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# A grey programming model for regional transit-oriented development planning*

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**Abstract.** This study presents a land-use design model for transit-oriented development planning at the regional level. The proposed model allocates a city's residential, employment and recreational activities according to four objectives and six groups of constraints. In order to handle adequately the presence of uncertainty and the flexibility needed in practical planning, the inputs and outputs of the model can be grey numbers. To solve the proposed model and other grey multi-objective programming models, the Grey TOPSIS approach can be adopted. The model is applied to Taipei City. Recommendations are made regarding the case study and a sensitivity analysis informs policy formulation and parameters setting.

**JEL classification:** C61, R41, R52

**Key words:** Transit-oriented development, land use planning, grey programming

## 1 Introduction

The spatial structure of urban development is principally shaped by major transportation systems. Automobile-oriented development causes urban sprawl, increases travelling distances and lowers resource-use efficiency. Since transit systems encourage a more efficient use of resources, planners are increasingly applying transit-oriented development (TOD) policies to re-shape the spatial structure of a city.

TOD base urban developmental plans on transit systems, thus increasing efficiency of land-use and transit operations. Theory and applications of TOD have been extensively studied, such as by Beimborn et al. (1991); Bernick and Cervero (1997); Corbett and Zykofsky (1999); Freilich (1998); and Moon (1990). The strategies discussed in these studies were classified into three categories by Cervero and Kockelman (1997): enhancing development density to raise

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transit ridership; diversifying land use to improve public transportation passenger convenience, and pedestrian-friendly design on walkways and transfer systems to increase the use of transit systems. A TOD has two planning levels. Regional planning comprehensively designs the spatial structure based on transit systems and focuses on the distribution of activities in a city. Local planning designs the detailed plan and concentrates on the precise contents of land use types, densities and facilities in a transit station area. Most studies have focused on local planning problems. Since urban spatial structure limits local development conditions and opportunities, recent studies, such as Kaneko and Fukuda (1999), have focused on regional TOD planning.

As a land use design problem (LDP), regional TOD planning is a mathematical programming model for land use planning in terms of type, size and location. Numerous LDPs have been developed for different scenarios. For instance, comprehensive land use planning has been discussed in Dokmeci et al. (1993); integrated planning of land use and networks in Lin and Feng (2003); land use expansion planning in Ridgley and Giambelluca (1992); urban renewal of transit station areas in Feng and Tsai (1988), and integrated planning of transportation and resource usage in Feng et al. (1989). Feng and Chang (1993) and Kaneko and Fukuda (1999) developed a regional TOD planning model by single objective programming, which addressed maximising transit ridership but ignored other important concerns of citizens and planners. The various goals of urban development often conflict with each other. For instance, locating more activities not only increases transit ridership, but also degrades the living environment. City planners should balance various aims, rather than emphasising only one goal. Lin and Gau (2005) developed a local TOD planning model by multi-objective programming which addressed the trade-offs among three aims, efficiency, environment and equity. However, regional TOD planning models that comprehensively consider urban development have seldom been developed.

Since the distribution of activities is a large-scale and general endeavour, planners must deal with uncertainty and ensure flexibility when progressing with a regional TOD plan. Uncertainty arises when planners only know the possible ranges, rather than precise values, of some planning conditions. For instance, the share of transit trips in Taipei City ranges between 30% and 50%. It varies by the hour (peak or off-peak), day (week or weekend), season (summer or winter), weather (sunny or rainy). The issue of flexibility emerges when planners want to respond flexibly to possible future scenarios, such as the lower and upper bounds of planned population. Traditional methods based on exact values of parameters cannot satisfy the planners' need for flexibility. Furthermore, stochastic and fuzzy methods, which are often used to handle uncertainty and flexibility, must specify probability and possibility distributions, which may be difficult to obtain in practice. Deng's (1982) grey numbers represent possible ranges (i.e., lower and upper bounds) of parameters or variables that are adequate to handle the practical needs mentioned above. Grey linear programming, in which parameters and variables are both grey numbers, was designed by Huang et al. (1992) and Huang and Moore (1993). This was followed by grey fuzzy programming (Huang and Baetz 1993), grey integer programming (Huang et al. 1995), grey nonlinear programming (Chang and Wang 1995) and grey multi-objective programming (Chang et al. 1996). These grey programming models were designed for waste management, and perform well with uncertain parameters (e.g., the possible range of the waste generation rate) and flexible planning results (e.g., the designed range of waste treatment capacity). Therefore, grey programming is adequate for designing regional TOD planning models.

This study presents a land use design model for regional TOD planning, in order to allocate the residential, employment and recreation activities in a city. The inputs and outputs of the model can be grey numbers to handle the uncertainty and flexibility in practical planning works. A Grey TOPSIS approach is designed to solve the developed model, and can also be used to solve other grey multi-objective programming models. Finally, the model is applied to Taipei

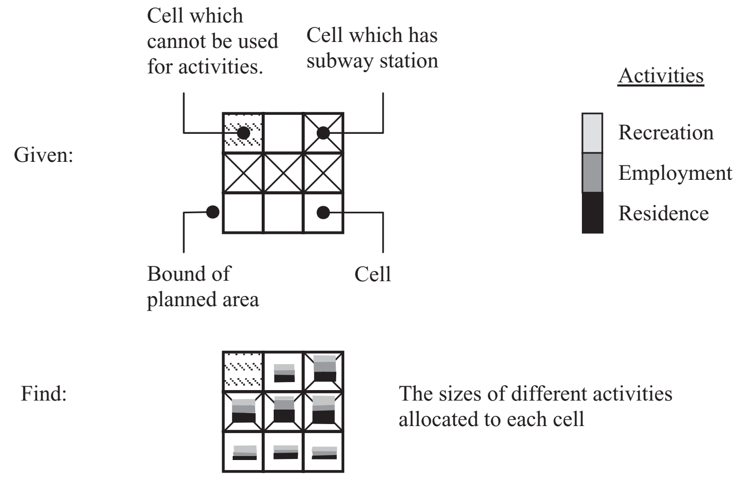



**Fig. 1.** Problem definitions

City TOD planning. The rest of this paper is organised as follows. Section 2 describes the modelling concepts, and Section 3 formulates the model. Section 4 then introduces the model-solving approach, and Section 5 illustrates the model application and analyses its sensitivity to parameters. Conclusions are finally drawn in Section 6, along with recommendations for future research.

## 2 Modelling concepts

When planning institutions decide to re-shape the urban structure using transit systems (this study only addresses the subway system), the regional TOD planning decision-making problem can be defined as in Fig. 1. The problem involves determining the optimal distributions of the sizes of urban activities under the following conditions: the planned area, the spatial division of cells in the planned area, the urban activities which need to be allocated (this study considers residence, employment and recreation), the cells which cannot be used for urban activities and the locations of subway stations. The decision variables should denote the types, sizes and locations of activities. Based on the planned distributions of activities, detailed and precise local infrastructure can then be addressed.

To identify the important planning objectives for TOD planning in Taiwan's cities, a survey was conducted in March 2003 among scholars, officers and planners in the fields of urban planning and transportation. Based on the survey results and the analytic hierarchy process (AHP), four objectives were selected for modelling in this study. The improvement of environmental quality in urban areas, in order to attract urban activities back to urban areas from suburbs or exurbs, had greatest priority. Infill and compact development benefits transit system operation.[1] The second objective was to enhance the convenience of interaction among activities. Removing impediments from interaction decreases the reliance on private vehicles and increases the likelihood of using alternative transportation modes, such as mass transit, walking and biking. Land-use variety enhances interaction convenience by shortening travel distances among activities. The third objective was to increase the subway's passengers, which benefits the operation of the subway system. Raising the development density in station areas signifi-

[1] Infill development refers to the creative recycling of vacant or underutilised lands within cities and suburbs.

cantly increases the transit usage according to empirical findings (e.g., Cervero and Kockelman 1997). The final objective was to enhance the accessibility of non-residential activities in order to decrease the travelling distance and the need to drive.

