Thinking Skills and Creativity 16 (2015) 40–50

Contents lists available at ScienceDirect

Thinking Skills and Creativity

journal homepage: http://www.elsevier.com/locate/tsc

Creative behaviours in mathematics: Relationships with abilities, demographics, affects and gifted behaviours

Yuh-Chyn Leu[a,1], Mei-Shiu Chiu[b,*]

[a] *Department of Mathematics and Information Education, National Taipei University of Education, 134, Ho-ping E. Road Sec. 2, Taipei, Taiwan, ROC*

[b] *Department of Education, National Chengchi University, 64, Zhinan Road Sec. 2, Taipei 11605, Taiwan, ROC*

ARTICLE INFO

Article history:
Received 20 August 2014
Received in revised form
26 November 2014
Accepted 9 January 2015
Available online 14 February 2015

Keywords:
Achievement
Affect
Creative behaviour
Mathematics learning

ABSTRACT

This study aims to identify key creative behaviours in mathematics and to investigate the relationships of the identified behaviours with mathematics achievements, demographics, mathematics affects, and general gifted behaviours. The research participants were 372 Grade 4 students. The results of exploratory and confirmatory factor analyses identify 5 key creative behaviours in mathematics: representation invention, component association, outcome improvement, alternative curiosity, and space imagination. The results of correlation analyses reveal that component association and outcome improvement relate to mathematics achievements, reasoning, and creativity. The 5 creative behaviours have few relationships with gender, gifted education experiences, and parent education, but many relationships with mathematics affects and general gifted behaviours. The identified key creative behaviours can be used to design effective pedagogies for promoting student creativity in mathematics.

1. Introduction

Creativity in mathematics is traditionally viewed as one of the most important characteristics of professional mathematicians but narrowly conceptualized by mathematics teachers (Bolden, Harries, & Newton, 2010). Recognizing student multiple creative behaviours based on the multiple characteristics of 'big' creators in mathematics may broaden the conception of 'mini' creators (students) in mathematics classrooms (Beghetto & Kaufman, 2009, p. 42). Carlton (1959) analyses the writings of 14 creative mathematicians during the period of 1790–1940 and identifies 21 characteristics of creative behaviours. The 21 creative behaviours may need to be empirically validated based on student responses and to be reduced in quantity in order to facilitate the development of operational pedagogy for mathematics education.

The purpose of this study therefore is to identify key creative behaviours in mathematics learning based on the perspective of multiplicity in creativity and drawing on the characteristics of creative mathematicians obtained by Carlton (1959). Further, this study will investigate the relationships between the identified behaviours and relevant student characteristics to deepen the understanding of the identified behaviours. The findings of this study are likely to advance the knowledge of multiple creative behaviours in mathematics learning and facilitate suitable pedagogies for cultivating student creativity in mathematics.

* Corresponding author. Tel.: +886 2 29393091x66135; fax: +886 2 29397717.
E-mail addresses: leu@tea.ntue.edu.tw (Y.-C. Leu), chium@nccu.edu.tw, meishiuchiu@gmail.com (M.-S. Chiu).
[1] Tel.: +886 2 27321104x53307.

http://dx.doi.org/10.1016/j.tsc.2015.01.001

1.1. Multiple creative behaviours in mathematics

Student or mathematician self-reports on creative behaviours in mathematics can be viewed as multiple creative dispositions or strategies towards mathematics with novel meanings, discourses, or stories that they construct based on their relationships or negotiations with mathematical experiences in their lived field (Di Martino & Zan, 2011; Nolan, 2012; Pepin, 2011). Most studies on creative behaviours in mathematics focus on creative problem solving and suggest that creative behaviours include diverse attributes such as divergent thinking, convergent thinking, motivation, and environment (Lin & Cho, 2011). Some studies extend the scope from problem solving to posing and view creative behaviours in mathematics as continuous, cognitive processes from problem-solving, mediated by transformation (decoding, representing, processing, and implementing), to problem posing and vice versa (Singer & Voica, 2013).

Creative behaviours in general creative thinking may include both cognitive (procedural and conceptual knowledge) and affective (emotions, moods, traits, and dispositions) components followed by the creative process of frame construction, situation consideration, learner understanding, and stream negotiation (Newton, 2013). The creative process may include attentional behaviours such as orienting sensitivity and effortful control (Lin, Hsu, Chen, & Chang, 2013) and can be categorized into fast (automatic) and effortful (logical) thinking (Allen & Thomas, 2011). Relatively few studies on creativity focus on cross-process strategies, skills, and behaviours such as cause analysis, association production (Acar & Runco, 2014), error identification, constraint management (Mumford, Medeiros, & Partlow, 2012), reproduction, creative imagination (Liang, Chang, & Hsu, 2013) and application preparation (Barrett et al., 2013). Thus, this study will focus on the issue of cross-process creative behaviours.

1.2. Diverse learning results in mathematics and relationships with creative behaviours

Student learning results need to be understood by different tasks such as closed-ended (one solution) tasks, reasoning tasks (Lithner, 2008), and open-ended (multiple solutions) creative tasks. Relationships between student learning results based on school achievement tests and creativity tests may vary dependent on learning contexts (Gralewski & Karwowski, 2012) and domains. For example, mathematics creativity relies on achievements (Sak & Maker, 2006) more than art creativity (Jeon, Moon, & French, 2011). Mathematics educators often assess cognitively creative results in mathematics with the components of general creativity such as fluency, flexibility, originality, and elaboration (Kwon, Park, & Park, 2006; Lev-Zamir & Leikin, 2011). Problem posing tasks are often used to assess student creativity in quantity (fluency) and quality (novelty and elaboration) (Bonotto, 2013; Van Harpen & Sriraman, 2013).

Creative behaviours in mathematics may have different relationships with diverse student learning results in mathematics. Livne and Milgram (2006) indicate that student general creative behaviours (thinking) relate to student creativity in mathematics but not to student achievement in mathematics. Mann's (2009) study, however, shows that student mathematics talents and general creative behaviours assessed by teachers relate to both student mathematics achievement and creativity.

