Nonlocal condensate model for QCD sum rules

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A R T I C L E   I N F O

A B S T R A C T

We include effects of nonlocal quark condensates into QCD sum rules (QSR) via the Källén–Lehmann representation for a dressed fermion propagator, in which a negative spectral density function manifests their nonperturbative nature. Applying our formalism to the pion form factor as an example, QSR results are in good agreement with data for momentum transfer squared up to $Q^2 \approx 10$ GeV^2. It is observed that the nonlocal quark condensate contribution descends like $1/Q^2$, different from the exponential decrease in Q^2 obtained in the literature, and contrary to the linear rise in the local-condensate approximation.

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In QCD sum rules (QSR) nonperturbative contributions are taken into account via vacuum expectation values of nonlocal operators, such as $\langle \bar{q}(0)q(z) \rangle$ and $\langle G(0)G(z) \rangle$ [1], where q is a quark field and G is the gluon field strength. In the standard approach vacuum effects are assumed to be sufficiently soft to allow the Taylor expansion of, for instance, the quark condensate $\langle \bar{q}(0)q(z) \rangle$, at $z = 0$ by means of local composite operators,

$$\langle \bar{q}(0)q(z) \rangle = \langle \bar{q}q \rangle + z^{\mu} \langle \bar{q}\partial_{\mu}q \rangle + \frac{z^{\mu_1} z^{\mu_2}}{2} \langle \bar{q}\partial_{\mu_1}\partial_{\mu_2}q \rangle + \cdots . \quad (1)$$

A local condensate $\langle \bar{q}q \rangle$, i.e., the first term of the above expansion, prohibits momentum flow. A loop diagram then turns into a tree diagram as shown in Fig. 1, when inserting the local quark condensate into the lower (nonperturbative) line. The external momentum q flows only through the upper (perturbative) line, and one has the loop integral approximated by the product of the propagator $1/q^2$ and the condensate $\langle \bar{q}q \rangle$ [2]. With this localization assumption, simple hadronic properties including masses, decay constants, moments of hadronic wave functions, and form factors have been calculated in QSR.

It has been known that nonperturbative contributions from local quark condensates grow with the momentum transfer squared Q^2 in form factor calculations, whereas perturbative contributions decrease [3,4]. This is the reason why the standard QSR approach
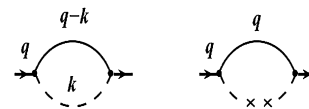
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Fig. 1. Loop diagram with the insertion of the local quark condensate, where k denotes the loop momentum.

encounters difficulty, when applied to form factors in the region with high $Q^2 > 3$ GeV^2 [5]. It has been observed that the Q^2 dependence of nonperturbative contributions is moderated by employing the nonlocal quark condensate $\langle \bar{q}(0)q(z) \rangle$ [5]. Moreover, using local quark condensates in QSR analysis of more complicated processes, such as Compton scattering [6] and the photon structure function in deeply inelastic scattering [7] which involve four-point correlation, infrared divergences appear. Consider the box diagram in Fig. 2, where a light hadron is scattered by an on-shell photon of momentum q_1. The external momentum q_1 flows through the upper horizontal quark line, when the local quark condensate is inserted into the left vertical quark line. The upper line then gives a divergent propagator proportional to $1/q_1^2 \to \infty$, and the evaluation of the Wilson coefficient associated with the quark condensate makes no sense. A resolution of the above difficulties is to relax the localization assumption. Including the nonlocal condensates, a finite loop momentum k is allowed to flow through the box diagram, and the above infrared divergence is smeared into $1/(q_1+k)^2$. This is our motivation to investigate effects of the nonlocal quark condensates in QSR. In this Letter we shall set up the framework by studying simpler processes like the pion form factor,
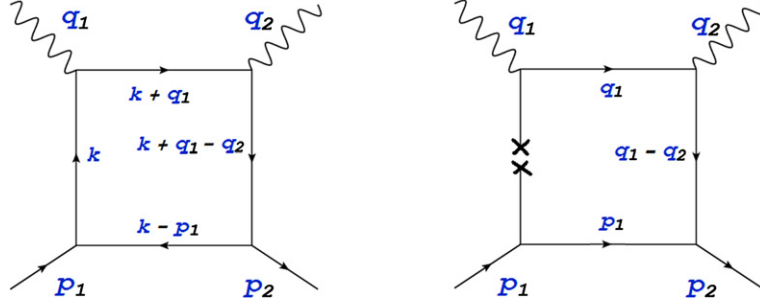
Fig. 2. Box diagrams for Compton scattering.

and compare the results with the local condensates and with the nonlocal condensates.