According to the modelling experiences of LDP and the considerations of TOD, the model should consider the following constraints. First, the sizes of the allocated activities in the entire planned area and each cell should be restricted to be within the given carrying capacities. Second, the differences within a cell between the present and planned activity sizes should be limited to ensure acceptance by citizens, which enhances the feasibility of realising the planned distribution. Third, the allocated size of an activity in a cell should be either greater than an economic threshold, or equal to zero. Fourth, the sizes of different activities allocated in the planned area and in a cell should be related to satisfy the requirements of specific activities. Fifth, the trips using the subway system should not exceed the system's capacity, and the volume conservation for trip ends should be constrained to sustain reasonable planning results. The final constraint relates to the value definitions of decision variables. This study used two decision variables: a binary variable equal to one when a specific activity locates in a specific cell and zero otherwise, and a non-negative variable to represent the size of a specific activity allocated to a specific cell. The next section formulates and describes parameters, decision variables, objectives and constraints.

## 3 Model formulation

This section begins by defining notations in 'white numbers' (i.e., precise values). The 'white model', whose parameters and variables are all white numbers, is then formulated based on the modelling concepts and notation. Finally, some parameters and decision variables of the white model are replaced by grey numbers (i.e. only upper and lower bounds are known) to create the grey model.

### *3.1 Notations definitions*

The white model has the following parameters:

$i$, $i'$, $l$, $k$: cells, where $i$ denotes the cell in the planned area; $i'$ represents a cell with a subway station; $l$ is the cell of the trip departure, and $k$ is the cell of the trip arrival;
$j$: denotes an activity: residence ($j = 1$), employment ($j = 2$) or recreation ($j = 3$);
$P_j^r$: amount of pollution $r$ formed by activity $j$ per year and per person, the pollution types include air ($r = 1$), water ($r = 2$) and solid waste ($r = 3$), measured in tons/person/year;
$C^r$: treatment cost for pollution $r$, measured in NT dollars/ton;
$K_{i'j}$: subway modal split rate of activity $j$ in cell $i'$;
$O_{ij}$: trip departure rate of activity $j$ in cell $i$, measured in person trips/person/day;
$D_{ij}$: trip arrival rate of activity $j$ in cell $i$, measured in person trips/person/day;
$T_{lk}$: trip distribution from cell $l$ to cell $k$ for work and recreation, measured in person trips/day;
$t_{lk}$: travel time from cell $l$ to cell $k$ for work and recreation, measured in hours;
$R_j$: floor space required by activity $j$, measured in 100 m$^2$/person;
$e_i^u$: upper bound of total floor space allocated in cell $i$, measured in 100 m$^2$;
$f_j$: clean water required by activity $j$, measured in 1000 m$^3$/person/year;
$\overline{F}_i$: upper bound of clean water supply for cell $i$ per year, measured in 1000 m$^3$/year;
$\overline{P}_i$: upper bound of residents in cell $i$ due to facilities' capacities, measured by number of persons;
$\overline{W}_i$: upper bound of job supply in cell $i$, measured by the number of jobs;

$q_c$: evaluation standard of the carrying capacity, later described in equation (6);
$A_{ij}$: upper bound rate of the difference between present and planned amounts for activity $j$ in cell $i$;
$N_{ij}$: present volume of activity $j$ in cell $i$, measured by the number of people;
$H_{ij}$: lower bound of economic scale for activity $j$ in cell $i$, measured by the number of people;
$m$: a very large number;
$N_p$: current number of residents in the planned area, as measured by the number of people;
$n_p$: lower bound rate of $N_p$ for the allocated residents in the planned area, later described in equation (11);
$b_p$: quantity ratio between residential and employment activities;
$s_p$: quantity ratio between residential and recreation activities;
$e_d$: quantity ratio for the relationship among activities in cells specified as type $d$ station areas;
$u_d$: relationship's direction of $e_d$, $\in \{1,-1\}$, later described in Eq. (15);
$N_d$: set of cells specified as type $d$ station areas;
$V_a$: total person trips using subway link $a$ in morning peak hour, as measured in person trips/hour;
$K_{l1}$: subway modal split rate of residential activity in cell $l$;
$C_a$: capacity of subway link $a$, measured in person trips/hour;
$P_a$: set of o-d pairs whose shortest paths use subway link $a$; and
$g$: ratio of morning peak hour trips to daily trips for subway system.

Two decision variables are defined: $Y_{ij}$ is a binary variable equal to one when activity $j$ locates at cell $i$ and zero otherwise and $X_{ij}$ is a non-negative variable representing the allocated size of activity $j$ in cell $i$.

### 3.2 *White model*

The model is based on the following assumptions. The transportation system and travel behaviour are both exogenously given: the travellers use only the subway stations located in the departure and arrival cells. The travel time between two cells is determined either by the subway system (in case of two cells both with subway stations), or by driving (via the shortest path). The subway system has the same capacity throughout the day. The trip distribution follows the rule of volume conservation, and only home-based travel for work and recreation are considered.[2] Second, the level of activity is determined by the number of individuals, i.e. the number of residents for the residential activity, the number of employees for the employment activity, and the number of customers for the recreational activity. Finally, the cost of re-allocating activities in the planned area is negligible, because the cost is spent over a long time frame, and the differences between present and planned activities in every cell are constrained within reasonable limits as described in Eq. (7).

#### 3.2.1 Objectives

The first objective is to improve environmental quality. Since the environmental quality relates to many indicators with different units and is difficult to measure in general, this study minimises the potential treatment cost of pollution using Eq. (1):

[2] Since the transportation system and travel behaviour are both exogenously given, interactions between transportation and land use are not endogenously analysed in the model. For example, there is no connection between modal split and travel times. Possible approaches for improving the model to address these limitations are proposed in the concluding section.

$$Z_1 = \sum_i \sum_j \sum_r C^r \times P_j^r \times X_{ij} \tag{1}$$

The term $C^r \times P_j^r \times X_{ij}$ measures the potential treatment cost of pollution $r$ for activity $j$ in cell $i$, and $Z_1$ summarises the cost in terms of pollution, activities and cells. More activities allocated in planned areas increase the potential cost and lower environment quality.

The second objective attempts to maximise the interaction convenience among activities. Land-use variety enhances this convenience and is measured by the number of allocated activity types as follows:

$$Z_2 = \sum_i \sum_j Y_{ij} \tag{2}$$

To ensure that the activity sizes allocated to cells are reasonable, the lower bound of economic scale for each cell is defined in Eqs. (9) and (10). The third objective attempts to maximise the number of subway passengers, which is determined by activity size, departure rate, arrival rate and subway modal split rate as follows:

$$Z_3 = \sum_{i'} \sum_j K_{i'j}\left(O_{i'j} + D_{i'j}\right) X_{i'j} \tag{3}$$

Note that $Z_3$ is approximately equal to twice the number of daily subway passengers, because the sum of daily departures is almost the same as the sum of daily arrivals for home-based travel. Of course, maximising $Z_3$ and maximising $(Z_3/2)$ result in the same optimal solutions.

The final objective attempts to increase the accessibility of non-residential activities, which is determined by the total travel time of working and recreation trips as follows:

$$Z_4 = \sum_l \sum_k T_{lk} t_{lk} \tag{4}$$

Minimising the result of Eq. (4) reduces pollution and energy consumption.

The above objectives should be considered separately for two reasons. First, the first and third objectives clearly conflict because allocating more activities increases subway passengers, but decreases environmental quality. Second, the four objectives have different implications and are measured in different units. If those objectives were transformed into one objective function, then an individual objective's performance would not be clearly identifiable, leading to insufficient information for evaluation and decision-making.

