Optimal nonlinear income taxation with productive government expenditure

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ABSTRACT

This paper develops an endogenous growth model with a production externality and nonlinear income taxation, and uses it to examine how the fiscal authority devises its nonlinear tax structure from the viewpoint of welfare maximization. It is found that, in the Barro (1990) model, Pareto optimality can be achieved if both policy instruments for the tax scalar and the extent of the tax progressivity/regressivity are set optimally.

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1. Introduction

In an influential paper, Barro (1990) sets up an endogenous growth model, which he uses to demonstrate that the government's investment in infrastructure is crucially related to economic growth and social welfare. On the one hand, Barro proposes, as in the case of physical capital, that the government's investment in infrastructure is helpful to private production. This specification introduces a distortion, namely, a production externality to the economy.[1] On the other hand, by means of the mechanism whereby the government's expenditure on infrastructure is financed by a proportional tax, this specification introduces a further distortion in income taxation to the economy. Given these two distortions, Barro finds that the first-best equilibrium (i.e., the Pareto optimality) cannot be achieved, even if the optimal income tax rate is optimally set by the fiscal policy. To be more specific, the welfare level under the first-best regime is definitely higher than that under the second-best regime.[2]

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[1] This distortion is named the production externality by Park and Philippopoulos (2003) and Dioikitopoulos and Kalyvitis (2008), while it is named the externality associated with the provision of public good by Barro (1990) and Irmen and Kuehnel (2009).

[2] Barro and Sala-i-Martin (1992, p. 649) propose that the Pareto optimality can be achieved if distorting income taxation is replaced by non-distorting lump-sum taxation. Turnovsky (1997) and Gómez (2004) specify that public capital is characterized by the degree of congestion and that the government balances its budget by adjusting lump-sum taxation. In their paper, the fiscal authority could choose the proportionate income tax rate that enables the welfare level under the second-best regime to be equal to that under the first-best regime.

This paper proposes a plausible vehicle to achieve the Pareto optimality. More concretely, by introducing a nonlinear tax structure developed by Slobodyan (2005) and Greiner (2006) into the Barro (1990) model, two policy instruments for the tax scalar and tax progressivity/regressivity are then available to the fiscal authority. It is found that a suitable package containing these two instruments can fully remedy the inefficiencies arising from the production externality and distortionary taxation, and, as a result, the Pareto optimality can be restored.

The reason why a suitable package containing the tax scalar and tax progressivity/regressivity can fully remedy market inefficiencies is as follows. In the Barro (1990) model, with a uniform tax on labor and capital incomes, the government is unable to restore the first-best allocation because a positive tax rate is required to finance productive public goods. However, a positive tax rate on capital income drives a wedge between the social and private returns to capital investment. In the Barro (1990) model with a linear income tax, this wedge prevents the government from achieving the first-best allocation. By allowing for tax regressivity that increases the private return to capital investment, the government is able to eliminate this wedge and achieve the first-best allocation by optimally coordinating the two fiscal instruments of tax scalar and tax regressivity. Intuitively, a regressive tax system encourages households to accumulate more capital because a higher capital income translates into a lower average tax rate under this tax system.

Our specification of a nonlinear tax structure is confirmed by a number of empirical studies, in which the tax rate is generally found to be progressive or regressive rather than proportional. For example, by using US data, Pintus (2008) provides some evidence to support tax progressivity and tax regressivity. His results reveal that estimated values of tax progressivity lie between 0.7% and 1.8% during the period 1940–80. Moreover, tax regressivity arises during the periods 1920–40 and 1980–90, and the estimated values in association with these two periods are about −2.4% and −0.7%, respectively. In addition, Kakwani (1977) finds that a progressive tax system arises in Australia, Canada and the US during the period 1968–70 and in the UK during the period 1964–67. Kakwani and Lambert (1998) and Van Ewijk and Tang (2007) indicate that the income tax system is regressive in Australia in 1984 and in the UK in 1994, respectively. With these empirical observations, it is fair to say that our specification of a nonlinear tax structure embodying both tax progressivity and tax regressivity is much better than a proportional tax system as a representation of reality.

It is curious that government expenditure financed by a nonlinear tax structure is very scarce in the literature on endogenous growth. Yamarik (2001) finds that in the AK model a higher tax progressivity does not affect the balanced growth rate although it can reduce the transitional growth rate. Li and Sarte (2004) find that, after the Tax Reform Act of 1986, a fall in the progressive tax rate results in a rise in US per capita GDP growth by 0.12 to 0.34 percentage points. To be more specific, Li and Sarte's (2004) analysis indicates that, with a production externality from the government's infrastructure investment, an increase in tax progressivity definitely depresses the balanced growth rate. However, none of these studies deal with the perspective of social welfare maximization. To fill the gap in this strand of the literature, this paper instead discusses whether the government can devise an optimal non-linear income tax structure to achieve the Pareto optimality.

The remainder of this paper proceeds as follows. Section 2 sets out an endogenous growth model featuring production externalities and nonlinear income taxation. Section 3 discusses the economy's balanced growth equilibrium. Section 4 compares the relative welfare level between the first-best regime and the second-best regime. Finally, concluding remarks are provided in Section 5.

2. The model

2.1. Households

The economy is populated by a large number of identical and infinitely-lived households. For simplicity, population is normalized to unity. The representative household derives utility from consumption C; its lifetime utility is given by

$$\int_0^\infty \frac{C^{1-\theta}-1}{1-\theta} e^{-\beta t} dt; \ \beta > 0, \ \theta > 1, \tag{1}$$

where β denotes the constant rate of time preference, θ is the inverse of the intertemporal elasticity of substitution, and labor supply L is inelastic and normalized to unity. The representative household holds physical capital K as its asset. At each instant of time, the representative household's flow budget constraint can then be expressed as

$$\dot{K} = (1-\pi)(w + rK) - C - \delta K; \ 0 \le \pi < 1, \ 0 \le \delta < 1, \tag{2}$$

where π is the income tax rate, w denotes the real wage, r represents the rate of return on physical capital, and δ is the capital depreciation rate.

