

# Social norms, costly punishment and the evolution of cooperation

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**Abstract** Within a co-evolutionary framework of reputations, strategies and social norms, we study the role of punishment in the promotion of cooperation. Norms differ according to whether they allow or do not allow the punishment action to be a part of the strategies, and, in the case of the former, they further differ in terms of whether they encourage or do not encourage the punishment action. In such a framework, depending on the applied social norm, players are first given different reputations based on their employed strategies. Players then update their strategies accordingly after they observe the payoff differences among different strategies. Finally, over a longer horizon, the evolution of the social norms may be driven by the average payoffs of all members of the society. The strategy dynamics are articulated under different social norms. It is found that costly punishment does contribute to the evolution toward cooperation. Not only does the attraction basin of the cooperative evolutionary stable state become larger, but the speed of convergence to the CESS also becomes faster. These two properties are further enhanced if the punishment action is encouraged by the social norm.

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## 1 Introduction

Cooperation is of utmost importance to human society, and our civilization is based upon the cooperation between genetically unrelated individuals in large groups (Axelrod 1984). This is obviously true for modern societies with large organizations and nation states, but it also holds for hunter-gatherer societies with sophisticated forms of hunting, warfare, and food sharing (Fehr and Fischbacher 2003). However, cooperation leads to a tension between what is best for the individual and what is best for the group. A group does better if everyone cooperates, but each individual is tempted to defect. Since neither the naive natural selection assumption in biology nor the pure self-interested individual assumption in economics can lead to cooperation directly (Olson 1965; Ostrom 2000; Henrich et al. 2005; Nowak 2006), there need to be some specific mechanisms for the emergence of cooperation in a population (Taylor and Nowak 2007).

Recently, the effect of *costly punishment* on cooperation has received considerable attention from various disciplines.<sup>1</sup> Costly punishment, which is also referred to as *altruistic punishment* (Fehr and Gächter 2002) or *sanctioning* (Falk et al. 2005) in some of the literature, means that people have the propensity to incur a cost in order to punish a social norm violator (Henrich et al. 2006). It is also a part of strong reciprocity which is a combination of voluntary cooperation in regard to cooperative, norm-abiding behaviors, and punishment in the case of non-cooperative, norm-violating behaviors (Gintis 2000; Fehr et al. 2002).

In the light of the behavioral experiments (Fehr and Gächter 2000, 2002; Gülerk et al. 2006; Rockenbach and Milinski 2006; Henrich et al. 2010) and ethnographic evidence (Knauff et al. 1991; Boehm 1993), costly punishment is prevalently seen (Falk et al. 2005). Furthermore, cross-cultural evidence in complex large-scale and small-scale societies around the globe (Oosterbeek et al. 2004; Henrich et al. 2005, 2006; Marlowe et al. 2008) suggests that the punishment of selfish behavior is “human universal” (Gächter and Herrmann 2009).

However, the role of costly punishment in promoting cooperation is ambiguous. In behavioral experiments, costly punishment has been shown to effectively enforce cooperation (Fehr and Gächter 2002; Fehr and Fischbacher 2003; Gülerk et al. 2006; Rockenbach and Milinski 2006), while some other experiments indicate that punishment is less efficient when the costs associated with punishment exceed the gains from increased cooperation (Dreber et al. 2008; Milinski and Rockenbach 2008; Egas and Riedl 2008; Wu et al. 2009). In particular, the recent theoretical work by

<sup>1</sup> There is a great amount of literature on this topic, for instance: Fehr and Gächter (2002), Boyd et al. (2003), Fowler (2005), Rockenbach and Milinski (2006), Henrich et al. (2006), Henrich (2006), Gülerk et al. (2006) and Ohtsuki et al. (2009) in general journals; Henrich and Boyd (2001), Gintis (2000) and Bowles and Gintis (2004) in biological journals; Fehr and Fischbacher (2004) in cognitive science journals; and Ostrom et al. (1992), Fehr and Gächter (2000), Andreoni et al. (2003), Falk et al. (2005) and Bochet et al. (2006) in economic and other social science journals.

Ohtsuki et al. (2009) shows that costly punishment can not lead to an efficient outcome in most situations, and a better alternative is to withhold help from defectors rather than punish them. This series of work makes the noticeable existence of costly punishment even more puzzling, as Milinski and Rockenbach (2008) point out, “costly punishment remains one of the most thorny puzzles in human social dilemmas (ibid, p. 298)”.

One possible reason for such a puzzle may be that these studies only focus on a short period of experience and omit the long history of cultural evolution. Obviously, the experimental works can only obtain the spot performance of subjects with cultivated culture, but not the process of the cultural cultivation of subjects. On the other hand, the analytical work of Ohtsuki et al. (2009) only analyzes the equilibrium (i.e., the Cooperative Evolutionary Stable State, CESS) but not the route to the equilibrium when the conclusion that costly punishment is less efficient is reached. If we turn our attention to the states far away from the equilibrium and study the route of the co-evolution of the social norms and individual strategies, costly punishment may play a different role in promoting cooperation.

In general, the research question that interests us can be endowed with a three-level evolutionary framework, starting from the level of individual reputation, then extending to the level of individual strategies, and finally coming to the level of social norms. This three-level evolutionary framework can extend the evolutionary selection within a society (Ohtsuki et al. 2009) to the evolutionary selection among societies. Consider a population of competing societies. To survive, each society has its social norm. This norm will determine the reputation of each individual based on how he/she behaves towards others. In the context of the donor-recipient game (to be detailed in Sect. 2), each individual when playing the role of a donor can take one of the following two or three actions toward the randomly matched individual (the recipient). The two basic actions are to cooperate (to give) and to defect (to ignore); some societies allow one additional action, i.e., to punish.

Under a given social norm, each individual has kept a reputation, either a good one or a bad one, and this reputation will be updated each time he plays the role of donor, depending on what he does and whom he meets. Norms can be different, depending on whether the society explicitly allows the punishment action. If it does, norms can be further differentiated by the way they treat the punishment action. Some norms do not care about it, but some norms require donors to punish ‘bad’ recipients and those who fail to comply with this requirement may earn for themselves a bad reputation.

In addition to the reputation, each individual also has a strategy which informs him of what to do (cooperate, defect, or punish) given the characteristic of the matched recipient (good reputation or bad reputation). Each individual will learn and update their strategies from the feedbacks received from the applied social norm in order to improve their fitness. With respect to a specific social norm, the strategies which can help individuals to gain higher payoffs will then become popular and vice versa. This within-society competition determines the evolution of the distribution of the strategies used in the society.

Over a longer horizon, due to the competition among societies, societies may evolve their social norms by comparing the average payoffs of all the social members that different social norms can provide. Such a social norm evolution may take the form of social transformation, a civil war, an external war, colonization, etc. This evolutionary

framework shares the same idea as the cultural group selection of [Bergstrom \(2002\)](#) and [Henrich \(2004\)](#), for a social norm may be regarded as a kind of culture ([Young 2008](#)).

In such a framework, the individual's strategy is adaptation to the social norm, and ultimately the surviving social norm determines how often the punishment actions are taken. If the fittest social norm is the one with a punishment option or one that even encourages punishment, individuals from such a cultural background will naturally exhibit the tendency to punish. This argument is also supported by cross-cultural experiments which demonstrate that punishments are substantially shaped by the cultural background across a range of diverse societies ([Gächter and Herrmann 2009](#)).

Instead of combining all levels of evolution into one equation as in [Henrich \(2004\)](#), we only model the dynamics of the evolution of individual strategies under several fixed social norms. This is because there are only a few social norms that can foster cooperation ([Henrich 2004](#)). We select three typical social norms including the non-punishment, punishment-optional and punishment-provoking social norms, and explicitly model the evolutionary dynamics of individual strategies under these three social norms. By comparing the cooperation propensity and average payoff in the dynamics under different social norms, we can gain a clear insight into the driving force behind the evolution of social norms.

It is found that costly punishment does contribute to the evolution toward cooperation. Once individuals have the choice of punishment, not only does the attraction basin of the cooperative evolutionary stable state (CESS) become larger, but also the speed of convergence to the CESS becomes faster in the social norms with a punishment option. These two properties are further enhanced if the punishment action is encouraged in the punishment-provoking social norm.

This result implies that costly punishment is necessary in at least two situations. The first is that where there are too many defectors in a society, for it will be stuck in a social dilemma in which defection is the best choice for each individual in the case of a non-punishment or even punishment-optional social norm and it can only struggle out of the social dilemma by encouraging punishment through providing individuals with the incentive to punish the defectors. The second is that, where the society is not sufficiently patient and wishes to reach the highly cooperative state quickly, a social norm with punishment or which even encourages punishment can increase the speed at which the cooperative evolutionary stable state is approached.

The remainder of the paper is organized as follows. Section 2 presents a model of an evolutionary donor-recipient game, which differs from the work of [Ohtsuki et al. \(2009\)](#) in that we explicitly model the dynamics of the strategy evolution. Section 3 compares the attraction basin of CESS and the speed of convergence to the CESS for three social norms. Section 4 gives the conclusion and discussions.

## 2 Model

### 2.1 Donor-recipient game

At each small time interval, a fraction of players is randomly sampled from a society with a large population to form pairs. In each pair, one player acts as a donor and the

other player as a recipient. The donor has two basic behavioral choices: cooperation ( $C$ ) and defection ( $D$ ). Cooperation involves a cost  $c$  for the donor and a benefit  $b$  for the recipient. Defection has no cost and yields no benefit. A donor may also have the choice of punishment ( $P$ ) for some social norms. Punishment has a cost  $\alpha$  to the donor and a cost  $\beta$  to the recipient. Here  $c$ ,  $b$ ,  $\alpha$  and  $\beta$  are all positive real numbers. Each individual is endowed with a binary reputation, which is either good ( $G$ ) or bad ( $B$ ). The donor can base his decision on the recipient's reputation. After each interaction, the reputation of the donor is updated according to the social norm of the population, while the reputation of the recipient remains the same. The reputation update process is susceptible to errors. With probability  $\mu$ , where  $0 \leq \mu \leq 0.5$ , an incorrect reputation is assigned, or, with probability  $1 - \mu$ , the correct reputation is assigned. The reputation of each individual is public information.

### 2.1.1 Strategies

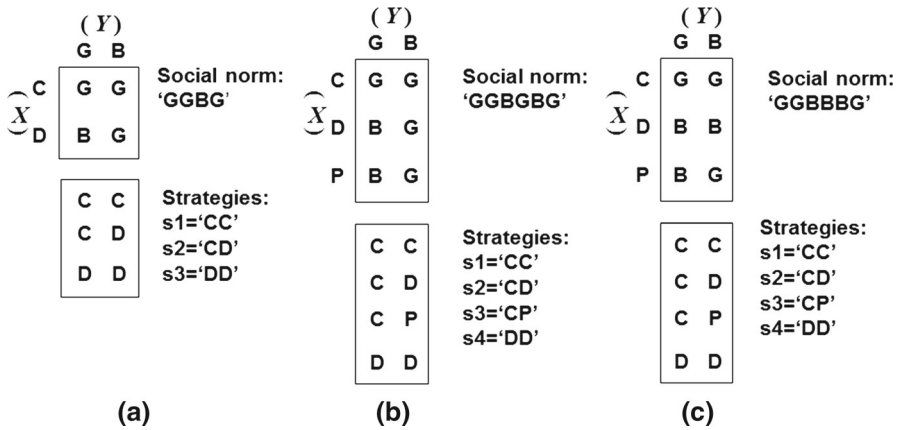
Each player has an action rule (or strategy),  $s$ , which depends on the recipient's reputation. A player with an action rule  $s$  takes the action  $s(G)$  toward a good recipient, and the action  $s(B)$  toward a bad one. Each of  $s(G)$  and  $s(B)$  can be either  $C$ ,  $D$ , or  $P$ . For social norms without a punishment option, there are  $2^2 = 4$  possible action rules:  $s(G)s(B) = CC, CD, DC, DD$ . For social norms with punishment being available, there are  $3^2 = 9$  possible action rules:  $s(G)s(B) = CC, CD, CP, DC, DD, DP, PC, PD$ , and  $PP$ . In the present study, we only consider a subset of these nine strategies, namely,  $CC, CD, CP$  and  $DD$ . These four are interesting and sufficient if one is only interested in the social norms which can facilitate cooperation, as the ones to be introduced in Sect. 2.1.2.