### 3.2.2 Constraints

The proposed model considers six groups of constraints. First, two constraints are determined by the carrying capacity as follows:

$$\sum_j R_j X_{ij} \le e_i^u, \forall i \tag{5}$$

$$\frac{\sum_j f_j X_{ij}}{\bar{F}_i} \le \frac{X_{i1}}{P_i} \le \frac{X_{i2}}{W_i} \le q_c, \forall i \tag{6}$$

Equation (5) states that the total floor space demanded by the allocated activities cannot exceed each cell's upper bound. Equation (6) stipulates that the size of allocated activities in each cell cannot exceed the carrying capacities of the clean water supply ($\bar{F}_i$), facilities supply ($\bar{P}_i$) and job supply ($\bar{W}_i$). The term $q_c$ is a given evaluation standard between 0 and 1. When $q_c = 1$, the activities' sizes are allowed to equal the carrying capacity of the three considerations above. Equation (6) indicates that the water supply is considered more seriously than the other two considerations, and that the facilities supply is considered more seriously than jobs supply. If the carrying capacities of different resources have various planning areas, then each resource should be formulated individually, thus replacing Eq. (6) with several constraints.

Second, the upper bounds of differences between present activities and planned activities for every cell are expressed as follows:

$$\left|X_{ij} - N_{ij}\right| \le A_{ij}N_{ij}, \forall i, j \tag{7}$$

where $A_{ij}$ denotes a given ratio to limit the difference. Equation (7) can be replaced by the following linear equation:

$$\left(1 - A_{ij}\right)N_{ij} \le X_{ij} \le \left(1 + A_{ij}\right)N_{ij}, \forall i, j \tag{8}$$

Third, the minimum size of an activity allocated to a cell is restricted as follows:

$$H_{ij} - X_{ij} \le m\left(1 - Y_{ij}\right), \forall i, j \tag{9}$$

$$X_{ij} \le mY_{ij}, \forall i, j \tag{10}$$

where $m$ denotes a very large number. If the activity $j$ is located at cell $i$ ($Y_{ij} = 1$), then the allocated size $X_{ij}$ exceeds the given minimum size $H_{ij}$ owing to Eq. (9), otherwise, if activity $j$ is not allocated at cell $i$ ($Y_{ij} = 0$), then the size $X_{ij} = 0$ owing to Eqs. (10) and (19).

Fourth, the relationships among activities are constrained at two levels. For the planned area, the allocated residents should exceed a given ratio ($n_p$) of the present residents as shown in Eq. (11), and the allocated employment and recreation activities should exceed the given ratios of the allocated residents as shown in Eqs. (12) and (13):

$$n_p N_p \le \sum_i X_{i1} \tag{11}$$

$$\sum_i X_{i2} \ge b_p \sum_i X_{i1} \tag{12}$$

$$\sum_i X_{i3} \ge s_p \sum_i X_{i1} \tag{13}$$

The above three equations are designed for the stability of city development. Moreover, to meet the requirements of activities and development goals for a cell, the relationship among activities for a cell is formulated as follows:

$$u_d(X_{i'2} + X_{i'3}) \ge u_d e_d X_{i'1}, \forall i' \in N_d \tag{14}$$

Equation (14) indicates that the size of allocated activities of employment and recreation should be greater (when $u_d = +1$), or smaller (when $u_d = -1$), than the size of allocated residential activity for the cells specified as type $d$ station areas. For example, $u_d = -1$ and $e_d = 1$ can be set for the cells where the non-residential activity should not exceed the residential activity.

Fifth, two considerations for the transportation system should be formulated: the capacity limitation of the subway system in Eqs. (15) and (16); and the volume conservation of trip ends in Eqs. (17) and (18). The volume and capacity are measured and restrained for each subway link. Only home-based working and recreation trips are considered in the volume conservation, of which Eqs. (17) and (18) limit the conservation of arrival and departure ends, respectively.

$$V_a = \sum_{(l,k)\in P_a} gK_{l1}T_{lk} \tag{15}$$

$$V_a \le C_a \tag{16}$$

$$\sum_l T_{lk} = D_{k2}X_{k2} + D_{k3}X_{k3}, \forall k \tag{17}$$

$$\sum_k T_{lk} = O_{l1}X_{l1}, \forall l \tag{18}$$

Finally, the possible values of decision variables are formulated as follows:

$$X_{ij} \ge 0, \forall i, j; \quad Y_{ij} \in \{0,1\}, \forall i, j; \quad T_{lk} \ge 0, \forall l, k; \quad V_a \ge 0, \forall a \tag{19}$$

where the variables of $T_{lk}$ and $V_a$ are defined for constraints.

### 3.3 *Grey model*

Since uncertainty and flexibility always exist in practical planning works, the model must include possible ranges, rather than precise values, of some parameters (planning conditions) and variables (planning results). This study replaces some parameters and variables in the white model with grey numbers, which can be defined as follows:

$$\otimes(a) = \left[\underline{\otimes}(a), \overline{\otimes}(a)\right] \tag{20}$$

where $\otimes(a)$ denotes a grey number with a lower bound of $\underline{\otimes}(a)$ and an upper bound of $\overline{\otimes}(a)$.

Some parameters in the white model are hard to determine precisely and have to be defined as grey numbers. First, the pollution generation rate, $\otimes\left(P_j^r\right)$, depends on environmental conditions. Second, the pollution treatment cost, $\otimes(C^r)$, depends on the technology used and the regulation standards. Third, the trip departure rate $\otimes(O_{ij})$, arrival rate $\otimes(D_{ij})$, floor space demand $\otimes(R_i)$, and amount of clean water $\otimes(f_i)$, all depend on people's backgrounds. For instance, a high-income resident usually makes more trips and consumes more floor space and water than a low-income resident. Fourth, the quantitative relationships among residential, employment and recreational activities, i.e. $\otimes(b_p)$ and $\otimes(s_p)$, are usually hard to be determined precisely in advance. Finally, the capacity of the subway system, $\otimes(C_a)$, usually varies by the time frame, vehicle's condition and operational manner.

After incorporating these grey parameters into the white model, the resulting grey model then can be presented as follows:

$$\begin{gathered}[\text{P1}]\ Min(1); Max(2); Max(3); Min(4)\\ \text{S.t.}\ (5); (6); (8) \sim (19)\end{gathered}$$

where $P_j^r$, $C^r$, $O_{ij}$, $D_{ij}$, $R_j$, $f_j$, $b_p$, $s_p$, $C_a$ and $X_{ij}$ are all replaced by the corresponding grey numbers. All the non-negative decision variables in [P1] are grey variables. The grey model can give planners flexible planning recommendations. For instance, planners can decide the most appropriate size of activity $j$ allocated in cell $i$ between the lower and upper bounds defined by $\otimes(X_{ij})$.

## 4 Problem solving approach

This study modified the technique for order preference by similarity to ideal solution (TOPSIS) approach to solve [P1]. The original TOPSIS was designed by Hwang and Yoon (1981) for multi-criteria evaluation. Lai et al. (1994) modified TOPSIS to solve multi-objective programming in two steps. The first step normalises all objectives and creates two equations for measuring the distances of solution to PIS (positive ideal solution) and NIS (negative ideal solution) individually. Second, a bi-objective programming of minimising the distance to PIS and maximising the distance to NIS is solved using Zimmermann's (1978) fuzzy programming approach to determine the compromise solution. Since TOPSIS is effective at transferring a large number of objectives into a bi-objective problem, which is solved more easily than the original problem, this approach can handle the four-objective programming in [P1]. However, the TOPSIS in Lai et al. (1994) was designed for the white model and needs further modification to solve [P1]. This study embedded the solving approach of grey linear programming (GLP) into the TOPSIS solving process to solve [P1], as well as other multi-objective GLP models.

The GLP approach was developed by Huang et al. (1992) and Huang and Moore (1993). The approach solves a GLP by solving two sub-problems, which are both standard linear programming problems and can be solved by conventional methods (such as Simplex) and packages (e.g., LINDO). By combining the solutions of two sub-problems, the optimum grey values of an objective and decision variables can be found. When solving GLP by this approach, the problem formulation should exactly match the form in Huang et al. (1992). For instance, the minimisation objective, or $\geq$ constraint, should be transformed into a maximisation objective, or $\leq$ constraint, by multiplying it by $-1$ (Liang 1997).

