

# Digital Solution of Lens Distortion

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**ABSTRACT:** Lens distortion causes imaged positions to be displaced from their ideal locations, and the geometry quality of the image is also deteriorated. To determine accurate spatial information from image, camera has to be carefully calibrated. In general, there are three kinds of methods being used to calibrate camera: laboratory methods, field methods, and stellar methods. A software package called PhotoModeler Pro 4.0 was used to calibrate cameras in this study.

The digital cameras of three different brands were calibrated separately by PhotoModeler Pro 4.0 several times. After getting the elements of interior orientation, the original images taken by the digital cameras were then rectified according to the solved parameters. The resultant images were shown to have good visual appearance.

## 1. INTRODUCTION

Because digital cameras were much more convenient and inexpensive, they were gradually employed to take images for further research in photogrammetry. However, lens of a nonmetric digital camera was often with relatively large distortions. To make compensation for these distortions, nonmetric digital cameras have to be calibrated prior to use. In this study, PhotoModeler Pro 4.0 has been used to calibrate digital camera.

Three different brands of digital cameras were calibrated separately. The elements of interior orientation of each digital camera were then determined. The stability of these resultant constants was estimated. The stable constants were introduced to the distortion-rectified program to mitigate lens distortions of the digital images.

## 2. METHODS

### 2.1 Calibration of digital cameras

The software, PhotoModeler Pro 4.0, was used to calibrate the digital cameras in this study. Calibration slide offered by PhotoModeler Pro 4.0 was first projected on the wall and taken by each digital camera in required positions. The resultant images were then imported to the computer. The grid pattern of the calibration slide in every resultant image would be marked automatically and the approach of self-calibrating bundle adjustment was applied to solve the elements of interior orientation, including calibrated focal length, principal point location, radial lens distortion, and decentering lens distortion. The steps to calibrate a digital camera were:

(1) To calculate the digital camera's format size, first set up a piece of paper whose size was known on a wall, and then the image of paper was taken straight on in a known distance.

(2) Project the calibration grid on a flat wall (as shown in Fig. 1).

(3) First turn off the flash of the digital camera, and keep it at wide angle. Then take eight images of the calibration grid from different angles and positions, including upper left position, middle left position, middle left vertical position, bottom left position, top right position, middle right position, middle right vertical position, and bottom right position. Note that the four control points have to come close to the edge of the image, but were never cut off.

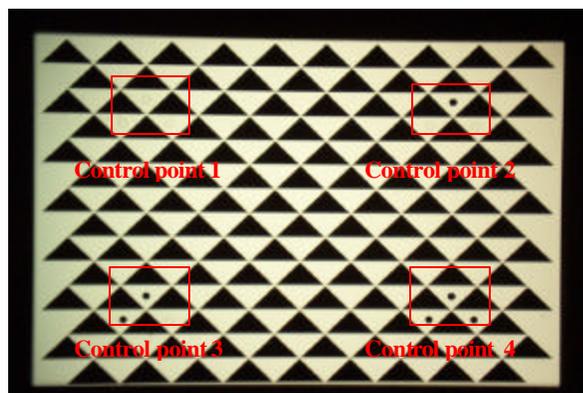


Figure 1 Calibration slide projected on the wall.

(4) Measure the distance between control point 1 and 4 on the projected pattern.

- (5) Import the eight images to the computer and start the camera calibrator program. Go on opening each image in turn and mark four control points on all eight images.
- (6) Start the auto-marking and calibration process. When the calibration processing was complete, the calibrated results were solved.

## 2.2 Calibration method

To find out the stability of the calibrated elements, calibration slide was projected on the wall in three positions<sup>1</sup>, and images were taken three times by three different brands of digital cameras<sup>2</sup> in each position. These resultant images were imported to the computer for later calibration. If the calibrated constants were stable, the average of each set of constants was introduced to the distortion-rectified program.

## 2.3 Image rectification

In the distortion-rectified program, the calibrated constants, including image format size, principal point location, and parameters of radial distortion and decentering distortion were used, and the nearest-neighbor resampling was introduced to remove lens distortions of the digital images.

## 3. RESULTS AND ANALYSIS

### 3.1 Formulas of lens distortion

The formulas of radial and decentering lens distortion employed in this study were:

$$dr = K_1 * r^2 + K_2 * r^4 \dots \dots \dots (1)$$

$$dp_x = P_1 * (r^2 + 2x^2) + 2P_2xy \dots \dots \dots (2)$$

$$dp_y = P_2 * (r^2 + 2y^2) + 2P_1xy \dots \dots \dots (3)$$

In the Eq. (1), dr was the amount of radial lens distortion, r was the radial distance from the principal point, and the K<sub>1</sub>, K<sub>2</sub> were coefficients of the polynomial (i.e. the radial lens distortion parameters). In the Eqs.(2) and (3), dp<sub>x</sub> and dp<sub>y</sub> were the amounts of decentering lens distortion to x and y, respectively.