### 2.1.2 Social norms

A social norm  $n$  is used for updating the reputations of players. A donor who has taken the action  $X$  ( $X = C, D, P$ ) toward a recipient whose reputation is  $Y$  ( $Y = G, B$ ) is assigned a reputation based on the social norm  $n$ ,  $n(Y, X)$  ( $= G, B$ ). Social norms of this type are based on a 'second-order assessment' (Nowak and Sigmund 2005), i.e., they depend on both the action of the donor and the reputation of the recipient. Figure 1 presents three typical social norms that we will study in this work with the related ordinary strategies.

Figure 1a represents the case where to punish a recipient is not feasible. A donor can only choose to cooperate or to defect. One social norm corresponding to this case is  $GGBG$ . Under this social norm, cooperators in relation to both good and bad recipients are assigned a good reputation. Defectors in regard to a bad recipient are also assigned a good reputation; however, defectors in regard to a good recipient are assigned a bad reputation. For convenience, we shall call this norm the *simple social norm*.<sup>2</sup>

<sup>2</sup> In addition to the matrix representation, social norms can also be represented by finite-state automata (see Fig. 8 in "Appendix A").



**Fig. 1** Typical social norms (**a** simple, **b** weakly augmented and **c** strongly augmented) with the related ordinary strategies. A social norm is used to update the donor's reputation taking into account both the donor's action ( $X$ ), cooperation ( $C$ ), defection ( $D$ ), or punishment ( $P$ ), and the recipient's reputation ( $Y$ ), good ( $G$ ) or bad ( $B$ ). A strategy specifies the action ( $C$ ,  $D$ , or  $P$ ) that the donor should take given the reputation of the recipients,  $G$  or  $B$

Figure 1b represents the case where to punish a recipient is an option. The social norms corresponding to this case extend the previous one by assigning a reputation value to the donor who chooses to punish the recipient. One example is  $GGBGBG$ . This norm, in addition to the simple social norm  $GGBG$ , further assigns a good reputation to the donor who punished a bad recipient, but a bad reputation to him/her if the one being punished is a good recipient. Again, for the convenience of further discussion, we shall refer to these kinds of norms as *augmented social norms*.

Figure 1c  $GBBBBG$  gives another example of the augmented social norms. It differs from the previous one by the value assigned to the defection action toward the bad recipient. The previous norm assigns a good reputation for this action, but this norm assigns the opposite. Since it is free to defect but costly to punish, the previous norm ( $GGBGBG$ ) results in a weaker incentive to punish the bad recipient than the current norm ( $GBBBBG$ ). Because of this subtle difference, we shall further distinguish the augmented social norms into the *weakly* augmented ones and *strongly* augmented ones; weak and strong are in the sense of their implied incentive for punishing the bad recipient.

In sum, three social norms are studied in this paper. There are *simple*, *weakly augmented*, and *strongly augmented* social norms. The main driver along this extension is the social emphasis on punishment. We will explicitly model the dynamics of individual strategies under these three social norms to study the role of punishment in promoting cooperation.

## 2.2 Evolutionary dynamics of strategies

In a society with social norm  $n$ , individuals interact with each other. Each of them has his own strategy that specifies what action he will take toward recipients with a good

or a bad reputation. Once a donor takes an action, a new reputation is assigned to him according to the social norm  $n$ . It is this reputation that will determine what action others may take toward him.

Individuals will learn and update their strategies to obtain a higher individual payoff by imitating a better strategy. Sometimes a player is given an opportunity to change his strategy. He randomly samples a player and compares the difference in payoffs. If a sampled player has a greater payoff, then the sampling player will imitate the sampled player's strategy with a probability proportional to the difference in payoffs. Otherwise the sampling player will retain the same strategy. Hence, similar to *replicator dynamics*, strategies with higher payoffs (fitness) tend to have more offspring.

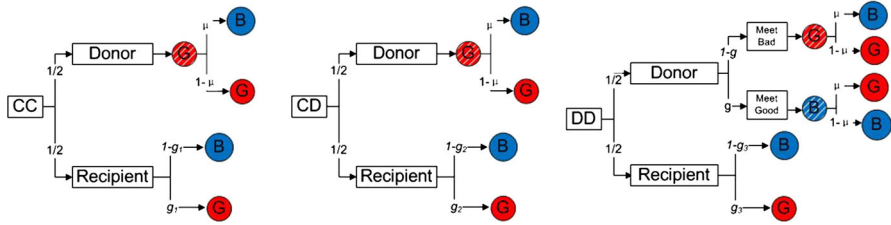
In this model, the payoff of a strategy relies not only on the relative size of the offspring (the fraction) of each strategy, but also on the fraction of *good* individuals, individuals with a good reputation. Because the reputation of individuals is ever changing, it is hard to provide a proper calculation of the payoff of a strategy. In a similar situation, Ohtsuki and Iwasa (2007) calculate the expected payoff of a strategy as the discounted total payoff along the infinitely long future of reputation evolution with the initial reputation of all individuals being good. However, there are two problems with this method: the first is that the calculation of the payoff along the infinitely long future is based on the fixed strategies frequency, while the individual strategy is also evolving although relatively slowly; the second is that arbitrarily assigning a good initial reputation to all individuals is not suitable, for the individual reputation should be inherited from one period to the next period.

Fortunately, we find that for a fixed strategy distribution (the relative size of each strategy), the reputation distribution (the frequencies of individuals with good and bad reputations) will quickly converge to a stable state mainly because the reputation is the instantaneous result of a donor's action.

We assume the following timeline for the evolution of the reputation distribution as well as the evolution of the strategy distribution. First, for a small time scale, only the reputation distribution evolves, while the underlying strategy distribution is fixed. This arrangement allows the reputation distribution to converge to a steady-state distribution (Sect. 2.2.1) and the expected payoff (fitness) of each strategy with respect to this steady-state distribution can then be derived (Sect. 2.2.2). Second, in a large time scale, the strategy distribution also evolves (Sect. 2.2.3). To distinguish between the two, we shall call the former the *short-term dynamics* and the latter the *long-term dynamics*. In the following, we shall first detail the operation of the short-term dynamics.

### 2.2.1 Stable reputation distribution

Given a fixed strategy distribution, a stable reputation distribution can be derived. For the simple social norm (*GGBG*), there are three strategies *CC*, *CD* and *DD*, with corresponding frequencies denoted by  $x_1$ ,  $x_2$  and  $x_3$ , and  $x_1 + x_2 + x_3 = 1$ . The percentages of players with good reputations in *CC*, *CD* and *DD* players are denoted by  $g_1$ ,  $g_2$  and  $g_3$ , respectively. Thus the percentage of good players in the entire population is  $g = x_1g_1 + x_2g_2 + x_3g_3$ .



**Fig. 2** Reputation dynamics of individuals adopting different strategies for the simple social norm (GGBG)

Figure 2 presents the reputation dynamics for *CC*, *CD* and *DD* players. A *CC* player has a  $\frac{1}{2}$  chance of being a donor, and takes cooperation action regardless of the reputation the recipient has, and this tends to give him a good reputation. Due to the assignment error, he obtains a good reputation with probability  $1 - \mu$  and a bad reputation with probability  $\mu$ .<sup>3</sup> The *CC* player also has a  $\frac{1}{2}$  chance of being a recipient; his reputation does not change and remains at the current frequency  $g_1$ . So the new frequency of a good reputation among *CC* players is  $g'_1 = \frac{1}{2}g_1 + \frac{1}{2}(1 - \mu)$ .

Similarly we obtain the new frequency of a good reputation among *CD* and *DD* players as  $g'_2 = \frac{1}{2}g_2 + \frac{1}{2}(1 - \mu)$  and  $g'_3 = \frac{1}{2}g_3 + \frac{1}{2}(1 - g)(1 - \mu) + \frac{1}{2}g\mu$  (See B.1 for the details). Since  $g = x_1g_1 + x_2g_2 + x_3g_3$ , we can solve the linear recursion and obtain the stable frequency of good reputation among players adopting each strategy,

$$g_1^* = g_2^* = 1 - \mu, \quad g_3^* = (1 - \mu) \left[ 1 - \frac{1 - 2\mu}{1 + (1 - 2\mu)x_3} \right].$$

Furthermore, the good reputation frequency among the entire population is

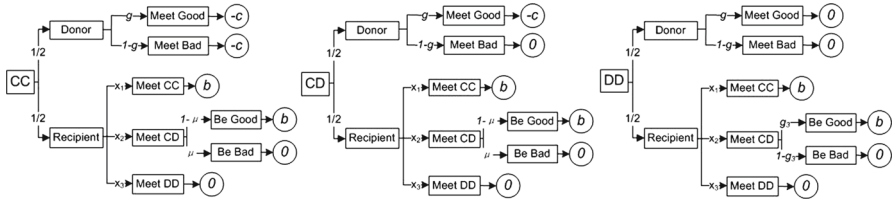
$$g^* = \frac{1 - \mu}{1 + (1 - 2\mu)x_3}.$$

For the weakly augmented norm (*GGBGBG*), we can also obtain the stable reputation frequency among *CC*, *CD* and *CP* players  $g_1^* = g_2^* = g_3^* = 1 - \mu$ , and the stable good reputation frequency among *DD* players as  $g_4^* = (1 - \mu) \left[ 1 - \frac{1 - 2\mu}{1 + (1 - 2\mu)x_4} \right]$ . The good reputation frequency among the entire population is  $g^* = \frac{1 - \mu}{1 + (1 - 2\mu)x_4}$  (see “Appendix B.2” for a derivation).

For the strongly augmented norm (*GGBBBG*), the stable good reputation frequencies among *CC* and *CP* players are  $g_1^* = g_3^* = 1 - \mu$ , while those of *CD* players are  $g_2^* = \mu + (1 - 2\mu)g^*$  and those of *DD* players are  $g_4^* = \mu$ . The good reputation frequency among the entire population is  $g^* = \frac{(1 - \mu)(x_1 + x_3)}{1 - (1 - 2\mu)x_2}$  (see “Appendix B.2” for a derivation).

<sup>3</sup> In the literature, this is known as the *reputation error* (Ohtsuki and Iwasa 2007). Errors and mistakes are prevalently seen in human behavior. “There will be mistakes in perceiving what the other player does, and mistakes in carrying out one’s reply. (Sigmund 1993, p. 192)” Hence, the donor has made a donation, but with a small probability of misunderstanding, it was not publicly acknowledged.





**Fig. 3** The calculation of the expected payoffs of strategies for the simple social norm (GGBG)

2.2.2 Fitness of strategies

We can then calculate a strategy’s expected payoff with respect to the stable reputation distribution (the steady-state distribution of the *good* people and the *bad* people in the society),  $g^*$ , and take it as the main driver for the strategy evolution. We shall detail the derivation process under the simple social norm. Then, the very similar derivation processes for the other two norms will be given in a compact manner.

*Simple Social Norm* The calculation of the expected payoffs of the *CC*, *CD* and *DD* strategies for the simple social norm (GGBG) is illustrated in Fig. 3. A *CC* player has a  $\frac{1}{2}$  chance of being a donor and cooperating with a cost  $c$ . With another  $\frac{1}{2}$  chance of being a recipient, he meets *CC*, *CD* and *DD* players with probabilities  $x_1$ ,  $x_2$  and  $x_3$ , respectively, and is expected to obtain  $b$ ,  $(1 - \mu)b$  and 0 revenue, respectively. So the expected revenue of strategy *CC* is  $p_1 = \frac{1}{2}(-c) + \frac{1}{2}[bx_1 + bx_2(1 - \mu)]$ . Similarly, the expected payoff of strategy *CD* and *DD* can also be calculated (see “Appendix C.1”). The expected revenues of all three strategies for the GGBG social norm are

$$\begin{cases} p_1 = \frac{1}{2}(-c) + \frac{1}{2}[bx_1 + bx_2(1 - \mu)], \\ p_2 = \frac{1}{2}g(-c) + \frac{1}{2}[bx_1 + bx_2(1 - \mu)], \\ p_3 = \frac{1}{2}(0) + \frac{1}{2}[bx_1 + bx_2g_3]. \end{cases} \tag{1}$$

In a similar vein as shown in C.2 and C.3, the expected revenues of strategies *CC*, *CD*, *CP* and *DD* for the weakly augmented social norm (GGBGBG) and the strongly augmented social norm (GBBBBG) can be derived as follows, respectively.

