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Investors' preference order of fuzzy numbers

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Abstract

Nowadays greater and greater realistic financial problems are modeled by using the stochastic programming in the fuzzy environment. Hence, ranking a set of fuzzy numbers that is consistent with the investors' preference becomes important for modelling a realistic problem. In this paper, we will provide a new ranking procedure that is consistent with the preference of the conservative investors. Our ranking procedure satisfies the axioms of three order relations for the separable fuzzy numbers or the triangle fuzzy numbers. We found that our ranking procedure has a better capability of discriminating the order of two fuzzy numbers. For the LR-type fuzzy numbers, our ranking procedure reduces the computational time substantially.

Keywords: Fuzzy ranking criterion; Fuzzy ranking procedure; Fuzzy numbers; Preference; Conservative investor

1. Introduction

A stochastic programming is called a fuzzy stochastic programming if the value in some scenarios is modeled as a fuzzy variable. To model a financial problem by using stochastic programming, we need to predict the possible outcomes for each scenario. It is unreasonable to estimate the future possible outcome as an exact value since the real value in each scenario cannot be predicted at sight. However, we could predict that the possible outcome of the scenarios is an interval with a possibility and model the financial problem as a fuzzy stochastic programming problem. To find out an optimal solution for the fuzzy stochastic programming, we first need to provide a ranking rule for the fuzzy numbers. The ranking rule is consistent with the investors' preference for a realistic financial problem.

Over the past two decades, a great deal of effort [1–12] has been made on ranking a set of fuzzy numbers. However, most methods are not developed for considering the financial problems. This implies that the fuzzy numbers ranked by these methods are not consistent with the preference of the investors. For example, two triangle fuzzy numbers cited by [2], $A_1 = (94/35, 46/7, 46/7, 10)$ and $A_2 = (2, 7, 7, 9)$ are given below in Fig. 1.

Chen and Lu [9] pointed out that these two fuzzy numbers ranked by the approaches of area measurement [3,7] are indifferent. However, a rational investor always prefers a portfolio with a greater possibility of higher expected rate of return. A rational investor, therefore, prefers A_2 to A_1 because A_2 has a greater possibility of the higher expected rate of return when these two fuzzy numbers are the values of the portfolios' rate of return.

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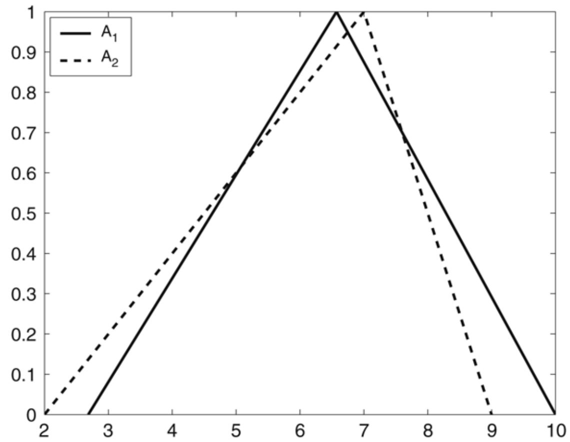

Fig. 1. The triangle fuzzy numbers cited by [2].

In addition, to coincide with the preference of the investors, Zhu and Lee [12] proposed that the complexity is one of the important criteria for the ranking method. Most methods [6–11] need to integrate the membership function of the fuzzy number. These ranking approaches will be used to give an order for the feasible solutions of a fuzzy stochastic programming. However, these complex ranking approaches will increase the complexity for solving the fuzzy stochastic programming problem. Therefore, providing a simple ranking procedure is important for solving a fuzzy stochastic programming.

A conservative investor may be a person who has an adversity toward the portfolio where the system cannot effectively predict the future movement. In this paper, we will provide a ranking procedure which is consistent with the preference of the conservative investors. Our ranking procedure, which does not need to integrate the membership function of a fuzzy number, reduces the computational time substantially. Moreover, the separable fuzzy numbers or the triangle fuzzy numbers ordered by our ranking procedure satisfies the axioms of the order relations. Finally, compared with the existing approaches, we found that our procedure provides a better capability of discriminating the order of two fuzzy numbers, especially for the case that the membership function of a fuzzy number is not integrable.

