科技部補助專題研究計畫成果報告
期末報告

一對多供應鏈在不確定性環境下之倒閉骨牌效應

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中華民國 104年09月30日

中文摘要：

本研究計畫主要探討在一個由單一上游廠商與多個下游廠商所組成的供應鏈中，彼此對於退場在環境不確定下的相互牽引機制。這類的產業結構相當常見，尤其是在高技術密集的產業，廠商都有一定的市場支配力(market power)與完全競爭的情形很不同。本研究結合了時間決策與價格決策且同時考慮了廠商間的策略性互動(strategic interaction)，這是非常貼切描述現實中廠商的決策方式。然而在學術研究上少有文獻在考慮策略性互動時同時討論時間決策與價格決策，所以這是這篇研究在文獻上的主要貢獻。

本研究發現上游廠商延遲下游廠商退出市場的策略會因為下游廠商數的不同而不同。在只有一家下游廠商的時候，上游廠商會「盡其所能」地延遲其下游廠商的退出，使得最後兩家會在新的退出時點同時退出市場。然而在多增加一個下游廠商的情況下，上游廠商不會將中間財的價格壓到像前者那般低以盡量延長下游廠商留在市場的時間，因為有另一家下游廠商會在持續留在市場裡。當在某一下游市場的獲利不是那麼划算的情況下，上游廠商會選擇讓對方退出，然後專心與另一家下游廠做生意。各家下游廠商佔上游廠商的總利潤比例也會影響上游廠商決定延緩的時間長短。這篇研究不止是用在上下游廠商間的供應鍊，亦可用在銀行與向其借貸的公司的關係中。

中文關鍵詞：策略性時間決策、不確定性、供應鍊、實選擇權

英文摘要：

This paper studies the spillover effect of exits in multiple vertical relationships under uncertainty using the real options approach. I expand the firms ‘ strategy space by allowing firms to make their price and quantity decisions along with timing decisions. I find that with the existence of an additional downstream firm will change the delaying strategy of the upstream firm. And the expected delaying time depends on how much proportion of the profit doing business with downstream firm i is of the upstream firms total profit. However, due to the asymmetry in vertical relationships, the downstream firms remain ‘‘helpless ‘ ‘ when facing the exit of the upstream firm.

英文關鍵詞：strategic timing, uncertainty, vertical, exits, supply chain, real options.

The Domino Effect in Multi-downstream Supply Chain under Uncertainty

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This is a very preliminary draft.

September 30, 2015

Abstract

This paper studies the spillover effect of exits in multiple vertical relationships under uncertainty using the real options approach. I expand the firms' strategy space by allowing firms to make their price and quantity decisions along with timing decisions. I find that with the existence of an additional downstream firm will change the delaying strategy of the upstream firm. And the expected delaying time depends on how much proportion of the profit doing business with downstream firm *i* is of the upstream firms total profit. However, due to the asymmetry in vertical relationships, the downstream firms remain "helpless" when facing the exit of the upstream firm.

1 Introduction

This paper studies the spillover effect of exits in multiple vertical relationship under uncertainty using the real options approach. I find that due to the asymmetry

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between the firms in vertical relationships, how a firm reacts to its counterparty's exit depends on the role of the firm in the supply chain. In this paper, I use the methodology of irreversible investment under uncertainty to study strategic exits in a vertical market structure. In a big bulk of the game-theoretical real options literature, the players' strategy space is simply their own respective timing decisions. However, to depict the strategic act of one firm delaying the exit of its counterparty and thus preventing its own exit, in this paper, the firms' strategy space is expanded to allow the firms to make price/quantity decisions along with their timing decisions. The methodological feature of the model is that it incorporates in the real-options framework the subgame perfect equilibrium concept.

The analysis of timing decisions is one of the prime importance in economic theory. While the process of entry into a market and investment in R&D under uncertainty have been extensively studied in recent years, the process of exit has not, especially the exit of asymmetric firms. Vertical relationships are particularly interesting and important not only because there exist various vertical relationships within a supply chain, but also because of the reliance between the upstream firms and the downstream firms. They rely on the existence of each other but at the same time compete in how to share the total profit of a good.

The model depicts a declining industry with one upstream firm and two downstream firms. The firms are independent of each other. I find this vertically not integrated industry structure particularly common among high-tech industries, such as smart phones. In the model of this paper, if one side of the vertical relationship exits, its counterparty must exit as well. When on firm is considering to exit, how will its counterparty react to this exit? Will the existence of third party change the firms' delaying strategy? I find that when a downstream firm wants to exit earlier than the upstream firm, the upstream firm strategically lowers the price of the intermediate good to delay the exit of its downstream but not as low as when there is only one downstream firm. However, due to the asymmetry in vertical relationships and the restricted strategy space, the downstream firms are "helpless" when facing the exit of the upstream firm.

During the past decade, many shocks, globally or regionally, struck various industries around the world. When the shock hit one side of the supply chain, the pain passes up and down the supply chain inevitably. For example, before the electronics and automotive industries could recover from the shock of the Japan earthquake in March 2011, they were struck again by a shortage of components produced by suppliers in central Thailand when the devastating flood struck Thailand in October 2011. Component shortages forced Honda to cut down production around the world and decreased its revenue until Honda's recovery in March 2012.