Nonlocal condensate models [8] have been applied to QSR for the pion wave function [9–12], whose outcome was then treated as an input of the perturbative QCD factorization formula for the pion form factor [13]. Recently, Bakulev, Pimikov and Stefanis calculated the space-like pion form factor based on QSR with nonlocal condensates [5]. They parameterized the nonlocal quark condensate as $\langle\bar{q}(0)q(z)\rangle = \langle\bar{q}q\rangle\exp(-|z^2|\lambda_q^2/8)$ [14], where λ_q^2 is related to the average virtuality of the condensed quarks. Our formalism is different, which starts from the Källén–Lehmann (KL) representation for a dressed propagator of the quark q [15],

$$\langle\Omega|T\big(q(z)\bar{q}(0)\big)|\Omega\rangle$$
$$= i\int\frac{d^4k}{(2\pi)^4}e^{-ik\cdot z}\int_0^\infty d\mu^2\frac{\slashed{k}\rho_1^q(\mu^2)+\rho_2^q(\mu^2)}{k^2-\mu^2+i\epsilon}, \tag{2}$$

where $|\Omega\rangle$ represents the exact QCD vacuum, T denotes the time ordering, the spectral density functions $\rho_{1,2}^q(\mu^2)$ describe the glutinous medium effect, and μ is the effective mass. The KL representation can be deemed as a superposition of free quark propagators for all mass eigenstates with the weights $\rho_{1,2}^q(\mu^2)$.

Eq. (2) is recast into

$$\langle\Omega|T\big(q(z)\bar{q}(0)\big)|\Omega\rangle = \frac{1}{16\pi^2}\int_0^\infty ds\exp\left(\frac{z^2}{4}s\right)\int_0^\infty d\mu^2\exp\left(-\frac{\mu^2}{s}\right)$$
$$\times\left[\frac{i\slashed{z}}{2}s\rho_1^q(\mu^2)+\rho_2^q(\mu^2)\right]. \tag{3}$$

We decompose the above matrix element into the perturbative and nonperturbative pieces

$$\langle\Omega|T\big(q(z)\bar{q}(0)\big)|\Omega\rangle \equiv iZS(z,m_q)+\langle\Omega|:q(z)\bar{q}(0):|\Omega\rangle, \tag{4}$$

respectively, with Z being a renormalization constant, $S(z,m_q)$ being the quark propagator in perturbation theory, m_q being the quark mass. The nonperturbative piece collects the contribution from large μ^2,

$$\langle\Omega|:q(z)\bar{q}(0):|\Omega\rangle = \frac{1}{16\pi^2}\int_0^\infty ds\exp\left(\frac{z^2}{4}s\right)\int_{\mu_c^2}^\infty d\mu^2\exp\left(-\frac{\mu^2}{s}\right)$$
$$\times\left[\frac{i\slashed{z}}{2}s\rho_1^q(\mu^2)+\rho_2^q(\mu^2)\right]. \tag{5}$$

The lower bound for the integration variable μ^2 is usually set to the multi-particle threshold m_γ^2 in the KL representation. Here we have modified it into

$$\mu_c^2 = \begin{cases} cs, & s > m_\gamma^2, \\ m_\gamma^2, & s \leqslant m_\gamma^2, \end{cases} \tag{6}$$

where the free parameter c of order unity will be fixed later. This modification respects the multi-particle threshold, and at the same time guarantees a finite integral in Eq. (5). Note that the integration over μ^2 in Eq. (5) develops a divergence as the variable s approaches infinity without the above modification. A negative spectral density function implies confinement [16], and we indeed have the property $\rho_1^q(\mu^2) < 0$ as shown in our formalism below.

We define the distribution functions

$$f_s(s) = \frac{-3}{4\pi^2\langle\bar{q}q\rangle}\int_{\mu_c^2}^\infty d\mu^2\exp\left(-\frac{\mu^2}{s}\right)\rho_2^q(\mu^2), \tag{7}$$

$$f_v(s) = \frac{3}{2\pi^2\langle\bar{q}q\rangle}\int_{\mu_c^2}^\infty d\mu^2\exp\left(-\frac{\mu^2}{s}\right)s\rho_1^q(\mu^2), \tag{8}$$

and parameterize the spectral density functions as

$$\rho_1^q(\mu^2) = N_1\exp(-a\mu^2)/\mu, \qquad \rho_2^q(\mu^2) = N_2\exp(-a\mu^2). \tag{9}$$