1.3. Relationships of creative behaviours in mathematics with demographics, mathematics affects, and gifted behaviour

Research has shown that student mathematics talents and general creative behaviours assessed by teachers are not related to gender (Mann, 2009). Domain-specific creative tasks, however, may slightly favour specific genders. For example, interpersonal and history tasks may favour females (Hong & Milgram, 2010; Hong, Peng, O'Neil, & Wu, 2013). Socio-economic status (SES) such as parent education may take a role, mediated by achievements and creativity-related psychological traits (e.g., confidence and motivation), in student creativity (Dai et al., 2012).

Student self-report affects in mathematics relate to student mathematics talents and general creative behaviours assessed by teachers (Mann, 2009). Research on general creativity also indicates likely relationships between the personality trait of openness and the interest in some domains (e.g., poems but not pictures) (Silvia & Sanders, 2010). Inducing positive affects, such as fun and happiness, may increase creative thinking and production (Fernández-Abascal & Díaz, 2013).

The relationships of gifted behaviours with creative behaviours appear to be an intuition because giftedness is normally conceptualized as including creativity (Kaufman, Plucker, & Russell, 2012). Student mathematics talents relate to their general creative behaviours although both of the 2 measures are assessed by teachers (Mann, 2009). Self-report creative behaviours and achievements also relate to self-report creative personality traits such as openness (Batey, Furnham, & Safiullina, 2010).

Based on the above review of literature, this study aims to answer the following research questions:

1. What are the key creative behaviours in student mathematics learning?
2. Are there relationships between student creative behaviours and mathematics abilities in terms of mathematics achievements, reasoning, and creativity?
3. Are there relationships of student creative behaviours with demographics, mathematics affects, and general gifted behaviours?

2. Methods

2.1. Participants and data collection

Data for this study were collected as part of a larger project on cultivating gifted students in mathematics. The research participants were 372 Grade 4 students (123 girls and 249 boys) from 111 primary schools around Taiwan. The students were at the top 50% of their classes in mathematics achievements in the previous year. The researchers contacted the education departments of all the local (city or county) governments, which then sent documents to all their primary schools to announce this survey. The class and mathematics teachers recommended the survey to the students by stating that the survey aimed to identify gifted students in mathematics for later special education services. Students voluntarily registered and attended the survey free of charge.

The students attended the survey based on their personal choice (39%), parent expectation (28%), teacher expectation (32%), and peer participation (1%), as the students indicated in the survey. The sampling procedure of teacher recommendation and student voluntary attendance may partially explain why more boys (N = 249) attended the survey than girls (N = 123). Another reason may be that the participants were relatively high achievers in mathematics. Gender differences favouring males in mathematics and science achievements appear to become large for high achievers (Reilly, Neumann, & Andrews, 2014). The survey was administered in a paper-and-pencil format. The students were informed before the survey that the results from self-report questionnaires were only for research and not for selection.

2.2. Measures

2.2.1. Creative Behaviours in Mathematics Questionnaire (CBMQ)

The initial 21 items of the CBMQ were adapted from the 21 characteristics of potentially creative thinkers or gifted children in mathematics posited by Carlton (1959, pp. 414–417) based on the analysis of creative mathematicians in history. The 15 items included in the final version of the CBMQ is presented in Table 1.

Table 1
Factor loadings of the items and Cronbach's αs for the 5 factors (creative behaviours in mathematics).

	Factor loadings by CFA/EFA					α
	Representation invention	Component association	Outcome improvement	Alternative curiosity	Space imagination	
1. You create other meanings for mathematical symbols taught by teachers.	.86/.87					.80
2. You invent new mathematical symbols.	.78/.90					
3. You find common clues or identify similar types of mathematical problems and ultimately find answers.		.73/.45				.73
4. You identify relationships and solve mathematical problems given the conditions of the problems.		.69/.69				
5. You quickly picture a complete answer or imagine final results.		.65/.68				
6. You like to create new mathematical solutions in addition to those made by your teachers or classmates.			.74/.42			.71
7. When you are not satisfied with your teacher's or classmates' mathematical solutions, you want to improve the solutions.			.65/.73			
8. When faced with a particularly difficult problem, you stick to it to find an answer.			.59/.36			
9. You produce many different solution methods for a mathematical problem.			.57/.37			
10. You appreciate and admire fast and beautiful mathematical solutions.			.36/.39			
11. You are curious about the mathematical symbols that you recognize and you find them fun.				.78/.53		.73
12. You continue thinking how to apply the answers obtained in class to other situations.				.68/.31		
13. You enjoy thinking about or guessing how an answer would change if one number in a mathematical problem is altered.				.61/.71		
14. You vividly imagine the paths of geometric figures during expansion, superposition, or rotation.					.78/.44	.71
15. You easily imagine things locating in space and relationships between things.					.71/.60	

Table 2
Scale score ranges, means, standard deviations (*SDs*), and correlations between creative behaviours in mathematics and mathematics test results, demographics, mathematics affects, and gifted behaviours.

Creative behaviours in mathematics	Scale score range	Mean	*SD*	Correlations				
				1	2	3	4	5
1. Representation invention	1–5	2.43	1.24					
2. Component association	1–5	3.80	.85	.44				
3. Outcome improvement	1–5	3.79	.77	.48	.67			
4. Alternative curiosity	1–5	3.78	.96	.48	.53	.68		
5. Space imagination	1–5	3.58	1.04	.48	.56	.56	.55	
Mathematics tests								
Achievement (total)	0–90	41.41	16.21	.08	.23	.24	.06	.14
Number	0–30	6.44	3.27	.06	.17	.20	.04	.10
Quantity	0–30	6.18	3.09	.09	.23	.21	.08	.14
Figure	0–30	8.09	2.82	.07	.20	.22	.05	.12
Reasoning (total)	0–100	42.83	20.36	.11	.33	.24	.08	.17
Creativity (total)	0–100	34.73	13.61	.00	.17	.16	.00	.05
Fluency	0–100	41.93	16.07	.00	.16	.13	.01	.06
Flexibility	0–100	31.53	12.17	.00	.17	.15	.00	.05
Originality	0–100	67.87	21.22	−.04	.10	.08	−.04	−.01
Elaboration	0–100	24.93	11.16	.00	.18	.15	.00	.04
Demographics								
Gender	0 = boy, 1 = girl	.33	.47	.01	−.06	−.04	.04	−.01
Gifted education	0 = no, 1 = yes	.40	.49	.12	.07	.04	−.01	.00
Father education	1–4	3.05	.76	.09	−.01	.05	.02	.04
Mother education	1–4	2.92	.69	.03	−.03	−.03	−.01	.04
Mathematics affect								
Anxiety	1–5	1.85	.87	−.13	−.26	−.23	−.16	−.17
Interest	1–5	4.06	.83	.32	.54	.58	.53	.40
Confidence	1–5	4.24	.72	.05	.32	.30	.27	.21
Gifted behaviour								
Passion	1–5	3.87	.71	.31	.47	.48	.45	.35
Creativity	1–5	4.07	.68	.43	.47	.46	.44	.39
Intelligence	1–5	4.14	.61	.34	.55	.50	.39	.35

