# Inflation and Capital Accumulation in a Two-Sector Cash-in-Advance Economy\*

This paper studies a dynamic two-sector model in which money is introduced through a cashin-advance constraint. A perfectly anticipated inflation changes the relative price between consumption goods, as they are differentially subject to the cash-in-advance constraint. Contrary to the conclusion of Stockman (1981), inflation may either increase or decrease capital accumulation, even though only consumption goods is subject to the cash-in-advance constraint.

# 1. Introduction

Does higher inflation induce more capital stock in the long run? This question was put forth by Tobin (1965) more than a quarter of a century ago. Still, it has been occupying the central stage of macroeconomic theory to date.

Tobin studied the inflation-accumulation issue in a non-optimizing framework and established a positive correlation between inflation and longrun capital. Using explicitly optimizing models, contemporary economists have extended and/or refuted Tobin's conclusion. Masterful surveys of these studies can be found in Orphanides and Solow (1990) and Haliassos and Tobin (1990, sec. 5). All models they survey are formulated in the tradition of one-sector, neoclassical growth theory. The standard one-sector growth model is useful in illustrating the intertemporal effect of inflation, yet it ignores its distributional effect.

Insofar as the literature is concerned, only Foley and Sidrauski (1970, 1971) used a two-sector (consumption-investment) model to demonstrate that a permanent inflation may have a distributional effect. Their work, which is close in spirit to Tobin's (1971, ch. 18) approach, presents a more carefully worked out view of investment. The rate of investment is, through

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the supply decisions of firms, an increasing function of the relative price of capital. Inflation depresses consumption demand, raises investment demand and its relative price, and hence promotes capital accumulation in the long run. While Foley and Sidrauski's conclusion might be compelling, they did not consider the underlying flow of factor inputs, and their analysis was hampered by the non-optimizing nature of the model.

The purpose of this paper is to introduce a two-sector cash-in-advance model to study the effects of inflation on capital accumulation. Our analysis and results, which might be regarded as a modern resurrection of the work of Foley and Sidrauski, use optimizing, general-equilibrium framework a la Lucas and Stokey (1983, 1987) to illustrate the distributional effects of inflation. We follow Lucas and Stokey in applying the cash-in-advance constraint to "cash goods" and not to "credit goods." A permanent inflation increases the relative demand for credit goods and its relative price, what in turn leads to an expansion in its production and hence on the demand for capital, given that the production of credit goods requires more capital than that of cash goods. It should be noted that this line of reasoning is independent of whether the investment is subject to the cash-in-advance constraint or not. Henceforth, some of the results here are contrary to what Stockman (1981) has obtained.

The paper is organized as follows. The model, the first-order conditions, and the steady-state characterizations are set out in the next section. In Section 3, the steady-state effects of a change in the monetary growth rate are examined. In Section 4, we explore other possible extensions and conclude with a brief summary.

#### 2. The Model

The model is a dynamic specific-factors model formulated after Jones (1971) and Roldos (1991, 1992).<sup>1</sup> Two consumption goods are produced: good one,  $c_1$ , and good two,  $c_2$ .  $c_1$  is produced by a constant-return-to-scale technology using labor, l, and capital, k. It can be either consumed or added to the existing capital stock. Capital does not depreciate.  $c_2$  is also produced by a constant-return-to-scale technology using labor and (fixed-quantity) land,  $\bar{n}$ . It can only be consumed. The technologies are summarized as follows:

$$\vec{k} + c_1 = F(l, k)$$
, (1)

$$c_2 = G(1 - l, \bar{n}), \qquad (2)$$

<sup>1</sup>For technical convenience we choose to formulate our model in continuous time.

where F and G are constant-return-to-scale production functions with standard properties. In the expression, we have normalized the total labor endowment to unity, so that l is the fraction of labor used in producing good one. Given (1) and (2), it is clear that  $c_1$  and k must sell for the same price in a competitive market, but  $c_1$  and  $c_2$  need not.

The economy consists of an infinitely-lived representative household maximizing an intertemporal utility function that is separable in  $c_1$  and  $c_2$ :<sup>2</sup>

$$\int_0^\infty [u(c_1(t)) \ + \ v(c_2(t))] e^{-\theta t} dt$$

where  $\theta > 0$  is the rate of time discount. The instantaneous utility functions  $u(\cdot)$  and  $v(\cdot)$  are strictly increasing, strictly concave, continuously differentiable, and satisfies the Inada condition, respectively.<sup>3</sup> The household can hold either cash, M, or capital. Cash is injected into the system through lump-sum transfers (withdrawn by lump sum taxes). The budget constraint and accumulation constraint are as follows:

$$p_1(t)(c_1(t) + i(t)) + p_2(t)c_2(t) + \dot{M}(t)$$
  
=  $p_1(t)F(l(t),k(t)) + p_2(t)G(1 - l(t), \bar{n}) + T(t)$ , (3)

$$\dot{k}(t) = i(t) , \qquad (4)$$

with k(0), M(0) given. In the expressions,  $p_1(t)(p_2(t))$  is the nominal price of  $c_1(c_2)$  at t, T(t) is the lump-sum nominal transfer from the government at t. It is assumed that cash is needed in order to purchase good two:

$$p_2(t)c_2(t) \le M(t) , \qquad (5)$$

where M(t) is the cash balance at time t. Thus,  $c_1$  is credit goods and  $c_2$  is cash goods as in Lucas and Stokey and related literature.<sup>4</sup>

Denoting by *H* the Hamiltonian of the problem,  $\alpha$ ,  $\beta$ , and  $\gamma$  the multipliers for (3)–(5), one can write *H* as:<sup>5</sup>

<sup>2</sup>Without separability, the comparative static exercises below become rather complicated.

<sup>3</sup>The household supplies its labor endowment inelastically. Therefore, labor supply does not appear as another argument in the utility function. Extending the model to include endogenous labor supply will complicate the analysis, but not change the substance of the results.

 $^{4}$ As it stands now, the literature has yet to agree on the empirical issue of which real sector is more cash constrained and which is less. A recent paper by Rogers and Wang (1993) may shed some light on this ambiguity.

<sup>5</sup>Time indices are omitted to conserve space.

$$\begin{split} H &= \{ u(c_1) + v(c_2) + \alpha [p_1 F(l,k) + p_2 G(1-l,\bar{n}) + T - p_1 (c_1+i) - p_2 c_2] \\ &+ \beta i + \gamma (M-p_2 c_2) \} e^{-\theta t} \; . \end{split}$$

The first-order conditions for an interior optimal path are given by (3)–(5) and

$$u'(c_1)/p_1 = \alpha , \qquad (6)$$

$$v'(c_2)/p_2 = \alpha + \gamma , \qquad (7)$$

$$p_1 F_1(l,k) = p_2 G_1(1-l,\bar{n}), \qquad (8)$$

$$\alpha = \beta/p_1 , \qquad (9)$$

$$\dot{\alpha} = \alpha \theta - \gamma , \qquad (10)$$

$$\dot{\beta} = \beta \theta - \alpha p_1 F_2(l, k) , \qquad (11)$$

and two transversality conditions:

$$\lim_{t \to \infty} \beta(t)k(t)e^{-\theta t} = 0 ,$$
  
$$\lim_{t \to \infty} \alpha(t)M(t)e^{-\theta t} = 0 .$$
(12)

Equation (6) equates the marginal utility of credit goods per dollar spent to the marginal utility of income; Equation (7) equates the marginal utility of cash goods per dollar spent to the sum of the marginal utility of income and the marginal utility of cash; Equation (8) equates the value of marginal product of labor in each sector; Equation (9) equates the marginal utility of investment per dollar spent to the marginal utility of income. Equation (10) describes the dynamic motion of the marginal utility of income, while Equation (11) describes the dynamic motion of the marginal utility of investment. Equations (12) are used to rule out Ponzi-game behavior in trading physical capital and in trading cash.

To simplify the system of (6)–(11), we first substitute (6)–(8) into (10) to obtain

$$\frac{\dot{\alpha}}{\alpha} = 1 + \theta - \frac{v'(c_2)G_1(1-l,\bar{n})}{u'(c_1)F_1(l,k)}.$$
(13)

Next, substitute (9) into (6) and (11) to obtain

$$u'(c_1) = \beta , \qquad (14)$$

$$\frac{\dot{\beta}}{\beta} = \theta - F_2(l, k) . \tag{15}$$

Combining (14) and (15) we obtain

$$\dot{c}_1 = \left[\frac{u'(c_1)}{u''(c_1)}\right] \left[\theta - F_2(l,k)\right].$$
(16)

The equilibrium conditions for the economy require that both goods markets clear and the money market clears. The cash-good market clearing condition implies that we can substitute (2) into (13) to obtain

$$\frac{\dot{\alpha}}{\alpha} = 1 + \theta - \frac{v'(G(1-l,\bar{n}))G_1(1-l,\bar{n})}{u'(c_1)F_1(l,k)}.$$
(17)

Assuming the cash-in-advance constraint is binding, we can substitute (2), (8) and (9) into (5) to obtain

$$\beta F_1(l, k) G(1 - l, \bar{n}) = \alpha G_1(1 - l, \bar{n}) M .$$
(18)

