Journal of Advanced Computational Intelligence and Intelligent Informatics, an Associate Editor of the WSEAS Transactions on Computers, an Editorial Board Member of the International Journal of Computational Intelligence and Applications, an Editorial Board Member of the Advances in Fuzzy Sets and Systems Journal, an Editor of the International Journal of Soft Computing, an Editor of the Asian Journal of Information Technology, an Editorial Board Member of the International Journal of Intelligence Systems Technologies and Applications, an Editor of the Asian Journal of Information Management, an Associate Editor of the International Journal of Innovative Computing, Information and Control, an Editorial Board Member of the International Journal of Computer Applications in Technology, an Associate Editor of the Journal of Uncertain Systems, and an Editorial Board Member of the Advances in Computer Sciences and Engineering Journal. He was an Editor of the Journal of the Chinese Grey System Association from 1998 to 2003. He is listed in *International Who's Who of Professionals*, *Marquis Who's Who in the World, and Marquis Who's Who in Science and Engineering*. He is an IET Fellow.