Note: The correlation coefficients underlined are significant at the .05 level.

2.2.2. Mathematics tests

Mathematics Achievement Test (*MAT*). The MAT was developed by Leu and Hou (2012), aiming to identify Grade 4 gifted students in mathematics. The problems were developed based on the content analysis of the national mathematics curriculum in Taiwan and was reviewed by 4 experts on mathematics and mathematics education, and gradually revised using 2 pilot studies. The MAT had 45 mathematical problems, with 15 problems on number, 15 on quantity, and 15 on figure. A correct answer for a problem could obtain 2 points and thus the total score of the MAT was 90. The results of Leu and Hou's validation study showed that all of the problems in the MAT could distinguish relatively high and low achievers for high achievers in school mathematics. The MAT also had a desirable internal reliability (Cronbach's $\alpha = .90$). Leu and Hou's norming study for the MAT recruited 318 Grade 4 students from ten normal classes randomly chosen from their respective ten schools in north Taiwan. The mean score obtained from the norming study is 27.28 ($SD = 11.90$; score range = 0–90). Students with scores higher than 51.08 (=27.28 + 11.90 * 2, i.e., Mean + SD * 2) were viewed as high achievers in mathematics and might receive mathematics gifted education. The difficulty of the MAT items were higher than that of the national curriculum for Grade 4 students given that the purpose of the MAT was to identify mathematically gifted students. Research has indicated that student mathematics creativity may partly reply on mathematics ability (Kattou, Kontoyianni, Pitta-Pantazi, & Christou, 2013; Sak & Maker, 2006). The high difficulty of the MAT items can be shown further by the mean score of 41.41 ($SD = 16.21$) (Table 2), slightly below 45 (the expected mean of the score range 0–90), for the present participants, who were in the top 50% of mathematics ability in their classes.

Mathematics Reasoning Test (*MRT*). The MRT had 10 mathematical problems assessing student higher-order ability of rule finding and problem solving. The problems were developed based on the national curriculum in Taiwan on mathematics deductive and inductive competences. Students were asked to answer each problem by providing detailed procedures and solutions. The total score was 100, each problem with 0–10 points. The MRT had acceptable internal reliability (Cronbach's $\alpha = .66$) and desirable inter-coder reliability (the percentage of items with the same coding = 97%). The coding discrepancies were resolved by discussion.

Mathematics Creativity Test (*MCT*). The MCT had 4 problems, 2 focusing on problem solving and 2 on problem posing, all of which were developed based on the national mathematics curriculum in Taiwan. The participants were encouraged to provide at least 3 answers to each problem and 6 answer spaces were provided for each problem. Participant answers were scored for their fluency, flexibility, originality, and elaboration, each with a score range of 0–100. The inter-coder reliability was 95% and the coding discrepancies were resolved by discussion.

The MRT and MCT were developed and first administered by this study and thus did not have norming data. Sample problems for the MAT, MRT, and MCT are presented in Appendix.

2.2.3. Demographics

The participants provided their demographic information regarding gender, whether they received gifted education at the time when they took the survey, and their fathers' and mothers' educational levels. Fifty percent of the participants received gifted education. The types of gifted education that they received included academic subjects (97%), art (2%), and music (1%). Most gifted educations for academic subjects are provided via resource programmes and those for art and music are via streaming teaching in some Taiwanese primary schools. In resource programmes, academically gifted students spend most of their time in a mix-ability class and some time in a gifted class taught by teachers of special education; around 2 periods/hour per week are allocated for teaching mathematics in the gifted class. By streaming teaching, students with giftedness in art and music spend all their school time in their gifted classes designed for cultivating their art and music talents, respectively. Students rated their parents' educational levels on a 4-point scale (1 = junior high school or below, 2 = senior high school, 3 = university or college, and 4 = graduate school).

2.2.4. Mathematics Affects Questionnaire (MAQ)

The MAQ used a 5-point scale (1 = strongly disagree to 5 = strongly agree) and included 3 subscales, partly taken from Huang's (2010) study. The subscale on anxiety included 3 items such as 'Mathematics class often makes me dizzy,' with the internal reliability of Cronbach's alpha (α) equal to .68. The subscale on interest had 5 items (α = .84) such as 'I think mathematics is very interesting.' The subscale on confidence had 5 items (α = .82) such as 'I do not have enough ability to solve difficult mathematics problem (reversed).'

2.2.5. Gifted Behaviours Questionnaire (GBQ)

The GBQ focused on general gifted behaviours, used a 5-point scale (1 = strongly disagree to 5 = strongly agree), and included 3 subscales (Chiu, 2008). The subscale on passion included 5 items (α = .84) such as 'I try my best to complete things.' The subscale on creativity had 5 items (α = .76) such as 'I often produce new and interesting ideas.' The subscale on intelligence had 6 items (α = .75) such as 'People often praise my cleverness.'