The choice of μ_c^2 in Eq. (6) then renders the integral in Eq. (7),

$$f_s(s) \propto \frac{s}{1+as}\exp\big(-\mu_c^2/s-a\mu_c^2\big), \tag{10}$$

exhibit the limiting behaviors $\exp(-m_\gamma^2/s)$ at small s and $\exp(-acs)$ at large s, consistent with $\exp(-m_\gamma^2/s)$ and the exponential ansatz $\exp(-\sigma_q s)$ postulated in the literature [12,17], respectively. Hence, the threshold mass m_γ is expected take a value of order of the constituent quark mass [20], and set to $m_\gamma \sim 0.36$ GeV[1] in this work. Comparing the Taylor expansion of the nonlocal quark condensates [2,17,21]

$$\langle\bar{q}(0)q(z)\rangle \equiv -\mathrm{Tr}\big[\langle\Omega|:q(z)\bar{q}(0):|\Omega\rangle\big]$$
$$= \langle\bar{q}q\rangle\left[1+\frac{z^2}{4}\left(\frac{\lambda_q^2}{2}-\frac{m_q^2}{2}\right)+\cdots\right],$$
$$\langle\bar{q}(0)\gamma_\mu q(z)\rangle \equiv -\mathrm{Tr}\big[\gamma_\mu\langle\Omega|:q(z)\bar{q}(0):|\Omega\rangle\big]$$
$$= -i\frac{z_\mu}{4}\langle\bar{q}q\rangle(m_q+\cdots) \tag{11}$$

with Eq. (5), we have the constraints

$$\int_0^\infty f_s(s)\,ds = 1, \qquad \int_0^\infty sf_s(s)\,ds = \frac{1}{2}\big(\lambda_q^2-m_q^2\big),$$

[1] There are other choices for the value of m_γ, for example, $m_\gamma \simeq 0.45$ GeV [12], $m_\gamma \simeq 0.50\pm 0.07$ GeV [18], $m_\gamma = 0.4$–0.6 GeV [19], etc.
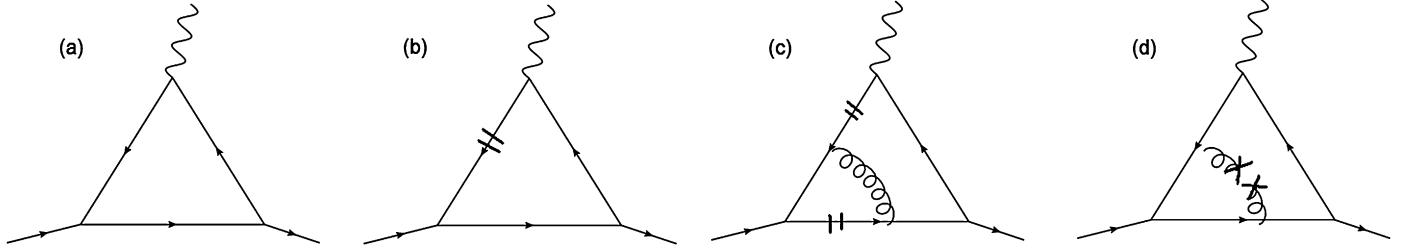
Fig. 3. (a) Perturbative contribution, (b) two-quark condensate contribution, (c) four-quark condensate contribution, and (d) gluon condensate contribution to the pion form factor.

$$\int\limits_0^\infty f_v(s)\,ds = m_q, \qquad (12)$$

which determine the free parameters a, N_1 and N_2 in Eq. (9), given values of λ_q and m_q.

The dressed propagator includes both the perturbative and non-perturbative contributions,

$$S^q(p) = \frac{\not{p} + m_q}{p^2 - m_q^2} - \frac{1}{2} i \frac{(\gamma^\alpha \not{p} \gamma^\beta G_{\alpha\beta} - m_q \gamma_\alpha G^{\alpha\beta} \gamma_\beta)}{(p^2 - m_q^2)^2}$$
$$- \frac{\pi \alpha_s \langle G_{\alpha\beta}^2 \rangle m_q \not{p}(m_q + \not{p})}{(p^2 - m_q^2)^4}$$
$$+ [\not{p} \hat{I}_1^q + \hat{I}_2^q] \frac{\exp[c(p^2 - \mu^2)/\mu^2]}{p^2 - \mu^2}, \qquad (13)$$

with the definitions

$$\hat{I}_{1,2}^q f(\mu) \equiv \int\limits_{\mu_c^2}^\infty d\mu^2 \, \rho_{1,2}^q(\mu^2) f(\mu). \qquad (14)$$

The second and third terms on the right-hand side of Eq. (13) arise from the background gluon field [22,23], and the forth term comes from the nonlocal quark condensates with the integrations over μ^2 and s being exchanged in Eq. (5). As stated before, local quark condensates lead to contributions linear in Q^2, which are more serious than the constant contributions from local gluon condensates at large Q^2 [3,4]. Gluon condensate contributions to the pion form factor are actually negligible. The contribution from the quark–gluon–antiquark condensate $\bar{q}Gq$ is smaller than that from Fig. 3(b) in our model, which is less than 5% of the four-quark condensate contribution. Therefore, only the nonlocal quark condensates are taken into account here.