*Weakly Augmented Social Norm*

$$\begin{cases} p_1 = \frac{1}{2}(-c) + \frac{1}{2}x_3\mu(-\beta) + \frac{1}{2}[bx_1 + b(x_2 + x_3)(1 - \mu)], \\ p_2 = \frac{1}{2}g(-c) + \frac{1}{2}x_3\mu(-\beta) + \frac{1}{2}[bx_1 + b(x_2 + x_3)(1 - \mu)], \\ p_3 = \frac{1}{2}g(-c) + \frac{1}{2}(1 - g)(-\alpha) + \frac{1}{2}x_3\mu(-\beta) + \frac{1}{2}[bx_1 + b(x_2 + x_3)(1 - \mu)], \\ p_4 = \frac{1}{2}[bx_1 + b(x_2 + x_3)g_4] + \frac{1}{2}x_3(1 - g_4)(-\beta). \end{cases} \tag{2}$$

*Strongly Augmented Social Norm*

$$\begin{cases} p_1 = \frac{1}{2}(-c) + \frac{1}{2}x_3\mu(-\beta) + \frac{1}{2}[bx_1 + b(x_2 + x_3)(1 - \mu)], \\ p_2 = \frac{1}{2}g(-c) + \frac{1}{2}x_3(1 - g_2)(-\beta) + \frac{1}{2}[bx_1 + bg_2(x_2 + x_3)], \\ p_3 = \frac{1}{2}g(-c) + \frac{1}{2}(1 - g)(-\alpha) + \frac{1}{2}x_3\mu(-\beta) + \frac{1}{2}[bx_1 + b(x_2 + x_3)(1 - \mu)], \\ p_4 = \frac{1}{2}[bx_1 + b(x_2 + x_3)\mu] + \frac{1}{2}x_3(1 - \mu)(-\beta). \end{cases} \tag{3}$$

*2.2.3 Replicator dynamics of strategies frequency*

We model the evolution of the strategy distribution using *replicator dynamics* (Hofbauer and Sigmund 1998). In replicator dynamics, the share of the population using each strategy, in our case,  $\{x_i\}_{i=1}^3$  or  $^4$ , grows at a rate proportional to that strategy’s payoff, in our case,  $\{p_i\}_{i=1}^3$  or  $^4$  [Eqs. (1)–(3)]. Mathematically, it can be written as follows.

$$\dot{x}_i = x_i(p_i - \bar{p}), i = 1, 2, 3, (4), \tag{4}$$

where

$$\bar{p} = \sum_{i=1}^{3(4)} x_i p_i \tag{5}$$

is the average payoff or the expected payoff per individual. In Eq. (4),  $i = 1, 2, 3$  for the simple social norm (*GGBG*) and  $i = 1, 2, 3, 4$  for the augmented social norms (*GGBGBG* and *GGBBBG*). These differential equations are defined on the simplex  $S_3 = \{(x_1, x_2, x_3)|x_1 + x_2 + x_3 = 1, x_i \geq 0\}$  for the simple social norm and  $S_4 = \{(x_1, x_2, x_3, x_4)|x_1 + x_2 + x_3 + x_4 = 1, x_i \geq 0\}$  for the augmented social norms. Each corner of the simplex is an equilibrium of the dynamics corresponding to a homogeneous population.<sup>4</sup>

The replicator dynamics as shown in Eq. (4) can be normalized by adding the same constant; in our case, we shall subtract the fitness of the last strategy from the first two (or three). Hence, for the simple social norm (*GGBG*), we define  $p'_1 = p_1 - p_3$ ,  $p'_2 = p_2 - p_3$  and  $\bar{p} = x_1 p'_1 + x_2 p'_2$  using the corresponding  $p_i$  ( $i = 1, 2, 3, 4$ ) in Eq. (1). Then the replicator dynamics of the first two strategies, *CC* and *CD*, can be derived as follows:

$$\begin{cases} \dot{x}_1 = x_1(p'_1 - \bar{p}) = -cx_1 + cx_1^2 \\ \quad + \frac{[(1 - 2\mu)b + c]x_1x_2 - (1 - 2\mu)bx_1^2x_2 - (1 - 2\mu)bx_1x_2^2}{2 - \frac{1-2\mu}{1-\mu}(x_1 + x_2)}, \\ \dot{x}_2 = x_2(p'_2 - \bar{p}) = cx_1x_2 \\ \quad + \frac{-cx_2 + [(1 - 2\mu)b + c]x_2^2 - (1 - 2\mu)bx_1x_2^2 - (1 - 2\mu)bx_2^3}{2 - \frac{1-2\mu}{1-\mu}(x_1 + x_2)}. \end{cases} \tag{6}$$

<sup>4</sup> This can be shown simply by inserting the vertex states into the replicator dynamics [Eq. (4)].

Likewise, for both weakly and strongly augmented social norms (*GGBGBG* and *GGBBBG*), we define  $p'_1 = p_1 - p_4$ ,  $p'_2 = p_2 - p_4$ ,  $p'_3 = p_3 - p_4$  and  $\bar{p} = x_1 p'_1 + x_2 p'_2 + x_3 p'_3$  with the corresponding  $p_i$  ( $i = 1, 2, 3$ ) in Eqs. (2) and (3), respectively. The replicator dynamics of the strategies of *CC*, *CD* and *CP*,

$$\begin{cases} \dot{x}_1 = x_1(p'_1 - \bar{p}), \\ \dot{x}_2 = x_2(p'_2 - \bar{p}), \\ \dot{x}_3 = x_3(p'_3 - \bar{p}), \end{cases} \tag{7}$$

are much longer; for brevity, we leave their explicit forms in “Appendix D” [see Eqs. (20) and (21)].

### 3 Analysis

The replicator dynamics of the three social norms, namely, Eqs. (6), (20), (21), provides us with the basis on which the contribution of costly punishment to cooperative behavior can be examined. Our evaluation will not be limited to the long-run equilibria or the evolutionary stable states (Sect. 3.1) as some studies have already done; instead, we will focus more on the *transition dynamics* in terms of the *basin of attraction* (Sect. 3.2) and the *speed of convergence* (Sect. 3.3).

#### 3.1 Evolutionary stable states

The long-term behavior of the replicator dynamics (the system of differential equations), Eqs. (6), (20) and (21), is characterized by their evolutionary stable states. For our purpose, it is useful to distinguish between the *non-cooperative state* and the *cooperative state*. For simplicity, let us define these two states by first confining them within the case of the *homogeneous agents*, i.e., all agents adopt the same strategy. With this confinement, the non-cooperative state refers to the state in which all agents would always defect regardless of the reputation of the matched recipient, i.e., all agents are *DD* players. In terms of the strategy distribution, the non-cooperative state corresponds to the degenerate distribution  $(x_1^*, x_2^*, x_3^*) = (0, 0, 1)$  for the simple social norm and the degenerate distribution  $(x_1^*, x_2^*, x_3^*, x_4^*) = (0, 0, 0, 1)$  for the augmented social norm.

The (homogeneous) cooperative states then refer to the state in which all agents would cooperate if the matched recipient has a good reputation, i.e., all agents are either *CC*, *CD* or *CP* players. Based on our notation, they correspond to the degenerate distribution  $x_1^* = 1, x_2^* = 1$  for the simple social norm, and plus  $x_3^* = 1$  for the augmented social norm, respectively.

We do not consider the *heterogeneous states* with a  $i$  where  $0 < x_i^* < 1$ . As we shall see from the analysis below, none of them is evolutionary stable. Hence, we only focus on the non-cooperative state and cooperative state and, accordingly, distinguish between the non-cooperative evolutionary stable state (NESS) and the cooperative

evolutionary stable state (CESS).<sup>5</sup> In the following, we shall present two propositions. These two propositions simply say that all the three social norms can support one non-cooperative evolutionary stable state (Proposition 1) and one cooperative evolutionary stable state (Proposition 2). Although the proposition is stated with only supportive intuition, a mathematical proof can be found in “Appendix E”.

**Proposition 1** *The non-cooperative state, i.e.,  $x_3^* = 1$  for the simple social norm and  $x_4^* = 1$  for the augmented social norm, is evolutionary stable.*

Proposition 1 simply says that, regardless of the ruling social norm, there is no incentive to cooperate if all other agents always defect. This is so because a good reputation will not do anything good for the recipient; whatever his reputation is, he will not be treated decently. It is then not cost-efficient to maintain a good reputation.

**Proposition 2** *1. The cooperative state  $x_2^* = 1$  is evolutionary stable for the simple social norm if  $\frac{c}{(1-2\mu)b} < 1$ .  
2. The cooperative state  $x_2^* = 1$  is evolutionary stable for the weakly augmented social norm if  $\frac{c}{(1-2\mu)b} < 1$ .  
3. The cooperative state  $x_3^* = 1$  is evolutionary stable for the strongly augmented social norm if  $\alpha < c$  and  $\frac{(1-\mu)c+\mu\alpha}{(1-2\mu)(b+\beta)} < 1$ .*

Proposition 2-(1) can be easily understood by rearranging the inequality condition as follows.

$$\frac{c}{(1-2\mu)b} < 1 \Rightarrow (1-\mu)b - (1-\mu)c > 2\mu(1-\mu)b. \quad (8)$$

When all other agents are *CD* players, the inequality (8) simply states that the net benefit from playing *CD* (the left-hand side of the second inequality) must be greater than the net benefit from playing *DD* (the right-hand side of the second inequality) when the agent meets the matched recipient.<sup>6</sup> Proposition 2-(2) is essentially the same as Proposition 2-(1). Adding *CP* does not alter our above argument much under the weakly augmented social norm since the additional cost of being a *CP* player, i.e.,  $\mu\alpha$ , is compensated by nothing under the *GGBGBG* norm.

Proposition 2-(3) can be understood in a similar vein. The inequality can be rearranged as

$$\frac{(1-\mu)c + \mu\alpha}{(1-2\mu)(b+\beta)} < 1 \Rightarrow (1-\mu)b - \mu\beta - (1-\mu)c - \mu\alpha > \mu b - (1-\mu)\beta \quad (9)$$

Assuming that the society has already evolved to the state in which all agents adopt *CP*, then as in inequality (8), the two sides of inequality (9) refer to the expected payoff from adopting the *CP* and the *DD* strategy under the strongly augmented

<sup>5</sup> Hereafter, we also remove the word “homogeneous” for brevity.

<sup>6</sup> In a similar but more complex setting, Ohtsuki and Iwasa (2007) also prove Proposition 2-(1). See Ohtsuki and Iwasa (2007), p. 522.

norm, respectively. In addition, at this point, the payoff from adopting  $CP$  is greater than that from  $CC$  as long as  $c > \alpha$ .

While all three norms have their own CESS, the stability condition of CESS for the simple and weakly augmented social norms is more stringent than that for the strongly augmented social norm. This is because under the normal case the cost of cooperation ( $c$ ) is greater than the cost of punishment ( $\alpha$ ), and, if so, it follows that

$$\frac{c}{(1 - 2\mu)b} > \frac{(1 - \mu)c + \mu\alpha}{(1 - 2\mu)(b + \beta)}.$$

This further implies that the CESS under the strongly augmented norm is more resilient to the error rate  $\mu$  than that for the other two norms.