2.3. Data analysis

The research questions were answered by statistical analysis using the R software Version 3.1.0 (2014-04-10) (R Core Team, http://www.R-project.org/). Research Question 1 was answered by exploratory factor analysis (EFA) and then confirmatory factor analysis (CFA). The CFA was used to validate the factor structure obtained by the EFA. EFA can obtain reliable factor solutions even with sample sizes below 50 if data are well structured with high factor loadings, low factor numbers, and high variable numbers (De Winter, Dodou, & Wieringa, 2009). CFA needs at least a sample of 200 cases to generate reliable solutions (Urbach & Ahlemann, 2010). As such, the 372 participants were randomly divided into two samples: 100 cases for EFA and 272 cases for CFA. In EFA, the factor extraction method was ordinary least squares for finding minimum residual solutions and the rotation method was Oblimin with Kaiser normalization. Both EFA and CFA can use goodness-of-fit indexes to help determine optimal numbers of factors although interpretable constructs based on theories are the major principle (Preacher, Zhang, Kim, & Mels, 2013). Good models normally have a non-significant chi-square (χ^2), near 1.00 (above .90) Tucker–Lewis Index (TLI), below .08 root mean square error of approximation (RMSEA), and below .08 root mean square of the residuals (RMR) (Hair, Black, Babin, & Anderson, 2010). The χ^2 value, however, may become significant because of large sample sizes (Bollen & Long, 1993; Browne & Cudeck, 1993).

Research Questions 2–3 were investigated using Pearson's bivariate correlation analysis. Gender and gifted education experience were dichotomous variables in this study (Table 2). Correlations between a dichotomous variable and a continuous one are called point-biserial correlations. Point-biserial correlation coefficients can be obtained using the same mathematical procedure as that used in Pearson's bivariate correlation analysis with the dichotomous variable being coded as a dummy variable (e.g., 0 = boy and 1 = girl; 0 = no and 1 = yes) (Fazli et al., 2012; Gerber & Finn, 2005, pp. 157–158). As a rule of thumb, a correlation coefficient (in absolute value) below or equal to .35 is regarded as being weak, from .36 to .67 moderate, and from .68 to 1.00 strong (Taylor, 1990).

3. Results

3.1. Key creative behaviours in mathematics

The results of EFA for Sample 1 (100 cases) show that 15 items, grouped into 5 constructs/factors, can be explained in a meaningful and theoretical way (Table 1) with desirable index values (maximum likelihood (MLE) $\chi^2(df)$ = 105.91 (99), p = .30; TLI = .98; RMSEA = .047; RMR = .038). The original CBMQ has 21 items, among which 6 items are deleted due to their (1) low factor loadings on all the factors or (2) high factor loadings on at least two factors using 'factor loading = .308' as the cutting point. The factor loadings leading from the 5 factors to their items range from .31 to .90. EFA is a data-driven statistical method. Finding interpretable multi-dimensional factor patterns tends to be important (Fabrigar, Wegener, MacCallum, &

Strahan, 1999) although extracting multiple factors may result in small factor loadings (Ledesma & Valero-Mora, 2007). Adequate cut-off criteria for factor loadings are not proposed by researchers to date but items with factor loadings larger than .30 appear to be acceptable (Costello & Osborne, 2005, p. 3; DiStefano, Zhu, & Mindrila, 2009, p. 3). Based on the contents of the items in the factors, the 5 factors (i.e., key creative behaviours in mathematics) are named representation invention, component association, outcome improvement, alternative curiosity, and space imagination. The 5 factors account for 11%, 10%, 10%, 8%, and 6% of the variance in student responses respectively, leading to a factor solution that totally accounts for 45% of the variance.

Sample 2 (272 cases) is analyzed using CFA for validating the structure of the 15 items grouped into 5 factors obtained by the above EFA (Table 1). The results of CFA validate the a priori structure with acceptable fit index values (MLE $\chi^2(df) = 161.17(95)$, $p < .0005$; TLI = .94; RMSEA = .051; RMR = .042). The factor loadings leading from the 5 factors to their items range from .36 to .86. The values of Cronbach's α for the 5 factors are .80, .73, .71, .73, and .71 respectively. A good psychological test normally has a Cronbach's α value of at least .70 (Kline, 2000, p. 15). The correlations between the 5 factors range from .44 to .68 (Table 2).

3.2. Relationships between the creative behaviours and test results in mathematics

Correlation analyses are used to investigate the relationships between the 5 key creative behaviours in mathematics (i.e., the 5 factors identified by the EFA and CFA) and mathematics achievements, reasoning, and creativity. Among the 5 creative behaviour, component association and outcome improvement have significant correlations with all of the total and subscales of mathematics achievements, reasoning, and creativity (r = .13–.33) except for originality (r = .10 and .08) (Table 2). Space imagination has significant correlations with total, quantity, and figure achievements (r = .14, .14, and .12), and reasoning (r = .17) and non-significant ones with the other. Representation invention has a significant correlation with reasoning (r = .11) and non-significant ones with the other. Alternative curiosity has no significant correlation with any of the test results in mathematics. A note to make is that the correlation values are small even for the significant ones.

3.3. Relationships of the creative behaviours in mathematics with demographics, mathematics affects, and gifted behaviours

The results of correlation analyses show that the 5 creative behaviours have no significant correlations with student gender, gifted education experiences, and father and mother educational levels (r = −.06 to .09) (Table 2); the only exception is the correlation between representation invention and gifted education experiences (r = .12). All of the correlation coefficients are very small (the highest r = .12).

Significant correlations occur between the 5 creative behaviours and the 3 subscales of mathematics affects except for the correlation between representation invention and confidence (r = .05). The significant correlations between the 5 creative behaviours and mathematics anxiety are negative (r = −.26 to −.13) and those between the 5 creative behaviours and both interest (r = .32–.58) and confidence (r = .21–.32) are positive. The sizes of the significant correlations are from small to moderate. The 4 moderate correlations occur between interest and 4 creative behaviours (component association, outcome improvement, alternative curiosity, and space imagination).

All of the correlations between the 5 creative behaviours and the 3 gifted behaviours (passion, creativity, and intelligence) are significant and positive (r = .31–.55). Most of the correlations are moderate (73% = 11/15) and the others are small.