### 3.2 Calibrated results

After calibrate, the elements of interior orientation of each digital camera were determined and filled up in the table 1 to table 3. Every set of element contained nine calibrated results, furthermore, its mean(i) and standard deviation(s) were computed. And then, if the calibrated results of each element lay inside the

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<sup>1</sup>The projector was separately arranged in the distance of 2.00m, 3.20m, and 4.40m from the wall.  
<sup>2</sup>The digital cameras used to calibrate were FUJIFILM FinePix 40i, Kodak DC280, and Nikon CoolPix 5000.

range of  $\pm 3s$ , they were treated as stable results.

Table 1 Calibrated results of FUJIFILM FinePix 40i.

session, position	f(mm)	format x(mm)	format y(mm)	pps x(mm)	pps y(mm)	radial distortion		decentering distortion	
						K <sub>1</sub>	K <sub>2</sub>	P <sub>1</sub>	P <sub>2</sub>
1, 2.0m	8.323	7.349	5.509	3.719	2.701	2.211E-03	-4.540E-05	2.988E-04	-2.484E-04
1, 3.2m	8.320	7.353	5.511	3.758	2.719	2.370E-03	-4.784E-05	5.993E-05	-1.123E-04
1, 4.4m	8.430	7.460	5.590	3.808	2.749	2.180E-03	-4.383E-05	3.484E-05	-1.560E-04
2, 2.0m	8.312	7.323	5.495	3.706	2.704	2.493E-03	-5.644E-05	3.871E-04	-3.337E-04
2, 3.2m	8.352	7.383	5.526	3.806	2.680	2.229E-03	-4.362E-05	1.022E-04	-2.083E-04
2, 4.4m	8.270	7.331	5.498	3.746	2.710	2.352E-03	-3.994E-05	2.020E-05	-2.719E-04
3, 2.0m	8.315	7.345	5.498	3.778	2.660	2.229E-03	-3.866E-05	5.080E-05	-2.675E-04
3, 3.2m	8.283	7.333	5.499	3.748	2.716	2.373E-03	-4.233E-05	7.252E-05	-1.587E-04
3, 4.4m	8.305	7.345	5.508	3.756	2.731	2.332E-03	-4.826E-05	2.584E-05	-1.369E-04
$\bar{i}$	8.323	7.358	5.515	3.758	2.708	2.308E-03	-4.515E-05	1.169E-04	-2.104E-04
s	0.047	0.042	0.030	0.035	0.026	1.019E-04	5.312E-06	1.324E-04	7.446E-05
$\bar{i} + 3s$	8.463	7.484	5.604	3.863	2.786	2.613E-03	-2.921E-05	5.142E-04	1.296E-05
$\bar{i} - 3s$	8.184	7.232	5.426	3.654	2.629	2.002E-03	-6.108E-05	-2.803E-04	-4.338E-04

Table 2 Calibrated results of Kodak DC280.

session, position	f(mm)	format x(mm)	format y(mm)	pps x(mm)	pps y(mm)	radial distortion		decentering distortion	
						K <sub>1</sub>	K <sub>2</sub>	P <sub>1</sub>	P <sub>2</sub>
1, 2.0m	29.219	34.177	22.652	16.957	11.820	7.334E-05	-1.060E-07	-1.768E-05	5.533E-05
1, 3.2m	29.332	34.110	22.613	16.917	11.786	1.013E-04	-1.874E-07	8.729E-06	3.600E-05
1, 4.4m	29.558	34.320	22.758	17.060	11.937	1.014E-04	-1.737E-07	-1.727E-05	3.371E-05
2, 2.0m	29.215	34.066	22.602	16.894	11.774	1.043E-04	-1.953E-07	-2.412E-06	4.589E-05
2, 3.2m	29.438	34.177	22.672	16.940	11.919	1.122E-04	-1.897E-07	-9.064E-06	6.176E-05
2, 4.4m	29.394	34.237	22.736	16.981	11.904	9.618E-05	-1.834E-07	2.009E-05	3.417E-05
3, 2.0m	29.201	34.144	22.609	17.059	11.526	9.574E-05	-2.416E-07	4.170E-05	5.609E-06
3, 3.2m	29.357	34.129	22.628	16.968	11.845	9.340E-05	-1.447E-07	-1.673E-05	6.653E-05
3, 4.4m	29.721	34.529	22.910	17.126	12.043	1.015E-04	-1.912E-07	-3.046E-06	6.107E-05
$\bar{i}$	29.382	34.210	22.687	16.989	11.839	9.771E-05	-1.792E-07	4.797E-07	4.445E-05
s	0.173	0.141	0.101	0.077	0.145	1.068E-05	3.720E-08	2.001E-05	1.932E-05
$\bar{i} + 3s$	29.901	34.632	22.989	17.219	12.273	1.298E-04	-6.762E-08	6.052E-05	1.024E-04
$\bar{i} - 3s$	28.862	33.787	22.384	16.759	11.406	6.566E-05	-2.908E-07	-5.956E-05	-1.350E-05