### 3.2 Basin of attraction

As we have said at the very beginning of the paper, our concern over the role of costly punishment has more to do with its transition dynamics rather than just the limit point. Hence, given that there are two evolutionary stable states under each social norm, one NESS and one CESS, the *basin of attraction* for each ESS is pertinent to our concern. The basins of attraction are the sets of all strategy distributions  $\{x_i\}_{i=1}^3$  or  $^4$  which lies on a portrait leading to the respective ESS. For an illustration, Fig. 4 presents the phase portraits for the three norms with the parameter setting:  $b = 3, c = 2, \alpha = 1, \beta = 4$  and  $\mu = 0.02$ . What particularly interests us is the relative size of the basin of attraction of the CESS and NESS. Calling the former  $\mathcal{B}_{CESS}$  and denoting its size (area, volume) by  $\#(\mathcal{B}_{CESS})$ , we try to estimate the following ratio under three different social norms:

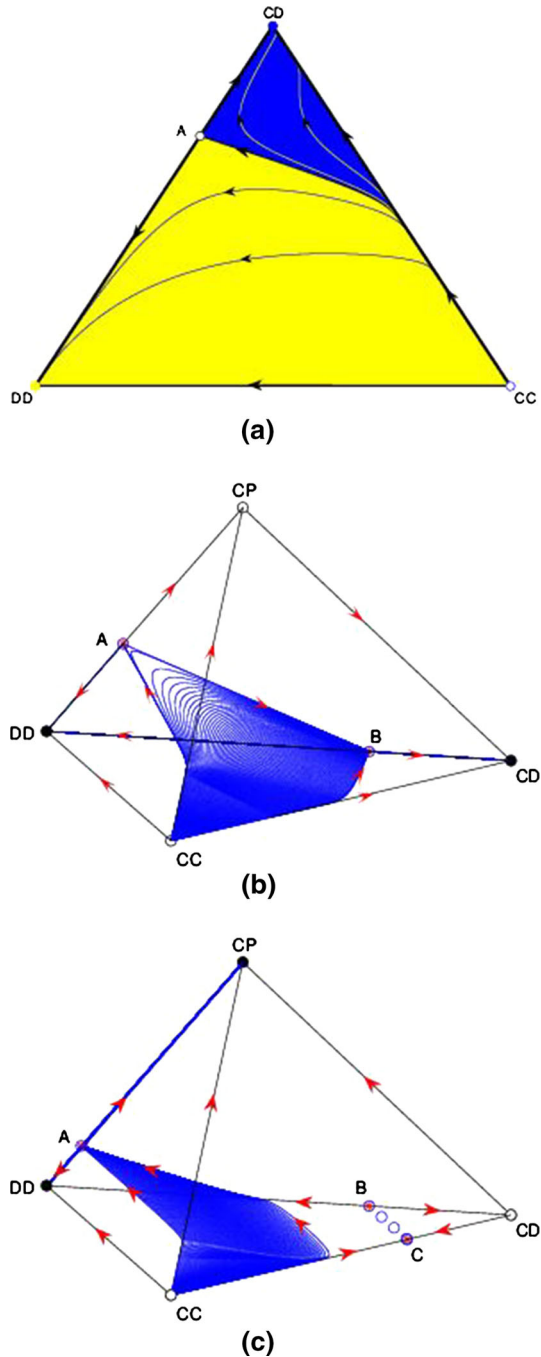
$$q = \frac{\#(\mathcal{B}_{CESS})}{\#(\mathcal{B}_{Simplex})}, \tag{10}$$

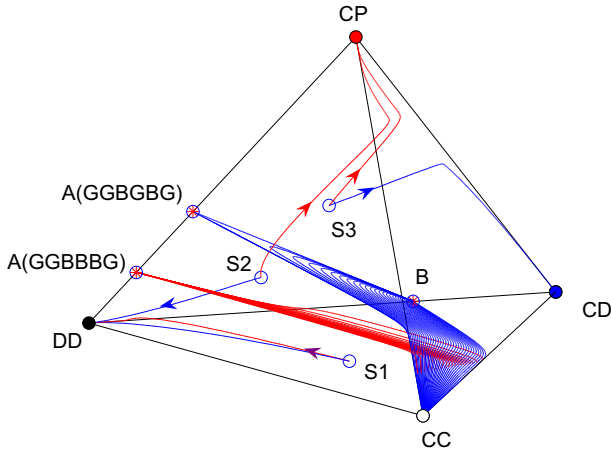
where  $\#(\mathcal{B}_{Simplex})$  is the size of the entire simplex.

Due to the lack of analytical tractability, numerical approaches are applied for this estimation. We first identify the critical points along each edge of the simplex. These points are where the two strategies, characterizing the edge, have the same expected payoffs. Of course, not all edges have these critical points, and what can be found are marked as “A” in Fig. 4a, “A” and “B” in Fig. 4b, and “A”, “B” and “C” in Fig. 4c. Then, in light of these critical points, one can use the fourth-order Runge–Kutta method and the bi-section method to identify the stable manifold (the separatrix line) dividing the plane into two regions, i.e.,  $\mathcal{B}_{CESS}$  and  $\mathcal{B}_{NESS}$ . Each of the two regions is further distinguished by its portrait, i.e., paths intensively sampled through the Runge–Kutta method.<sup>7</sup> With this identified  $\mathcal{B}_{CESS}$  and  $\mathcal{B}_{NESS}$ , we find that the relative size of the

<sup>7</sup> In Fig. 4a, Point A,  $(x_1, x_2) = (0, \frac{1}{1-2\mu} \frac{c}{b})$ , is a saddle node whose unstable manifold is along the CD-DD line. In Fig. 4b, Point A,  $(x_1, x_2, x_3) = (0, 0, \frac{1}{1-2\mu} \frac{(1-\mu)(\alpha+c)}{(1-\mu)(b+\beta)+\alpha})$  is a saddle node with a one-dimensional stable manifold and a two-dimensional unstable manifold. Point B,  $(x_1, x_2, x_3) = (0, \frac{1}{1-2\mu} \frac{c}{b}, 0)$  is a saddle node with a one-dimensional unstable manifold along the CD-DD line and a two-dimensional

**Fig. 4** Phase portrait of the three social norms (**a** simple, **b** weakly augmented and **c** strongly augmented) with  $b = 3$ ,  $c = 2$ ,  $\alpha = 1$ ,  $\beta = 4$  and  $\mu = 0.02$ . Each vertex represents a state with individuals adopt the same corresponding strategy, such as points  $DD$  in all three norms representing the state in which all individuals adopting the  $DD$  strategy. The arrows indicate the direction of the flow. For the simple norm  $GGBG$  (**a**), the blue part is the attraction basin of the cooperative evolutionary stable state  $CD$  and the yellow part is the attraction basin of the non-cooperative evolutionary stable state  $DD$ . The separatrix line is the stable manifold of saddle point  $A$ . State  $CC$  is unstable. For the weakly augmented norm  $GGBGBG$  (**b**), a separatrix surface which is the stable manifold of saddle point  $A$  divides the phase space into two parts. The part over the surface is the attraction basin of the cooperative evolutionary stable state  $CD$  and the nether part is the attraction basin of the stable state  $DD$ . States  $CC$  and  $CP$  are unstable. For the strongly augmented norm  $GGBBBG$  (**c**), there is also a separatrix surface dividing the phase space into two parts. The upper and nether regions are the attraction basins of states  $CD$  and  $CP$ , respectively. States  $CC$  and  $CD$  are unstable. The attraction basin of the cooperative evolutionary stable state occupies 15, 60 and 81 % of the entire simplex for the simple, weakly augmented and strongly augmented social norms, respectively. **a**  $GGBG$ . **b**  $GGBGBG$ . **c**  $GGBBBG$





**Fig. 5** Typical trajectories under the weakly augmented (blue lines) and strongly augmented (red lines) norms. The simplex is divided into three regions. The bottom region under the separatrix surface of the weakly augmented norm, exemplified by the point  $S1$ , is also the attraction basin of the non-cooperative evolutionary state  $DD$  under the strongly augmented norm. The middle region between the separatrix surfaces of the two norms, exemplified by the point  $S2$ , is the attraction basin of the non-cooperative evolutionary state  $DD$  under the weakly augmented norm but the attraction basin of the cooperative evolutionary state  $CP$  under the strongly augmented norm (color figure online)

basin of attraction of CESS, or simply the  $q$ , increases from 0.15 (simple social norm), to 0.60 (weakly augmented social norm), and further up to 0.81 (strongly augmented social norm).<sup>8</sup>

Obviously, in terms of  $q$ , the social norms augmented with costly punishment increase the likelihood of the emergence of the cooperative evolutionary stable state. Even though the CESS of the weakly augmented norm does not directly involve punishment ( $CD$  rather than  $CP$  being the CESS), the feasibility of costly punishment ( $P$ ) has sharply increased  $q$  from a value of 0.15 to 0.60. To see the further enlargement of the basin of attraction of the CESS when costly punishment is explicitly incorporated into the social norm, Fig. 4b, c are drawn together into Fig. 5. In Fig. 5, the same initial point  $S2$  follows the blue trajectory to converge to  $DD$  under the weakly augmented norm, but then follows the red trajectory to converge to  $CP$  under the strongly augmented norm.

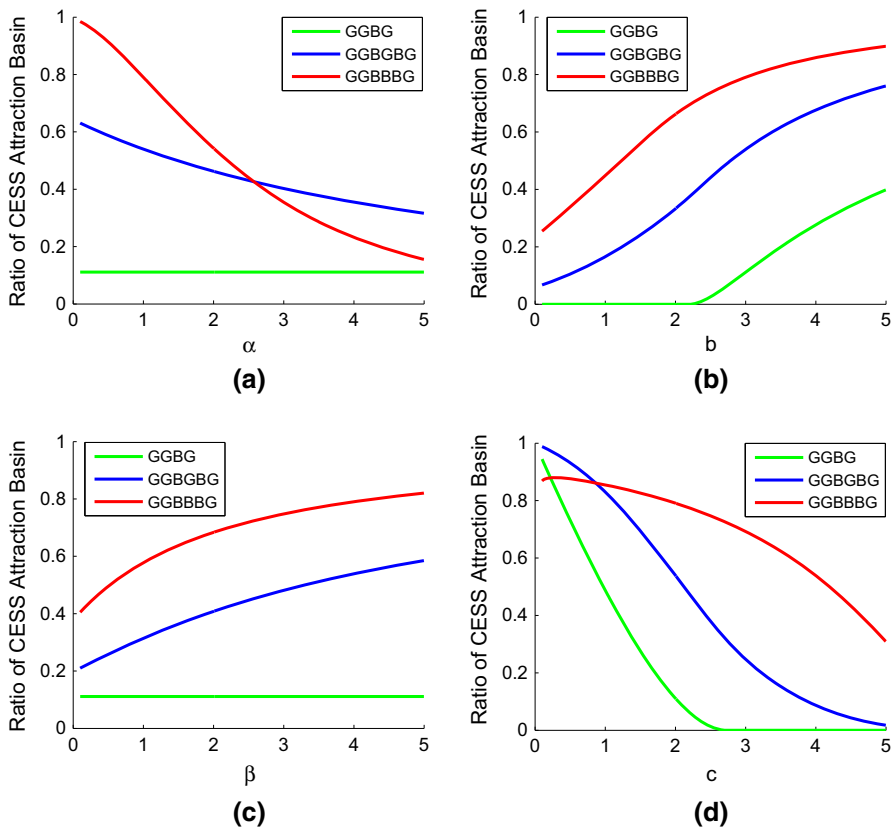
Footnote 7 continued

stable manifold ( the separatrix surface). In Fig. 4c, Point A,  $(x_1, x_2, x_3) = (0, 0, \frac{1}{1-2\mu} \frac{(1-\mu)\alpha+\mu c}{\alpha+\beta+b-c})$  is a saddle node with a one-dimensional unstable manifold along the  $CP$ - $DD$  line and a two-dimensional stable manifold (the separatrix surface). In the facet of  $CC$ - $CD$ - $DD$ , there is a line connecting B,  $(x_1, x_2, x_3) = (0, \frac{1}{1-2\mu} \frac{c}{b}, 0)$  and C,  $(x_1, x_2, x_3) = (\frac{1}{1-2\mu} \frac{c}{b}, \frac{1}{1-2\mu} \frac{c}{b}, 0)$  which consists of equilibria which are Lyapunov stable.