Taking the logarithm of both sides of  $\left(18\right)$  and differentiating with respect to time:

$$A\dot{l} + \frac{\dot{\beta}}{\beta} + \left[\frac{F_{12}(l,k)}{F_1(l,k)}\right]\dot{k} = \frac{\dot{\alpha}}{\alpha} + \frac{\dot{M}}{M}, \qquad (19)$$

where

$$A \equiv -\frac{G_1(1-l,\bar{n})}{G(1-l,\bar{n})} + \frac{G_{11}(1-l,\bar{n})}{G_1(1-l,\bar{n})} + \frac{F_{11}(l,k)}{F_1(l,k)} < 0 \; .$$

Money supply is assumed to follow a constant rate of growth,  $\mu$ ,

$$\frac{\dot{M}}{M} = \mu , \qquad (20)$$

with

$$\mu \ge -\theta . \tag{21}$$

Condition (21) is generally known to ensure the existence of a monetary steady state. Substituting (1), (15), (17) and (20) into (19), we have

$$A\dot{l} = 1 + \mu + F_2(l, k) - \left[\frac{v'(G(1 - l, \bar{n}))G_1(1 - l, \bar{n})}{u'(c_1)F_1(l, k)}\right] - \left[\frac{F_{12}(l, k)}{F_1(l, k)}\right] [F(l, k) - c_1].$$
(22)

The equilibrium motions of  $(c_1, l, k)$  are thus completely characterized by (1), (16) and (22).

In the steady state,  $\dot{c}_1 = \dot{l} = \dot{k} = 0$ . Given the conditions imposed, it is straightforward to verify that a unique steady state exists. Before the formal comparative analysis, it is noted that Equation (16) implies that the steady-state marginal productivity of capital is tied to the rate of time preference:

$$\theta = F_2(\bar{l}, \bar{k}) , \qquad (23)$$

where a bar over the variable denotes its steady-state value. The intuition underlying (23) is straightforward. As the cash-in-advance constraint does not apply to either  $c_1$  or  $\dot{k}$ , the household can reduce  $c_1$  and add to the investment at t directly, hence, the steady-state real rate of return of capital is determined by the Modified Golden Rule (without population growth) from the traditional optimal growth theory.

Our model also exhibits an inefficiency à la Friedman's (1969) Optimum Quantity of Money. To see that, note first that Equations (22) and (23) may be jointly rewritten as, in the steady state,

$$1 + \mu + \theta = \frac{v'(\bar{c}_2)G_1(1 - \bar{l}, \bar{n})}{u'(\bar{c}_1)F_1(\bar{l}, \bar{k})}.$$
 (24)

Given condition (21), Equation (24) implies

$$\frac{u'(\bar{c}_1)}{v'(\bar{c}_2)} \le \frac{G_1(1-\bar{l},\bar{n})}{F_1(\bar{l},\bar{k})} \,. \tag{25}$$

The LHS of (25) is the marginal rate of substitution of  $c_1$  for  $c_2$ , while the RHS is the corresponding marginal rate of transformation. When  $\mu > -\theta$ , the (private) opportunity cost of consuming  $c_2$  is more than its (social) opportunity cost of production. It is well known that this wedge of monetary inefficiency exists because the contemporaneous income cannot, while only cash can, purchase  $c_2$  (Lucas 1982, 356).

## 3. The Steady-State Analysis

To study the effects of a perfectly anticipated inflation on the steadystate capital and consumption goods, total differentiate (23), (24) and (26):<sup>6</sup>

$$\bar{c}_1 = F(\bar{l}, \bar{k}) . \tag{26}$$

We have

$$\frac{d\bar{k}}{d\mu} = \frac{(u'F_1)^2 F_{12}}{D} > 0 , \qquad (27)$$

$$\frac{d\bar{l}}{d\mu} = \left(\frac{-F_{22}}{F_{12}}\right) \left(\frac{d\bar{k}}{d\mu}\right) > 0 , \qquad (28)$$

$$\frac{d\bar{c}_1}{d\mu} = \left(\frac{-F_1 F_{22}}{F_{12}} + F_2\right) \left(\frac{d\bar{k}}{d\mu}\right) > 0 , \qquad (29)$$

where

$$\begin{split} D &\equiv \left[ u'v'F_1F_{22}G_{11} \right] + \left[ u'v''F_1F_{22}(G_1)^2 \right] + \left[ u''v'(F_1)^2F_{22}G_1 \right] \\ &- \left[ u''v'F_1F_2F_{12}G_1 \right] > 0 \;, \end{split}$$

and we have made use of the fact that  $F_{11}F_{22} = (F_{12})^2$ .