To solve [P1], this study developed the Grey TOPSIS approach to handle grey numbers when adopting Lai et al.'s (1994) method. Figure 2 shows the Grey TOPSIS process. In the following description of the steps of Grey TOPSIS, the definitions of grey algebraic operations (+, −, ×, ÷, <, =, >) are all based on Huang et al. (1995).

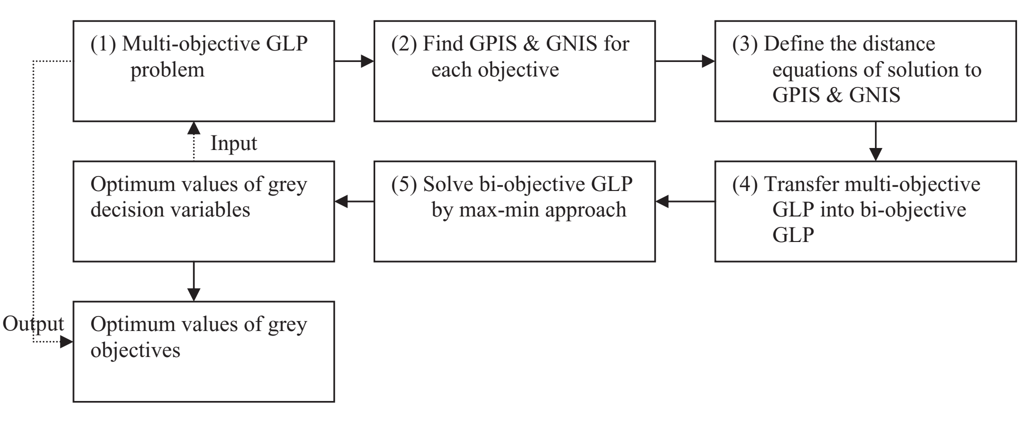



**Fig. 2.** Grey TOPSIS process

(1) Define a multi-objective GLP with $k$ objectives, $n$ decision variables and $m$ constraints as follows:

$$[\mathrm{P2}]\ Max/Min\,[\otimes Z_1(\otimes x), Z_2(\otimes x), \ldots, Z_k(\otimes x)]$$
$$\text{s.t.} \otimes x \in \otimes X = \{\otimes x | \otimes g_h(\otimes x)\,(\leq, =, \geq)\,0, h = 1, 2, \ldots, m\}$$

where $\otimes Z_j(\otimes x)$ denotes a maximisation objective, $j \in J$; $\otimes Z_i(\otimes x)$ denotes a minimisation objective, $i \in I$; $\otimes x = [\otimes x_1, \otimes x_2, \ldots, \otimes x_n]$ denotes the vector of grey decision variables, and $\otimes X$ denotes the grey feasible solution set. For instance, in problem [P1], $k = 4$, $J = \{2,3\}$, $I = \{1,4\}$ and the objective vector is $[Min\otimes Z_1(\otimes x); Max\otimes Z_2(\otimes x); Max\otimes Z_3(\otimes x); Min\otimes Z_4(\otimes x)]$.

(2) Find the grey PIS (GPIS) and grey NIS (GNIS). Let

$$\otimes Z^* = \{Max \otimes Z_j (\text{or } Min \otimes Z_i), \forall j\,(\text{or } i)\}$$

and

$$\otimes Z^- = \{Min \otimes Z_j (\text{or } Max \otimes Z_i), \forall j\,(\text{or } i)\}$$

where $j \in J$ and $i \in I$. Accordingly, $\otimes Z^* = \{\otimes Z_1^*, \otimes Z_2^*, \ldots, \otimes Z_k^*\}$ denotes the GPIS, and $\otimes Z^- = \{\otimes Z_1^-, \otimes Z_2^-, \ldots, \otimes Z_k^-\}$ denotes the GNIS. This step must solve $2k$ GLP problems. For example, in problem [P1],

$$\otimes Z^* = \{Min \otimes Z_1; Max \otimes Z_2; Max \otimes Z_3; Min \otimes Z_4\}$$

and

$$\otimes Z^- = \{Max \otimes Z_1; Min \otimes Z_2; Min \otimes Z_3; Max \otimes Z_4\},$$

for which in total $2 \times 4 = 8$ GLP problems have to be solved using the approach developed by Huang et al. (1992).

(3) Define the distance equations of solution to GPIS and GNIS individually as follows:

$$\otimes d_p^{GPIS} = \left\{ \sum_{j \in J} w_j^p \left[ \frac{\otimes Z_j^* - \otimes Z_j(\otimes x)}{\otimes Z_j^* - \otimes Z_j^-} \right]^p + \sum_{i \in I} w_i^p \left[ \frac{\otimes Z_i(\otimes x) - \otimes Z_i^*}{\otimes Z_i^- - \otimes Z_i^*} \right]^p \right\}^{1/p} \tag{21}$$

$$\otimes d_p^{GNIS} = \left\{ \sum_{j \in J} w_j^p \left[ \frac{\otimes Z_j(\otimes x) - \otimes Z_j^-}{\otimes Z_j^* - \otimes Z_j^-} \right]^p + \sum_{i \in I} w_i^p \left[ \frac{\otimes Z_i^- - \otimes Z_i(\otimes x)}{\otimes Z_i^- - \otimes Z_i^*} \right]^p \right\}^{1/p} \tag{22}$$

where $w_t (t = 1, 2, \ldots, k)$ denotes the weight of objective $t$, and $p = 1, 2, \ldots$ denotes the exponent of $w_t$ and indicates the distance to measure. For instance, $p = 1$ measures the grid street distance, and $p = 2$ measures Euclidean distance. According to Lai et al. (1994), increasing the $p$ value raises the influences of objectives with large weights on measuring distances. Therefore, the value of $p$ relates to the decision makers' preference. This study set $p = 1$ to maintain the problem at linear programming. For example in [P1], two grey distances can be defined as follows:

$$\otimes d_1^{GPIS} = w_1\left[\frac{\otimes Z_1(\otimes x) - \otimes Z_1^*}{\otimes Z_1^- - \otimes Z_1^*}\right] + w_2\left[\frac{\otimes Z_2^* - \otimes Z_2(\otimes x)}{\otimes Z_2^* - \otimes Z_2^-}\right] + w_3\left[\frac{\otimes Z_3^* - \otimes Z_3(\otimes x)}{\otimes Z_3^* - \otimes Z_3^-}\right] + w_4\left[\frac{\otimes Z_4(\otimes x) - \otimes Z_4^*}{\otimes Z_4^- - \otimes Z_4^*}\right]$$

$$\otimes d_1^{GNIS} = w_1\left[\frac{\otimes Z_1^- - \otimes Z_1(\otimes x)}{\otimes Z_1^- - \otimes Z_1^*}\right] + w_2\left[\frac{\otimes Z_2(\otimes x) - \otimes Z_2^-}{\otimes Z_2^* - \otimes Z_2^-}\right] + w_3\left[\frac{\otimes Z_3(\otimes x) - \otimes Z_3^-}{\otimes Z_3^* - \otimes Z_3^-}\right] + w_4\left[\frac{\otimes Z_4^- - \otimes Z_4(\otimes x)}{\otimes Z_4^- - \otimes Z_4^*}\right]$$

If the preference on the $p$ value differs significantly from 1 and the grey non-linear problem can be solved efficiently, then other $p$ values can be set in further studies.