<sup>8</sup> The area or the volume of  $\mathcal{B}_{CESS}$ ,  $\#\mathcal{B}_{CESS}$ , is calculated based on the intensive sampling of points in the simplex. We take a step size of 0.001 in each of the three, in the case of Fig. 4a, or four, in the case of Fig. 4b, c, dimensions and obtain a total of 500,500 and 20,958,500 points, respectively. We then trace the flow of each of these paths emanating from these points and see where they converge, CESS or NESS, then use the fraction of the converging paths to CESS to approximate the size (area or volume) of  $\#\mathcal{B}_{CESS}$ .

To obtain an overview of the effect of parameters ( $\alpha$ ,  $\beta$ ,  $b$ ,  $c$ ) on the CESS attraction basin, we calculate the ratio  $q$  under different parameter settings numerically and present the results in Fig. 6. The  $q$  under the simple, weakly augmented and strongly augmented norms are green-colored, blue-colored and red-colored, respectively. In most regular parameter settings, such as  $\alpha < c$ , the strongly augmented norm ( $GGBBBG$ ) tends to have the largest basin of attraction of the CESS ( $q$ ), followed by the weakly augmented norm ( $GGBGBG$ ), with that of the simple social norm ( $GGBG$ ) being the smallest. Only in some situations, such as when the inequality  $\alpha < c$  is violated, do we see exceptions, as shown in the right part of Fig. 6a ( $\alpha$  is relatively too large) or the left part of Fig. 6d ( $c$  is relatively too small).

Very loosely, let us characterize the good society as a society consisting of good people, i.e., people with good intentions. They help strangers and are also helped when they become strangers to others. Then what is shown in our analysis above is that the emergence of this good society can critically depend on the ruling social norm, particularly, whether this norm would punish those “bad” people and those who are tolerant with them.



**Fig. 6** Ratios of the CESS attraction basin of different social norms under different parameter settings ( $b = 3$ ,  $c = 2$ ,  $\alpha = 1$ ,  $\beta = 4$  if not specified, and  $\mu = 0.02$ )



### 3.3 Convergence speed of the three social norms

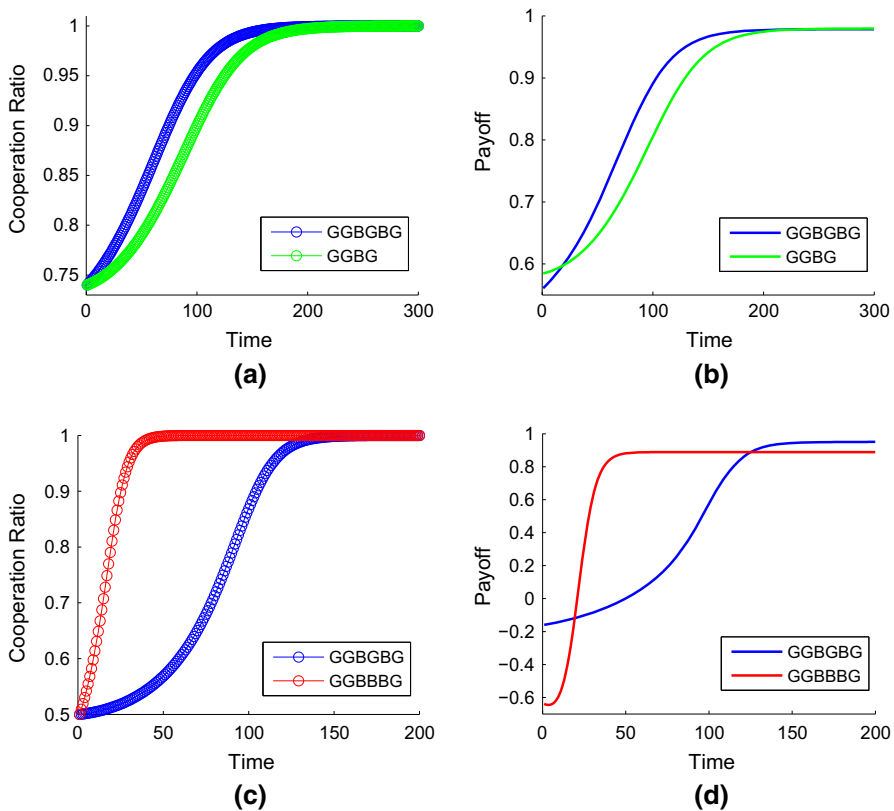
The expanded basin of attraction as shown above sheds light on the contribution of punishment in cooperation-enhancing norms. It simply differentiates what a society can possibly achieve with or without the incorporation of punishment as part of the norm. The difference between being able to converge to CESS or the alternative, NESS, is permanent. Hence, we have shown that norms which emphasize punishment can lead to a non-trivial positive effect for the society. In this section, we shall consider a secondary and short-term effect, i.e., even though within the same basin of attraction to CESS, depending on the presence of punishment or not, the convergence speed can still be different. In other words, even if it starts from such an initial strategy distribution,  $\{x_i(0)\}_{i=1}^3$  or  $^4$ , whereby a society with any of the three social norms will approach cooperative evolutionary stable states, a society may converge to the cooperative states with different speeds for different social norms.

To see this, we first start with two concrete numerical illustrations, which are then followed by a more general result based on the intensive simulation. Figure 7a, b provide an illustrative comparison of the transition dynamics of societies with a simple (green) and augmented (blue) social norm, starting from the very close initial states,  $x_1(0) = 0.02$ ,  $x_2(0) = 0.72$  for the simple social norm (*GGBG*) and  $x_1(0) = 0.02$ ,  $x_2(0) = 0.71$ ,  $x_3(0) = 0.01$  for the weakly augmented social norm (*GGBGBG*). In Fig. 7a, the horizontal axis is time, and the vertical axis is the cooperation ratio, defined as the percentage of individuals adopting the *CC*, *CD* or *CP* strategies. We can easily observe that a society with a weakly augmented social norm will converge to the cooperative evolutionary stable states more rapidly than a society with a simple social norm.

The economic significance of this faster convergence to a cooperative state is shown in Fig. 7b, where the expected payoffs, as defined in Eq. (5), of the two social norms are drawn in contrast. By contrast, when both societies converge to their cooperative state, with all individuals adopting either the *CC* or *CD* state, and the expected payoff being the same, the norm which can lead the society to converge faster to it can bring additional gains during the transition time.

Figure 7d, e provide another illustrative comparison of the transition dynamics in societies with a weakly augmented (blue) and strongly augmented (red) social norm, starting from an identical initial strategy distribution,  $x_1(0) = 0.05$ ,  $x_2(0) = 0.15$ ,  $x_3(0) = 0.3$ . As shown in Fig. 7d, a society with a strongly augmented social norm will converge to the cooperative evolutionary stable state more rapidly than a society with a weakly augmented norm. Consequently, the expected payoff of a society with a strongly augmented norm increases more quickly than that of a society with a weakly augmented norm, except for a very short period of time in the beginning. After the society reaches the cooperative evolutionary stable state, i.e., with all individuals adopting *CD* (for the *GGBGBG* norm) or *CP* (for the *GGBBBG* norm) strategy, the expected payoff of the latter will be slightly smaller than that of the former for there are errors in reputation assignment and punishment will be meted out to ‘bad’ individuals with cost to both the punisher and punished.

What has been shown here is that, even though in some cases norms would not matter from a long-run perspective, they still matter for the transition dynamics and



**Fig. 7** Illustrative comparisons of cooperative behavior and economic efficiency for the three social norms. **a** and **b** are the dynamics of the cooperative population and the expected payoffs realized using the initial strategy distribution  $x_1(0) = 0.02$ ,  $x_2(0) = 0.72$  for the simple social norm (green line) and  $x_1(0) = 0.02$ ,  $x_2(0) = 0.71$ ,  $x_3(0) = 0.01$  for the weakly augmented social norm (blue line). **c** and **d** are the counterparts using  $x_1(0) = 0.05$ ,  $x_2(0) = 0.15$ ,  $x_3(0) = 0.3$  for both weakly and strongly augmented social norms, blue for the weakly one and red for the stronger one (color figure online)

can determine how fast the society becomes rich and prospers. As indicated in our illustrations, the differences due to this transition dynamics can be so prolonged that it takes quite a long while to see its disappearance. Hence, unlike some earlier analysis mainly focusing on the long-term performance (Ohtsuki et al. 2009), we find that the norm which promotes costly punishment can have a non-trivial effect. Nevertheless, their key argument remains valid when the society has already come to a cooperative state and most people have a good reputation most of the time. At this stage, the norm which values costly punishment can incur more social loss than social gain. Cases like that shown in our Fig. 7e demonstrate the undesirability of the strongly augmented norm after the society has been “fully developed” and hence suggest the desirability of a norm change or a regime change.<sup>9</sup>

<sup>9</sup> Here, we come to a point to see that the social norm is not just exogenous in terms of shaping human behavior, but should, more generally, co-evolve with human behavior. While the current paper does not

## 4 Concluding remarks

### 4.1 Findings and implications

Costly punishment is widespread in human society although both theoretical analysis and laboratory experiments show that punishment provides little efficiency and can hardly increase the overall benefit of a population. In this paper, we reconcile the gap between reality and theory. We study the role of costly punishment in a “developing” society characterized by her process of evolution toward a fully cooperative state. The analysis is conducted in a co-evolutionary framework of reputations, strategies, and social norms. A social norm is a globally shared rule used to update the reputation of agents based on what they did and to whom. It is the collective choice of a population and evolves gradually according to the total benefit of all members in a society for a considerably long period of time.<sup>10</sup> Agents are embedded in a certain social norm and they choose their strategies to maximize their individual benefits given the current social environment that includes the strategies of other agents and the ruling social norm.

We explicitly model the evolutionary dynamics of the three different social norms which give different support for the “altruistic” punishment behavior, and find that the social norm which acknowledges “altruistic” punishment would matter for a “developing” society. It makes it easier for a developing society to transit faster into a fully cooperative state or a “good society”, easier in the sense of the size of the basin of attraction to the CESS, and faster in the sense of the speed of convergence.

To sum up, costly punishment is inevitable in the process of evolution to cooperation in two situations. The first situation is that the current state, under a social norm which does not value punishment, has no hope to get to the cooperative evolutionary stable state (not in the basin of attraction). One can bring hope to this society by changing the norm to the one which values punishment (enlarging the basin of attraction). The second situation is that even if the current state is moving toward the cooperative evolutionary stable state, the society is not sufficiently patient for its slow progress. A social norm which supports punishment can speed up this transition.

### 4.2 Direction for further studies

Our focus on the transition dynamics justifies why social norms may value costly punishment or altruistic punishment. In the long-run when the society has already come to the CESS, we concur with [Ohtsuki et al. \(2009\)](#) on the social undesirability of costly punishment since it may result in a lower social efficiency given the noises of the operational social norm, i.e., the reputation error probability  $\mu$ . A desirable social

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Footnote 9 continued

deal with this co-evolutionary framework, it does suggest that if the selection mechanism for norms also exists, probably with an even longer horizon and at a slower pace, the ruling norm may change with the development of society.

<sup>10</sup> We do not directly model the evolution of social norms, but we do argue that different norms may serve the society differently at her different developing stages. See Sect. 3.3.

norm at this stage should lead individuals to withhold help from defectors rather than to punish them. However, the CESS only serves as a point of reference. It may be difficult to reach if more realistic concerns of social development are taken into account. In this section, we will point out a number of directions for further research.