We can also observe the interactions between upstream and downstream firms in the smartphone industry. The growth of the smartphone market has slowed down, but the competition between smartphone brands has become more competitive. In the order to survive the cut-throat competition, a downstream firm, such as HTC and Samsung, must lower its marginal cost to be competitive in prices, otherwise they may have to exit the market. This competition pressure will move up along the supply chain to their upstream IC(Integrated Circuit) suppliers. The pressure of “cost down” is something that the entire smartphone supply chain faces. Though we do not straight forwardly observe the renegotiation between downstream smartphone brand and its upstream IC design houses, we do observe the IC design houses lowering their cost so that they can sell their chips to their downstream firms at a lower price. This compex relationships and competition process have been repeated again and again, from the PC industry to notebooks, and now to smartphones. Therefore studying the strategic exit behavior in supply chains under uncertainty is a very important issue.

This paper builds on several existing strands of literature. Below I review the most relevant literature and discuss the contribution of this paper.

The theory of irreversible investment under uncertainty considers problems in which a firm must choose the optimal timing of investment when the decision cannot be reversed and the value of the project evolves stochastically. The real options approach improves the traditional investment theory[1] by allowing the value of delay and the importance of flexibility to be quantified and incorporated explicitly into the analysis. A thorough review is given in Dixit and Pindyck (1994).

In reality, many decisions made by firms take into consideration the actions of others, but the basic real-options models do not account for the strategic interactions between firms. There is another strand of literature that incorporates game-theoretic concepts in the real-options framework. Examples of models in discrete time are Smit and Ankum (1993), and Kulatilaka and Perotti (1998). Grenadier (1996, 2002), Lambrecht (2000, 2001, 2004), Weeds (2002), and Mason and Weeds (2010) modeled investment decisions using diffusion processes. Grenadier (1996) and Weeds (2002) modeled the strategic interactions in leader-follower games under complete information. Lambrecht and Perraudin (2003), Pawlina and Kort (2006), Thijssen (2010, 2015), Boyarchenko and Levendorskiĭ (2014) study preemption games with irreversible investments under uncertainty.

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[1] The traditional investment theory is based on the rule that an investment project should be undertaken whenever its net present value is positive. However, this decision rule neglects the comparison of the value of investing today and sometime in the future.

de Villemeur, Ruble, and Versaevel (2014) studied irreversible investments under uncertainty in vertical relationships. There are also other papers on strategic investment or entry decisions, such as Perotti and Rossetto (2000); Gryglewicz, Huisman, and Kort (2008); Grenadier and Wang (2007); Thijssen (2011); Thijssen, Huisman, and Kort (2006, 2012). There are also papers about strategic exit under uncertainty, Lambrecht (2001) and Murto (2004). Huisman et al. (2005) provided a survey of game theoretic real options models.

This paper naturally belongs to this strand of literature. I model uncertainty using the geometric Brownian motion, which is standard in the continuous-time real-options models. While the majority of this literature considers horizontal competition, I differ from them by considering exit in vertical relationships. In vertical relationships, the firms are not only asymmetric but also rely on the existence of each other. Instead of entry, this paper models the strategic exit in vertical relationships. This paper is mostly related to Murto (2004) which adds uncertainty into a model based on Ghemawat and Nalebuff (1985). In a vertical relationship, the follower in the exit game suffers from losing its foothold. Since the survival of the firm relies on its counterparty, it is only optimal to exit simultaneously in the structure of both firms are monopolists in its own market.

Most static entry and exit models are generally solved by backward induction starting from the terminal period of the game. But in my stochastic framework, I cannot work backward from a fixed-time moment. I use the state-dependent Markov strategies, which are expressed as stopping sets in the state space such that the firm exits when the state variable hits the corresponding stopping set for the first time. At each state level, the firm has the optimal pricing or quantity strategy that maximizes its life-time expected present value and at the same time there is a corresponding stopping set. The firm exits when the state level falls in its corresponding stopping set for the first time. The equilibrium in my model is subgame perfect.

The contribution of this paper can be seen from two aspects. First, it extends the real-options literature by studying strategic interactions associated with abandonment options in vertical relationships. In most game-theoretic real-options papers, the only strategy the firms decide is the exercise threshold. I enrich the firms strategy space by allowing the firms to decide on price and quantities to maximize their life-time expected present values. Pawlina and Kort (2010) expand the firms strategy space by allowing the firms to decide the level of quality once. Second, this paper studies the interdependence of firms in vertical relationships during hard times whereas best part of the literature concentrate on vertical integration and controls. For both the game-theoretic real-options literature and vertical relationship literature, by introducing uncertainty into the supply chain, this model brings a new view to the interdependence relationship in supply chains. In reality, there exist non-integrated firms in supply chains especially in high technology concentrated industries which have high entry barriers. As mentioned in the two motivation examples, unexpected exogenous shock has been continuously bringing large impact on the vertical relationships in a supply chain. Therefore, this is an important and interesting area to explore.

This paper is organized as follows. In Section 2, I give a brief summary of the results of my previous paper that this paper builds on. In section 3, I introduce the benchmark model. In section 4, I analyze the equilibrium in the case that the upstream strategically delays the exit of the downstream firms.

2 Brief Summary of Previous Result

In this section, I give a brief summary of the results in Chiang (2015). As the results in Chiang (2015) will be the first step of backwards induction in this paper.