The rest of this paper is organized as follows. In Section 2, we propose a fuzzy stochastic programming model for a portfolio selection problem. This model is a motivation for providing a ranking approach which is consistent with the preference of the investors. A ranking criterion and a ranking procedure are provided in Section 3. We show that our ranking approaches satisfy the axioms for the order relations for the separable fuzzy numbers and the triangle fuzzy numbers. In Section 4, we compare our ranking procedure with the existing approaches and find that the fuzzy numbers ranked by our procedure are consistent with the conservative investor's preference. In Section 5, we will provide a concise conclusion and the directions for future studies.

2. Motivation

In this section, we will consider a portfolio selection problem. Suppose that there are N distinct tradable assets in the market and model this market on a single period scenario tree. The terminal rate of return for asset i, denoted as R_i, is assumed as a fuzzy random variable. Roughly speaking, an interval equipped with a possibility is called a fuzzy variable and a random variable is called a fuzzy random variable if its outcomes are fuzzy variables. The formal definitions of fuzzy numbers and fuzzy random variables are listed in the Appendix.

An investor constructs a portfolio by holding y_i stocks of asset i. The portfolio's terminal rate of return, which is calculated by

$$\sum_{i=1}^{N} y_i R_i,$$

is also a fuzzy random variable. The goal of a portfolio selection problem is to select an optimal trading strategy $\mathbf{y} = (y_1, y_2, \ldots, y_N)$, which maximizes the expected rate of return for the portfolio. The expected rate of return for a portfolio is always measured by the expectation. As a result, the portfolio selection problem can be modeled as the following fuzzy stochastic program:

$$\max_{\mathbf{y}} \mathrm{E}\left[\sum_{i=1}^{N} y_i R_i\right]$$

for some given constraints. By usual definition, the expectation of a fuzzy random variable is a fuzzy variable. In order to solve the above fuzzy stochastic program, we will provide a rule for ranking a set of fuzzy numbers in the next section.

3. Preference order of conservative investor

Each individual's investment decision is characterized as if he will determine the investment policy to maximize the expected rate of return. However, the investors will have different preference for the optimal policies when the future expected rate of return is uncertain. We now consider the preference of a conservative investor for the uncertain future expected rate of return.

Each investor can be classified as either risk-seeking or conservative. A conservative investor may be a person who has an adversity toward the portfolio where the system cannot effectively predict the future movement. More precisely, the adversity comes from future downward movement of the return. Therefore, a conservative investor prefers a portfolio without possibility to get the lower expected rate of return. On the other hand, the upward movement of the return is a benefit to the investors. Hence, the conservative investor is attracted by the portfolio with a nonzero possibility of the higher expected rate of return.

Therefore, when the uncertainty of the expected rate of return is described by a fuzzy number, a conservative investor prefers a fuzzy number which satisfies the following conditions.

Preference of a conservative investor

(1) The value with possibility 1 is as great as possible.
(2) The possibility of the small value occurred is as small as possible.
(3) The possibility of the large value occurred is nonzero.

Before proposing our ranking procedure to the fuzzy numbers, we first consider the relation between a fuzzy number and a real number.

Definition 3.1. For $F \in \mathcal{F}(F)$, $a \in \mathbb{R}$, and $\alpha \in [0, 1]$

(1) $F \leq_\alpha a$ if and only if $\max_{x\in(-\infty,a]} \mu_{\tilde{F}}(x) \geq \alpha$.
(2) $F \geq_\alpha a$ if and only if $\max_{x\in(a,\infty]} \mu_{\tilde{F}}(x) \geq \alpha$.