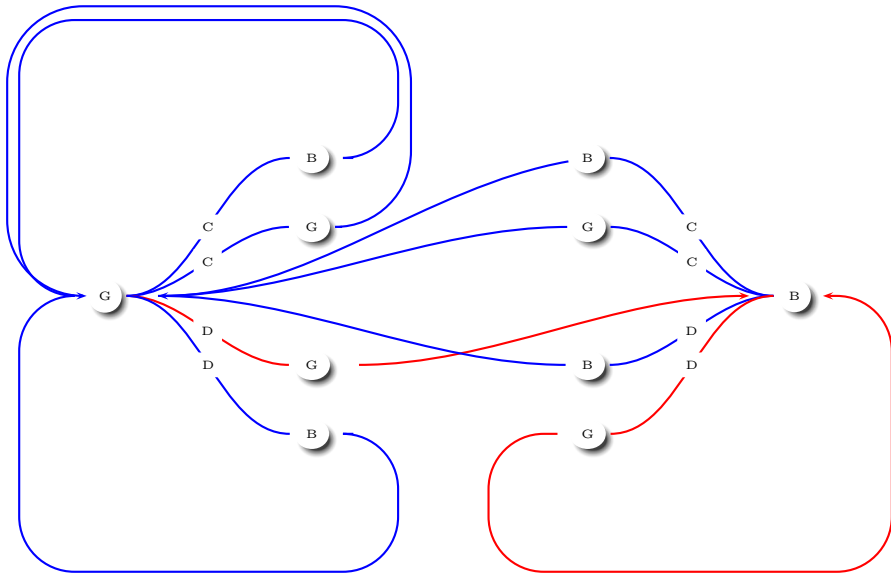
First, one important variable which is not considered in our current model is the constant change in demographic characteristics related to migration, mortality, and fertility. This changing demographic structure may impact both the domain of attraction and the convergence path to the CESS. Technically, one direction in which to extend our current model is to introduce an appropriate *birth-and-death* process to characterize the demographic change. This added birth-and-death process may constantly perturb the transition dynamics, including the strategy distribution. Hence, in this more unsettled environment, a social norm which acknowledges costly punishment behavior may enhance the stability of CESS.

Second, in this paper, we do not consider a larger set of strategies. While the four selected are typical and enough if one is only interested in the social norms which can facilitate cooperation, some other strategies, from a different point of view, may also be worth an effort. For example, a series of recent experiments based on the public good game shows that while stingy subjects were punished, exceedingly generous subjects were also punished (Herrmann et al. 2008; Parks and Stone 2010; Irwin and Horne 2013). A later phenomenon known as *antisocial punishment* may motivate a design of the strategies that is finer than the one considered in this paper.

Third, while our proposed model can be considered to be a co-evolutionary model of reputations, strategies, and norms, due to analytical concerns, in this paper we only consider one of them at a time. Hence, the reputation distribution evolves with respect to a given strategy distribution, and strategy distribution evolves with respect to a given reputation distribution. Likewise, the evolution of social norms will be considered only after the strategy distribution reaches its steady state. Nonetheless, in a more realistic situation, the three can evolve in parallel, despite their different time frames. Extending the current model to this truly co-evolutionary framework may make the model analytically intractable, and one has to rely on simulation. In this regard, agent-based models would help. In fact, agent-based models may do more. The conventional literature simply assumes that the reputation of any agent is freely publicly known and is homogeneously recognized by all other agents. Using agent-based modeling, we can relax both assumptions and study the effect of social norms on the emergence of a ‘good society’ when both interactions and reputation are local.<sup>11</sup>

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<sup>11</sup> See Ma et al. (2014) for recent progress in this direction.



**Fig. 8** The simple social norm (*GGBG*) in the form of a finite-state automaton

**Appendix A: Social norms in the form of finite-state automata**

The social norm as demonstrated in the matrix form (Fig. 1) can also be demonstrated in the familiar finite state automata. Figure 8 gives such an example.<sup>12</sup> What is shown in this figure corresponds to the simple social norm “*GGBG*”. There are two states in the automaton. State “G” refers to the state of a good reputation, and state “B” refers to the state of a bad reputation. The transition rules are depicted by various arrowed paths. For example, if the agent starting with the “G” state (the left-hand side of the figure) takes an action “D” (defection) toward a “G” agent, then by the transition rule his state will be changed to the “B” state (the right-hand of the figure), as shown by the red-color arrowed path leading from state “G” to state “B”.

**Appendix B: Short-term dynamics: stable reputation distribution**

In the main text, we only provide a simple introduction regarding how to obtain a stable reputation distribution under the simple social norm. The details for obtaining the stable reputation distribution for all three social norms are provided here.

**B.1 Simple social norm**

We have shown the reputation dynamics (the difference equation) of the *CC* players. The reputation dynamics of the *CD* and *DD* players are detailed here.

<sup>12</sup> We are grateful to Cyrille Piatecki for this suggestion and his generosity of providing us with his latex source code to generate Fig. 8.

A *CD* player has a  $\frac{1}{2}$  chance of being a donor, and he takes cooperative action toward good recipients and defection action toward bad recipients; both should bring him a good reputation under the simple social norm *GGBG*. Due to the assignment error, he obtains a good reputation with a probability of  $1 - \mu$  and a bad reputation with a probability of  $\mu$ . The *CD* player also has a  $\frac{1}{2}$  chance of being a recipient and his reputation does not change and remains as good with probability  $g_2$ . So after one iteration the percentage of good reputation for *CD* players is  $g'_2 = \frac{1}{2}(1 - \mu) + \frac{1}{2}g_2$ .

A *DD* player has a  $\frac{1}{2}$  chance of being a donor, and he takes defection action regardless of the reputation of the recipient. He has a chance of  $1 - g$  of meeting a bad recipient. According to the simple social norm *GGBG*, he should obtain a good reputation. With the reputation assignment error, he will obtain a good reputation with probability  $(1 - \mu)(1 - g)$  and a bad reputation with probability  $\mu(1 - g)$ . He also has a chance of  $g$  of meeting a good recipient and obtaining a bad reputation with probability  $(1 - \mu)g$  and a good reputation with probability  $\mu g$ . The *DD* player also has a  $\frac{1}{2}$  chance of being a recipient and his reputation does not change and remains as good with probability  $g_3$ . So the new percentage of a good reputation among *DD* players is  $g'_3 = \frac{1}{2}(1 - g)(1 - \mu) + \frac{1}{2}g\mu + \frac{1}{2}g_3$ .

In sum, what we have is a system of difference equations (11):

$$\begin{cases} g'_1 = \frac{1}{2}(1 - \mu) + \frac{1}{2}g_1, \\ g'_2 = \frac{1}{2}(1 - \mu) + \frac{1}{2}g_2, \\ g'_3 = \frac{1}{2}(1 - g)(1 - \mu) + \frac{1}{2}g\mu + \frac{1}{2}g_3, \end{cases} \quad (11)$$

where  $g = x_1g_1 + x_2g_2 + x_3g_3$ .

By solving equation (11), we obtain the steady state solution, i.e., the limit distribution of *good* people for each strategy, as in Eq. (12):

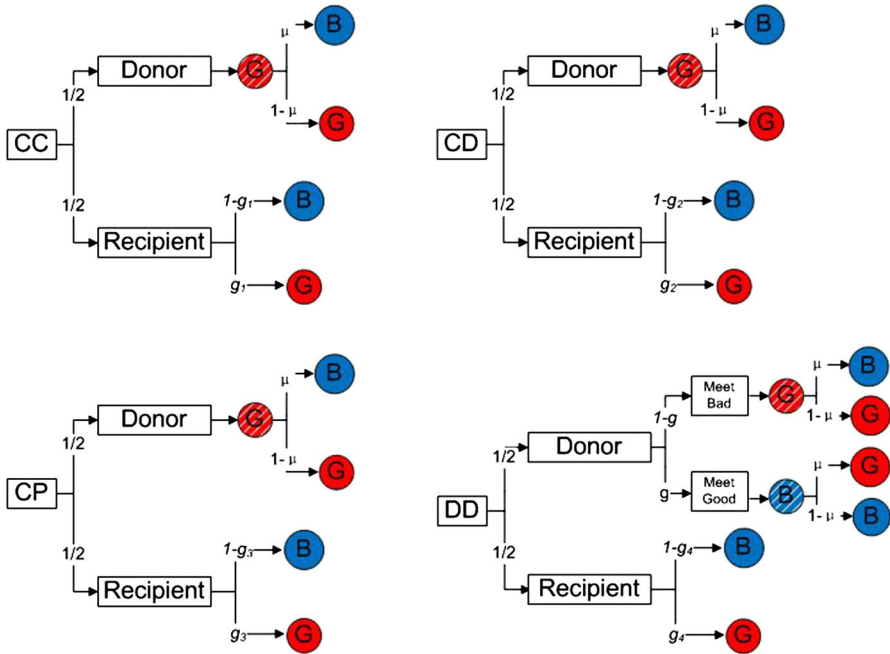
$$\begin{cases} g_1^* = 1 - \mu, \\ g_2^* = 1 - \mu, \\ g_3^* = (1 - \mu) \left[ 1 - \frac{1 - 2\mu}{1 + (1 - 2\mu)x_3} \right]. \end{cases} \quad (12)$$

The percentage of *good* people for the whole population is  $g^* = \frac{1 - \mu}{1 + (1 - 2\mu)x_3}$ .

## B.2 Weakly augmented social norm

For the weakly augmented social norm (*GGBGBG*), there are four strategies *CC*, *CD*, *CP* and *DD*, with corresponding frequencies denoted by  $x_1$ ,  $x_2$ ,  $x_3$  and  $x_4$ , and  $x_1 + x_2 + x_3 + x_4 = 1$ . The ratios of players with good reputation among *CC*, *CD*, *CP* and *DD* players are denoted by  $g_1$ ,  $g_2$ ,  $g_3$  and  $g_4$ , respectively. Thus the percentage of good players for the entire population is  $g = \sum_{i=1}^4 x_i g_i$ . The reputation dynamics of the four kinds of players is illustrated by Fig. 9.

By making the same inference as we did in B.1, it is easy to see that  $g'_i = \frac{1}{2}(1 - \mu) + \frac{1}{2}g_i$  ( $i = 1, 2, 3$ ), mainly because this weakly augmented norm will always assign the players who do *nice* things for *nice* guys a good reputation, and does not care what they do for the *bad* guys. Similarly, it is easy to see that the dynamics of  $g_4$  has the same form of  $g_3$  in the simple norm (11): defection toward nice guys shall be assigned



**Fig. 9** Reputation dynamics of individuals adopting different strategies under a weakly augmented social norm (GGBGBG)

a bad reputation. This similarity can also be seen by comparing Fig. 2 with Fig. 9. Hence, in sum, we have the following system of difference equations, (13).

$$\begin{cases} g'_1 = \frac{1}{2}(1 - \mu) + \frac{1}{2}g_1, \\ g'_2 = \frac{1}{2}(1 - \mu) + \frac{1}{2}g_2, \\ g'_3 = \frac{1}{2}(1 - \mu) + \frac{1}{2}g_3, \\ g'_4 = \frac{1}{2}(1 - g)(1 - \mu) + \frac{1}{2}g\mu + \frac{1}{2}g_4. \end{cases} \tag{13}$$

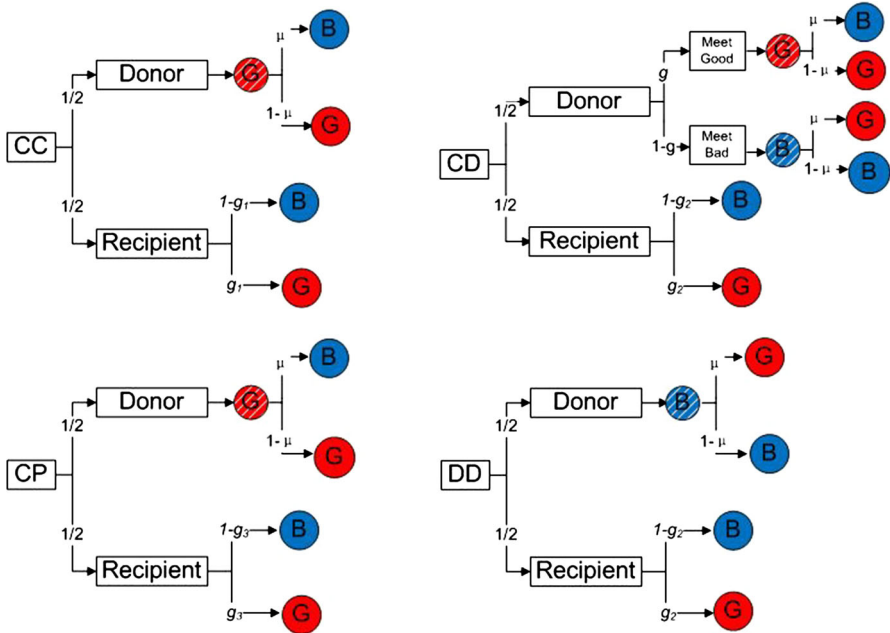
The steady-state solution for Eq. (13) is:

$$\begin{cases} g_1^* = 1 - \mu, \\ g_2^* = 1 - \mu, \\ g_3^* = 1 - \mu, \\ g_4^* = (1 - \mu) \left[ 1 - \frac{1-2\mu}{1+(1-2\mu)x_4} \right]. \end{cases} \tag{14}$$

Furthermore, the good reputation ratio for the entire society is  $g^* = \frac{1-\mu}{1+(1-2\mu)x_4}$ .