(4) Transfer [P2] into the following bi-objective problem:

$$[\text{P3}]\ Min \otimes d_p^{GPIS}(\otimes x); \quad Max \otimes d_p^{GNIS}(\otimes x)$$
$$\text{s.t.} \otimes x \in \otimes X$$

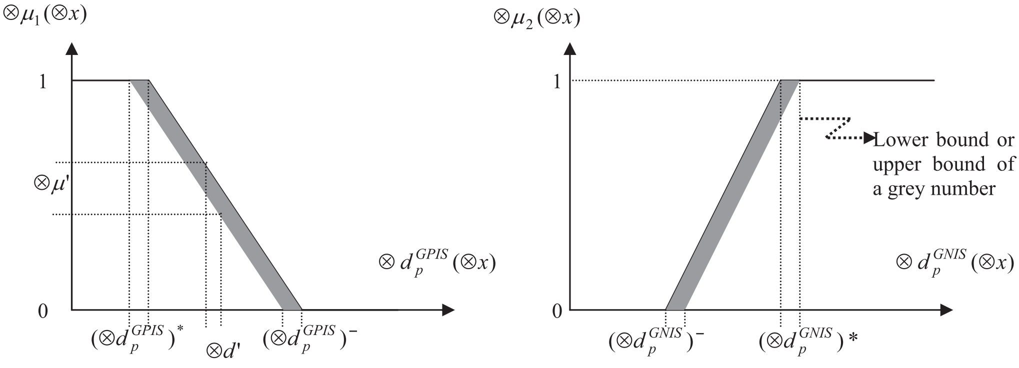
The problem [P3] attempts to minimise the distance to the GPIS and maximise the distance to the GNIS. These two objectives usually conflict with each other and cannot be achieved simultaneously. To determine the achievement levels of objectives, the fuzzy set membership function is applied. Define the following two linear membership functions as follows:

$$\otimes \mu_1(\otimes x)\begin{cases} 1 & \text{if } \otimes d_p^{GPIS}(\otimes x) < \left(\otimes d_p^{GPIS}\right)^* \\ 1 - \dfrac{\otimes d_p^{GPIS}(\otimes x) - \left(\otimes d_p^{GPIS}\right)^*}{\left(\otimes d_p^{GPIS}\right)^- - \left(\otimes d_p^{GPIS}\right)^*} & \text{if } \left(\otimes d_p^{GPIS}\right)^- \geq \otimes d_p^{GPIS}(\otimes x) \geq \left(\otimes d_p^{GPIS}\right)^* \\ 0 & \text{if } \otimes d_p^{GPIS}(\otimes x) > \left(\otimes d_p^{GPIS}\right)^- \end{cases} \tag{23}$$

$$\otimes \mu_2(\otimes x)\begin{cases} 1 & \text{if } \otimes d_p^{GNIS}(\otimes x) > \left(\otimes d_p^{GNIS}\right)^* \\ 1 - \dfrac{\left(\otimes d_p^{GNIS}\right)^* - \otimes d_p^{GNIS}(\otimes x)}{\left(\otimes d_p^{GNIS}\right)^* - \left(\otimes d_p^{GNIS}\right)^-} & \text{if } \left(\otimes d_p^{GNIS}\right)^- \leq \otimes d_p^{GNIS}(\otimes x) \geq \left(\otimes d_p^{GNIS}\right)^* \\ 0 & \text{if } \otimes d_p^{GNIS}(\otimes x) < \left(\otimes d_p^{GNIS}\right)^- \end{cases} \tag{24}$$

where $\left(\otimes d_p^{GPIS}\right)^* = \underset{\otimes x \in \otimes X}{\text{Min}} \otimes d_p^{GPIS}(\otimes x)$ denotes the optimum value of the first objective in [P3], of which the optimum decision variable vector is $\otimes x^P$; $\left(\otimes d_p^{GNIS}\right)^* = \underset{\otimes x \in \otimes X}{\text{Max}} \otimes d_p^{GNIS}(\otimes x)$ denotes the optimum value of the second objective in [P3], of which the optimum decision variable vector is $\otimes x^N$, $\left(\otimes d_p^{GPIS}\right)^- = \otimes d_p^{GPIS}(\otimes x^N)$ and $\left(\otimes d_p^{GNIS}\right)^- = \otimes d_p^{GNIS}(\otimes x^p)$. Figure 3 shows the membership function expressions. A grey distance, such as $\otimes d'$ in Fig. 3(A), creates a grey membership degree, such as $\otimes \mu'$ in Fig. 3(A).

Other studies have defined membership functions by the lower and upper bounds of a grey number, and performed well in their applications, such as Chang et al. (1997). However, this




(A) Equation (23) (B) Equation (24)

**Fig. 3.** Expressions for membership functions

study defines the membership functions by grey numbers, as grey distances to GPIS or GNIS, to meet the TOPSIS process.

(5) Based on the definitions of Eqs. (23) and (24), solve [P3] using the max-min approach in Zimmermann (1978) as the following problem:

$$[\text{P4}]\; Max \otimes a$$
$$\text{s.t} \otimes \mu_1(\otimes x) \geq \otimes a; \otimes \mu_2(\otimes x) \geq \otimes a; \otimes a \geq 0; \otimes x \in \otimes X$$

Problem [P4] is a GLP problem and can be solved by the approach described in Huang et al. (1992). The solved optimum values of the grey decision variables can then be used to calculate the optimum values of the grey objective functions in the original problem, i.e. [P2] or [P1].

## 5 Applications

The proposed model was applied to re-shape the distribution of activities in Taipei City for regional TOD planning. This section begins with a description of the case study, the parameters used and the illustration of the model's planning results. Finally, the sensitivity analyses of shifting the subway modal split rate and the degree of information uncertainty are presented.

### *5.1 Case and parameters*

Taipei's subway system has been in service in different phases since 1996. The system has currently 71 km of track and 65 stations serving Taipei City and five satellite cities in Taipei County. Another 65 km in length and 56 stations are currently under construction. To alleviate traffic congestion, reduce air and noise pollutions and restrain urban sprawl development, re-shaping of the urban spatial structure based on the subway system, that constitutes a major new transportation infrastructure project in Taipei, became an important policy in the city's comprehensive development plan.

Figure 4 illustrates the planned area, Taipei City, where 2.6 millions residents live in a land area of 272 km$^2$. The planned area has 82 cells, of which 47 cells have subway stations based on the current network in operation. Moreover, four dummy cells (D1 to D4) denote the satellite

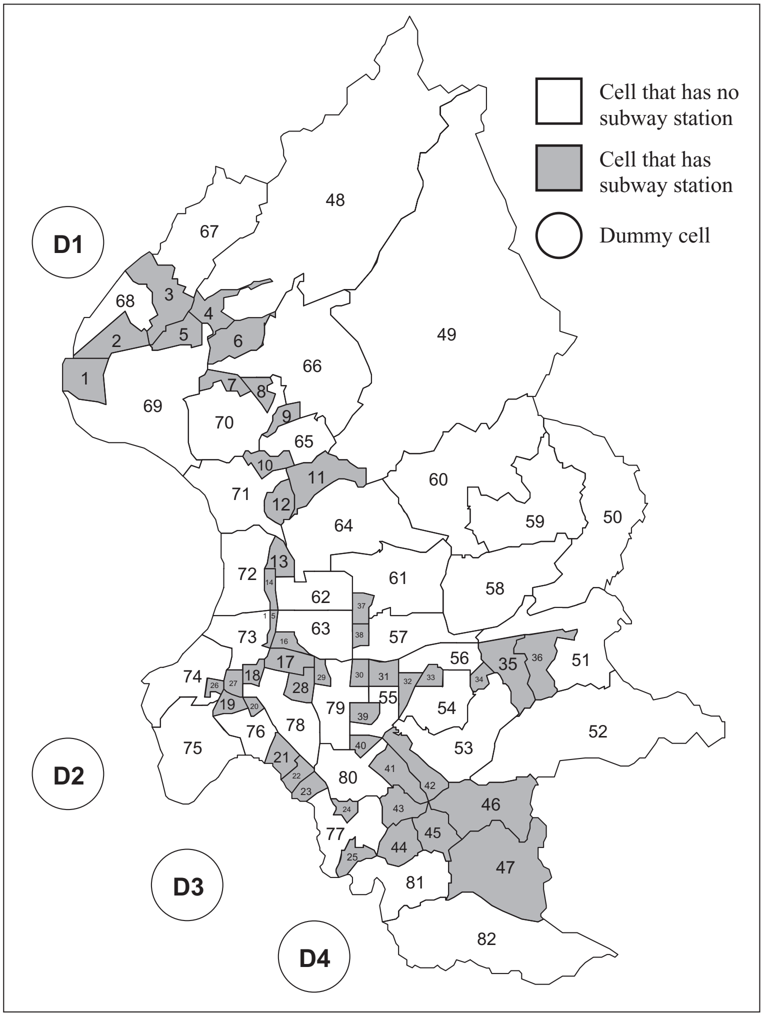



**Fig. 4.** Planned area and cells

cities connected with the planned area by subway system.[3] Based on the time frame stated in Department of Transportation of Taipei City (2001), i.e., the major source of data used in this study, the base year and target year in model analyses are set at the years 2000 and 2025, respectively.