Example 1. Let $F = (-1, 0, 0, 1)$ be a triangle fuzzy number. Given $\alpha = 0.8$, we have $F \geq_{0.8} a$ for all $a \in (-\infty, 0.2]$ and $F \leq_{0.8} a$ for all $a \in [-0.2, \infty)$.

In order to capture the preference of a rational investor, the fuzzy numbers could be ranked by the following criterion.

Ranking criterion. *For $F_1, F_2 \in \mathcal{F}(F)$, we say $F_1 \succ F_2$ if there exists a real number a such that $F_1 >_1 a$ and $a >_1 F_2$. If there do not exist a real number a such that $F_1 >_1 a$ and $a >_1 F_2$, these two fuzzy numbers are called indifference, denoted as $F_1 \approx F_2$.*

Axioms for the order relations. For any F_1, F_2 and F_3 in $\mathcal{F}(F)$,

(reflexive law) $F_1 \preceq F_1$.

(antisymmetric law) If $F_1 \preceq F_2$ and $F_2 \preceq F_1$, then $F_1 \approx F_2$.

(transitive law) If $F_1 \preceq F_2$ and $F_2 \preceq F_3$, then $F_1 \preceq F_3$.

Definition 3.2. A set of fuzzy numbers is called separable if for any two fuzzy numbers F_1 and F_2 there is a real number a such that $F_1 <_1 a$ and $a <_1 F_2$.

Theorem 3.3. *Let $\mathcal{F}(F)$ be a set of separable fuzzy numbers. Then the ordered set $\langle \mathcal{F}(F), \succeq \rangle$ satisfies the axioms for the order relations. Furthermore, $\succeq$ is a linear ordering on $\mathcal{F}(F)$.*

Proof. It suffices to show the transitive law. Let F_1, F_2 and F_3 be three fuzzy numbers in $\mathcal{F}(F)$ with $F_1 \prec F_2$ and $F_2 \prec F_3$. This implies that there exist $a, b \in \mathbb{R}$ such that $F_1 <_1 a$, $a <_1 F_2$ and $F_2 <_1 b$, $b <_1 F_3$. Hence, we have $a < b$ and $F_1 <_1 b$. Consequently, $F_1 \prec F_3$. □

In Section 4, we will demonstrate several examples which point out that some approaches of area measurement may not work for the separable fuzzy numbers. This implies that our ranking criterion provides a more powerful method than the approaches of area measurement for ranking a set of separable fuzzy numbers.

Unfortunately, this ranking criterion dose not work for the nonseparable set of fuzzy numbers. For example, when the possibility 1 of two fuzzy numbers occurs at the same value, the two fuzzy numbers are indifference by using the ranking criterion. However, these two fuzzy numbers play different roles for a conservative investor if the possibility 0 of these two occurs at the different area.

In order to overcome the disadvantage while remaining consistent with the preference of conservative investors, we now provide a procedure to rank two fuzzy numbers. For any fuzzy number F with membership function μ_F, we first extend the membership function to all the real line by

$$\tilde{\mu}_F(x) = \begin{cases} \mu_F(x) & \text{if } \mu_F(x) \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

Ranking procedure. *For $F_1, F_2 \in \mathcal{F}(F)$, we say $F_1 \succ F_2$ if they satisfy the following ranking procedure:*

if

There exists a real number a such that $F_1 >_1 a$ and $a >_1 F_2$;

else if

$$\inf\{x \in \mathbb{S} | \tilde{\mu}_{F_1}(x) = \tilde{\mu}_{F_2}(x)\} > \inf\{x \in \mathbb{S} | \tilde{\mu}_{F_1}(x) - \tilde{\mu}_{F_2}(x) \leq 0\};$$

else if

$$\sup\{x \in \mathbb{S} | \tilde{\mu}_{F_1}(x) - \tilde{\mu}_{F_2}(x) \geq 0\} > \sup\{x \in \mathbb{S} | \tilde{\mu}_{F_1}(x) = \tilde{\mu}_{F_2}(x)\}.$$

Here, $\mathbb{S} = \{x | \mu_{F_1}(x) > 0\} \cup \{x | \mu_{F_2}(x) > 0\}$.