4. Discussion

4.1. Key creative behaviours in mathematics

This study identifies 5 key creative behaviours in mathematics based on a questionnaire (the CBMQ) initially with the 21 characteristics (items) of potentially creative thinkers in mathematics posited by Carlton (1959). The results of EFA, CFA, and reliability analysis show that the CBMQ has desirable construct validity and internal reliability and include 5 validated factors (with 15 items): representation invention, component association, outcome improvement, alternative curiosity, and space imagination. The 5 key creative behaviours in mathematics are interpreted with the content of their items and related literature as follows.

Representation invention. Creative students add new meanings for existing symbols or directly produce new symbols in mathematics, showing a non-conforming style (Wechsler, Vendramini, & Oakland, 2012).

Component association. Creative students produce answers by linking components in problems (Allen & Thomas, 2011). Component association appears to capture the essence of mathematics in beauty and elegance (Mann, 2006).

Outcome improvement. Creative students find what the problem really is and what its better solutions/applications are (Allen & Thomas, 2011; Barrett et al., 2013). Outcome improvement appears to capture the essence of time and experience in mathematics practices (Mann, 2006).

Alternative curiosity. Creative students enjoy recognizing, applying, and guessing in going beyond present mathematical conditions. Positive affects appear to be part of the intuitive and creative process (Eubanks, Murphy, & Mumford, 2010).

Space imagination. Creative students go beyond the present, static, and seemingly irrelevant 1- or 2-dimensional things and form, operate, and rotate 3-dimensional images in a meaningful whole (Liang et al., 2013).

In sum, the 5 creative behaviours in mathematics indicate the key tasks and cross-process strategies in creativity activities and are consistent with the characteristics of mathematicians and creative students in mathematics posited by related researchers. The 5 creative behaviours provide clear objects for teaching programmes to operate to increase student mathematics creativity. For example, pedagogy for cultivating student mathematics creativity may follow a flexible 5-step process. First, teachers can raise student alternative curiosity by introducing mathematics tasks or concepts in non-conventional ways such as metaphors (Abrahamson, Gutiérrez, & Baddorf, 2012), novel stories (Sack, 2013), and embodiment (De Freitas & Sinclair, 2012). Second, teachers invite students to associate components in mathematics problems with diverse environments beyond schools (Davies et al., 2013). Third, students are encouraged to go beyond the problem-environment association and imagine in multi-dimensional systems such as objects, spaces (Pitta-Pantazi & Christou, 2010), and narratives (Cremin, Chappell, & Craft, 2013). Fourth, students actualize their virtual imaginations by inventing meta-representations (i.e., models) (Dong & Sarkar, 2011) such as drawings with element descriptions (Van der Veen, 2012), gestures, and digital works (De Freitas & Sinclair, 2013). Five, students aim to improve the outcomes of their representations by reviewing the whole process and to form 'shared representations' (De Vries & Masclet, 2013, p. 46) regulated, co-constructed, and understood by the community.

The 5 creative behaviours, however, may still contain unknown and unconscious processes (Ritter, van Baaren, & Dijksterhuis, 2012). Further, student diverse creative behaviours may interact with creative mathematics teaching practices such as ambiguous problems or situations (Amit & Gilat, 2012).

4.2. Creative behaviours related to mathematics abilities

The results of correlation analyses show that the 5 identified creative behaviours have some significant but weak relationships with student mathematics. Component association and outcome improvement relate to most subscales of mathematics achievements, reasoning, and creativity, space imagination to mathematics achievements and reasoning, representation invention to reasoning, and alternative curiosity to none.

Past studies show that creative behaviours may have different relationships with diverse mathematics abilities (Livne & Milgram, 2006). This study further shows that some creative behaviours (component association and outcome improvement) tend to relate more to diverse mathematics abilities than others (space imagination, representation invention, and alternative curiosity, in a descending order). Component association and outcome improvement may be well cultivated in present student learning contexts, while space imagination and representation invention may be less. The aim of primary mathematics education, as presented in the national curriculum, is to cultivate student competence to understand, calculate, and apply the knowledge of (1) number and quantity, (2) geometry, (3) algebra, and (4) statistics and probability, in descending order (Ministry of Education in Taiwan, 2008). Mathematics textbooks focus on well-structured, close-ended, and non-creative tasks with a minor focus on ill-structured, open-ended, and creative tasks (e.g., problem posing) (Chiu, 2009). Space imagination and representation invention delve into the active, holistic, constructivist, intuitive, and aesthetic part of creativity, widely evidenced by great scientists' use of mathematics representations (including symbols) to recognize and demonstrate patterns (De Cruz & De Smedt, 2013; Hong, 2013). The national curriculum may include space imagination and representation invention if educators view children as active, interactive, and creative learners (Glăveanu, 2011; Tall, 2011) for pure (de-contextualized) or impure (contextualized) mathematics (Luitel, 2013). Alternative curiosity fails to show any relationships with diverse abilities in this study but positive affects may be a basis for student later creative process (Newton, 2013) and production (Zenasni & Lubart, 2011). These findings have advanced the knowledge of different relationships between diverse creative behaviours and diverse abilities in mathematics. The findings may help educators develop creativity pedagogies for increasing student diverse abilities in mathematics.

The relationships between creative behaviours and abilities may interact with student schooling years and learning experiences (Sak & Maker, 2006). For example, students at Grade 4 may have had sufficient experiences and abilities of component association and outcome improvement, which can be transformed to relate to mathematics achievements, while students at lower grades may not. Space imagination and representation invention may relate to mathematics abilities if the mathematics curriculum allows students to experience relevant learning experiences before Grade 4. Future research can clarify these issues further.

4.3. Few relationships of creative behaviours with demographics and many relationships with mathematics affects and general gifted behaviours

The 5 creative behaviours are not related to demographics (gender, gifted education experiences, and parent education) except for a weak relationship between representation invention and gifted education experiences. Past studies tend to indicate that creative behaviours are not related to gender (Mann, 2009) but imply relationships between creative behaviours and SES because creative behaviours may mediate the role of SES in creativity (Dai et al., 2012). The use of relatively high achievers in mathematics in this study may partially influence the result of few relationships between creative behaviours and demographics because SES may relate to achievements (Berliner, 2006). Future research can investigate similar issues using the sample of students with different levels of mathematics achievements. The significant relationship between representation invention and gifted education experiences suggests the likely role of gifted education in increasing student creative behaviour of representation invention.