Chiang (2015) considers a declining final good market with an upstream monopolist and a downstream monopolist which are initially not integrated and remain independent of each other. Under the price signed in the contract there could be three possible cases for the ordering of non-strategic exit thresholds: (i) both firms exit simultaneously; (ii) the downstream firm exits prior to the upstream firm; (iii) the upstream firm exits before the downstream firm. At the beginning of time, both firms agree to the intermediate good price function that maximizes their profits for every time moment t. Since the firms have an option to make one time change to the intermediate good price function in the future, this is naturally the optimal strategy for both firms as they are profit maximizing up to the date a non-strategic exit threshold is reached. In case (i), both firms optimally exit simultaneously at their exit thresholds without the price change option being exercised. When the last two cases occur, the firm which has a lower optimal exit threshold has the incentive to strategically delay its counterparty's exit, and therefore prevent its own exit. In case (ii), the upstream firm will propose to lower the price of the intermediate good once the higher non-strategic exit threshold is reached. This new price function is such that that they would both exit simultaneously later at an optimal time under the new price function. However, in case (iii), though the downstream firm would like to delay the exit, it cannot not do anything to prevent the exit. Therefore, the downstream firm would exit right after the exit of the upstream firm.

This paper is an extension of Chiang (2015). It increases the competition in the downstream of the vertical relationship. I find that with an additional downstream player in the game, the upstream firm has a different delaying strategy than in the one-to-one vertical relationship.

3 The Model

This paper studies a vertical structure that contains one upstream firm and two downstream firms that operate in two disconnected regions. The firms face declining markets, or say there is negative shocks constantly hitting the market. The model is in continuous time with an infinite time horizon. The firms are labelled U for the upstream firm, and D_1 and D_2 for the two downstream firms. All three firms discount the future at rate γ. This is the benchmark model and for simplicity of the benchmark model, the two downstream firms are asymmetric only in their operational cost that they pay every time t. Without loss of generosity, let $F_2 > F_1$. The two downstream firms are different to firm U in how much they account for firm U's cost. Firm U's operating cost is F_U when it fully operates and makes business with both of the downstream firms. To be more specific, selling intermediate goods to firm D_i incurs cost $\alpha_i \in (0, 1/2)$, where $i = 1, 2$.

At time t, the downstream firms face a constant price elasticity demand function:

$$D(p_t; X_t) = e^{X_t} p_t^{\delta},$$

where p_t is the price of the final good, $\delta < -1$ is the constant price elasticity and the demand shock $\{X_t\}_{t \geq 0}$ is a Brownian motion increment defined on a filtered probability space

$$(\Omega, \mathscr{F}, (\mathscr{F}_t)_{0 \leq t < \infty}, P)$$

satisfying the usual properties[2]. With drift $\mu < 0$[3] and variance σ^2, the dynamic of $\{X_t\}_{t \geq 0}$ is expressed by

$$dX_t = \mu dt + \sigma dW_t,$$

and W_t is the standard Wiener process. The inverse demand function is

$$P(q_t; X_t) = \left(q_t / e^{X_t}\right)^{\frac{1}{\delta}},$$

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[2] Namely, $\mathscr{F}_0$ contains all the $P-$null sets of $\mathscr{F}$, and filtration $(\mathscr{F}_t)_{0 \leq t < \infty}$ is right continuous.

[3] This assumption guarantees that the firms will exit in finite time and corresponds to a declining industry.

where q_t is the amount of final good sold at time t. The downstream firms' marginal costs of producing the final good are normalized to zero. Observing the demand at time t, the downstream firm buys the intermediate good from the upstream firm at price p_{ut} and then sells the good to the consumers. The marginal cost of the upstream firm to produce one unit of the intermediate good sold to the downstream firm is a constant, mc.

At the beginning of time $(t = 0)$ when a downstream firm and the upstream firm were signing the contract, both firms agree that the upstream firm may charge the intermediate good according to a certain price function, say $p_u(X_t)$, and one of the firms may propose an alternative price function in the future when the time calls[4]. That is to say, the intermediate good price function could only be changed once after the contract is signed and this change in the price function is irreversible [5]. For every time moment, firm U sells the intermediate good to firm D_i according to the price function agreed in the contract or according to the new price function if the change has been made. Firm D_i then decides how much quantity of the final good to sell given the input price and realized demand shock, $i \in \{1,2\}$.

Other than deciding when to offer the change in the price function and what new price function to offer, the firms have the option to exit their markets, *i.e.* the firms also have an exit decision to make and this is an irreversible decision. Given the current shock x, the strategy of firm i, $i = \{D_1, D_2, U\}$, is to decide whether to continue operating or to exit or to change the price of the intermediate good. The firms have different strategies, here I explain part of the firms' strategy space and leave the remaining strategy space till later of the paper. Part of the firms' strategies are two stopping sets: when to change the intermediate good price (this may be empty for some firms) and when to exit. A firm's strategy is a stopping rule specifying a threshold or "trigger point" for the stochastic variable X at which the firm exits, *i.e.* firm i chooses an exit threshold $h_i \in \mathbb{R}$ to exit its market, $i = \{D_1, D_2, U\}$. To be more explicit, the statement is that firm i exits the first time when the stochastic process X_t crosses the value h_i, crossing this threshold from above. Since the state variable is stochastic, the time when the state variable first crosses an exit threshold is also a random variable. Therefore,

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[4] For example, when one party wants to exit but the counterparty wishes to stay in the market longer, keeping in mind that the two firms are vertically related and operated independently.