Let $\overline{Y}$ stand for the economy-wide average income and $Y(= w + rK)$ be the household's income. Following Slobodyan (2005) and Greiner (2006), we assume that the income tax rate π takes the following form:

$$\pi = 1 - (1-\tau)\left(\frac{\overline{Y}}{Y}\right)^\phi; \ 0 < \tau < 1. \tag{3}$$

Along the lines of Li and Sarte (2004), the parameters τ and ϕ reflect the scalar in the tax schedule and the extent of tax progressivity/regressivity, respectively.

The tax schedule reported in Eq. (3) has the following features. First, total income tax is defined as $T = \pi Y$; hence the marginal income tax rate π_m can be expressed as $\pi_m = \partial T / \partial Y$. To match the observation reality, the restriction $\pi_m \geq 0$ is imposed.[3] As a consequence, from Eq. (3) we have

$$\pi_m = \frac{\partial T}{\partial Y} = \frac{\partial(\pi Y)}{\partial Y} = 1 - (1-\tau)(1-\phi)\overline{Y}^\phi Y^{-\phi} \geq 0. \tag{4}$$

Second, the average tax rate π_a can be defined as $\pi_a = T/Y = \pi Y/Y = \pi$. It is straightforward to infer from this definition and Eq. (3) that

$$\frac{\partial \pi_a}{\partial Y} = \phi(1-\tau)\overline{Y}^\phi Y^{-(1+\phi)} \gtreqless 0; \quad if \quad \phi \gtreqless 0. \tag{5}$$

Eq. (5) indicates that whether the average tax rate is increasing, unchanged, or decreasing depends on the sign of ϕ. Based on this feature, the tax schedule is said to be progressive, flat, or regressive when the average tax rate is increasing, unchanged, or decreasing, respectively. It should be noted that Barro (1990) confines his analysis to the flat tax schedule, and hence is only concerned with the situation associated with $\phi = 0$.

Third, if the current income equals the economy-wide average income, then the average tax rate degenerates to $\pi = \tau$. This is the reason why the parameter τ is referred to as "the scalar of tax schedule".[4]

The representative household maximizes Eq. (1) subject to Eqs. (2) and (3) by choosing a sequence $\{C, K\}_{t=0}^{\infty}$. By letting λ be the shadow value of the physical capital stock K, the optimum conditions are as follows:

$$C^{-\theta} = \lambda, \tag{6a}$$

$$(1-\phi)(1-\tau)\overline{Y}^\phi Y^{-\phi} r - \delta = -\frac{\dot{\lambda}}{\lambda} + \beta. \tag{6b}$$

Eq. (6a) indicates that the household equates the marginal utility of consumption with the marginal utility of physical capital. Eq. (6b) reveals that the net rate of return on physical capital equals the rate of return on consumption. Finally, to ensure that the representative household's intertemporal budget constraint is met, the transversality condition of K must be imposed as follows:

$$\lim_{t \to \infty} \lambda K e^{-\beta t} = 0. \tag{7}$$

2.2. Firm

There is a continuum of identical competitive firms with the total number normalized to one. Output of the representative firm Y is produced by using physical capital K, labor L, and the government's infrastructure expenditure G. In line with Greiner (2006), assume that the government's infrastructure expenditure raises the productivity of labor input, implying that it is labor augmenting. Accordingly, the production function can be expressed as follows:

$$Y = K^{1-\alpha}(LG)^\alpha; \quad 0 < \alpha < 1. \tag{8}$$

In line with Barro (1990), in Eq. (8) the government's infrastructure, which is nonrival and nonexclusive, enters as additional input in the technology available to firms. This provision of the government's infrastructure generates an externality to the private production, meaning that all firms benefit from the provision of public goods by producing more output without incurring any cost for using the government's infrastructure.

The producers act competitively by treating the output and input prices as given and taking the government's infrastructure as given. The demand functions for labor and physical capital are respectively given by

$$w = \alpha\, K^{1-\alpha} L^{\alpha-1} G^\alpha, \tag{9a}$$

$$r = (1-\alpha)K^{-\alpha}L^\alpha G^\alpha. \tag{9b}$$

Eqs. (9a) and (9b) indicate that the firm employs both inputs until the marginal product of each input equals its market price.

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[3] See, for example, Diamond (1998) for a survey on the nonnegative marginal income tax rate.
[4] Alternatively, the parameter τ is referred to as "the level of tax schedule" by Guo and Lansing (1998) and Greiner (2006).

2.3. Government

The public expenditure is financed by the income tax. Thus, the government's flow budget constraint can be written as follows:

$$G = \pi(w + rK) = \pi Y = Y - (1 - \tau)\overline{Y}^{\phi} Y^{1 - \phi}. \tag{10}$$

By putting the inelastic labor supply ($L = 1$), the representative household's budget constraint in Eq. (2), the demand function for production labor in Eq. (9a), the demand function for physical capital in Eq. (9b), and the government's budget constraint in Eq. (10) together, the social resource constraint is given by

$$\dot{K} = Y - C - G - \delta K. \tag{11}$$

3. Competitive equilibrium

We confine the analysis to a symmetric equilibrium where the representative household income and economy-wide average income are equivalent, i.e., $Y = \overline{Y}$. Under such a situation, it follows from Eq. (3) that the income tax is independent of the extent of tax progressivity (regressivity)

$$\pi = \tau. \tag{12}$$

Based on Eqs. (6a), (6b), (9b), (10) and (12), we can derive the following expression for an optimum as follows:

$$\frac{\dot{C}}{C} = \frac{1}{\theta} \left\{ (1 - \phi)(1 - \alpha)(1 - \tau)\tau^{\frac{\alpha}{1 - \alpha}} - \delta - \beta \right\}. \tag{13}$$

Eq. (13) is known as the Keynes–Ramsey rule. It is quite clear in Eqs. (12) and (13) that the extent of tax progressivity/regressivity ϕ affects the representative household's intertemporal consumption decision even though it does not affect the income tax in equilibrium.