Table 3 Calibrated results of Nikon CoolPix 5000.

session, position	f(mm)	format		pps		radial distortion		decentering distortion	
		x(mm)	y(mm)	x(mm)	y(mm)	K <sub>1</sub>	K <sub>2</sub>	P <sub>1</sub>	P <sub>2</sub>
1, 2.0m	26.995	30.838	23.127	15.608	11.650	1.880E-04	-1.965E-07	-1.074E-06	-1.630E-05
1, 3.2m	27.037	30.994	23.219	15.688	11.646	1.500E-04	-2.069E-08	3.732E-06	2.375E-05
1, 4.4m	27.149	31.002	23.214	15.777	11.487	1.870E-04	-2.074E-07	-4.396E-06	-1.048E-05
2, 2.0m	27.159	31.078	23.268	15.789	11.611	1.895E-04	-1.795E-07	1.442E-05	3.033E-06
2, 3.2m	27.451	31.310	23.445	15.948	11.645	1.532E-04	-1.245E-07	1.717E-06	-2.865E-05
2, 4.4m	26.901	30.836	23.101	15.597	11.631	1.492E-04	-6.332E-08	1.494E-05	1.652E-05
3, 2.0m	26.852	30.719	23.036	15.604	11.558	1.358E-04	-3.164E-08	1.494E-05	-1.066E-05
3, 3.2m	26.970	30.879	23.140	15.636	11.583	1.630E-04	-1.254E-07	2.124E-05	-7.591E-06
3, 4.4m	26.991	30.895	23.154	15.684	11.622	1.576E-04	-8.091E-08	3.332E-06	2.018E-05
ī	27.056	30.950	23.189	15.703	11.604	1.637E-04	-1.144E-07	7.650E-06	-1.133E-06
s	0.179	0.172	0.119	0.116	0.053	1.976E-05	7.001E-08	8.855E-06	1.806E-05
ī +3s	27.594	31.467	23.545	16.052	11.764	2.230E-04	9.560E-08	3.421E-05	5.305E-05
ī -3s	26.518	30.433	22.834	15.355	11.444	1.044E-04	-3.245E-07	-1.891E-05	-5.531E-05

As listed in the tables above, no calibrated result was outside the range of  $\pm 3s$ . So the calibrated results were stable. The average of each set of elements was then introduced to the distortion-rectified program.

### 3.3 Image rectification

The image taken by FUJIFILM FinePix 40i was employed to be rectified in this study.

3.3.1 Elements introduced to the distortion-rectified program:

• Image format size: 7.3581mm × 5.5149mm

• Principal point location: (3.7583mm, 2.7077mm)

• Parameters of radial distortion:  $K_1 = 2.308E-03$ ,  $K_2 = -4.515E-05$

• Parameters of decentering distortion:  $P_1 = 1.169E-04$ ,  $P_2 = -2.104E-04$

3.3.2 Original image

The original image for rectification was shown in Fig. 2. Its image size was 1280 pixels × 960 pixels. The radial and decentering lens distortion of the original image were rectified separately and collectively to get the resultant images. To see the effect of lens distortion, the resultant images were first grayscaled and replaced by red band, then combined each of them with the original image respectively. Consequently,



Figure 2 Original image.

the red parts in the combined images indicated the deformation caused by the lens distortion.



Figure 3 Part of the radial distortion rectified image.



Figure 4 Part of the decentering distortion rectified image.



Figure 5 Part of the radial and decentering distortion rectified image.



Figure 6 Part of the radial distortion rectified image. Figure 7 Part of the decentering distortion rectified image.



Figure 8 Part of the radial and decentering distortion rectified image.

Fig. 3 to 5 were the magnified upper left part, and Fig.6 to 8 were the magnified bottom right part of the lens distortion rectified image. From these two set of images, radial lens distortion could be found that had more

significant influence on the image than decentering lens distortion. Besides, the rectification of radial lens distortion was rectified along radial lines from the principal point.

#### **4. SUMMARY**

In this study, the elements of interior orientation of digital cameras were established by PhotoModeler Pro 4.0. And the results were stable enough to introduce to the rectification of lens distortion of the image. A distortion free image should have benefit for the subsequent application of a digital camera.

Lens distortion was related to the distance from exposure station to the object, however, the difference of object distance was not applied in the image rectification in this study. This point was available for the further research.

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