[5] In reality, firms face the cost of renegotiating and pay the transaction cost of changing the price, so they do not change price that often despite that there is volatility in production. Therefore, it is a truer portrait of the real world that the firms can only adjust the intermediate good price finitely. Here in this model, the price change could only occur once. This assumption was made for the sake of tractability. I thank Frank Riedel for this suggestion.

instead of choosing a calendar exit time, the firms choose thresholds of the state variable. To summarize, firm i exits if the state variable drops below h_i for the first time and the stopping set, also called the "exit region" in this paper, takes the form of $(-\infty, h_i]$.

At time t, firms observe the realization of the demand shock, x. Firm U charges firm D_i p_{uit} per unit of the intermediate good and firm D decides q_{it}, the amount of the final good to sell. Firms receive their revenue net of input costs, π_U and π_{Di} respectively, and pay their respective operational cost, F_U, F_1 and F_2. The exact expression of p_{uit} and π_i will be shown later. After observing the realization of the current shock, both firms decide whether or not to exit. The scrap values of all firms are normalized to zero. Should one firm want to exit later than its counterparty, it may be able to delay the counterparty's exit and thus prevent its own exit by exercising the option to change the intermediate good price function from $p_{uit}(x)$ to $\hat{p}_{uit}(x)$.

Now I discuss how the intermediate price schedule in the contract is decided. I first discuss the profit that the firms earn at time t. At time t, given p_{uit} and the realization of the state variable $X_t = x$, firm D_i maximizes its profit at time t by solving the following problem:

$$\max_{q_{it}} \quad q_{it}(p_i - p_{uit}) - F_i, \tag{3.1}$$

where $p(q_{it};x) = (q_{it}/e^x)^{1/\delta}$. Firm i incurs a fixed operating cost F_i, $i = D_1, D_2, U$, at each moment in time if firm i is in business at time t. By solving (3.1), firm D_i maximizes its profit at

$$q_{it} = \left(\frac{\delta}{\delta+1}\right)^{\delta} p_{uit}^{\delta} e^x. \tag{3.2}$$

Firm U takes (3.2) as given and maximizes its current profit:

$$\max_{\{p_{uit}\}_{i=1}^2} \quad \sum_{i=1}^{2} (p_{uit} - mc) \cdot q_{it} - F_U. \tag{3.3}$$

The price that maximizes (3.3) is

$$\bar{p}_{uit} = \bar{u} \cdot mc, \tag{3.4}$$

where $\bar{u} = \delta/(\delta+1)$. Recall that $\delta < -1$ is the constant price elasticity, thus $\bar{u} > 1$. $\bar{u}$ can be interpreted as the optimal markup of the upstream firm, Firm U.

This markup $\bar{u}$ increases as δ increases. This matches the intuition of the markup of firms increases when their market power increases. When δ is closer to -1, the firms have greater market power. One can observe that though the quantity of the final good is state dependent, the price of the intermediate good which maximizes firm U's time t profit, $\bar{p}_{uit}$, is a constant and is independent of the state variable, X_t[6].

Lemma 3.1. *At time t, given the realization of the stochastic state variable, x, the firms' time t profits are are maximized by*

$$\bar{p_{uit}} = \bar{u} \cdot mc \quad \textit{and} \quad \bar{q}_{it} = \left(\frac{\delta}{\delta+1}\right)^{\delta} (\bar{u} \cdot mc)^{\delta} e^{x}, \tag{3.5}$$

where $\bar{u} = \delta/(\delta+1) > 1$ *with* $\delta < -1$.

As mentioned earlier in this section, the firms have an irreversible option to change the price function from the original price function specified in the contract, $p_{ui}(u, X_t)$ to another price function, $\hat{p}_{ui}(u, X_t)$. For simplicity, it is assumed that the price functions take the form $p_{ui} = p_u(u_i, X_t) = u_i \cdot mc$, where $u_i \in [1, \infty)$. The markup u is greater or equal to 1 because it is not reasonable for firm U to sell the intermediate good at a price lower than its marginal cost. For the time being, I am going to drop all the subscripts of the prices for D_1 and D_2 as in this part they are symmetric. The only notations that will contain 1 and 2 to tell apart firm D_1 from D_2 are those that are affected by their operating costs F_1 and F_2. With $p_u = u \cdot mc$ and $u \in [1, \infty)$, at time t, the dowstream firms will receive profits of the form

$$\pi_i(u, x) = W_i(u) e^{x} - F_i, \quad i = \{D_1, D_2\}, \tag{3.6}$$

where

$$W_{D1}(u) = W_{D2}(u) = W_D(u) = \left(\frac{-1}{\delta+1}\right) (u \cdot mc)^{\delta+1} \left(\frac{\delta}{\delta+1}\right)^{\delta}. \tag{3.7}$$

The upstream firm's profit flow at time t is more complicated as it depends on how many downstream firms are in the market. And for each profit flow, firm U has an

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[6] If firm U produces the intermediate good with a decreasing return to scale production function with a single input y, for example $f(y) = y^{\alpha}$ and $\alpha < 0$, then the p_{ut} that maximizes firm U's time t profit would be state dependent. The model of this paper is a special case with $\alpha = 1$.This is robust and does not change the result of the paper.