Inserting Eq. (13) into the triangle diagrams for the three-point correlation function, we derive the perturbative and nonperturbative contributions to the pion form factor $F_\pi(Q^2)$,

$$-f_\pi^2 F_\pi(Q^2) \exp\left(-\frac{2m_\pi^2}{M^2}\right)$$
$$= \frac{1}{\pi^2} \left\{ \int\limits_0^{s_0} ds_1\, ds_2\, \rho^{pert}(s_1, s_2, Q^2) \exp\left(-\frac{s_1 + s_2}{M^2}\right) \right.$$
$$\left. + \Delta^{quark} + \Delta^{gluon} \right\}. \qquad (15)$$

In the above expression f_π is the pion decay constant, m_π is the pion mass, M is the Borel mass, and s_0 is the duality interval. The calculation of the spectral function ρ^{pert} associated with the perturbative contribution, and of the quark (gluon) condensate

contribution Δ^{quark} (Δ^{gluon}) involves four types of diagrams displayed in Fig. 3. The perturbative spectral function and the gluon condensate contribution are given by [3,4]

$$\rho^{pert} = \frac{N_c(e_d - e_u)}{2\lambda^{7/2}} Q^4 \{s_1(Q^2 + s_1)^3 + s_2(Q^2 + s_2)^3$$
$$- s_1 s_2 [2Q^4 + Q^2(s_1 + s_2) - 2(s_1^2 + s_2^2) + 6s_1 s_2]\}, \qquad (16)$$
$$\Delta^{gluon} = -\frac{\alpha_s}{12\pi M^2} \langle G_{\alpha\beta}^2 \rangle, \qquad (17)$$

respectively, with N_c being the number of colors, e_u (e_d) being the charge of the u (d) quark, and the variable

$$\lambda = (s_1 + s_2 + Q^2)^2 - 4s_1 s_2. \qquad (18)$$

We compute the quark condensate contribution, obtaining

$$\Delta^{quark} = \langle \bar{q}q \rangle \int\limits_0^{s_0} ds_1\, ds_2 \left[(e_u \hat{I}_1^u - e_d \hat{I}_1^d) \rho_{2qc}^a \right.$$
$$\left. + (e_u \hat{I}_1^d - e_d \hat{I}_1^u) \rho_{2qc}^b \right] \exp\left(-\frac{s_1 + s_2}{M^2}\right) + \alpha_s \langle \bar{q}q \rangle^2 \Delta_{4qc}^{quark}. \qquad (19)$$

The two-quark condensate spectral functions $\rho_{2qc}^{a,b}$ and the four-quark condensate function Δ_{4qc}^{quark} are written as

$$\rho_{2qc}^a = \frac{N_c}{\lambda^{7/2}} \left[Q^6 \mu^2 (Q^2 + \mu^2)^2 - (Q^4 - Q^2 \mu^2 + \mu^4)(s_1^4 + s_2^4) \right.$$
$$- Q^2(3Q^4 + 2Q^2\mu^2 - 5\mu^4)(s_1^3 + s_2^3)$$
$$- 3Q^2(Q^6 + 2Q^4\mu^2 - 2\mu^6)(s_1^2 + s_2^2)$$
$$- Q^4(Q^6 + 2Q^4\mu^2 + 4Q^2\mu^4 + 3\mu^6)(s_1 + s_2)$$
$$- 2(Q^4 + 2Q^2\mu^2 - 2\mu^4)(s_1^3 s_2 + s_2^3 s_1)$$
$$+ Q^2(Q^4 + 2Q^2\mu^2 - 5\mu^4)(s_1^2 s_2 + s_2^2 s_1)$$
$$+ 6(Q^4 + Q^2\mu^2 - \mu^4)s_1^2 s_2^2$$
$$\left. - 2Q^2(Q^6 + 5Q^4\mu^2 - 2Q^2\mu^4 - 6\mu^6)s_1 s_2 \right],$$
$$\rho_{2qc}^b = -\frac{N_c Q^4}{2\lambda^{7/2}} \left[2Q^2\mu^2(3Q^4 - 12Q^2\mu^2 + 10\mu^4) \right.$$
$$+ (s_1^4 + s_2^4) + 3(Q^2 - 2\mu^2)(s_1^3 + s_2^3)$$
$$+ 3(Q^4 - 2Q^2\mu^2 + 2\mu^4)(s_1^2 + s_2^2)$$
$$+ Q^2(Q^4 + 6Q^2\mu^2 - 18\mu^4)(s_1 + s_2)$$
$$+ 2(s_1^3 s_2 + s_2^3 s_1) - (Q^2 - 6\mu^2)(s_1^2 s_2 + s_2^2 s_1)$$
$$\left. - 6s_1^2 s_2^2 - 2(Q^4 - 12Q^2\mu^2 + 6\mu^4)s_1 s_2 \right], \qquad (20)$$