This study determined the parameters of [P1] in two steps. First, all of the parameters were determined in terms of white values. The grey parameters were then set at the estimated white values $\pm 5.26\%$. This range denotes that the information uncertainty has a grey degree of 10%, and is determined by the following two equations: $D = \left(\bar{\otimes}a - \underline{\otimes}a\right) / \bar{\otimes}a$ and $M = \left(\bar{\otimes}a + \underline{\otimes}a\right)/2$, where $\bar{\otimes}a$ and $\underline{\otimes}a$ denote the upper and lower bounds of grey number $\otimes a$, $D$ is the degree of greyness and positively related to information uncertainty, and M denotes the whitened value of the grey parameter, determined in the first step. The value of $D$ can be determined by planners based on different scenarios in planning works: the higher the information uncertainty, the larger

[3] The dummy cells denote the volume conservation constraints; D1 denotes Tamsui Town; D2 denotes Banqiao City, D3 denotes Yonghe and Zhonghe Cities, and D4 denotes Xindian City.

the $D$ value. Combining the equations for $D$ and $M$ yields $\bar{\otimes}a = M + \left(\frac{D}{2-D}\right)M$ and $\underline{\otimes}a = M - \left(\frac{D}{2-D}\right)M$. If D = 10%, then $\left(\frac{D}{2-D}\right) = 5.26\%$.

Because [P1] has too many parameters to list here, only the estimations and sources of parameters are briefly described as follows:

(1) White parameters in objective functions. The subway modal split rate ($K_{i'j}$) and travel time ($t_{lk}$) were both estimated according to the investigations in Department of Transportation of Taipei City (2001).
(2) White parameters in constraint equations. First, the upper bound of floor space in a cell ($e_i^u$) was estimated by multiplying the total floor space limit in Department of Urban Development of Taipei City (1996) by the current ratio of the cell's floor space to the city's floor space. Three carrying capacities were determined: the upper bound of water supply ($\bar{F}_i$) used the supply volume of the base year, assuming that the supply volume would not change until the target year; the facilities service capacity ($\bar{P}_i$) was based on the Ministry of Interior in Taiwan's regulation on facilities supply, and the job supply ($\bar{W}_i$) was predicted by functions calibrated in Department of Transportation of Taipei City (2001). The evaluation standard ($q_c$) was set at 1.4 based on the test experiences, and the difference between present and allocated amounts ($A_{ij}$) was set at 20%. Both of these two parameters can be set at another value according to planning needs. Second, the current levels of activities ($N_{ij}$) were determined from two investigations: the Department of Transportation of Taipei City (2001) for residents and employees, and the Department of Urban Planning of Taipei City (1991) for recreation activity. Third, the lower bound of economic scale ($H_{ij}$) was determined by the smallest current amount among cells for each activity. Fourth, the lower bound ratio of planned residents to the present residents ($n_p$) was set at 95% because the total number of residents in Taipei City has been stable in the past decade. The cell's quantity ratios among different activities ($e_d$) were determined by the present ratios. Five station area types were specified by Department of Urban Development of Taipei City (2001): central business district (CBD) ($d = 5$), transfer centre ($d = 4$), office-commerce mix ($d = 3$), residence-retail mix ($d = 2$) and residence ($d = 1$). The lower bound ratios of non-residential activities to residential activity ($u_d = 1$) for $d = 2$ to 5, and the upper bound ratio of non-residential activities to residential activity, ($u_d = -1$) for $d = 1$, were considered. Finally, the ratio of morning-peak-hour trips to daily trips ($g$) in the base year was 8.85% based on records from Taipei Mass Rapid Transit Company (TMRTC).
(3) Grey parameters in objective functions. The pollution generation rate ($\otimes(P_j^r)$) and treatment cost ($\otimes(C^r)$) were estimated from investigations of the Council for Economic Planning and Development (2000) and the Department of Environment of Taipei City (2001). The trip departure and arrival rates ($\otimes(O_{ij})$ and $\otimes(D_{ij})$) were based on figures from the Department of Transportation of Taipei City (2001).
(4) Grey parameters in constraint equations. The demand rates of clean water ($\otimes(f_j)$) and floor space ($\otimes(R_j)$), and the city's quantity ratios among activities ($\otimes(b_p)$ and $\otimes(s_p)$), were all determined by the base year values and can be changed according to planning needs. The Taipei subway system has two sub-systems in terms of median capacity and mass capacity, and its capacity data ($\otimes(C_a)$) was obtained from TMRTC.

### *5.2 Planning results*

The planned distribution of activities in all cells is available in Li (2003). Figure 5 shows the allocated activity levels in eleven cells, located in Daan and Hsinyi Districts, with six subway