Substituting Eqs. (8), (10) and (12) into Eq. (11), the resource constraint can be expressed as follows:

$$\frac{\dot{K}}{K} = (1 - \tau)\tau^{\frac{\alpha}{1 - \alpha}} - \frac{C}{K} - \delta. \tag{14}$$

By defining $z = C/K$, it follows from Eqs. (13) and (14) that the transitional dynamics can be expressed in terms of z as follows:

$$\frac{\dot{z}}{z} = \frac{1}{\theta} \left\{ (1 - \phi)(1 - \alpha)(1 - \tau)\tau^{\frac{\alpha}{1 - \alpha}} - \delta - \beta \right\} - (1 - \tau)\tau^{\frac{\alpha}{1 - \alpha}} + z + \delta. \tag{15}$$

Let $\tilde{z}$ denote the stationary value of z. At the balanced growth equilibrium, the economy is characterized by $\dot{z} = 0$ and $z = \tilde{z}$ in Eq. (15). Moreover, given that at the balanced growth equilibrium $\tilde{z}$ is constant, the growth rate of physical capital is identical to that of consumption at a common growth rate $\tilde{\gamma}$. Then, from Eq. (13) the common growth rate $\tilde{\gamma}$ is given by

$$\tilde{\gamma} = \frac{1}{\theta} \left\{ (1 - \phi)(1 - \alpha)(1 - \tau)\tau^{\frac{\alpha}{1 - \alpha}} - \delta - \beta \right\}. \tag{16}$$

It should be noted that to satisfy the nonnegative marginal income tax rate and the positive balanced growth rate condition, the following restriction on the extent of tax progressivity/regressivity ϕ should be imposed[5]:

$$\frac{-\tau}{1 - \tau} \leq \phi < 1 - \frac{\delta + \beta}{(1 - \alpha)(1 - \tau)\tau^{\alpha/(1 - \alpha)}}. \tag{17}$$

The first inequality in Eq. (17) is the condition for the non-negative marginal income tax rate, while the second inequality in Eq. (17) is the condition for a positive balanced growth rate.

Differentiating Eq. (16) with respect to ϕ and τ gives rise to the following:

$$\frac{\partial \tilde{\gamma}}{\partial \phi} = \frac{-(1 - \alpha)(1 - \tau)\tau^{\frac{\alpha}{1 - \alpha}}}{\theta} < 0, \tag{18a}$$

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[5] Appendix A provides a detailed derivation.

$$\frac{\partial \tilde{\gamma}}{\partial \tau} = \frac{(1-\phi)(1-\alpha)\tau^{\frac{\alpha}{1-\alpha}}}{\theta}\left[\frac{\alpha-\tau}{\tau(1-\alpha)}\right] \gtreqless 0; \ \text{if} \ \tau \lesseqgtr \alpha. \tag{18b}$$

Moreover, by integrating Eqs. (17), (18a) and (18b), we obtain an important result that the balanced growth rate attains its maximum when tax progressivity/regressivity and the tax scalar are respectively given by

$$\phi^g = -\frac{\alpha}{1-\alpha}, \tag{19a}$$

$$\tau^g = \alpha. \tag{19b}$$

Intuitively, a rise in the tax scalar will directly reduce the net-of-tax marginal product of physical capital. In response to a fall in the net-of-tax marginal product of physical capital, the household is inclined to raise consumption and reduce investment. Hence, the balanced economic growth rate is lowered as a response. On the other hand, a higher tax scalar is associated with an expansion in the government's infrastructure expenditure. This will boost the net-of-tax marginal product of physical capital, and hence will tend to stimulate the balanced growth rate.[6] Therefore, it balances these two conflicting forces when the growth-maximizing tax scalar is $\tau^g = \alpha$ as shown in Eq. (19b). Furthermore, a rise in the extent of tax progressivity depresses the net-of-tax marginal product of physical capital. In response to a fall in the net-of-tax marginal product of physical capital, the household tends to boost consumption and reduce saving. Hence, the balanced growth rate is lowered in response as shown in Eq. (18a). As a result, given $\tau^g = \alpha$, the balanced growth rate could increase again as tax progressivity(regressivity) approaches its lower bound as indicated in Eq. (19a) if the policy for tax progressivity(regressivity) is available.

4. Welfare

4.1. Second-best optimum

Along the balanced growth equilibrium, with a given initial private capital stock K_0, private consumption grows at a common rate $\tilde{\gamma}$ (which is a function of ϕ and τ). Then, the time path of private consumption can be expressed as follows:

$$C_t = \tilde{C}_0 e^{\tilde{\gamma}t}, \tag{20}$$

where $\tilde{C}_0 = [(1-\tau)\tau^{\alpha/(1-\alpha)} - \tilde{\gamma} - \delta]K_0.$[7]

Inserting Eq. (20) into Eq. (1), the welfare function in association with the second-best equilibrium (i.e., the indirect lifetime utility function) is given by

$$W = \frac{1}{1-\theta}\left\{\frac{\left(\tilde{C}_0\right)^{1-\theta}}{[\beta-(1-\theta)\tilde{\gamma}]} - \frac{1}{\beta}\right\}. \tag{21}$$