exit threshold to decide. To be more specific, firm U's profit flows are

$$\begin{aligned}
&\pi_u = \pi_u(u,u,x) = 2W_u(u)e^x - F_U, && \text{if no exit occurred;}\\
&\hat{\pi}_u = \pi_u(u,\hat{u},x) = [W_u(\hat{u}) + W_u(u)]e^x - F_U, && \text{if price changed between } D_2 \text{ and } U;\\
&\bar{\pi}_u = \pi_u(u,.,x) = W_u(u)e^x - (1-\alpha_2)F_U, && \text{if firm } D_2 \text{ exited.}\\
&\tilde{\pi}_u = \pi_u(\tilde{u},.,x) = W_u(\tilde{u})e^x - (1-\alpha_2)F_U, && \text{if price changed between } D_1 \text{ and } U,
\end{aligned}$$

where

$$W_U(u) = (u-1)u^\delta mc^{\delta+1}\left(\frac{\delta}{\delta+1}\right)^\delta. \tag{3.8}$$

The corresponding exit thresholds for each profit flow are h_u, $\hat{h_u}$, $\bar{h_u}$ and $\tilde{h_u}$ respectively.The price signed in the contract is the best price that firm U would prefer (please see Chiang (2015)). Therefore intuitively $\hat{h_u} > h_u$ and $\tilde{h_u} > \bar{h_u}$. The relationship between $\bar{h_u}$ and $\hat{h_u}$ depends on α_2. If $\alpha_2 < 1/2$, then $\bar{h_u} > \hat{h_u}$.

The revenue part of the profits π_i contains a deterministic components, W_i, and a stochastic component, e^x. The deterministic components are independent of the state variable and are functions of the constant elasticity, marginal cost and markup u.

The expected present value (EPV) of the flow $\mathbb{E}[\int_0^\infty e^{-\gamma t}W_i(u)e^{X_t}dt]$ is finite iff $\mathbb{E}[e^{X_t}] < \infty$ and the no-bubble condition $\gamma - \Psi(1) > 0$ holds. Here Ψ is the Lévy exponent of the Brownian motion definable from $\mathbb{E}[e^{AX_t}] = e^{t\Psi(A)}$. Indeed, if $\Psi(1) < \gamma$, then by Fubini's theorem,

$$\begin{aligned}
&\mathbb{E}^x\left[\int_0^{+\infty} e^{-\gamma t}W_i(u)e^{X_t}dt\right] \equiv \mathbb{E}\left[\int_0^{+\infty} e^{-\gamma t}W_i(u)e^{X_t}dt|X_0 = x\right]\\
&= \int_0^{+\infty} e^{-\gamma t}W_i(u)\mathbb{E}^x[e^{X_t}]dt = W_i(u)\int_0^{+\infty} e^{-\gamma t+z+t\Psi(1)}dt = \frac{W_i(u)e^x}{\gamma - \Psi(1)}.
\end{aligned}$$

The value functions are well-defined if and only if

$$\gamma - \Psi(1) > 0, \tag{3.9}$$

where $\Psi(z) = \mu z + \frac{\sigma^2}{2}z^2$. This is the no-bubble condition for the value functions.

4 Value Functions and Thresholds

Before setting up a firm's problem, I first need to know the exiting order of the firms under the initial contract. In order to do that I first find the exit thresholds of the firms if no price change was possible. This is crucial in understanding how the timing thresholds are derived.

4.1 Optimal Exit Timing without Changing the Price Function

In this subsection, I investigate which firm would want to exit first given that there is no change in the price function. Since there is nothing the firms could do to delay the exit of their counterparty, it is simply the standard real options problem applied to exit decisions. The firms first calculate their own exit threshold as if independent of their counterparty, then they see who wants to exit first. The firm who wants to exit later would have to exit with the firm who will exit earlier in this framework. The optimal exit timing of the firms without price change is derived by solving the following system of equations, $i = \{1, 2, U\}$:

$$\begin{aligned} \left(\gamma - \mu \frac{\partial}{\partial x} - \frac{\sigma^2}{2}\frac{\partial^2}{\partial x^2}\right) V_i(u,x) &= W_i(u)e^x - F_i && \text{if } x > h_i \\ V_i(u,x) &= 0 && \text{if } x \leq h_i \end{aligned}$$

The above second order differential equation is a standard real-options problem, please refer to Dixit and Pindyck (1994) for details. The closed form solution to h_i is

$$e^{h_i} = \frac{F_i}{\kappa^+(1)W_i(u)}, \tag{4.1}$$

where $\kappa^+(1) = \beta^+/(\beta^+ - 1)$ and $\beta^\pm$ are the roots of $\gamma - \Psi(z) = 0$. By the no bubble condition in (3.9), $\beta^+ > 1 > 0 > \beta^-$. Please note that the firms are considering strategic interactions when it comes to making profits for every time moment t but are nonstrategic about exit timing in this section.The assumption of $F_2 > F_1$ implies that $h_2 > h_1$, which means that with the same price signed in their contracts, firm D_2 will exit earlier than firm D_1.

Under the price signed in the contract, there could be three possible cases: (i) $h_2 > h_u > h_1$; (ii) $h_2 > h_1 > h_u$, and (iii) $h_u > h_2 > h_1$.

From Chiang (2015), case (iii) is exactly the case in which the downstream firms cannot do anything to delay the exit of the upstream firm due the restriction of the strategy space. Expanding the strategy space to allow lump-sum transfers between the firms may allow the downstream firms to delay the exit of the upstream firm, however that is not the focus of this research. I leave it for future research. Apparently, the first two cases are more interesting. Would the upstream's delaying strategy be different? How long would the firm postpone the exit? Before answering these question, one must understand how the exit thresholds of the firms behave.

Theorem 4.1. $\forall u \in [1,\infty)$, $\bar{u} = \underset{u}{\operatorname{argmin}} \quad h_U(u)$.