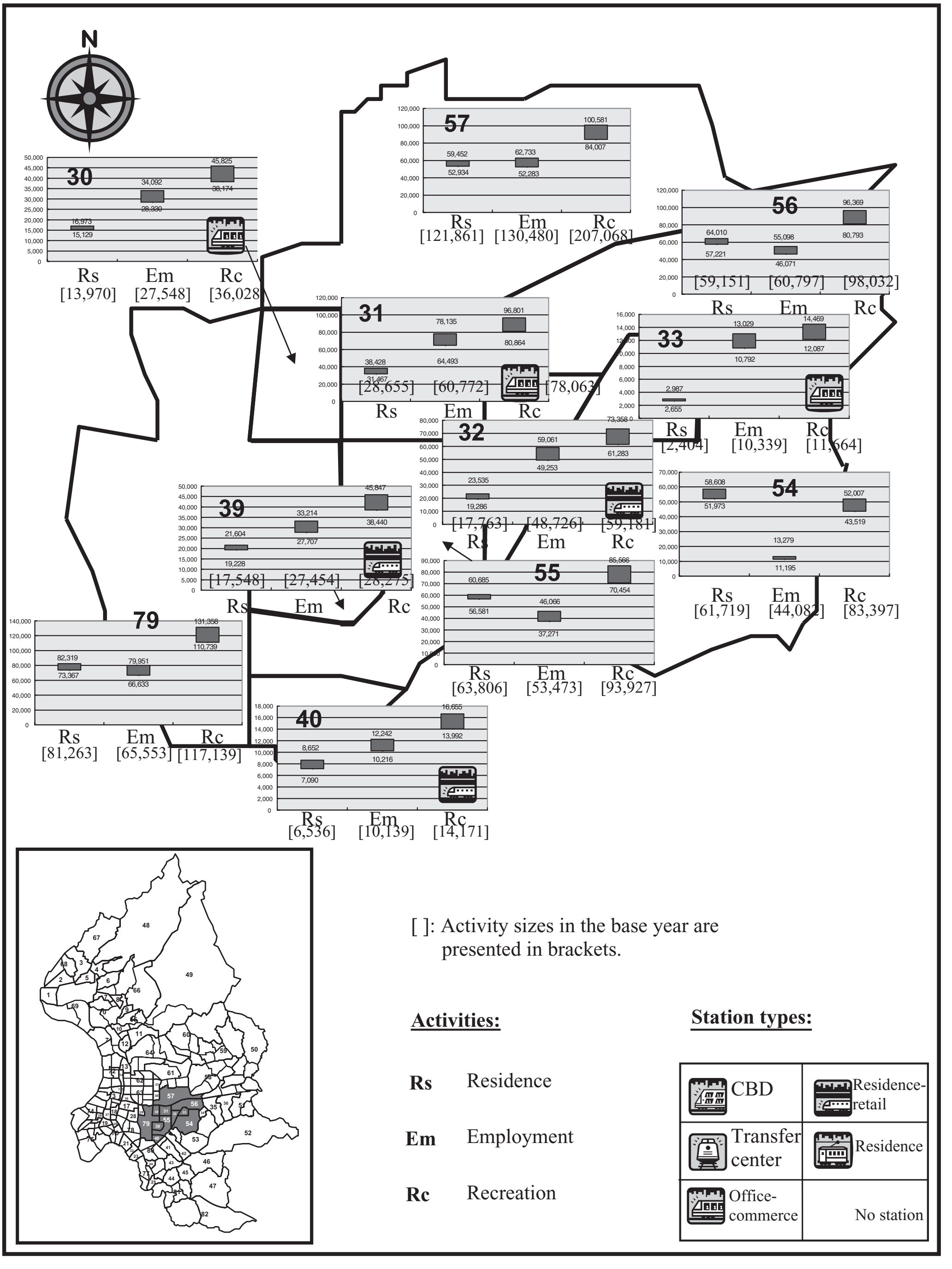



**Fig. 5.** Planned distribution for part of cells

stations including two station areas: office-commerce mix and residence-retail mix. The planning results are grey numbers for three activities located in each cell. For instance, cell 40, a residence-retail mix station area, is recommended to locate 7,000–8,600 residents for residential activity; 10,000–12,000 employees for employment activity, and 14,000–16,000 customers for

**Table 1.** Comparisons between present distribution and planned distribution

| Items / Objectives (unit) | Objective values for the present distribution | Average objective values for the present distribution* | Average objective values for the planned distribution* |
|---|---|---|---|
| Min $\otimes(Z_1)$ (NT dollar) | 7,361,537,880 | 908 | [643,1103] |
| Max $\otimes(Z_2)$ (none) | 246 | 246** | 246** |
| Max $\otimes(Z_3)$ (person trip) | 732,353 | 0.09 | [0.11,0.17] |
| Min $\otimes(Z_4)$ (hour) | 970,374 | 0.37 | [0.13,0.19] |

* Average objective value = objective value/amount of relative activities; residential, employment and recreation activities for $\otimes(Z_1)$ and $\otimes(Z_3)$, residential activity for $\otimes(Z_4)$.
** Objective value rather than average objective value.

recreation activity. Based on the recommended ranges from the modelling, planners can determine adequate activity levels by considering other essential issues and goals.

Figure 5 also shows the current activity levels. All the cells with subway stations have more planned activities than current levels, and most of the cells without stations have fewer planned activities than current levels. The planned distribution presents two features of TOD in regional scale planning. First, the densities of activities in station areas are higher than those in non-station areas. The average densities are in the range of [118, 185] persons/hectare for residential activity, [81, 98] persons/hectare for employment activity and [152, 182] persons/hectare for recreation activity. The activities density in cells with subway stations are double those in cells without subway stations. Additionally, activities are balanced either in the cell itself or among neighbouring cells. For example of Fig. 5, the numbers of residents and jobs are balanced in cell 79 and between cells 32 and 54, and the level of recreation activity meets the demands of the other activities for each cell.

Table 1 compares the present and planned distributions, showing that although the two distributions have almost the same performance on the first and second objectives, the planned distribution performs better than the present distribution on the third and fourth objectives. Accordingly, the spatial structure re-shaped by the developed model dominates the present structure in terms of the objectives for regional TOD planning.

### *5.3 Sensitivity analysis*

This sub-section discusses two sensitivity analyses, for policy analysis and for information uncertainty.

#### 5.3.1 Raising subway modal split rate ($k_{i'j}$)

The average subway modal split rate in the base year was about 20%. If the administration decided to promote strategies for increasing subway ridership in the future, then what planning changes would result? This study raised the subway modal split rate and studied the changes in the model's objective values as shown in Fig. 6. The second objective, to maximise the total number of allocated activity types, is easy to optimise (82 cells × 3 activity types per cell = 246 activity types) in both present and planned distribution, and was found to be unrelated to modal split rate, while the other three objectives presented significant changes with the rise in subway ridership. The second objective seems to be redundant, and can be ignored in further studies and practical planning because mixed use of land is common in Taiwan. The second objective is automatically maximised when considering the constraints of Eqs. (8) and (14). The increase in

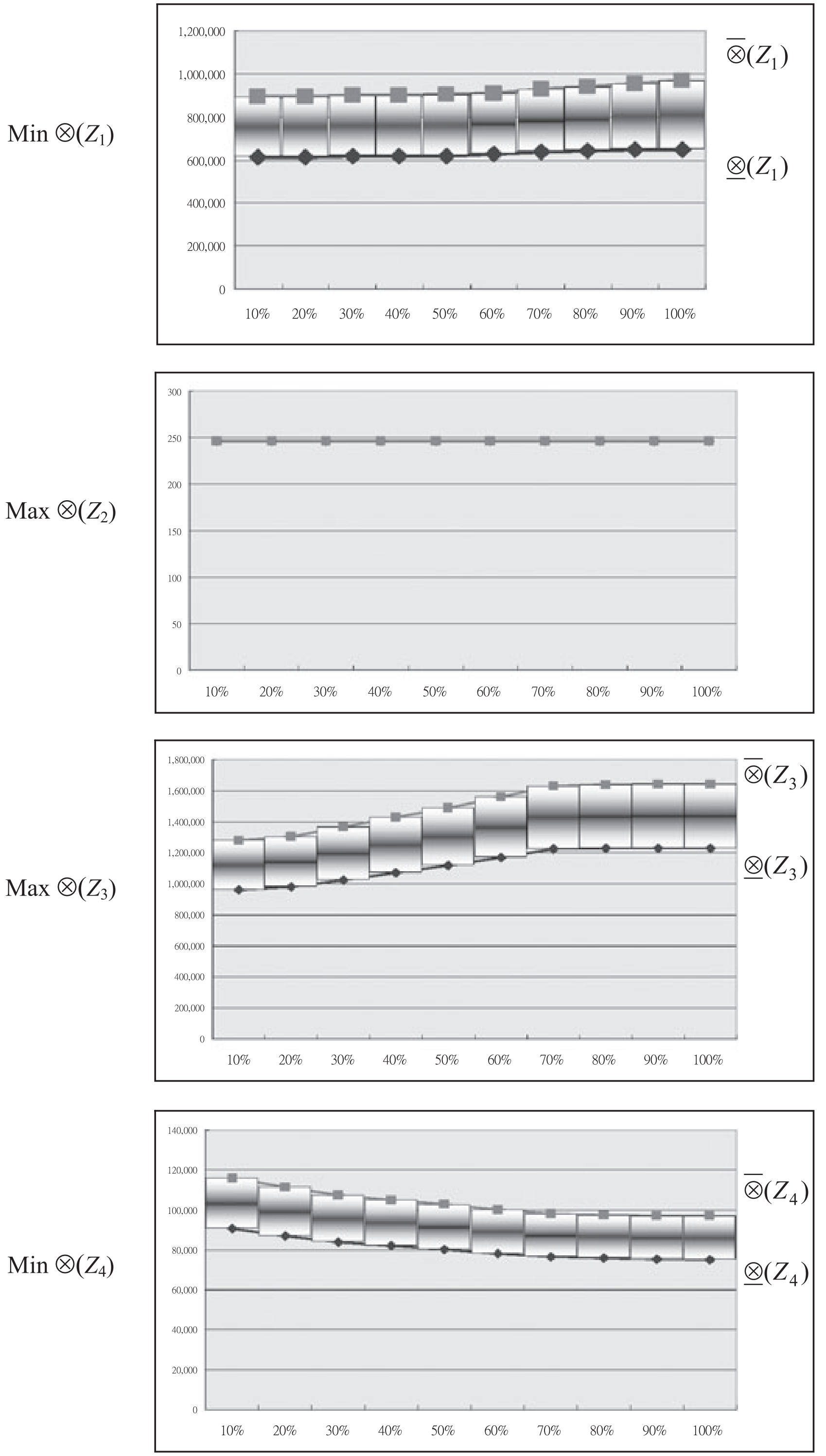



**Fig. 6.** Sensitivity analysis of raising subway modal split rate

*Note*: For each chart, the horizontal axis represents the raising percentage of the subway modal split rate in each cell; and the vertical axis denotes the objective value with the same unit in Table 1.