Note that two fuzzy numbers F_1 and F_2 are called indifference, if the equivalence relation holds for all the three criteria in our ranking procedure.

Proposition 3.4. *The ranking procedure is consistent with the preference of the conservative investors.*

Proof. Let F_1 and F_2 be two fuzzy numbers with $F_1 \succ F_2$, then one of the following cases must be held.

(1) If there exists a real number a such that $F_1 >_1 a >_1 F_2$, this implies that the value with possibility 1 of F_1 is greater then the the value with possibility 1 of F_2.
(2) If $\inf\{x \in \mathbb{S} | \tilde{\mu}_{F_1}(x) = \tilde{\mu}_{F_2}(x)\} > \inf\{x \in \mathbb{S} | \tilde{\mu}_{F_1}(x) - \tilde{\mu}_{F_2}(x) \leq 0\}$, this implies that F_2 has a greater possibility of small value.
(3) If $\sup\{x \in \mathbb{S} | \tilde{\mu}_{F_1}(x) - \tilde{\mu}_{F_2}(x) \geq 0\} > \sup\{x \in \mathbb{S} | \tilde{\mu}_{F_1}(x) = \tilde{\mu}_{F_2}(x)\}$, this implies that F_1 has a greater possibility of greater value.

Hence, we found that the ranking procedure is consistent with the preference of the conservative investors. □

When the fuzzy numbers are given as LR-type, the ranking procedure can be simplified as follows.

Ranking procedure for LR-type fuzzy number. *For $F_1 = (l_1, c_1, d_1, r_1)$, $F_2 = (l_2, c_2, d_2, r_2)$, we say $F_1 \succ F_2$ if they satisfy the following ranking procedure:*

if $c_1 > d_2$; else if $l_1 > l_2$; else if $c_1 > c_2$; else if $r_1 > r_2$; else if $d_1 > d_2$.

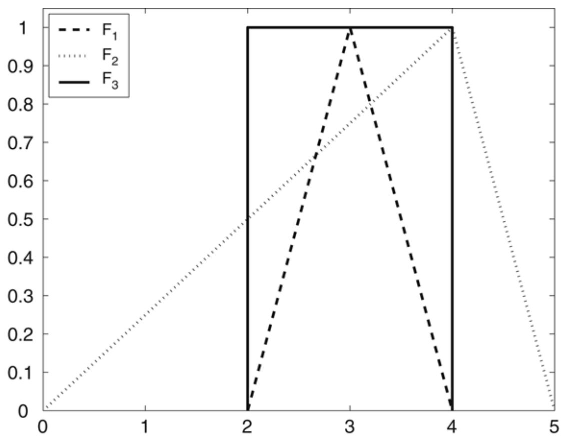

Fig. 2. The preference of the conservative investors does not satisfy the transitive law.

Remark 3.5. Ranking two LR-type fuzzy numbers by the above ranking procedure, we do not calculate the integral value of their membership function and only compare the peaks and the spreads of two fuzzy numbers. Moreover, the ranking procedure is consisted only of five logical expressions, which can be calculated by hand. So, compared with the existing approaches that calculate the integral value of the membership function, our ranking procedure reduces the computational time substantially for the LR-type fuzzy numbers.

Example 2. We consider the following three fuzzy variables $F_1 = (2, 3, 3, 4)$, $F_2 = (0, 4, 4, 5)$ and $F_3 = (2, 2, 4, 4)$ (see Fig. 2.) Applying our ranking procedure, we have $F_1 \preceq F_2$, $F_2 \preceq F_3$ and $F_3 \preceq F_1$. This does not satisfy the transitive law, but is consistent with the preference of the conservative investors. For F_1, F_2, the investors prefer F_2 to F_1, because F_2 has a greater possibility to attain the higher expected rate of return. For F_2 and F_3, the investors prefer F_3 to F_2 because F_2 has a possibility to attain the lower expected rate of return. For F_1 and F_3, they have the same possibility to attain the same expected rate of return. However, F_3 has a greater possibility to attain the lower expected rate of return. Therefore, the conservative investors prefer F_1 to F_3.