### B.3 Strongly augmented social norm

The same four strategies can also be operated by the strongly augmented social norm (GBBBBG), hence the notations used above (B.2) are kept here, including  $x_1, x_2, x_3$  and  $x_4$ , and  $g_1, g_2, g_3, g_4$  and  $g$ . The reputation dynamics of the four kinds of players is illustrated by Fig. 10.



**Fig. 10** Reputation dynamics of individuals adopting different strategies under a strongly augmented social norm (GGBBBG)

By comparing Fig. 10 with Fig. 9, one can see that the main differences between the two norms are their effects on the dynamics of  $g_2$  and  $g_4$  (the right half of the figures), corresponding to the percentage of *good* people among the *CD* and *DD* players. This is so because taking defection action toward *bad* recipient means that they shall no longer be assigned a good reputation. However, by reasoning along the tree depicted in Fig. 10, one can easily work out the new dynamics for  $g_2$  and  $g_4$ . They, together with  $g_1$  and  $g_3$ , are given in Eq. (15). Notice that the dynamics of  $g_1$  and  $g_3$  are the same as those in Eq. (13).

$$\begin{cases} g'_1 = \frac{1}{2}(1 - \mu) + \frac{1}{2}g_1 \\ g'_2 = \frac{1}{2}[(1 - \mu)g + \mu(1 - g)] + \frac{1}{2}g_2 \\ g'_3 = \frac{1}{2}(1 - \mu) + \frac{1}{2}g_3 \\ g'_4 = \frac{1}{2}\mu + \frac{1}{2}g_4 \end{cases} \tag{15}$$

The steady-state solution for the system of difference equations, (15), is shown in Eq. (16).

$$\begin{cases} g_1^* = 1 - \mu, \\ g_2^* = \mu + (1 - 2\mu)g^*, \\ g_3^* = 1 - \mu, \\ g_4^* = \mu. \end{cases} \tag{16}$$

The percentage of *good* people for the entire society is  $g^* = \frac{(1-\mu)(x_1+x_3)+\mu(x_2+x_4)}{1-(1-2\mu)x_2}$ .



### Appendix C: Expected payoff

In the main text, we have provided a simple introduction on how to calculate the expected payoff of strategies with respect to an arbitrary given strategy distribution using the example of the ‘*CC*’ strategy under the simple social norm (*GGBG*). The detailed process of calculating the expected payoff from all strategies for a given strategy distribution  $\{x_i\}_{i=1}^3$  or  $^4$  for all three social norms is provided here.

#### C.1 Simple social norm

For the simple social norm (*GGBG*), the calculation of the expected payoff of the *CC*, *CD* and *DD* strategies is illustrated in Fig. 3 in the main text. The expected payoff of a *CC* player is already given in Sect. 2.2.2. In the following, we only derive the expected payoffs of a *CD* and a *DD* player.

The expected payoff of a *CD* player differs from that of a *CC* player only in the case when they are donors. This is so because the *CD* player will cooperate only when the matched recipient is good; otherwise, there will be no donation (no cost). When they both become recipients the expected payoff is the same since the chance of their reputations being good are the same ( $g_1^* = g_2^* = 1 - \mu$ , Sect. 2.2.1). Therefore, the expected payoff of a player with strategy *CD* is  $p_2 = \frac{1}{2}g^*(-c) + \frac{1}{2}[bx_1 + bx_2(1 - \mu)]$ .

The *DD* player never donates. His payoff is, therefore, zero when he plays the role of donor. With a  $\frac{1}{2}$  chance, the *DD* player will also be a recipient. Based on the payoff trees shown in Fig. 3, one can see that the difference between the *CD* player and the *DD* player lies in the branch when they meet a *CD* player, who only donates to the good recipient. Hence, by simply replacing  $g_2^*$  with  $g_3^*$ , we can derive the expected payoff for the *DD* player,  $p_3 = \frac{1}{2}(0) + \frac{1}{2}[bx_1 + bx_2g_3^*]$ .

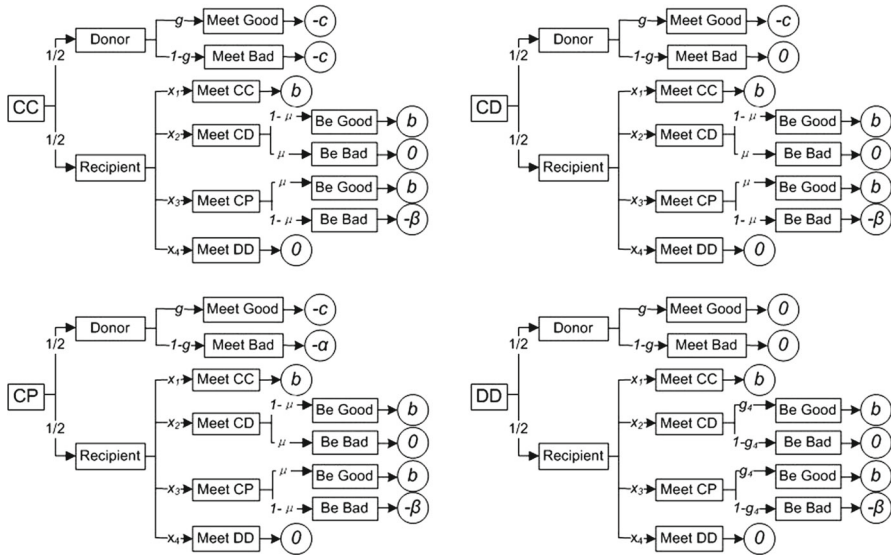
To sum up, the expected payoffs of all three strategies for the simple social norm (*GGBG*) are

$$\begin{cases} p_1 = \frac{1}{2}(-c) + \frac{1}{2}[bx_1 + bx_2(1 - \mu)], \\ p_2 = \frac{1}{2}g(-c) + \frac{1}{2}[bx_1 + bx_2(1 - \mu)], \\ p_3 = \frac{1}{2}(0) + \frac{1}{2}[bx_1 + bx_2g_3^*], \end{cases} \tag{17}$$

where  $g_3^* = (1 - \mu)[1 - \frac{1-2\mu}{1+(1-2\mu)x_3}]$ .

#### C.2 Weakly augmented social norm

The calculation of the expected payoff for the *CC*, *CD*, *CP* and *DD* strategies under the weakly augmented social norm (*GGBGBG*) can be facilitated using the payoff trees in Fig. 11. As for other similar ones, each of these trees has an upper branch (the donor branch, with a  $\frac{1}{2}$  chance) and a lower branch (the recipient branch, with another  $\frac{1}{2}$  chance). The donor branch has a simpler structure, namely, to donate or not. The recipient branch is more expansive because the payoff will depend on the strategies adopted by the matched donor and the reputation of the recipient himself. For the encounter with a *CC* or a *DD* player, this reputation is irrelevant, while it matters for the encounter with the *CD* or *CP* players. When reputation matters, the stable



**Fig. 11** Calculation of the expected payoff of strategies for the weakly augmented social norm (GGBGBG)

reputation distribution of each kind of player  $g_i^*$  ( $i = 1, 2, 3, 4$ ) (Sects. 2.2.1 and B.2) will be applied to figure out the expected treatment received by the respective recipient.

By harnessing the structure of the payoff tree as outlined above, one can skip all cumbersome algebra and easily see the expected payoff of the four strategies under the weakly augmented social norm (GGBGBG), which is summarized as follows.

$$\begin{cases} p_1 = \frac{1}{2}(-c) + \frac{1}{2}x_3\mu(-\beta) + \frac{1}{2}[bx_1 + b(x_2 + x_3)(1 - \mu)], \\ p_2 = \frac{1}{2}g^*(-c) + \frac{1}{2}x_3\mu(-\beta) + \frac{1}{2}[bx_1 + b(x_2 + x_3)(1 - \mu)], \\ p_3 = \frac{1}{2}g^*(-c) + \frac{1}{2}(1 - g^*)(-\alpha) + \frac{1}{2}x_3\mu(-\beta) + \frac{1}{2}[bx_1 + b(x_2 + x_3)(1 - \mu)], \\ p_4 = \frac{1}{2}[bx_1 + b(x_2 + x_3)g_4^*] + \frac{1}{2}x_3(1 - g_4^*)(-\beta), \end{cases} \tag{18}$$

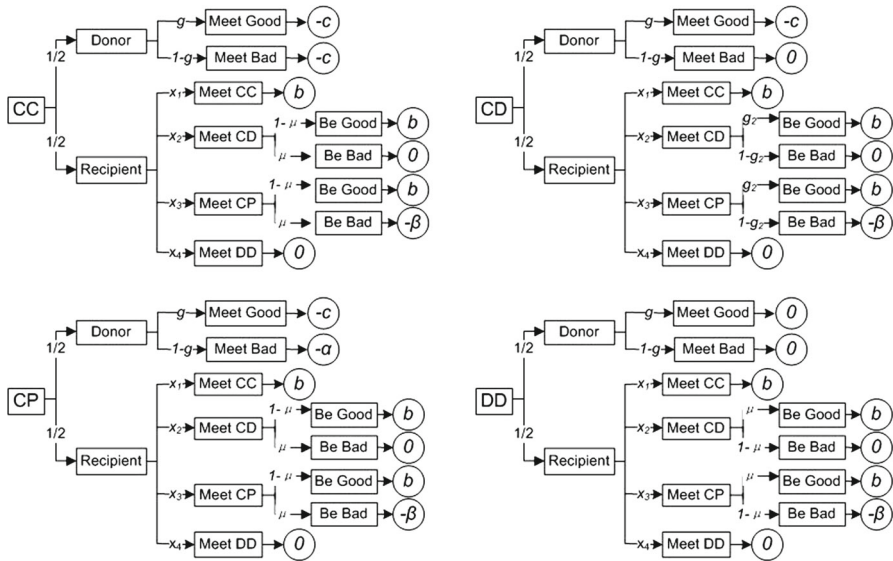
where  $g^*$  and  $g_4^*$  are given in Eq. (14).

### C.3 Strongly augmented social norm

For the strongly augmented social norm (GGBBBG), the calculation of the expected payoffs of the CC, CD, CP and DD strategies is facilitated by using Fig. 12. With the same basic understanding of these tree structures as described in C.2, one can easily figure out the expected payoffs as summarized in Eq. (19).

$$\begin{cases} p_1 = \frac{1}{2}(-c) + \frac{1}{2}x_3\mu(-\beta) + \frac{1}{2}[bx_1 + b(x_2 + x_3)(1 - \mu)], \\ p_2 = \frac{1}{2}g^*(-c) + \frac{1}{2}x_3(1 - g_2^*)(-\beta) + \frac{1}{2}[bx_1 + bg_2^*(x_2 + x_3)], \\ p_3 = \frac{1}{2}g^*(-c) + \frac{1}{2}(1 - g^*)(-\alpha) + \frac{1}{2}x_3\mu(-\beta) + \frac{1}{2}[bx_1 + b(x_2 + x_3)(1 - \mu)], \\ p_4 = \frac{1}{2}[bx_1 + b(x_2 + x_3)\mu] + \frac{1}{2}x_3(1 - \mu)(-\beta), \end{cases} \tag{19}$$

where  $g^*$  and  $g_2^*$  are given in Eq. (16).