Theorem 4.2. *$h_1(u)$ and $h_2(u)$ are strictly decreasing in u.*

The deterministic component of firm U's profit $W_U(u)$ is concave in $u < (\delta - 1)/(\delta + 1)$ and is globally maximized at $\bar{u}$, therefore the lowest exit threshold possible for firm U is $h_U(\bar{u})$. Whereas the deterministic component of firm D_i's profit $W_D(u)$ is strictly decreasing in u. When u becomes higher, the cost for firm D_i increases.

5 Case (i): Firm U Exits Between the Two Downstream Firms

To formalize the firms' objective functions of the optimal exit timing , I define $\tau_k = inf\{t > 0 : X_t \leq h_k(X_t)\}$ as the time when the state variable hits the exit region of firm k for the first time. Firm U's objective function is

$$
\begin{aligned}
(5.1)\quad \max_{\hat{u},\tilde{u}} \quad & \mathbb{E}^x \left[\int_0^{\tau_2} e^{-\gamma t} \pi_u(u,u,X_t)dt \right] + \mathbb{E}^x \left[\int_{\tau_2}^{\hat{\tau}_u \wedge \hat{\tau}_2} e^{-\gamma t} \pi_u(u,\hat{u},X_t)dt \right] \\
& + 1_{\{\hat{\tau}_u > \hat{\tau}_2\}} \cdot 1_{\{\bar{\tau}_u > \hat{\tau}_2\}} \cdot \mathbb{E}^x \left[\int_{\hat{\tau}_2}^{\bar{\tau}_u \wedge \tau_1} e^{-\gamma t} \pi_u(u,.,X_t)dt \right] \\
& + 1_{\{\bar{\tau}_u > \hat{\tau}_1\}} \cdot \mathbb{E}^x \left[\int_{\hat{\tau}_2}^{\tilde{\tau}_u \wedge \tau_1} e^{-\gamma t} \pi_u(\tilde{u},.,X_t)dt \right].
\end{aligned}
$$

where $\hat{u}$ means changing the intermediate good price between firm D_2 and firm U from $p_u = u \cdot mc$ to $\hat{p_u} = \hat{u} \cdot mc$ and $\tilde{u}$ means changing the intermediate good price between firm D_1 and firm U from $p_u = u \cdot mc$ to $\tilde{p_u} = \tilde{u} \cdot mc$. Also the exit timing are defined as followed: $\hat{\tau}_i = inf\{t > 0 | X_t \leq \hat{h_i}\}, i = 2, u,$ $\tilde{\tau}_j = inf\{t > 0 | X_t \leq \hat{h_j}, j = 1, u\}$ and $\bar{\tau}_u = inf\{t > 0 | X_t \leq \bar{h_u}\}$.

The downstream firms are the same to the upstream firm except that they cost firm U differently in doing the business with them.This difference is accounted in α_1 and α_2. In this paper the information structure is complete information. That is to say that all firms know what is signed in the contracts and know the costs of all firms.

To delay the exit of firm D_2, firm U must offer the new price before the exit of firm D_2, $h_s \geq h_2$, otherwise firm D_2 will exit and this exit is irreversible, thus firm U will loose one stream of profit. For the same reasoning, firm U will propose the price change between itself and firm D_1 at h_1 if firm D_1 exits prior to firm U.

The upstream firm's maximization problem (5.1) is solved by backwards induction. Note here that the time horizon is infinite, so there is no last period to start working backwards. By backwards induction, I mean that work back from the second stage of the game where firm D_2 already exited and there is only firm U and firm D_1. This stage is exactly the scenario discussed in Chiang (2015). The goal of this paper is to discuss what will firm U do to delay the exit of firm D_2. The question of when firm D_1 sees that firm U lowered the intermediate good price to delay the exit of firm D_2, will firm D_1 ask for the same favor? Though intuitively it is better for firm D_1, however firm U will refuse. The threat of exiting earlier than h_1 is not credible as h_1 is the optimal exit threshold for firm D_1 to exit under the contract price. And by lowering the price between firm D_1 and firm U will make firm U exit even earlier.In the case that we are discussing in this section, firm U is already exiting earlier than firm D_1. Hence, it is not beneficial for firm D_1 to ask for a price change as well.

The value function can be derived by using the expected present value operators [7] in Boyarchenko and Levendorskiĭ (2007), or by solving the corresponding second order differential equation as showed in section 4.1.

5.1 Numerical Example

In this subsection, I show a numerical solution in solving the case that before any price changes firm U exits between the two downstream firms. From Table 5.1,

---

[7] The EPV operators for Brownian motion: Define the operators $\varepsilon^{\pm}$ by

$$\varepsilon^{+}g(x) = \beta^{+}\int_{0}^{\infty} e^{-\beta^{+}y}g(x+y)dy, \text{ and}$$
$$\varepsilon^{-}g(x) = \beta^{-}\int_{0}^{\infty} e^{-\beta^{-}y}g(x-y)dy;$$
$$\varepsilon = \varepsilon^{+}\varepsilon^{-} = \varepsilon^{-}\varepsilon^{+},$$

where $\beta^{\pm}$ are roots to $\gamma - \psi(\beta) = 0$. And by the no bubble assumption: $\beta^{+} > 1 > 0 > \beta^{-}$. Also, let $\kappa^{+}(1) = \frac{\beta^{+}}{\beta^{+}-1}$ and $\kappa^{-}(1) = \frac{\beta^{-}}{\beta^{-}-1}$.