Theorem 3.6. *Let $\mathcal{F}(F)$ be a set of triangle fuzzy numbers. Then the ordered set $\langle \mathcal{F}(F), \succeq \rangle$ satisfies the axioms for the order relation.*

Proof. It suffices to show the transitive law. Let $F_1 = (l_1, c_1, r_1)$, $F_2 = (l_2, c_2, r_2)$ and $F_3 = (l_3, c_3, r_3)$ be three triangle fuzzy numbers with $F_1 \prec F_2$ and $F_2 \prec F_3$. We now verify the following three cases:

case $c_1 < c_2$:
 If $c_2 \leq c_3$ then $c_1 < c_3$.
case $c_1 = c_2$ and $l_1 < l_2$:
 If $c_2 < c_3$ then $c_1 < c_3$.
 If $c_2 = c_3$ and $l_2 \leq l_3$ then $c_1 = c_3$ and $l_1 < l_3$.
case $c_1 = c_2$, $l_1 = l_2$ and $r_1 < r_2$:
 If $c_2 < c_3$ then $c_1 < c_3$.
 If $c_2 = c_3$ and $l_2 < l_3$ then $c_1 = c_3$ and $l_1 < l_3$.
 If $c_2 = c_3$, $l_2 = l_3$ and $r_2 < r_3$ then $c_1 = c_3$, $l_1 = l_3$ and $r_1 < r_3$.

This implies that $F_1 \prec F_3$. □

4. Comparative examples

1. Two triangle fuzzy numbers cited by [2], $A_1 = (94/35, 46/7, 46/7, 10)$ and $A_2 = (2, 7, 7, 9)$. There exists a real number 6.7 such that $A_1 \leq_1 6.7$ and $6.7 \leq_1 A_2$. By our ranking criterion, we have $B_1 \preceq B_2$. However, these two fuzzy numbers are indifference by using area measurement.

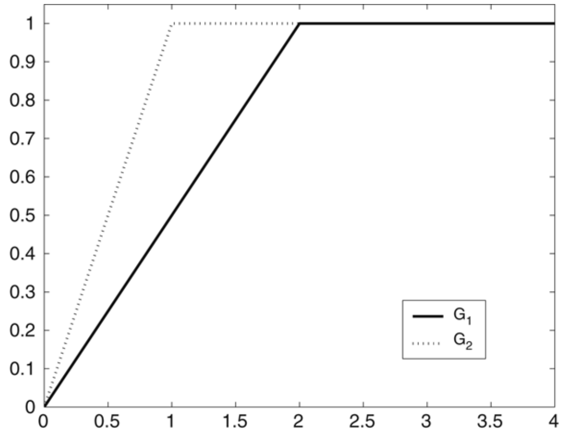

Fig. 3. Unbounded fuzzy numbers.