By using Eq. (21), we can deal with how the fiscal authority chooses a package consisting of both ϕ and τ to maximize welfare. Differentiating the welfare function with respect to ϕ and τ yields the following:

$$\frac{\partial W}{\partial \phi} = \frac{\left(\tilde{C}_0\right)^{-\theta}\left\{\tilde{C}_0 - [\beta-(1-\theta)\tilde{\gamma}]K_0\right\}}{[\beta-(1-\theta)\tilde{\gamma}]^2} \times \frac{\partial \tilde{\gamma}}{\partial \phi}, \tag{22a}$$

$$\frac{\partial W}{\partial \tau} = \frac{\left(\tilde{C}_0\right)^{-\theta}}{[\beta-(1-\theta)\tilde{\gamma}]^2} \times \left\{\tau^{\frac{\alpha}{1-\alpha}}[\beta-(1-\theta)\tilde{\gamma}]\left[\frac{\alpha-\tau}{\tau(1-\alpha)}\right]K_0 + \left[\tilde{C}_0 - (\beta-(1-\theta)\tilde{\gamma})K_0\right]\frac{\partial \tilde{\gamma}}{\partial \tau}\right\}. \tag{22b}$$

The first-order conditions for welfare maximization require that $\partial W/\partial \phi = \partial W/\partial \tau = 0$. Based on Eqs. (18a), (18b), (22a) and (22b), we can derive the second-best optimal policies.

$$\phi^* = \frac{-\alpha}{1-\alpha}, \tag{23a}$$

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[6] Using symmetric cross-section regressions, Wahab (2011) finds that the growth of government expenditure exerts a positive effect on output growth.

[7] Based on $z = C/K$ and Eq. (14) with $\tilde{\gamma} = \dot{K}/K$ along the balanced growth path, we can derive $\tilde{C}_0 = [(1-\tau)\tau^{\alpha/(1-\alpha)} - \tilde{\gamma} - \delta]K_0$.

$$\tau^* = \alpha. \tag{23b}$$

Then, by substituting Eqs. (23a) and (23b) into Eqs. (16) and (20), we can infer the balanced growth rate and the initial private consumption associated with second-best optimal policies.

$$\tilde{\gamma}^* = \frac{1}{\theta}\left[(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} - \delta - \beta\right], \tag{24}$$

$$\tilde{C}_0^* = \left[(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} - \tilde{\gamma}^* - \delta\right]K_0. \tag{25}$$

We then check whether the optimal package of $\phi^* = -\alpha/(1-\alpha)$ and $\tau^* = \alpha$ satisfies the second-order conditions for the welfare-maximizing problem. The second-order conditions for welfare maximization require that $\partial^2 W/\partial \phi^2 < 0$, $\partial^2 W/\partial \tau^2 < 0$ and $(\partial^2 W/\partial \phi^2)(\partial^2 W/\partial \tau^2) - (\partial^2 W/\partial \tau \partial \phi)^2 > 0$. Differentiating Eqs. (22a) and (22b) with respect to ϕ and τ yields the following:

$$\frac{\partial^2 W}{\partial \phi^2} = -\frac{\theta\left(\tilde{C}_0^*\right)^{-\theta}}{\left[\beta - (1-\theta)\tilde{\gamma}^*\right]^2} \times \left[\left(\frac{\partial \tilde{\gamma}}{\partial \phi}\right)^*\right]^2 < 0, \tag{26a}$$

$$\frac{\partial^2 W}{\partial \tau^2} = -\frac{K_0(1-\alpha)\left(\tilde{C}_0^*\right)^{-\theta}\alpha^{\frac{1}{1-\alpha}}}{\left[\beta - (1-\theta)\tilde{\gamma}^*\right][\alpha(1-\alpha)]^2} < 0, \tag{26b}$$

$$\frac{\partial^2 W}{\partial \tau \partial \phi} = 0. \tag{26c}$$

Eqs. (26a)–(26c) indicate that the second-best optimal policies of $\phi^* = -\alpha/(1-\alpha)$ and $\tau^* = \alpha$ definitely satisfy the second-order conditions for welfare maximization.

Based on Eqs. (21), (24) and (25), the welfare level in association with the second-best optimal policies, namely, W_{2nd}, can be expressed as follows:

$$W_{2nd} = W\left(\tau^* = \alpha, \phi^* = \frac{-\alpha}{1-\alpha}\right) = \frac{1}{1-\theta}\left\{\frac{\left(\tilde{C}_0^*\right)^{1-\theta}}{\left[\beta - (1-\theta)\tilde{\gamma}^*\right]} - \frac{1}{\beta}\right\}. \tag{27}$$

A graphical apparatus is helpful to our understanding of how the welfare level is related to the second-best optimal policies. In Fig. 1, in association with the second-best tax scalar $\tau^*(=\alpha)$, the locus $WW(\tau^* = \alpha)$ depicts the relationship between W and ϕ reported in Eq. (21). Moreover, it is clear from Fig. 1 that the welfare-maximizing tax regressivity is $\phi^*(= -\alpha/(1-\alpha))$, and the welfare level associated with the second-best optimal policies is designated as W_{2nd}. In his pioneering paper, Barro (1990) imposes the restriction $\phi = 0$. As exhibited in Fig. 1, the welfare level associated with the Barro analysis is designated as $W_{Barro}(= W(\tau^* = \alpha, \phi = 0))$. As is evident, W_{2nd} is greater than W_{Barro}, thereby enabling the fiscal authority to boost the welfare level by adjusting ϕ from 0 to $-\alpha/(1-\alpha)$.[8]

By assuming that only the tax scalar is available to the fiscal authority, Barro (1990) concludes that the growth-maximizing optimal proportional tax rate is equivalent to the welfare-maximizing one. This result is dubbed "the Barro result" by Futagami, Morita, and Shibata (1993). This paper devises a nonlinear tax structure, and hence makes both the tax scalar and tax progressivity/regressivity available to the fiscal authority. By so doing, Eqs. (19a) and (19b) reveal that the growth-maximizing tax scalar and tax progressivity/regressivity are $\tau^g = \alpha$ and $\phi^g = -\alpha/(1-\alpha)$, respectively. Moreover, Eqs. (23a) and (23b) indicate that the welfare-maximizing tax scalar and tax progressivity/regressivity are given by $\tau^* = \alpha$ and $\phi^* = -\alpha/(1-\alpha)$, respectively. A comparison of τ^g with τ^* and ϕ^g with ϕ^* leads to $\tau^g = \tau^*$ and $\phi^g = \phi^*$. This result implies that the growth-maximizing package of the tax scalar and tax progressivity/regressivity is coincident to the welfare-maximizing package.