**Fig. 12** Calculation of the expected payoff of strategies for the strongly augmented social norm (*GGBBBG*)

**Appendix D: Replicator dynamics for the augmented social norms**

In Sect. 2.2.3, we provide the full expression of the replicator dynamics for the three kinds of players under the simple social norm (*GGBG*). The full expressions of that under the other two social norms are quite lengthy and hence are articulated as follows.

Based on the normalization described in Sect. 2.2.3, the replicator dynamics associated with the weakly augmented social norm (*GGBBGG*) is

$$\left\{ \begin{array}{l}
 \dot{x}_1 = -cx_1 + cx_1^2 + \alpha x_1 x_3 \\
 + \left[ \begin{array}{l}
 [(1 - 2\mu)b + c]x_1 x_2 \\
 + [(1 - 2\mu)(b + \beta) + (c - \alpha)]x_1 x_3 \\
 - (1 - 2\mu)b[x_1 + x_2]x_1 x_2 \\
 + (1 - 2\mu)(b + \beta)[x_1 + x_3]x_1 x_3 \\
 + (1 - 2\mu)(2b + \beta)x_1 x_2 x_3
 \end{array} \right] \Big/ \left[ 2 - \frac{1 - 2\mu}{1 - \mu} \sum_{i=1}^3 x_i \right] \\
 \dot{x}_2 = -cx_1 x_2 + \alpha x_2 x_3 \\
 + \left[ \begin{array}{l}
 -cx_2 + [(1 - 2\mu)b + c]x_2^2 \\
 + [(1 - 2\mu)(b + \beta) + (c - \alpha)]x_2 x_3 \\
 - (1 - 2\mu)b[x_1 + x_2]x_2^2 \\
 - (1 - 2\mu)(b + \beta)[x_1 + x_3]x_2 x_3 \\
 - (1 - 2\mu)(2b + \beta)x_2^2 x_3
 \end{array} \right] \Big/ \left[ 2 - \frac{1 - 2\mu}{1 - \mu} \sum_{i=1}^3 x_i \right] \\
 \dot{x}_3 = -\alpha x_3 + cx_1 x_3 + \alpha x_3^2 \\
 + \left[ \begin{array}{l}
 -(c - \alpha)x_3 + [(1 - 2\mu)b + c]x_2 x_3 \\
 + [(1 - 2\mu)(b + \beta) + (c - \alpha)]x_3^2 \\
 - (1 - 2\mu)b[x_1 x_2 + x_2^2 + x_2 x_3]x_3 \\
 - (1 - 2\mu)(b + \beta)[x_1 x_3 + x_3^2]x_3 \\
 - (1 - 2\mu)(2b + \beta)x_2 x_3^2
 \end{array} \right] \Big/ \left[ 2 - \frac{1 - 2\mu}{1 - \mu} \sum_{i=1}^3 x_i \right]
 \end{array} \right. \quad (20)$$

The replicator dynamics associated with the strongly augmented social norm (GBBBBG) is

$$\left\{ \begin{array}{l}
 \dot{x}_1 = -cx_1 + cx_1^2 + [(1 - 2\mu)(b + \beta) + \alpha]x_1x_3 \\
 \quad + (1 - 2\mu)b[1 - x_1 - x_3]x_1x_2 - (1 - 2\mu)(b + \beta)[x_1 + x_3]x_1x_3 \\
 \quad - \left[ \begin{array}{l}
 -\frac{\mu c}{1-2\mu}x_1x_2 + \frac{\mu(\alpha-c)}{1-2\mu}x_1x_3 - cx_1^2x_2 \\
 +\mu bx_1x_2^2 + [\mu(b + \beta) - c]x_1x_2x_3 \\
 +(\alpha - c)[x_1 + x_3]x_1x_3 \\
 +(1 - 2\mu)b[x_1 + x_3]x_1x_2^2 \\
 +(1 - 2\mu)(b + \beta)[x_1 + x_3]x_1x_2x_3^2
 \end{array} \right] \Big/ \left[ \frac{1}{1 - 2\mu} - x_2 \right] \\
 \dot{x}_2 = cx_1^2 + \alpha x_2x_3 - (1 - 2\mu)bx_1x_2^2 \\
 \quad - (1 - 2\mu)(b + \beta)[x_1 + x_3]x_2x_3 - (1 - 2\mu)bx_2^2x_3 \\
 \quad + \left[ \begin{array}{l}
 -\frac{\mu c}{1-2\mu}x_2 - cx_1x_2 + \mu[b + \frac{c}{1-2\mu}]x_2^2 \\
 +[(\mu b + \mu\beta - c) + \frac{\mu(c-\alpha)}{1-2\mu}]x_2x_3 \\
 +[(1 - 2\mu)b + c]x_1x_2^2 - \mu bx_2^3 \\
 +[(1 - 2\mu)(b + \beta) + (c - \alpha)]x_1x_2x_3 \\
 +[(1 - 2\mu)b - \mu(b + \beta) + c]x_2^2x_3 \\
 +[(1 - 2\mu)b + (c - \alpha)]x_2x_3^2 \\
 -(1 - 2\mu)b(x_1 + x_3)x_2^3 \\
 -(1 - 2\mu)(b + \beta)(x_1 + x_3)x_2^2x_3
 \end{array} \right] \Big/ \left[ \frac{1}{1 - 2\mu} - x_2 \right] \\
 \dot{x}_3 = -\alpha x_3 + cx_1x_3 + [(1 - 2\mu)(b + \beta) + \alpha]x_3^2 \\
 \quad + (1 - 2\mu)b[1 - x_1 - x_3]x_2x_3 - (1 - 2\mu)(b + \beta)[x_1 + x_3]x_3^2 \\
 \quad - \left[ \begin{array}{l}
 \frac{\mu(c-\alpha)}{1-2\mu}x_3 - \frac{\mu c}{1-2\mu}x_2x_3 + \mu bx_2^2x_3 \\
 -(c - \alpha)x_1x_3^2 + [\mu(b + \beta) - c]x_2x_3^2 \\
 -(c - \alpha)x_3^3 + (c - \alpha)(x_1 + x_3)x_3 \\
 +(1 - 2\mu)b(x_1 + x_3)x_2^2x_3 - cx_1x_2x_3 \\
 +(1 - 2\mu)(b + \beta)(x_1 + x_3)x_2x_3^2
 \end{array} \right] \Big/ \left[ \frac{1}{1 - 2\mu} - x_2 \right]
 \end{array} \right. \quad (21)$$

## Appendix E: Proofs of propositions one and two

**Proposition 1** To prove Proposition 1, first, we realize that the origin,  $(x_1, x_2) = (0, 0)$  (for the simple and weakly augmented social norms) and  $(x_1, x_2, x_3) = (0, 0, 0)$  (for the strongly augmented social norm), is a fixed point of the differential equations (6), (20) and (21), respectively. Secondly, it can be shown that the eigenvalues of the Jacobian matrix ( $J$ ) of the above three differential equations evaluated at the origin are all negative. Hence, these origins are also locally stable.

The first point is straightforward. We simply evaluate the above three differential equations at their origins and then obtain  $\dot{x}_1 = \dot{x}_2 = 0$  under Eqs. (6) and (20) and  $\dot{x}_1 = \dot{x}_2 = \dot{x}_3 = 0$  under Eq. (21).

To see the second point, the Jacobian matrix ( $J$ ) of the three differential equations evaluated at the origin is given as follows.

$$J \Big|_{\substack{x_1=0 \\ x_2=0}} = \begin{bmatrix} -c & 0 \\ 0 & -c/2 \end{bmatrix} \tag{22}$$

$$J \Big|_{\substack{x_1=0 \\ x_2=0 \\ x_3=0}} = \begin{bmatrix} -c & 0 & 0 \\ 0 & -c/2 & 0 \\ 0 & 0 & -(c + \alpha)/2 \end{bmatrix} \tag{23}$$

$$J \Big|_{\substack{x_1=0 \\ x_2=0 \\ x_3=0}} = \begin{bmatrix} -c & 0 & 0 \\ 0 & -\mu c & 0 \\ 0 & 0 & -\mu c - (1 - \mu)\alpha \end{bmatrix} \tag{24}$$

Equations (22), (23) and (24) are the Jacobian matrix of the differential equations (6), (20) and (21), respectively. The eigenvalues of the three matrices are

$$\begin{aligned} \lambda_{1,1} &= -c, \lambda_{1,2} = -c/2, \\ \lambda_{2,1} &= -c, \lambda_{2,2} = -c/2, \lambda_{2,3} = -(c + \alpha)/2 \\ \lambda_{3,1} &= -c, \lambda_{3,2} = -\mu c, \lambda_{3,3} = -\mu c - (1 - \mu)\alpha, \end{aligned}$$

where  $\lambda_{i,j}$  denotes the the  $j$ th eigenvalue of the (D.i)th equation. It is easy to see that they are all negative given the sings of the parameters involved.  $\square$

*Proposition 2* To prove Proposition 2, we follow a similar procedure to that for the proof of Proposition 1. The only difference is that we no longer use the origin as the evaluation point. Instead,  $(x_1, x_2) = (0, 1)$ ,  $(x_1, x_2, x_3) = (0, 1, 0)$ , and  $(x_1, x_2, x_3) = (0, 0, 1)$  are chosen as the evaluation points for Eqs. (6), (20) and (21), respectively. The rest is the same. First, these points are fixed points under the respective dynamics, and, second, they are locally stable.

We skip the first as it is straightforward, and simply show the second by providing the Jacobian matrix of each differential equation by evaluating the chosen points. The three Jacobian matrices are

$$J \Big|_{\substack{x_1=0 \\ x_2=1}} = \begin{bmatrix} -\mu c & 0 \\ c - (1 - \mu)(1 - 2\mu)b & (1 - \mu)[c - (1 - 2\mu)b] \end{bmatrix} \tag{25}$$

$$J \Big|_{\substack{x_1=0 \\ x_2=1 \\ x_3=0}} = \begin{bmatrix} -\mu c & 0 & 0 \\ c - (1 - \mu)(1 - 2\mu)b & (1 - \mu)[c - (1 - 2\mu)b] & \mu\alpha + (1 - \mu)[c - (1 - 2\mu)b] \\ 0 & 0 & -\mu\alpha \end{bmatrix} \tag{26}$$

$$J \Big|_{\substack{x_1=0 \\ x_2=0 \\ x_3=1}} = \begin{bmatrix} -\mu(c - \alpha) & 0 & 0 \\ 0 & \mu\alpha - \mu(1 - 2\mu)(b + \beta) & 0 \\ c - \mu(1 - 2\mu)(b + \beta) & (1 - \mu)[c - \mu(1 - 2\mu)(b + \beta)] & (1 - \mu)c + \mu\alpha - (1 - 2\mu)(b + \beta) \end{bmatrix} \tag{27}$$

The eigenvalues of the three Jacobian matrices are given as follows:

$$\begin{aligned} \lambda_{4,1} &= -\mu c, \lambda_{4,2} = (1 - \mu)[c - (1 - 2\mu)b], \\ \lambda_{5,1} &= -\mu c, \lambda_{5,2} = (1 - \mu)[c - (1 - 2\mu)b], \lambda_{5,3} = -\mu\alpha, \\ \lambda_{6,1} &= -\mu(c - \alpha), \lambda_{6,2} = \mu\alpha - \mu(1 - 2\mu)(b + \beta), \\ \lambda_{6,3} &= (1 - \mu)c + \mu\alpha - (1 - 2\mu)(b + \beta) \end{aligned}$$

Among these eigenvalues, the signs of  $\lambda_{4,1}$ ,  $\lambda_{5,1}$ ,  $\lambda_{5,3}$ ,  $\lambda_{6,1}$  are obviously negative given our basic assumptions regarding the involved parameters. What, however, needs to be further delineated is  $\lambda_{4,2}$ , which requires  $\frac{1}{1-2\mu} \frac{c}{b} < 1$ ,  $\lambda_{5,2}$ , which requires  $\frac{1}{1-2\mu} \frac{c}{b} < 1$ ,  $\lambda_{6,3}$ , which requires  $\frac{(1-\mu)c+\mu\alpha}{(1-2\mu)(b+\beta)} < 1$ , and, finally,  $\lambda_{6,2}$ , which is automatically negative if  $\lambda_{6,3} < 0$ .  $\square$

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