2. Two triangle fuzzy numbers cited by [1], $B_1 = (0.5, 0.5, 0.5, 1.0)$ and $B_2 = (0.15, 0.7, 0.7, 0.8)$. There exists a real number 0.6 such that $B_1 \leq_1 0.6$ and $0.6 \leq_1 B_2$. By our ranking criterion, we have $A_1 \preceq A_2$.
3. Two triangle fuzzy numbers cited by [8], $C_1 = (1.9, 2, 2, 2.1)$ and $C_2 = (2.1, 3, 3, 4)$. There exists a real number 2.5 such that $C_1 \leq_1 2.5$ and $2.5 \leq_1 C_2$. By ranking criterion, we have $C_1 \preceq C_2$. However, Yong et al. [11] pointed out that the ranking order is $C_2 \preceq C_1$ by Cheng's CV index [4], which are not consistent with the preference of the investors.
4. The three triangle fuzzy numbers cited by [4], $D_1 = (0.4, 0.5, 0.5, 1)$, $D_2 = (0.4, 0.7, 0.7, 1)$ and $D_3 = (0.4, 0.9, 0.9, 1)$. There exist two real numbers 0.6 and 0.8 such that $D_1 \leq_1 0.6$ and $0.6 \leq_1 D_2$, and $D_2 \leq_1 0.8$ and $0.8 \leq_1 D_3$. By ranking criterion, we have $D_1 \preceq D_2 \preceq D_3$. However, Yong et al. [11] pointed out that the ranking order is $D_1 \preceq D_3 \preceq D_2$ by Cheng's CV index.

The above four examples are all separable fuzzy numbers. Our ranking criterion, which does not need to calculate the integration, provides a simple method to rank these fuzzy numbers.

5. The three triangle fuzzy numbers cited by [3], $E_1 = (5, 7, 7, 9)$, $E_2 = (3, 7, 7, 9)$ and $E_3 = (3, 4, 7, 9)$. There do not exist real numbers to distinguish these three fuzzy number. Hence, the ranking criterion do not work for these fuzzy numbers. Now, we apply our ranking procedure and have $E_3 \prec E_2 \prec E_1$ without calculating the integral values of their membership function. The same results are obtained by [3].
6. Two fuzzy numbers cited by [4], $F_1 = (0, 1, 1, 2)$ and $F_2 = (1/5, 1, 1, 7/4)$. Applying the ranking procedure for the LR-type fuzzy number, we find that $F_1 \prec F_2$ because $1 = 1$ and $0 < 1/5$. The same result is also obtained by Cheng's CV index.

The above two examples show that our ranking procedure have the same results as the existing ranking approaches without integrating the membership function of the fuzzy numbers. This implies that our ranking procedure is more efficient then the existing approaches and can be calculated by hand.

7. Two fuzzy numbers displayed in Fig. 3.

   The membership function's support of these two fuzzy numbers are unbounded. This implies that the approaches which integrate the membership function do not work for these fuzzy numbers. By using our ranking procedure, we find that

$$\inf\{x \in \mathbb{S} | \tilde{\mu}_{G_1}(x) = \tilde{\mu}_{G_2}(x)\} = 2 > 0 = \inf\{x \in \mathbb{S} | \tilde{\mu}_{G_1}(x) - \tilde{\mu}_{G_2}(x) \leq 0\},$$

   where $\mathbb{S} = \{x \in \mathbb{R} | x > 0\}$. Consequently, we get $G_1 \succ G_2$.

When the membership function of a fuzzy number is not integrable, the ranking approaches which integrate the membership function could not work for ranking these fuzzy numbers. Fortunately, our ranking procedure overcomes this disadvantage.

5. Conclusion

In this paper, we have provided a ranking criterion which is consistent with the preference of the conservative investor. This ranking criterion has a better capability of discriminating the order of two separable fuzzy numbers. This ranking criterion do not work for ranking the nonseparable fuzzy numbers. In order to overcome the disadvantage of ranking criterion, we develop a ranking procedure. Our ranking procedure provides a better way to rank two fuzzy numbers when the membership functions of these fuzzy numbers are not integrable. Furthermore, a set of the triangle fuzzy numbers ranking by our ranking procedure satisfies the axioms for the order relation. Compared with the existing ranking approaches, our ranking procedure reduces the computational time substantially.

Our ranking procedure is designed to be consistent with the preference of conservative investor. However, the investors are not all conservative. One of the future studies is to develop a ranking procedure which is consistent with the risk-seeking investors. On the other hand, ranking a set of fuzzy numbers is only a beginning for solving a fuzzy stochastic programming. In crisp environment, many solution methods or approximation methods, such as the decomposition method or the Monte Carlo method, have been developed for solving a stochastic programming. So far these methods have not been extended to the fuzzy stochastic programming yet. One of our future studies is to provide a fuzzy Monte Carlo method for solving a fuzzy stochastic programming.