Most of the 5 creative behaviours negatively and weakly relate to mathematics anxiety, positively and moderately to interest, and positively and weakly to confidence. All of the 5 creative behaviours in mathematics positively and most moderately relate to the 3 gifted behaviours (passion, creativity, and intelligence). The findings are generally consistent with past research findings that creative behaviours or production relate to mathematics affects, general affects, and general gifted behaviours in a positive direction (Fernández-Abascal & Díaz, 2013; Mann, 2009). This study uses self-report measures to assess creative behaviours, mathematics affects, and general gifted behaviours, which may be part of the reasons for relatively high relationships between them (cf. Batey et al., 2010).

The general trend of moderate correlations between self-report creative behaviours in mathematics and self-report mathematics interests and gifted behaviours suggests that creativity may partially depend on active and holistic positive self-views (Reiter-Palmon, Robinson-Morral, Kaufman, & Santo, 2012). The positive self-views may be developed in a learning environment where teachers respect students, notice student multiple intelligences, take a long-term view on student potentials, and use teaching time and resources in flexible and diverse ways (Davies et al., 2013).

5. Conclusion

This study identifies 5 key creative behaviours in mathematics (representation invention, component association, outcome improvement, alternative curiosity, and space imagination) using a questionnaire developed based on the characteristics of creative mathematicians in history. Component association and outcome improvement generally relate to mathematics achievements, reasoning, and creativity, space imagination to mathematics achievements and reasoning, and representation invention to reasoning. The 5 creative behaviours have few relationships with gender, gifted education experiences, and parent education, but many relationships with mathematics affects and general gifted behaviours. The 5 key creative may be further validated and serve as domain-specific operational conceptions for future teaching programmes to cultivate student creativity in mathematics.

Acknowledgement

This study is partially supported by the Ministry of Science and Technology, Taiwan (NSC 99-2511-S-152-003-MY3; NSC 101-2511-S-004-001; MOST 102-2511-S-004-004).

Appendix. Sample problems for the 3 mathematics tests

1. Sample problems from the Mathematics Achievement Test (MAT)

Number: 「 $57.2 \div 0.32 = \square \div 320$ 」, what should be the number inside □ ?

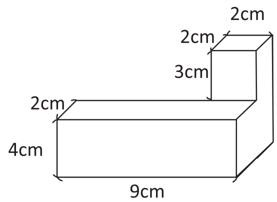
① 572
② 5720
③ 57200
④ 57.2

Quantity: Robert rides bike at the speed of 4 meters per second. How many kilometres per hour can Robert ride?

① 240 kilometres
② 24 kilometres
③ 144 kilometres
④ 14.4 kilometres

Figure: What is the volume that the right figure shows?

① 84 cm^3
② 72 cm^3
③ 100 cm^3
④ All of the above are wrong.



2. A sample problem from the Mathematics Reasoning Test (MRT)

Please find A in Figure 2 based on the rule in Figure 1.

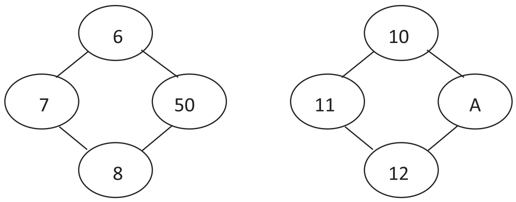
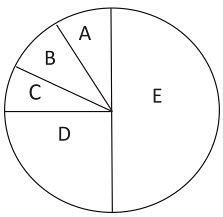


Figure 1 Figure 2

3. Sample problems from the Mathematics Creativity Test (MCT)

Problem solving: There are sweets for 68 people. If each person can obtain 17 sweets, the remaining is 76 sweets. If each person obtains 19 sweets, how many more sweets are needed? Please write at least 3 different solution methods. It is better to have different solution methods.

Problem posing: The radius of the circle is 10 cm. The circle is divided into 5 parts: A, B, C, D, and E. Use the above description to pose at least 3 different mathematical problems with your mathematics knowledge. It is better that you pose different mathematical problems. (There is no need to write solution methods.)



References

Abrahamson, D., Gutiérrez, J. F., & Baddorf, A. K. (2012). Try to see it my way: The discursive function of idiosyncratic mathematical metaphor. *Mathematical Thinking and Learning*, *14*, 55–80.

Acar, S., & Runco, M. A. (2014). Assessing associative distance among ideas elicited by tests of divergent thinking. *Creativity Research Journal*, *26*, 229–238.

Allen, A. P., & Thomas, K. E. (2011). A dual process account of creative thinking. *Creativity Research Journal*, *23*, 109–118.

Amit, M., & Gilat, T. (2012). Reflecting upon ambiguous situations as a way of developing students' mathematical creativity. In T. Y. Tso (Ed.), *Proceedings of the 36th conference of the international group for the psychology of mathematics education* (Vol. 2) (pp. 191–198). Taipei, Taiwan: PME.

Barrett, J. D., Peterson, D. R., Hester, K. S., Robledo, I. C., Day, E. A., Hougen, D. P., et al. (2013). Thinking about applications: Effects on mental models and creative problem-solving. *Creativity Research Journal*, *25*, 199–212.

Batey, M., Furnham, A., & Safiullina, X. (2010). Intelligence, general knowledge and personality as predictors of creativity. *Learning and Individual Differences*, *20*, 532–535.

Beghetto, R. A., & Kaufman, J. C. (2009). Do we all have multicreative potential? *ZDM Mathematics Education*, *41*(1), 39–44.

Berliner, D. C. (2006). Our impoverished view of educational reform. *Teachers College Record*, *108*, 949–995.

Bolden, D. S., Harries, T. V., & Newton, D. P. (2010). Pre-service primary teachers' conceptions of creativity in mathematics. *Educational Studies in Mathematics*, *73*, 143–157.

Bollen, K. A., & Long, J. S. (1993). *Testing structural equation models*. Newbury Park, CA: Sage.

Bonotto, C. (2013). Artifacts as sources for problem-posing activities. *Educational Studies in Mathematics*, *83*, 37–55.