Parameters	F_1	F_2	F_U	α
	200	180	80	0.3
thresholds	h_2	h_u	h_1	$\bar{h}_u$
	-5.8657	-6.5588	-6.7820	-6.2223
thresholds	$\hat{h}_2$	$\hat{h}_u$	$\hat{u}^*$	u_2
	-6.2522	-6.5025	1.6044	1.0445

Table 1: Numerical example of case (i): $h_2 > h_u > h_1$.

it is shown that the optimal new price is $\hat{u}^* = 1.0644 > u_2$, where u_2 is such that for $\hat{u} \in (u_2, \bar{u}]$, $\hat{h}_u < \hat{h}_2$; for $\hat{u} \in [1, u_2]$, $\hat{h}_u \geq \hat{h}_2$.Based on the results in Chiang (2015), if there is only one upstream and one downstream, say firm D_2, then firm U will offer the price change to u_2 so that the two firms will simultaneously exit at $\hat{h}_u = \hat{h}_2$. However as shown in Table 5.1, firm U will indeed offer a price change, but not as low as u_2 because it takes into account the existence of the other downstream firm. In the example given above, after delaying the exit of firm D_2, firm U can receive a higher payoff longer than letting firm D_2 exit and only receive a single profit flow from firm D_1, *i.e.* $\hat{h}_2 < \bar{h}_u$. This means that all three firms will simultaneously exit at $\hat{h}_2$. This result depends on the size of α_2. If α_2 increases, then eventually there will be longer time passing by between the exit of firm D_2 and the price change between firm U and firm D_1. Parameters of the stochastic stat variable is not included in the table as they will remain fixed.

5.2 The Optimal Switching Time

5.3 The Optimal New Price

5.4 Expected Delay in Exit

6 Conclusion

Increasing the competition on one side of the supply chain changes the delaying strategy of the upstream firm. When there is only one downstream firm, the upstream firm will do all its might to delay the exit of the downstream firm, so that it can at least exit at the same time of the downstream after the price change. However, when there is an increase in numbers of the downstream firms, the upstream firm no longer rely that much on a single downstream firm, and thus will not offer a price as low as in the one-to-one structure. And the expected delaying time depends on how much proportion of the profit doing business with downstream firm *i* is of the upstream firms total profit.

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Piin-hueih Chiang

在這次的學術會議中，有幸有這個機會聽到亞洲不同國家的研究發展方向，也有機會聽些其他領域的研究。以下是本次參加會議印象最深亦是最有想法的三篇文章:

New Tests of Granger Causality for two Groups of Time Series by Ying-Chao Hung
此篇的主題是屬於本人較為不熟悉的計量領域，在此演講中學了不少新知識作為將來研究發展可能方向的基石。本篇文章延續了 VAR (Vector Autoregression) model 探討著 multivariate time series，並結合改良過去的 Granger Causality. 其方法可應用於多種領域。主要的問題是 Y 變數是否可以有效地預測 X 變數。其中之 Y 變數為一組的時間序列變數，並假設與 X 為 n correlated. 文中定義 Granger Causality: Yt causes Xt up to horizon *c*. 目標為估計 correlation matrix. 作者用 power β 來比較不同的測驗發現並沒有單一方法 dominate. 其研究所面臨到的問題: How to find the critical value? However, Big Data causes computational complexity.

Do Consumers Trust Government or Business? A Case Study of Organic Vegetable by Thanee Chaiwat and Nisachon Leerattanakorn
這是一篇想法相當有趣的文章，主要探討消費者在購物時是相信政府的認證還是廠商的品牌品質。作者提出說消費者某一程度而言等同於投票者，所以政府和廠商是面臨同一組需求，因而可以做比較。本篇論文的數據來自在泰國清邁針對有機蔬菜做的調查。作者發現在政府較沒效率的情況下，有 55%的消費者選擇相信廠商的品牌(private brand)，有 22%的消費者選擇相信政府核發的認證(public certificate)。

在這個有趣又有意義的主題背後，或許是時間短暫不足討論細節，對這一篇論有下幾個想法：

1. 有機蔬果的標示，每一個國家標示的方法不同，有的時候未必能清楚知道是哪一個廠商供應。許多時候蔬果是散裝地賣，確實有標示不易與不易辨視的問題。或許用農產品以外的其他商品更為理想，例如乾貨。

2. 本篇只做了在泰國單一城市的 case study，在樣本上恐有 Bias 問題。至少在研討會上作者並沒有提出清邁是否能代表全泰國的論述，只說明了清邁是政府有意扶植的 green province。如果可以擴大範圍為泰國各大城市，是不是會有不同的結果?甚至是可以做不同國家的資料，並且有系統性與指標性的合理量化政府效率，將會是更有趣且完整的研究。也更能支持作者最後的結語：當政府愈沒效率，消費者會愈相信廠商品牌。

3. 由於本篇研究並未提出其使用的迴歸式，且從 presentation 中並沒提到使用的變數，因此即便結果合乎常理，但需要有更嚴謹的計量方法。消費者是否相信廠商因為在乎自己的商譽而會在品質上比政府更小心?大體來說這是一個相當有趣且可以發展的題目，但需要構思的細節還很多。

Comparing Economic Developments in the Greater China through Building Chinese Consumer Confidence Indexes by Geoffrey Tso and Jin Li