Summing up the above discussion, we can establish the following proposition:

Proposition 1. If both the tax scalar and the extent of tax progressivity/regressivity are available to the fiscal authority, the growth-maximizing package for both policies is equivalent to the welfare-maximizing ones.

4.2. First-best optimum

We are now in a position to deal with the first-best equilibrium, and to examine the difference between the first-best equilibrium and the second-best equilibrium in the presence of productive government expenditure. In the existing literature on

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[8] Appendix B provides a brief discussion as to whether the second-best optimal policies reported in Eqs. (23a) and (23b) are robust if the Cobb–Douglas production function is replaced by the CES production function. This question was brought to our attention by an anonymous referee, to whom we are most grateful.

endogenous growth, to sustain an equilibrium with balanced growth the government's infrastructure expenditure G must itself be tied to the specific growing macro variable. This can be achieved most conveniently by assuming that the social planner sets government expenditure as a fixed proportion g of output

$$G = gY. \tag{28}$$

This specification is a standard one in the endogenous growth literature; see, for example, Turnovsky (1997) and Irmen and Kuehnel (2009).

The social planner maximizes Eq. (1), subject to Eqs. (8), (11) and (28) by choosing C and K. As is evident, an important implication emerges from this first-best mathematical implementation. That is, in allocating resources among consumption, government spending, and social saving, the social planner possesses the relevant information including that the government's infrastructure generates an externality to the private production expressed in Eq. (8) and the income tax rate is not related to his decision expressed in Eq. (11).[9] By letting μ be the co-state variable associated with Eq. (11), the optimum conditions are

$$C^{-\theta} = \mu, \tag{29a}$$

$$\mu\left[(1-g)\,g^{\frac{\alpha}{1-\alpha}} - \delta\right] = -\dot{\mu} + \mu\beta. \tag{29b}$$

From Eqs. (11), (28), (29a), and (29b), we can solve the common growth rate at the Pareto optimum $\overline{\gamma}$

$$\overline{\gamma} = \frac{1}{\theta}\left[(1-g)\,g^{\frac{\alpha}{1-\alpha}} - \delta - \beta\right]. \tag{30}$$

Given the initial capital stock K_0 and that consumption grows at a common rate $\overline{\gamma}$, the welfare function in association with the first-best equilibrium, W_F, can be expressed as follows:

$$W_F = \frac{1}{1-\theta}\left\{\frac{\left(\overline{C}_0\right)^{1-\theta}}{[\beta - (1-\theta)\overline{\gamma}]} - \frac{1}{\beta}\right\}, \tag{31}$$

where $\overline{C}_0 = [(1-g)\,g^{\alpha/(1-\alpha)} - \overline{\gamma} - \delta]K_0$.

From Eq. (31) we can infer how the social planner chooses g to maximize welfare. Differentiating the welfare function with respect to g yields the following:

$$\frac{\partial W_F}{\partial g} = \frac{\overline{C}_0^{1-\theta}g^{\frac{\alpha}{1-\alpha}}}{[\beta - (1-\theta)\overline{\gamma}]^2}\left[\frac{\alpha - g}{g(1-\alpha)}\right]. \tag{32}$$

From Eq. (32) we can derive the optimal ratio of government expenditure.

$$g^* = \alpha. \tag{33}$$

In fact, an alternative approach to the social planner's optimization problem is to maximize Eq. (1), subject to Eqs. (8), (11) and (28) by choosing C, K, and G. In such a way, the optimal condition for G is $G = \alpha Y$. This result is consistent with the conclusion of our first-best solution, which specifies that $G = gY$ and solves the optimal ratio of government expenditure $g^* = \alpha$. In his previous study on endogenous growth, Turnovsky (1997, p. 620) confirms this result. He mentions that "g is set optimally… is equivalent to optimizing directly with respect to G".[10]

Substituting Eq. (33) into Eqs.(30) and (31), the welfare level in association with $g^* = \alpha$ is given by

$$W_{1st} = W_F(g^* = \alpha) = \frac{1}{1-\theta}\left\{\frac{\left(\overline{C}_0^*\right)^{1-\theta}}{[\beta - (1-\theta)\overline{\gamma}^*]} - \frac{1}{\beta}\right\}, \tag{34}$$

where $\overline{\gamma}^* = \frac{1}{\theta}\left[(1-\alpha)\,\alpha^{\alpha/(1-\alpha)} - \delta - \beta\right]$ and $\overline{C}_0^* = [(1-\alpha)\alpha^{\alpha/(1-\alpha)} - \overline{\gamma} - \delta]K_0$.

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[9] Our analysis adopts the two-stage approach proposed by Turnovsky (1997). The first-stage decision involves the optimum condition for C and K, and the second-stage decision concerns the optimum choice for government spending.

[10] The social planner chooses G so as to maximize Eq. (1) subject to Eqs. (8) and (11). The optimality for government expenditure is $G = \alpha Y$. This result reveals that the social planner's optimal decision for $g^* = \alpha$ in our two-stage approach is equivalent to optimizing directly with respect to G.
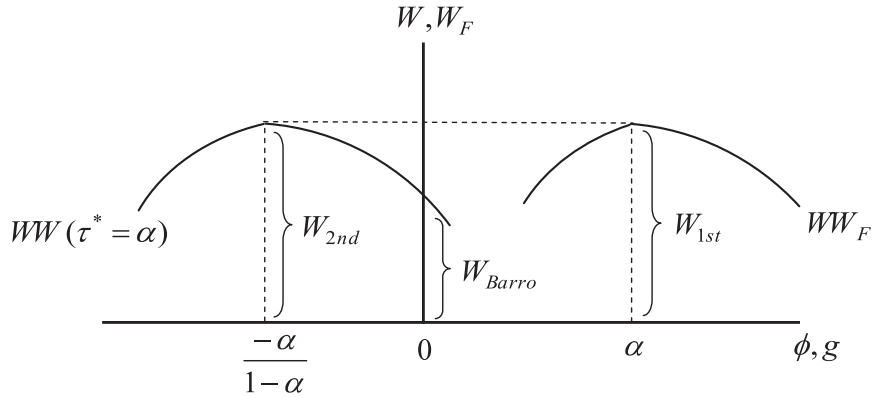
Fig. 1. The welfare level: first best vs. second best.