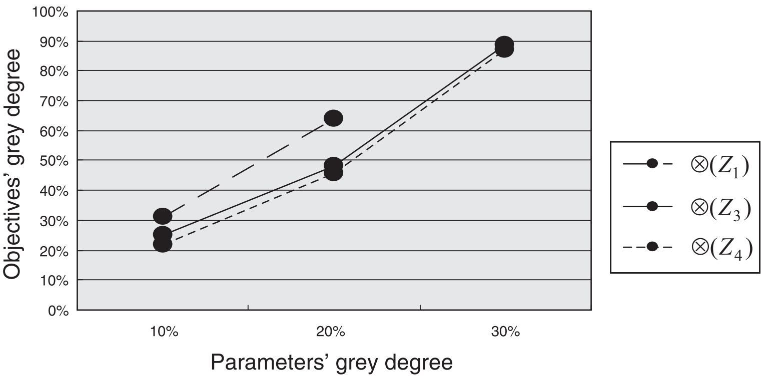



**Fig. 7.** Sensitivity analysis of increasing parameters' grey degree

the subway modal split rate slightly worsens the first objective and significantly improves the third and fourth objectives because activity levels rise in subway station areas. However, owing to capacity constraints, the improvements in the last two objectives cease when the subway modal split rate is increased by more than 70%. Therefore, this study recommends raising the subway modal split rate for each cell by up to 70%.

### 5.3.2 Increasing the degree of greyness of the parameters ($D$)

The uncertainty of information was set at a 10% degree of greyness in the case study. If the uncertainty of information increased, then how would the planning results change? This study raised the degree of greyness of all grey parameters simultaneously from 10% to 20% and then 30%. Figure 7 shows the consequent changes of the grey degree of objective values. The grey degree of the second objective remained at 0% and can be ignored in the discussion. Figure 7 reveals the following findings for the model. First, the grey degrees of objectives are three times of the grey degrees of parameters. When information uncertainty rises, the model enlarges the activity distribution ranges to handle the risk caused by uncertainty. Second, comparing the first objective and the last two objectives shows that the multiplication and division operations of grey numbers in the objective function create larger grey degrees in objective values than does the summation operation. Third, setting the parameters' grey degree too high causes unreasonable objective values. For instance, if the parameters' grey degree was 30%, then the grey degree of the first objective exceeds 100%, and its value range includes both negative and positive values, which is unreasonable in practical planning exercises.

## 6 Conclusion

Although the TOD paradigm has been increasingly studied, an analytical model that can help planners evaluate new infrastructure efficiently and systematically is still under development. This study develops a TOD planning model by applying grey programming to deal with the uncertainty and flexibility in practical applications, and by developing a model at a regional, rather than local, scale to re-shape the distribution of activities in a city for TOD. Furthermore, the problem solving approach developed in this study, Grey TOPSIS, was found to be applicable to a case study. However, the proposed approach still requires theoretical proof of stability, defined as follows. The resulting grey value of the objective function based on the final set of grey optimal solutions is expected to be equal to the grey value of the objective function in the final optimal solution. Stability is obtained when this condition holds (Chang et al. 1999).

Based on the results of the case study, we can recommend a spatial structure for activity distribution as part of the sketch (or comprehensive) planning of Taipei City. The present distribution was found to be dominated by the planned distribution in terms of TOD objectives. Sensitivity analysis results reflect the experiences of model application for policy analysis and parameters setting.

Moreover, this study makes the following three recommendations for model application. First, the [P1] model was developed for regional planning of activity distribution, rather than for detailed local planning in a local area. Second, the grey programming approach applied in the model is appropriate in that only parameter ranges are known. When the probability distributions of parameters can be identified, then stochastic or fuzzy programming is recommended to replace the grey programming component of the model. Third, the spatial division of cells in the planned area should match with the spatial division of existed data or statistics required for estimating parameters.

As for development of the model, this study recommends the following areas for further study. First, the transportation system and travel behaviour must be endogenously designed or analysed in the developed model to reach better and more convincing solutions. Like the traditional LDP model, this study designs the distribution of activities under given transportation system. Many studies, such as Lin and Feng (2003), have developed models to design transportation and land use systems simultaneously according to interactions between transportation and land use. Moreover, since many studies, such as Crane (2000), have found that land use patterns significantly affects travel behaviour, the travel demand must be endogenously analysed in a land use model, as in the Integrated Transportation and Land-Use Package (Putman, 1983), combined trip distribution/assignment model (Evans 1976; Sheffi 1985) and the spatial computable general equilibrium model in Anas and Xu (1999). The model building methods mentioned above are useful for improving the model developed in this study.

Second, more comprehensive analyses can be considered in the model. For instance, the link volumes of the subway system in the model only address peak-hour volumes and trip end volume conservation. Handling off-peak volumes and subway trip assignment is essential to analyzing volumes adequately. In addition to residence, employment and recreation, more activities and trip purposes must be considered to develop further the model. Third, the market mechanism must be embedded in the model to make the planning results feasible for implementation. For instance, the equilibrium between supply and demand of the property market for each cell should be considered. Finally, since the real world usually operates with non-linear relationships, the model formulation should be extended to a non-linear problem.

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# Un modelo de programación Grey para la planificación de desarrollo regional orientado al tránsito

**Jen-Jia Lin, Chia-Nung Li**

**Abstract.** Este estudio presenta un modelo de diseño de uso del suelo para la planificación de desarrollo regional orientado al tránsito de ámbito regional. El modelo propuesto asigna las actividades residenciales, laborales y de recreo de acuerdo con cuatro objetivos y seis grupos de restricciones. Para poder manejar adecuadamente la presencia de incertidumbre y la flexibilidad necesaria en la planificación práctica, los *inputs* y *outputs* del modelo pueden ser números *grey*. Para resolver el modelo propuesto y otros modelos de programación multiobjetivo *grey*, se puede adoptar el método Grey TOPSIS. El modelo se aplica para Taipei City. Se ofrecen recomendaciones sobre el estudio de caso y un análisis de sensitividad asesora en la formulación de políticas y el establecimiento de parámetros.

**JEL classification:** C61, R41, R52

**Palabras clave:** Desarrollo orientado al tránsito, planificación del uso del suelo, programación *grey*

要旨： 本調査では、地域レベルでの交通志向型の開発計画に関する土地利用計画モデルを提示する。提案モデルでは、４つの目的及び６つの制約要件グループに基づいて、都市の居住、雇用、娯楽活動を割り当てる。実用的な計画における不確実性の存在や計画に必要な柔軟性に適切に対処するため、モデルのインプットとアウトプットは灰数とすることが可能である。提案モデル及びその他の灰色多目的プログラミング・モデルを解くために、灰色TOPSISアプローチを採用することができる。モデルは台北市に適用した。ケーススタディについての提言を示すとともに、感度分析により政策形成及びパラメータ設定に関する情報を提供する。