$$\Delta_{4qc}^{quark} = (e_u \hat{I}_2^u - e_d \hat{I}_2^d)$$

$$\times \lim_{m^2 \to 0} \frac{\partial}{\partial m^2} \left(\int_{m^2}^{s_0} ds_1 \int_{\alpha}^{s_0} ds_2 + \int_0^{m^2} ds_1 \int_0^{\alpha} ds_2 \right)$$

$$\times \left(e^{-s_1/M^2} - 1 \right) g_1 e^{-s_2/M^2}$$

$$+ (e_u \hat{I}_2^d - e_d \hat{I}_2^u)$$

$$\times \lim_{m^2 \to 0} \frac{\partial}{\partial m^2} \left(\int_{\mu^2}^{s_0} ds_1 \int_{\beta}^{s_0} ds_2 + \int_0^{\mu^2} ds_1 \int_0^{\beta} ds_2 \right)$$

$$\times \left(e^{-s_2/M^2} - 1 \right) g_2 e^{-s_1/M^2}, \tag{21}$$

with the functions

$$g_1 = \frac{8\pi N_c}{3\lambda^{5/2} s_1} \Big\{ Q^4 \big[-6m^4 - \mu^2(Q^2 + \mu^2) + m^2(4Q^2 + 6\mu^2) \big]$$

$$+ (Q^2 + \mu^2)s_1^3 + (Q^2 - \mu^2)s_2^3$$

$$- (2m^2 Q^2 - 2Q^4 - 3Q^2\mu^2 + \mu^4)s_2^2$$

$$- (2m^2 Q^2 - 2Q^4 - Q^2\mu^2 + \mu^4)s_1^2$$

$$+ Q^2 \big[Q^4 - Q^2\mu^2 - 2\mu^4 + 2m^2(Q^2 + 3\mu^2) \big] s_1$$

$$+ Q^2 \big[Q^4 + 3Q^2\mu^2 + 4\mu^4 + 2m^2(Q^2 - 3\mu^2) \big] s_2$$

$$- (Q^2 + 3\mu^2)s_1^2 s_2 - (Q^2 - 3\mu^2)s_2^2 s_1$$

$$+ 2s_1 s_2 (2m^2 Q^2 - Q^4 - 2Q^2\mu^2 + \mu^4) \Big\},$$

$$g_2 = \frac{8\pi N_c}{3\lambda^{5/2} s_2} \Big\{ Q^4 \big[Q^4 + m^4 - 6\mu^2(Q^2 - \mu^2)$$

$$+ 2m^2(Q^2 - 3\mu^2) \big] + (m^4 - 4m^2 Q^2 + Q^4)s_1^2$$

$$+ (Q^2 + m^2)^2 s_2^2$$

$$+ 2Q^2 s_2 (Q^2 + m^2)(Q^2 + m^2 - 3\mu^2)$$

$$- 2Q^2 s_1 \big[2m^4 - (Q^2 - m^2)(Q^2 - 3\mu^2) \big]$$

$$- 2s_1 s_2 (m^4 - m^2 Q^2 - 2Q^4) \Big\}, \tag{22}$$

and the variables

$$\alpha = (m^2 Q^2 + \mu^2 s_2) \left(\frac{1}{Q^2 + \mu^2} + \frac{1}{s_2 - m^2} \right),$$

$$\beta = (\mu^2 Q^2 + m^2 s_1) \left(\frac{1}{Q^2 + m^2} + \frac{1}{s_1 - \mu^2} \right). \tag{23}$$

Note that the singularity from $s_1 \to 0$ ($s_2 \to 0$) in the function g_1 (g_2) is removed by the factor $(e^{-s_1/M^2} - 1)$ $[(e^{-s_2/M^2} - 1)]$ in Eq. (21). It is observed that the contributions from the nonlocal quark condensates must be power-like in Q^2 in the asymptotic limit, no matter how to parameterize $\rho_{1,2}^q(\mu^2)$. The dominant contribution Δ_{4qc}^{quark} descends like $1/Q^2$ as $Q^2 \to \infty$, which is different from the exponential decrease in Q^2 obtained in [5], and contrary to the linear rise in the local condensate approximation [3,4].