The results reported in Eqs. (31), (33) and (34) can be illustrated in Fig. 1. The locus WW_F traces all pairs of W_F and g that satisfy Eq. (31). As indicated in Eq. (33), the first-best policy that maximizes welfare is to set the ratio of government expenditure equal to α, and hence, as exhibited in Fig. 1, the welfare level in association with the first-best policy is W_{1st}.

4.3. Welfare comparisons

Fig. 1 reveals the fact that $W_{1st} > W_{Barro}$. This result indicates that, even if the tax scalar policy is set optimally (i.e., $\tau^* = \alpha$), the welfare level in the first-best situation is higher than that in the Barro (1990) second-best situation. More importantly, Fig. 1 also reveals the fact that $W_{1st} = W_{2nd}$. This result has an important implication for the Barro analysis, i.e., the welfare level of the first-best equilibrium can be achieved if the policy for tax progressivity (regressivity) is available to the fiscal authority. Based on the above discussion, the following proposition is established:

Proposition 2. The second-best optimal welfare and the first-best optimal welfare are equivalent if the two policy instruments for the tax scalar and tax progressivity (regressivity) are available to the fiscal authority.

The economic intuition behind Proposition 2 can be explained as follows. Based on Eqs. (16) and (30), we can infer that a comparison between the balanced growth rate under the second-best regime and the first-best regime leads to two important results. First, Barro (1990) assumes that the government can only choose the policy of the tax scalar to remedy the distortion stemming from productive government externalities, i.e., $\phi = 0$ is imposed and $\tau^* = \alpha$ is chosen under the second-best regime. Moreover, as indicated in Eq. (33), the social planner sets $g^* = \alpha$ under the first-best regime. Under such a situation, we can infer the following inequality:

$$\overline{\gamma}(g^* = \alpha) = \frac{1}{\theta}\left[(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} - \delta - \beta\right] > \frac{1}{\theta}\left[(1-\alpha)(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} - \delta - \beta\right] = \tilde{\gamma}(\phi = 0, \tau^* = \alpha). \tag{35}$$

Eq. (35) indicates that the balanced growth rate under the first-best regime is definitely greater than that under the second-best regime. Given that the welfare level is governed by the balanced growth rate, the welfare level is higher under the first-best regime compared with that under the second-best regime.

Second, when two policy instruments for tax ϕ and τ are available to the fiscal authority, the package consisting of $\phi^* = -\alpha/(1-\alpha)$ and $\tau^* = \alpha$ is chosen under the second-best regime. Meanwhile, as indicated in Eq. (33), the social planner still sets $g^* = \alpha$ as its best choice under the first-best regime. Under such a situation, we can infer the following expression:

$$\overline{\gamma}(g^* = \alpha) = \frac{1}{\theta}\left[(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} - \delta - \beta\right] = \tilde{\gamma}\left(\phi^* = \frac{-\alpha}{1-\alpha}, \tau^* = \alpha\right). \tag{36}$$

Eq. (36) reveals that the balanced growth rate under the first-best regime is equal to that under the second-best regime. As a result, the welfare level under the first-best regime is equal to that under the second-best regime.

The economic intuition underlying the result that the first-best optimum can be decentralized by setting an optimal package of the tax scalar and the progressivity/regressivity parameter can be explained as follows. Let PRK denote the "private" return to physical capital (i.e., $\partial \dot{K} / \partial K$ in the "private" budget constraint, Eq. (2)) in the de-centralized economy and SRK denote the "social"

return to physical capital (i.e., $\partial \dot{K} / \partial K$ in the "social" resource constraint, Eq.(11)) in the centralized economy. Based on Eqs. (2), (3), (8), (9b) and (11), we can deduce the following two expressions:

$$PRK = (1-\phi)(1-\tau)(1-\alpha)K^{-\alpha}G^{\alpha} - \delta, \tag{37a}$$

$$SRK = (1-\alpha)K^{-\alpha}G^{\alpha} - \delta. \tag{37b}$$

It should be noticed that in deriving Eqs. (37a) and (37b) both the symmetric equilibrium condition $Y = \overline{Y}$ and the inelastic labor supply condition $L = 1$ are used.

Three points related to the difference between PRK and SRK should be mentioned here. First, the wedge between PRK and SRK is crucially related to the two policy instruments for the tax scalar and tax progressivity (regressivity). This implies that these two policy instruments are effective in correcting the inefficiency caused by distortions. Second, in his pioneering analysis, Barro (1990) assumes that only the instrument for the tax scalar is available to the fiscal authority (i.e., $\phi = 0$) and finds that the optimal tax scalar is equal to the extent of the production externalities $\tau^* = \alpha$. Inserting $\phi = 0$ and $\tau^* = \alpha = G/Y$ into Eqs. (37a) and (37b), we have the following expression linking both PRK and SRK:

$$PRK + \delta = (1-\phi)(1-\alpha)(SRK + \delta). \tag{38}$$

This implies that $PRK < SRK$, meaning that an optimal tax scalar $\tau^* = \alpha$ is not enough to remedy the distortions caused by both production externalities and income taxation. Third, when both instruments for the tax scalar and tax progressivity/regressivity are available to the fiscal authority, it is quite easy to deduce from Eq. (38) with $\tau^* = \alpha$ that the optimal tax regressivity is $\phi^* = -\alpha/(1-\alpha)$. To be more specific, the package of $\tau^* = \alpha$ and $\phi^* = -\alpha/(1-\alpha)$ can fully remedy inefficiencies stemming from both distortions of production externalities and income taxation.