Browne, M. W., & Cudeck, R. (1993). Alternative ways of assessing model fit. In K. A. Bollen, & J. S. Long (Eds.), *Testing structural equation models* (pp. 136–162). Newbury Park, CA: Sage.

Carlton, L. V. (1959). *An analysis of the educational concepts of fourteen outstanding mathematicians, 1790–1940, in the areas of mental growth and development, creative thinking and symbolism and meaning* (Unpublished doctoral dissertation). IL: Northwestern University.

Chiu, M.-S. (2009). Approaches to the teaching of creative and non-creative mathematical problems. *International Journal of Science and Mathematics Education*, *7*, 55–79.

Chiu, Y.-F. (2008). *A study on co-cognitive traits, gifted behaviour traits and self-efficacy of elementary students* (Unpublished master thesis). Taiwan: Department of Special Education, National Taipei University of Education (in Chinese)

Costello, A. B., & Osborne, J. W. (2005). Best practices in exploratory factor analysis: Four recommendations for getting the most from your analysis. *Practical Assessment, Research & Evaluation*, *10*(7), 1–9.

Cremin, T., Chappell, K., & Craft, A. (2013). Reciprocity between narrative, questioning and imagination in the early and primary years: Examining the role of narrative in possibility thinking. *Thinking Skills and Creativity*, *9*, 135–151.

Dai, D. Y., Tan, X., Marathe, D., Valtcheva, A., Pruzek, R. M., & Shen, J. (2012). Influences of social and educational environments on creativity during adolescence: Does SES matter? *Creativity Research Journal*, *24*(2/3), 191–199.

Davies, D., Jindal-Snape, D., Collier, C., Digby, R., Hay, P., & Howe, A. (2013). Creative learning environments in education: A systematic literature review. *Thinking Skills and Creativity*, *8*, 80–91.

De Cruz, H., & De Smedt, J. (2013). Mathematical symbols as epistemic actions. *Synthese*, *190*, 3–19.

De Freitas, E., & Sinclair, N. (2012). Diagram, gesture, agency: Theorizing embodiment in the mathematics classroom. *Educational Studies in Mathematics*, *80*, 133–152.

De Freitas, E., & Sinclair, N. (2013). New materialist ontologies in mathematics education: The body in/of mathematics. *Educational Studies in Mathematics*, *83*, 453–470.

De Vries, E., & Masclet, C. (2013). A framework for the study of external representations in collaborative design settings. *International Journal of Human–Computer Studies*, *71*, 46–58.

De Winter, J. C. F., Dodou, D., & Wieringa, P. A. (2009). Exploratory factor analysis with small sample sizes. *Multivariate Behavioural Research*, *44*, 147–181.

Di Martino, P., & Zan, R. (2011). Attitude towards mathematics: A bridge between beliefs and emotions. *ZDM Mathematics Education*, *43*(4), 471–482.

DiStefano, C., Zhu, M., & Mindrila, D. (2009). Understanding and using factor scores: Considerations for the applied researcher. *Practical Assessment, Research & Evaluation*, *14*(20), 1–11.

Dong, A., & Sarkar, S. (2011). Unfixing design fixation: From cause to computer simulation. *Journal of Creative Behaviour*, *45*, 147–159.

Eubanks, D. L., Murphy, S. T., & Mumford, M. D. (2010). Intuition as an influence on creative problem-solving: The effects of intuition, positive affect, and training. *Creativity Research Journal*, *22*, 170–184.

Fabrigar, L. R., Wegener, D. T., MacCallum, R. C., & Strahan, E. J. (1999). Evaluating the use of exploratory factor analysis in psychological research. *Psychological Methods*, *4*, 272–299.

Fazli, S., Mehnert, J., Steinbrink, J., Curio, G., Villringer, A., Müller, K. R., et al. (2012). Enhanced performance by a hybrid NIRS–EEG brain computer interface. *Neuroimage*, *59*, 519–529.

Fernández-Abascal, E. G., & Díaz, M. D. M. (2013). Affective induction and creative thinking. *Creativity Research Journal*, *25*, 213–221.

Gerber, S. B., & Finn, K. V. (2005). *Using SPSS for windows: Data analysis and graphics*. Springer.

Glăveanu, V. P. (2011). Children and creativity: A most (un)likely pair? *Thinking Skills and Creativity*, *6*, 122–131.

Gralewski, J., & Karwowski, M. (2012). Creativity and school grades: A case from Poland. *Thinking Skills and Creativity*, *7*, 198–208.

Hair, J. F., Jr., Black, W. C., Babin, B. J., & Anderson, R. E. (2010). *Multivariate data analysis: A global perspective* (7th ed.). Upper Saddle River, NJ: Pearson Education.

Hong, E., & Milgram, R. M. (2010). Creative thinking ability: Domain generality and specificity. *Creativity Research Journal*, *22*, 272–287.

Hong, E., Peng, Y., O'Neil, H. F., & Wu, J. (2013). Domain-general and domain-specific creative-thinking tests: Effects of gender and item content on test performance. *Journal of Creative Behaviour*, *47*, 89–105.

Hong, F. T. (2013). The role of pattern recognition in creative problem solving: A case study in search of new mathematics for biology. *Progress in Biophysics and Molecular Biology*, *113*, 181–215.

Huang, J.-S. (2010). *The study of perfectionism and mathematics self-efficacy of gifted students in elementary school* (Unpublished master thesis). Taiwan: Department of Science Education, National Taipei University of Education (in Chinese).

Jeon, K. N., Moon, S. M., & French, B. (2011). Differential effects of divergent thinking, domain knowledge, and interest on creative performance in art and math. *Creativity Research Journal*, *23*, 60–71.

Kattou, M., Kontoyianni, K., Pitta-Pantazi, D., & Christou, C. (2013). Connecting mathematical creativity to mathematical ability. *ZDM Mathematics Education*, *45*, 167–181.

Kaufman, J. C., Plucker, J. A., & Russell, C. M. (2012). Identifying and assessing creativity as a component of giftedness. *Journal of Psychoeducational Assessment*, *30*, 60–73.

Kline, P. (2000). *The handbook of psychological testing* (2nd ed.). London: Routledge.