A remark is in order. As calculating condensate contributions in the conventional QSR approach, the upper bound of the integration variable s is usually extended to infinity. In our formalism both the perturbative and condensate contributions are calculated in the same framework with the dressed quark propagators. Hence, it is more natural to parameterize the continuum contribution to the spectral function on the hadronic side of the sum rule as that on the operator-product-expansion side for $s > s_0$, which includes the condensate terms. After cancelling the continuum contributions from both sides of the sum rule, the upper bound s_0 appears in Eq. (19). This is a difference between our formalism and the conventional QSR approach.

The local condensates appearing in Eqs. (17) and (19) are taken to be [24]

$$\frac{\alpha_s}{\pi} \langle G_{\alpha\beta}^2 \rangle = 0.005 \pm 0.004 \text{ GeV}^2,$$

$$\langle \bar{q}q \rangle = -(1.65 \pm 0.15) \times 10^{-2} \text{ GeV}^3,$$

$$\alpha_s \langle \bar{q}q \rangle^2 = (1.5 \pm 0.2) \times 10^{-4} \text{ GeV}^6. \tag{24}$$

The duality interval $s_0(Q^2)$ at a given Q^2 is determined by the requirement that the form factor is least sensitive to the Borel mass M. The average virtuality λ_q and lower bound c, being not known with certainty, are fixed by fits to the data of the pion form factor $F_\pi(Q^2) = 0.179 \pm 0.021$ at $Q^2 = 1.99 \text{ GeV}^2$ [25–28]. In Fig. 4(a) we display the allowed values of c and λ_q as a curve in the c–λ_q plane. The range of λ_q is consistent with $\lambda_q = 0.63$ GeV from QSR [29] and $\lambda_q = 0.85$ GeV from the instanton analysis [30]. Below we adopt $\lambda_q = 0.75$ GeV and $c = 0.3$ to produce the central values of our predictions for the pion form factor. Choosing the light quark masses $m_u = 4.2$ MeV and $m_d = 7.5$ MeV, we solve for the free parameters a, N_1 and N_2 from the constraints in Eq. (12), whose results are listed in Table 1. The product $ac \approx 6.8 \text{ GeV}^{-2}$ is in agreement with the value of $\sigma_q \approx 10 \text{ GeV}^{-2}$ postulated in [31]. The opposite signs of N_1 and N_2 imply the violation of positivity, which can be interpreted as a manifestation of confinement [16].

Fig. 4(b) indicates the best choice of $s_0 = 0.715 \text{ GeV}^2$ with $\lambda_q = 0.75$ GeV and $c = 0.3$, at which the pion form factor $F_\pi(Q^2 = 1.99 \text{ GeV}^2)$ becomes independent of M for $M > 1.5$ GeV. In the calculation below, we simply set the Borel mass to $M = 1.5$ GeV. In Fig. 4(c) we present the Q^2 dependence of the best choice $s_0(Q^2)$ with the same inputs, whose curve is close to a straight line:

$$s_0(Q^2) = 0.6 + 0.06Q^2 - 0.0014Q^4, \tag{25}$$

for $Q^2 > 1 \text{ GeV}^2$. It is seen that s_0 drops rapidly in the region of low $Q^2 < 1 \text{ GeV}^2$, where QSR are supposed to be inapplicable. s_0 in Fig. 4(c), increasing from 0.65 GeV^2 to 1.05 GeV^2 for $1 \text{ GeV}^2 < Q^2 < 10 \text{ GeV}^2$, shows a bit stronger Q^2 dependence compared to that in [5]. Nevertheless, its range obeys the postulation [5] that it should not be lower than the middle point 0.6 GeV^2 of the interval between the meson masses $m_\pi^2 = 0$ and $m_{A_1}^2 = 1.6 \text{ GeV}^2$. Besides, we have confirmed that the pion decay constant squared takes the value $f_\pi^2 \simeq 0.0171 \text{ GeV}^2$ for $s_0 \approx 0.7 \text{ GeV}^2$ in our formalism with the nonlocal quark condensates.