這篇研究是作者一系列與中國大陸、香港、澳門及台灣四地四所大學的學者合作中的一篇。標題中的Greater China即指中國大陸、香港、澳門及台灣。整個數據資料是四地的time series data，為季資料。本文主要研究議題是-這四地的消費者信心指數是否會引起消費者對未來的期望。這裡的Consumer Confidence indexes (簡稱CCI) 乃指消費者對經濟現況的滿意指數值(Satisfaction)。根據他們的數據，從2008年的金融海嘯後，台灣的CCI一直是四地最低，不過卻有最大的標準差。港澳兩地的CCI相近，變化差不多。大陸則是最高且變動最少。作者先使用簡單的correlation探討CCI與消費者的未來期望的關係，但correlation本身有其限制，因此作者進而使用Granger Causality Test 來測試一個短期中的the statistical precedence of CCI / economic statistics over economic statistics / CCI。他們得到結論是無法證明CCI會引起Predicted consumption expectation.這個結果有點令人意外，就直覺而言，現今的狀況(消費者對經濟的現況的信心)是會影響到消費者對未來的經濟與消費期望。

科技部補助計畫衍生研發成果推廣資料表

日期:2015/09/07

科技部補助計畫	計畫名稱: 一對多供應鏈在不確定性環境下之倒閉骨牌效應
	計畫主持人: 江品慧
	計畫編號: 103-2410-H-004-002- 學門領域: 產業組織與政策

無研發成果推廣資料

103 年度專題研究計畫研究成果彙整表

計畫主持人：江品慧	**計畫編號：**103-2410-H-004-002-
計畫名稱：一對多供應鏈在不確定性環境下之倒閉骨牌效應	

成果項目			量化			單位	備註（質化說明：如數個計畫共同成果、成果列為該期刊之封面故事...等）
			實際已達成數（被接受或已發表）	預期總達成數(含實際已達成數)	本計畫實際貢獻百分比		
國內	論文著作	期刊論文	0	0	100%	篇	
		研究報告/技術報告	0	0	100%		
		研討會論文	2	2	100%		
		專書	0	0	100%		
	專利	申請中件數	0	0	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力（本國籍）	碩士生	1	1	100%	人次	
		博士生	0	0	100%		
		博士後研究員	0	0	100%		
		專任助理	0	0	100%		
國外	論文著作	期刊論文	0	0	100%	篇	
		研究報告/技術報告	0	0	100%		
		研討會論文	1	1	100%		
		專書	0	0	100%	章/本	
	專利	申請中件數	0	0	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力（外國籍）	碩士生	1	1	100%	人次	
		博士生	0	0	100%		
		博士後研究員	0	0	100%		
		專任助理	0	0	100%		

其他成果 (無法以量化表達之成果如辦理學術活動、獲得獎項、重要國際合作、研究成果國際影響力及其他協助產業技術發展之具體效益事項等，請以文字敘述填列。)	無

	成果項目	**量化**	**名稱或內容性質簡述**
科教處計畫加填項目	測驗工具(含質性與量性)	0	
	課程/模組	0	
	電腦及網路系統或工具	0	
	教材	0	
	舉辦之活動/競賽	0	
	研討會/工作坊	0	
	電子報、網站	0	
	計畫成果推廣之參與（閱聽）人數	0	

科技部補助專題研究計畫成果報告自評表

請就研究內容與原計畫相符程度、達成預期目標情況、研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）、是否適合在學術期刊發表或申請專利、主要發現或其他有關價值等，作一綜合評估。

1. 請就研究內容與原計畫相符程度、達成預期目標情況作一綜合評估

□達成目標

■未達成目標（請說明，以 100 字為限）

□實驗失敗

□因故實驗中斷

■其他原因

說明：

邏輯的推導與理論的證明過程複雜，每一步都是下一步驟的基石，因此每一步都是經過多次反覆的推演證明確認無誤後才進行下一步，故目前只完成初步目標。計畫主持人將會持續完成剩餘的目標。

2. 研究成果在學術期刊發表或申請專利等情形：

論文：□已發表 □未發表之文稿 ■撰寫中 □無

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其他：（以 100 字為限）

3. 請依學術成就、技術創新、社會影響等方面，評估研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）（以 500 字為限）

本研究計畫主要探討在一個由單一上游廠商與多個下游廠商所組成的供應鏈中，彼此對於退場在環境不確定下的相互牽引機制。這類的產業結構相當常見，尤其是在高技術密集的產業，廠商都有一定的市場支配力(market power)與完全競爭的情形很不同。本研究結合了時間決策與價格決策且同時考慮了廠商間的策略性互動(strategic interaction)，這是非常貼切描述現實中廠商的決策方式。然而在學術研究上少有文獻在考慮策略性互動時同時討論時間決策與價格決策，所以這是這篇研究在文獻上的主要貢獻。

本研究發現上游廠商延遲下游廠商退出市場的策略會因為下游廠商數的不同而不同。在只有一家下游廠商的時候，上游廠商會「盡其所能」地延遲其下游廠商的退出，使得最後兩家會在新的退出時點同時退出市場。然而在多增加一個下游廠商的情況下，上游廠商不會將中間財的價格壓到像前者那般低以盡量延長下游廠商留在市場的時間，因為有另一家下游廠商會在持續留在市場裡。各家下游廠商佔上游廠商的總利潤比例也會影響上游廠商決定延緩的時間長短。

這篇研究不止是用在上下游廠商間的供應鍊，亦可用在銀行與向其借貸的公司的關係中。在實證上有相當的發展空間。