Before ending this section, two points should be noted here. First, in an AK growth model Yamarik (2001) specifies a nonlinear tax structure and assumes that tax revenues are rebated back to the household as lump-sum transfers. Under such a setting, Yamarik stresses that a higher tax progressivity does not govern the balanced growth rate although it can affect the transitional growth rate. However, in our paper both the balanced growth rate and the level of social welfare are crucially related to the extent of tax progressivity (regressivity). This is the reason why our paper can deal with the issue of how the fiscal authority sets an optimal extent of progressivity (regressivity) to remedy market inefficiency.

Second, it should be noted that, as proposed by Slobodyan (2005), in Eq. (3) the economy-wide average income $\overline{Y}$ is treated as a base level of income in the tax schedule. It is a common belief that the usual motivation for a progressive tax is that it creates an income redistribution effect among individuals. In our specification of a nonlinear tax system in Eq. (3), the representative household treats $\overline{Y}$ as a constant, and recognizes that the average tax rate is increasing if a progressive tax (i.e., $\phi > 0$) is levied. This implies that our nonlinear tax system is characterized by an income redistribution mechanism among individuals. However, given the fact that our paper assumes that all households are homogeneous, the income redistribution mechanism thus does not work. It is certain that the income redistribution mechanism would be present once heterogeneous agents are brought into the picture. See Li and Sarte (2004) for a detailed discussion on nonlinear taxation with heterogeneous agents.

5. Concluding remarks

This paper sets up an endogenous growth model featuring a production externality and nonlinear income taxation, and uses it to examine how the fiscal authority devises its nonlinear tax structure from the viewpoint of welfare maximization. It is found that, in the face of two distortions arising from the production externality and income taxation, the fiscal authority can select an optimal package of the tax scalar and the extent of tax progressivity/regressivity to remedy inefficiencies stemming from these two distortions. It is also found that the Pareto optimality can be restored if the two policy instruments for the tax scalar and tax progressivity (regressivity) are available to the fiscal authority.

Since the mid-1980s, the importance of market imperfections has long been recognized in the macroeconomics literature, e.g., Blanchard and Kiyotaki (1987), Heijdra and van der Ploeg (1996), Devereux, Head, and Lapham (2000), Lubik and Marzo (2007), and Lai, Chin, and Chang (2010). Under an imperfectly competitive economy, firms have monopoly power in the product market in light of the demand curves. As a consequence, they set higher levels of price and produce lower levels of output than in a Walrasian economy. It would be an interesting extension to examine whether the fiscal authority can devise a nonlinear taxation structure to remedy the distortions stemming from a production externality, income taxation, and imperfect competition.[11,12] Moreover, it would be worthwhile to quantify the welfare cost in the presence of the three above-mentioned distortions.[13]

[11] Yakita (2004) develops an endogenous growth model that features monopolistic competition in both private consumption and public investment. His analysis finds that a welfare-maximizing income tax rate may be greater than the growth-maximizing tax rate, depending on whether the elasticity of substitution in public investment exceeds that in private consumption or not. However, the Yakita (2004) result is based on a linear taxation structure.

[12] Aronsson and Sjögren (2004) develop a partial equilibrium model featured with union wage setting (i.e., imperfect competition in the labor market) and progressive taxation on labor income, and use it to examine the optimal degree of progressivity of the labor income tax.

[13] For a detailed analysis of the welfare cost in association with monopolistic competition, see Matheron and Maury (2004).

Acknowledgments

Appendix A

This appendix presents a proof of Eq. (17). First of all, to match with the reality, the restriction for the nonnegative marginal income tax rate, i.e., $\tau_m \geq 0$, is imposed. Given that the symmetric equilibrium requires $Y = \overline{Y}$, the restriction for $\tau_m \geq 0$ reported in Eq. (4) can be expressed as follows:

$$\tau_m \geq 0; \ if \ \phi \geq \frac{-\tau}{1-\tau}. \tag{A1}$$

Next, to sustain an equilibrium with balanced growth, an endogenously growing economy is required to exhibit a positive balanced growth rate. From Eq. (14) we can infer that the restriction that an endogenously growing economy has a positive balanced growth rate can be expressed as

$$\tilde{\gamma} > 0; \ if \ \phi < 1 - \frac{\delta + \beta}{(1-\alpha)(1-\tau)\tau^{\alpha/(1-\alpha)}}. \tag{A2}$$

Integrating Eqs. (A1) and (A2), we impose the restriction for both the nonnegative marginal income tax rate and the positive growth rate as follows:

$$\frac{-\tau}{1-\tau} \leq \phi < 1 - \frac{\delta + \beta}{(1-\alpha)(1-\tau)\tau^{\alpha/(1-\alpha)}}. \tag{A3}$$

Eq. (A3) is identical to Eq. (17) in the main text.