Kwon, O. N., Park, J. H., & Park, J. S. (2006). Cultivating divergent thinking in mathematics through an open-ended approach. *Asia Pacific Education Review*, *7*, 51–61.

Ledesma, R. D., & Valero-Mora, P. (2007). Determining the number of factors to retain in EFA: An easy-to-use computer program for carrying out parallel analysis. *Practical Assessment, Research & Evaluation*, *12*(2), 1–11.

Leu, Y.-C., & Hou, C.-L. (2012). The compilation of mathematics achievement test for elementary mathematics: Grade 4 gifted students as the appraisement tool. *Bulletin of Eastern-Taiwan Special Education*, *14*, 303–326 (in Chinese).

Lev-Zamir, H., & Leikin, R. (2011). Creative mathematics teaching in the eye of the beholder: Focusing on teachers' conceptions. *Research in Mathematics Education*, *13*(1), 17–32.
Liang, C., Chang, C. C., & Hsu, Y. (2013). Personality and psychological factors predict imagination: Evidence from Taiwan. *Learning and Individual Differences*, *27*, 67–74.
Lin, C. Y., & Cho, S. (2011). Predicting creative problem-solving in math from a dynamic system model of creative problem solving ability. *Creativity Research Journal*, *23*, 255–261.
Lin, W. L., Hsu, K. Y., Chen, H. C., & Chang, W. Y. (2013). Different attentional traits, different creativities. *Thinking Skills and Creativity*, *9*, 96–106.
Lithner, J. (2008). A research framework for creative and imitative reasoning. *Educational Studies in Mathematics*, *67*, 255–276.
Livne, N. L., & Milgram, R. M. (2006). Academic versus creative abilities in mathematics: Two components of the same construct? *Creativity Research Journal*, *18*, 199–212.
Luitel, B. C. (2013). Mathematics as an im/pure knowledge system: Symbiosis, (w)holism and synergy in mathematics education. *International Journal of Science and Mathematics Education*, *11*, 65–87.
Mann, E. L. (2006). Creativity: The essence of mathematics. *Journal for the Education of the Gifted*, *30*, 236–260.
Mann, E. L. (2009). The search for mathematical creativity: Identifying creative potential in middle school students. *Creativity Research Journal*, *21*, 338–348.
Ministry of Education in Taiwan. (2008). *The national curriculum for primary and junior high schools*. (in Chinese).
Mumford, M. D., Medeiros, K. E., & Partlow, P. J. (2012). Creative thinking: Processes, strategies, and knowledge. *Journal of Creative Behaviour*, *46*, 30–47.
Newton, D. P. (2013). Moods, emotions and creative thinking: A framework for teaching. *Thinking Skills and Creativity*, *8*, 34–44.
Nolan, K. (2012). Dispositions in the field: Viewing mathematics teacher education through the lens of Bourdieu's social field theory. *Educational Studies in Mathematics*, *80*, 201–215.
Pepin, B. (2011). Pupils' attitudes towards mathematics: A comparative study of Norwegian and English secondary students. *ZDM Mathematics Education*, *43*(4), 535–546.
Pitta-Pantazi, D., & Christou, C. (2010). Spatial versus object visualisation: The case of mathematical understanding in three-dimensional arrays of cubes and nets. *International Journal of Educational Research*, *49*, 102–114.
Preacher, K. J., Zhang, G., Kim, C., & Mels, G. (2013). Choosing the optimal number of factors in exploratory factor analysis: A model selection perspective. *Multivariate Behavioural Research*, *48*(1), 28–56.
Reilly, D., Neumann, D. L., & Andrews, G. (2014). Sex differences in mathematics and science achievement: A meta-analysis of national assessment of educational progress assessments. *Journal of Educational Psychology*, http://dx.doi.org/10.1037/edu0000012
Reiter-Palmon, R., Robinson-Morral, E. J., Kaufman, J. C., & Santo, J. B. (2012). Evaluation of self-perceptions of creativity: Is it a useful criterion? *Creativity Research Journal*, *24*, 107–114.
Ritter, S. M., van Baaren, R. B., & Dijksterhuis, A. (2012). Creativity: The role of unconscious processes in idea generation and idea selection. *Thinking Skills and Creativity*, *7*, 21–27.
Sack, J. J. (2013). Development of a top-view numeric coding teaching–learning trajectory within an elementary grades 3-D visualization design research project. *Journal of Mathematical Behaviour*, *32*, 183–196.
Sak, U., & Maker, C. J. (2006). Developmental variation in children's creative mathematical thinking as a function of schooling, age, and knowledge. *Creativity Research Journal*, *18*, 279–291.
Silvia, P. J., & Sanders, C. E. (2010). Why are smart people curious? Fluid intelligence, openness to experience, and interest. *Learning and Individual Differences*, *20*, 242–245.
Singer, F. M., & Voica, C. (2013). A problem-solving conceptual framework and its implications in designing problem-posing tasks. *Educational Studies in Mathematics*, *83*, 9–26.
Tall, D. (2011). Crystalline concepts in long-term mathematical invention and discovery. *For the Learning of Mathematics*, *31*, 3–8.
Taylor, R. (1990). Interpretation of the correlation coefficient: A basic review. *Journal of Diagnostic Medical Sonography*, *6*(1), 35–39.
Urbach, N., & Ahlemann, F. (2010). Structural equation modelling in information systems research using partial least squares. *Journal of Information Technology Theory and Application*, *11*(2), 5–40.
Van der Veen, J. (2012). Draw your physics homework? Art as a path to understanding in physics teaching. *American Educational Research Journal*, *49*, 356–407.
Van Harpen, X. Y., & Sriraman, B. (2013). Creativity and mathematical problem posing: An analysis of high school students' mathematical problem posing in China and the USA. *Educational Studies in Mathematics*, *82*, 201–221.
Wechsler, S. M., Vendramini, C. M. M., & Oakland, T. (2012). Thinking and creative styles: A validity study. *Creativity Research Journal*, *24*, 235–242.
Zenasni, F., & Lubart, T. (2011). Pleasantness of creative tasks and creative performance. *Thinking Skills and Creativity*, *6*, 49–56.