Appendix. Some basic definitions

In this paper, the LR-type fuzzy numbers are defined as follows.

Definition A.1. A fuzzy number F is of LR-type if there exist reference function L (for left), R (for right), and scalars $\alpha > 0$, $\beta > 0$ with $F = (l, c, d, r)$, if its membership function has the following forms:

$$\mu_F(x) = \begin{cases} L\left(\dfrac{c-x}{l}\right) & \text{if } x \le c, \\ 1 & \text{if } c < x < d, \\ R\left(\dfrac{x-d}{r}\right) & \text{if } x \ge d, \end{cases}$$

where c and d, called the peak of F, are real numbers and l and r are called the left and right spreads, respectively. Symbolically F is denoted by (l, c, d, r).

Let $\mathcal{F}(F)$ be a collection of all the convex, symmetric fuzzy number, that is $\mathcal{F}(F) = \{(a, r) | a \in [-\infty, \infty], r \in [0, \infty]\}$. There are two operations on $\mathcal{F}(F)$, namely fuzzy addition and scalar multiplication.

Lemma A.2. *Let $\tilde{F}_1 = (a_1, r_1)$ and $\tilde{F}_2 = (a_2, r_2)$ be any two fuzzy numbers in $\mathcal{F}(F)$ and λ be any real number. Then*

(a) $\tilde{F}_1 + \tilde{F}_2 = (a_1 + a_2, r_1 + r_2)$.
(b) $\lambda \tilde{F}_1 = (\lambda a_1, |\lambda| r_1)$.

For example, let $F = (c, r) \in \mathcal{F}(F)$. Then $-F = (-c, r)$ and $F - F = (0, 2r)$.

Let $\mathcal{F}$ be a collection of fuzzy variables defined on the possibility space $(\Theta, \mathcal{P}(\Theta), \text{Pos})$. Then fuzzy random variable is defined as follows.

Definition A.3. Let $(\Omega, \mathcal{F}, \Pr)$ be a probability space. A fuzzy random variable is a function $X : \Omega \to \mathcal{F}$ such that for any Boral set B of $\mathbb{R}$,

$$X(B)(\omega) = \text{Pos}(X(\omega) \in B)$$

is a measurable function of ω.

Now, we define a discrete random variable with fuzzy number outcomes. Let $\Omega = \{\omega_1, \omega_2, \ldots, \omega_K\}$ be a discrete finite sample space and $\mathcal{F}$ be a σ-field generated by Ω, then we define the fuzzy discrete random variable on Ω.

Definition A.4 (*Fuzzy Expectation*). Let X be a fuzzy discrete random variable on the probability space $(\Omega, \mathcal{F}, P)$. The expectation of X is defined as

$$\mathrm{E}[X] = \sum_{i=1}^{K} F_i p_i = \left(\sum_{i=1}^{K} p_i c_i, \sum_{i=1}^{K} p_i r_i \right).$$

Example 3. Let $\Omega = \{\omega_1, \omega_2, \omega_3\}$ and $F_i = (c_i, r_i)$ be fuzzy variables in $\mathcal{F}(F)$. Then the function

$$X(w) = \begin{cases} (c_1, r_1) & \text{for } \omega = \omega_1, \\ (c_2, r_2) & \text{for } \omega = \omega_2, \\ (c_3, r_3) & \text{for } \omega = \omega_3, \end{cases}$$

is a fuzzy random variable. Suppose that the probability of each event is given as $p_i, i = 1, 2, 3$. The fuzzy expectation of X can be calculated as

$$\mathrm{E}(X) = \sum_{i=1}^{3} p_i (c_i, r_i) = \left(\sum_{i=1}^{3} p_i c_i, \sum_{i=1}^{3} p_i r_i \right).$$

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