It should be noted that the restriction for ϕ reported in Eq. (A3) also satisfies the transversality condition. To show this point, we first make use of Eqs. (6a) and (13) to obtain the following:

$$\lambda = \lambda_0 \exp\left\{\left[(\beta + \delta) - (1-\phi)(1-\alpha)(1-\tau)\tau^{\alpha/(1-\alpha)}\right]t\right\}, \tag{A4}$$

$$K = K_0 \exp\left\{\left[\left((1-\phi)(1-\alpha)(1-\tau)\tau^{\alpha/(1-\alpha)} - (\beta + \delta)\right)/\theta\right]t\right\}, \tag{A5}$$

where λ is the endogenously-determined initial marginal utility and K_0 is the given initial stock of physical capital. Given Eqs. (A4) and (A5), the transversality condition of K stated in Eq. (7) can be rearranged as follows:

$$\lim_{t\to\infty} = \lambda_0 K_0 \exp\left\{-\left[\left((1-\theta)\delta + \beta - (1-\theta)(1-\phi)(1-\alpha)(1-\tau)\tau^{\alpha/(1-\alpha)}\right)/\theta\right]t\right\}. \tag{A6}$$

Based on Eq. (A6) with $\theta > 1$, the restriction that the transversality condition is satisfied can be expressed as follows:

$$\phi < 1 + \frac{(1-\theta)\delta + \beta}{(\theta-1)(1-\alpha)(1-\tau)\tau^{\alpha/(1-\alpha)}}. \tag{A7}$$

It is quite easy to infer the following result.

$$1 + \frac{(1-\theta)\delta + \beta}{(\theta-1)(1-\alpha)(1-\tau)\tau^{\alpha/(1-\alpha)}} > 1 + \frac{(1-\theta)(\delta + \beta)}{(\theta-1)(1-\alpha)(1-\tau)\tau^{\alpha/(1-\alpha)}}$$
$$= 1 - \frac{(\delta + \beta)}{(1-\alpha)(1-\tau)\tau^{\alpha/(1-\alpha)}}. \tag{A8}$$

Based on Eqs. (A3), (A7) and (A8), we can conclude that the restriction for ϕ in Eq. (A3) also satisfies that in Eq. (A7). This result implies that the transversality condition is satisfied when we impose the restriction for both the nonnegative marginal income tax rate and the positive balanced growth rate.

Appendix B

To make a clear comparison with the Barro (1990) analysis, in the main text the production function we consider is of the Cobb-Douglas form. It is natural to raise the question of whether the second-best optimal policies reported in Eqs. (23a) and (23b) are robust if the Cobb-Douglas production function is replaced by the CES production function. This Appendix tries to provide a brief discussion of the question.

The CES production function can be expressed as follows:

$$Y = \left[(1-\alpha)K^{\frac{\sigma-1}{\sigma}} + \alpha(LG)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}; \ \sigma > 0, \ 1 > \alpha > 0, \tag{B1}$$

where the parameter σ denotes the elasticity of substitution between physical capital and augmenting labor. Based on Eq. (B1), the demand functions for the labor input and physical capital are respectively given by the following:

$$w = \alpha \left[(1-\alpha) + \alpha \left(\frac{LG}{K} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}-1} \left(\frac{LG}{K} \right)^{\frac{\sigma-1}{\sigma}-1}, \tag{B2}$$

$$r = (1-\alpha) \left[(1-\alpha) + \alpha \left(\frac{LG}{K} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}-1}. \tag{B3}$$

Similar to the analysis in the main text, we confine the analysis to a symmetric equilibrium, i.e., $Y = \overline{Y}$. From Eqs. (10), (B1) and the labor-market clearing condition $L = 1$ (which remind us that the labor supply is inelastic and equals unity) we have the following:

$$\left(\frac{G}{K} \right)^{\frac{\sigma-1}{\sigma}} = \frac{(1-\alpha)\tau^{(\sigma-1)/\sigma}}{1-\alpha\tau^{(\sigma-1)/\sigma}}. \tag{B4}$$

Based on Eqs. (6a), (6b), (12) and (B4), the common growth rate $\tilde{\gamma}$ is given by the following:

$$\tilde{\gamma} = \frac{1}{\theta} \left\{ (1-\phi)(1-\tau) \left[1-\alpha\tau^{\frac{\sigma-1}{\sigma}} \right] \left[\frac{1-\alpha}{1-\alpha \ \tau^{(\sigma-1)/\sigma}} \right]^{\frac{\sigma}{\sigma-1}} - (\delta + \beta) \right\}. \tag{B5}$$

By an analogous analysis as implemented in Subsection 4.1, the welfare function in association with the second-best equilibrium (i.e., the indirect lifetime utility function) can be expressed as follows:

$$W = \frac{1}{1-\theta} \left\{ \frac{\left(\tilde{C}_0 \right)^{1-\theta}}{\left[\beta - (1-\theta) \tilde{\gamma} \right]} - \frac{1}{\beta} \right\}, \tag{B6}$$

where $\tilde{C}_0 = \left\{ (1-\tau) \left[(1-\alpha)/(1-\alpha\tau^{(\sigma-1)/\sigma}) \right]^{\sigma/(\sigma-1)} - \tilde{\gamma} - \delta \right\} K_0$. By using Eqs. (B5) and (B6), we can derive the second-best optimal policies as follows:

$$\phi^* = \frac{-\alpha^\sigma}{1-\alpha^\sigma}, \tag{B7}$$

$$\tau^* = \alpha^\sigma. \tag{B8}$$

Eqs. (B7) and (B8) have two important implications. First, when $\sigma \to 1$, the CES production function degenerates to the Cobb–Douglas production function specified by Barro (1990), and the optimal tax scalar and the optimal extent of tax progressivity (regressivity) reported in Eqs. (B7) and (B8) change to $\phi^* = -\alpha/(1-\alpha)$ and $\tau^* = \alpha$, respectively. This result confirms our inference in the main text. Second, the second-best package of the tax scalar and tax progressivity (regressivity) is crucially related to the elasticity of substitution between physical capital and augmenting labor. Differentiating Eqs. (B7) and (B8) with respect to σ, we can infer the linkage regarding how ϕ^* and τ^* are related to σ as follows:

$$\frac{\partial |\phi^*|}{\partial \sigma} = \frac{\ln \alpha(\alpha^\sigma)}{(1-\alpha^\sigma)^2} < 0, \tag{B9}$$

$$\frac{\partial \tau^*}{\partial \sigma} = \ln \alpha(\alpha^\sigma) < 0. \tag{B10}$$

Eqs. (B9) and (B10) tell us that, in response to a lower elasticity of substitution between physical capital and augmenting labor, the best response for the government is to raise both the tax scalar and the tax regressivity.

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