Our results of the pion form factor $F_\pi(Q^2)$ are displayed in Fig. 5(a) for three values of $\lambda_q = 0.8$, 0.75, and 0.7 GeV with $c = 0.3$, corresponding to the curves from top to bottom, respectively. Their difference indicates the theoretical uncertainty of our analysis. It is obvious that all three curves are well consistent with the experimental data for $Q^2 > 1 \text{ GeV}^2$, the region where QSR are applicable. Inputting a smaller value of $\lambda_q \simeq 0.63$ GeV [5,29] into our formalism directly, a curve lower than the data is obtained as shown in Fig. 5(a). However, if increasing the parameter c accordingly up to $c = 0.39$ for this different λ_q, the result will become consistent with the data. We investigate the perturbative and condensate contributions to the pion form factor $F_\pi(Q^2 = 1.99 \text{ GeV}^2)$ at different Borel mass M, as exhibited in Fig. 5(b). It is observed that the former increases with M, and the latter decreases with M for $M > 1$ GeV. The gluon condensate contribution becomes negligible for $M > 1$ GeV, justifying the sole modification from the nonlocal quark condensates. Although the magnitudes of different contributions vary with M, their sum is almost constant

Table 1
Parameters associated with the quarks u and d in our formalism.

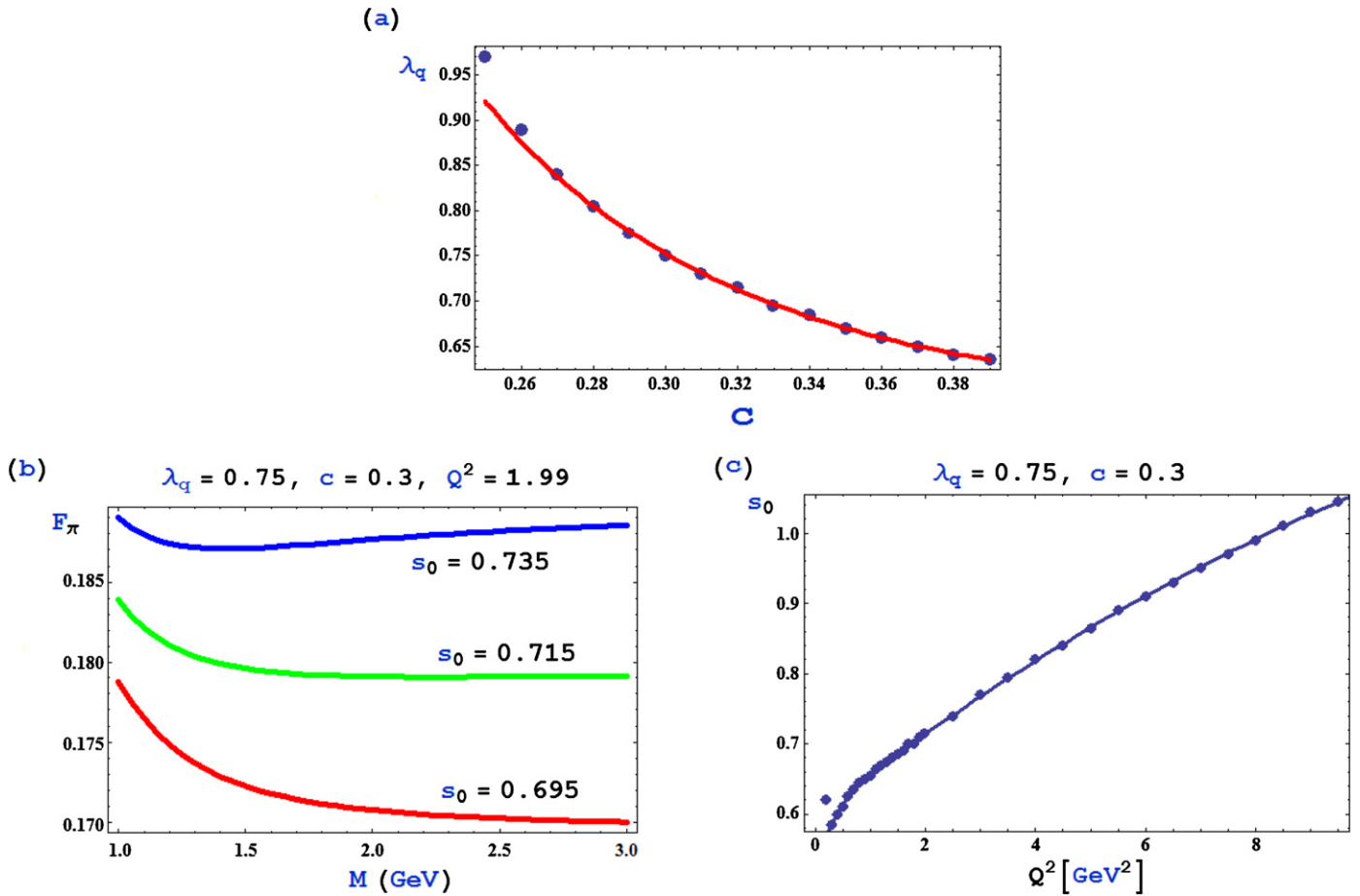|	λ_q (GeV)	m_q (MeV)	a (GeV^{-2})	$N_1/\langle \bar{q}q \rangle$ (GeV^{-4})	$N_2/\langle \bar{q}q \rangle$ (GeV^{-4})
u quark	0.75	4.2	22.7	20.54	−7784.54
d quark	0.75	7.5	22.7	36.70	−7789.33

Fig. 4. (a) Curve for the allowed values of c and λ_q that produce the data of $F_\pi(Q^2 = 1.99 \text{ GeV}^2)$. (b) M dependence of $F_\pi(Q^2 = 1.99 \text{ GeV}^2)$ for different s_0 with $\lambda_q = 0.75$ GeV and $c = 0.3$. (c) Q^2 dependence of s_0 for the pion form factor. The function of the fitting curve is presented in Eq. (25).
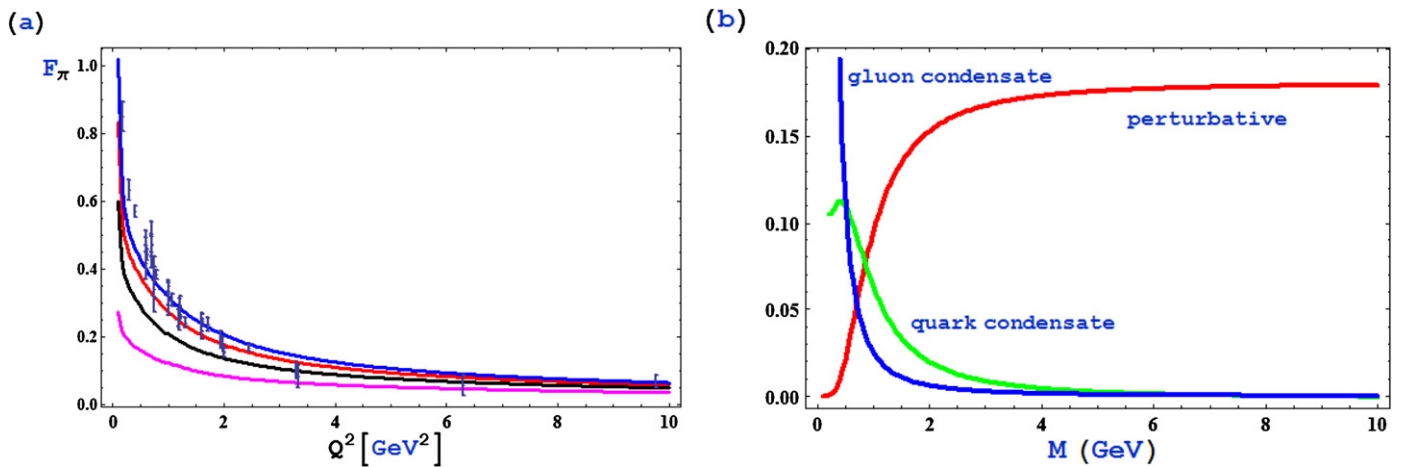

Fig. 5. (a) Q^2 dependence of F_π for, from top to bottom, $\lambda_q = 0.8$, 0.75, 0.7 and 0.63 GeV with $c = 0.3$. The data points are referred to [25–28]. (b) M dependence of the perturbative and condensate contributions to $F_\pi(Q^2 = 1.99 \text{ GeV}^2)$.

for $M > 1.5$ GeV. The quark condensates contribute 23% of the pion form factor $F_\pi(Q^2 = 1.99$ GeV$^2)$ at $M = 1.5$ GeV, which is slightly higher than the percentage 17% in the localization approximation [3].

In this Letter we have included the nonlocal quark condensates into QSR via the KL parametrization for a dressed fermion propagator, which is decomposed into the perturbative and nonperturbative pieces. The negative spectral density function implies that the contribution from higher effective quark masses is nonperturbative. The parametrization of the spectral density functions leads to the known exponential ansatz for the nonlocal condensate model in our formalism. We have analyzed the pion form factor as an example, and the results are in good agreement with the data for Q^2 between 1–10 GeV^2. The fitted ranges of the virtuality λ_q and of the duality interval $s_0(Q^2)$, presented in Fig. 4, are also consistent with those reported in the literature. The nonlocal quark condensates remedy the improper dependence of the nonperturbative contribution in the localization approximation at large Q^2: the quark condensate effects decrease like $1/Q^2$, which is different from the exponential decrease obtained in the literature. Viewing the success of this approach to the pion form factor, we shall extend it to more complicated processes, including Compton scattering and two-photon hadron production [32].

Acknowledgements

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