The reasoning for the results in (27)-(29) can be explained as follows.<sup>7</sup> Suppose that the economy is in a steady state initially. Higher inflation raises the opportunity cost of purchasing  $c_2$  relative to  $c_1$  (since  $c_2$  is subject to the cash-in-advance constraint), this induces a substitution of demand for  $c_1$ instead of  $c_2$ . It also results in a shift of labor supply from the cash-good sector to the credit-good sector, henceforth, the supply is less (more) in the former (latter) sector. However, (25) implies that there will be excess supply (demand) in the cash-good (credit-good) sector. Hence, the relative price  $p_1/p_2$  must rise in order to clear both markets. In the long run,  $p_1/p_2$  is higher to justify more (less) production of credit (cash) goods, what in turn leads to more capital accumulation.

Although the above model is of the specific-factors variety, it should be noted that if both sectors are allowed to use capital and labor as in the more general Heckscher-Ohlin model, a condition similar to (25) still holds.

<sup>6</sup>The following comparative static exercises are incomplete without first verifying the stability of the system. In the appendix, it is shown that the system is saddle-point stable.

<sup>7</sup>And they are verified analytically in the appendix.

Our results would go through if the credit-good sector is capital intensive.<sup>8</sup> Higher inflation raises the demand for credit goods as well as redistributes factor inputs to that sector. Because of (25) and the assumed capital intensities, there will be excess demand (excess supply) in the credit-good (cash-good) sector. Again, this leads to a higher relative price, and an expansion of production in the credit-good sector, what in turn induces more capital accumulation in the long run.<sup>9</sup>

The welfare implication of alternative monetary rules can also be easily demonstrated. The conventional estimates of welfare costs of inflation, e.g. Bailey (1956), Lucas (1981), and Cooley and Hansen (1989, 1991) have been focusing on steady-state comparisons. All those studies have produced the result that higher inflation reduces economic welfare. But, this is not the case for the present model. To see that, let U be  $u(\bar{c}_1) + u(\bar{c}_2)$  and differentiate with respect to  $\mu$ ,

$$\frac{dU}{d\mu} = u' \left( \theta + (\mu + \theta) \frac{F_1 F_{22}}{F_{12}} \right) \left( \frac{d\bar{k}}{d\mu} \right).$$

The sign of the above expression is uncertain, but for  $\mu + \theta = 0$ , higher inflation enhances steady-state welfare!<sup>10</sup> Such a perverse result should not be viewed as a denial of the theory of Optimum Quantity Money, but a serious qualification for steady-state welfare comparison. In a companion paper, Huo (1995) shows that when one takes into account the welfare loss on the dynamic adjustment path, higher inflation reduces overall economic welfare with certainty.

### 4. Extensions and Conclusion

The reasoning given in the previous section can be applied to alternative formulations regarding the cash-in-advance constraint. We will now examine five such formulations.<sup>11</sup>

(i) The cash-in-advance constraint is applied to  $c_1$  only:

<sup>9</sup>I am grateful to an anonymous referee on this point.

<sup>10</sup>The intuition for this result can be roughly sketched as follows: When  $\mu$  changes, there are two steady-state welfare effects. The first is the welfare loss resulted from the distorted production pattern; the second is the welfare gain resulted from higher capital stock. When  $\mu + \theta = 0$ , that is, money grows exactly according to Friedman's OQM, a marginal change of  $\mu$  does not create any distortion, so the first effect is absent; but the second effect is strictly welfare enhancing since more capital stock produces more  $c_1$ .

<sup>11</sup>Analytical derivations are available upon request.

<sup>&</sup>lt;sup>8</sup>Assuming investment is still produced in the credit-good sector.

$$p_1(t)c_1(t) \le M(t)$$

This is the opposite to the case in previous sections. Inflation lowers the relative price  $p_1/p_2$  and drives labor input from the credit-good to the cash-good sector. Due to the relative price effect alone,  $d\bar{k}/d\mu < 0$ . Note that if one ignores  $c_2$ , this formulation of the present model is identical to Stockman's one-sector formulation. Thus, the result that  $d\bar{k}/d\mu = 0$ , which he obtained, is due to neglecting the distributional effect of inflation.

(ii) The cash-in-advance constraint is applied to both  $c_1$  and  $c_2$  jointly:

$$p_1(t)c_1(t) + p_2(t)c_2(t) \le M(t)$$
.

In this case, a permanent inflation does not affect the relative price  $p_1/p_2$ . Hence,  $d\bar{k}/d\mu = 0$ .

(iii) The cash-in-advance constraint is applied to both  $c_1$  and i jointly:

$$p_1(t)[c_1(t) + i(t)] \le M(t)$$
.

In this case, a permanent inflation has two effects in the steady state. On the one hand, it reduces the relative price  $p_1/p_2$  as  $c_2$  is not subject to the cash-in-advance constraint. On the other hand, since cash is used in investment purchase, we have the Stockman effect. Since both effects are known to decrease capital accumulation,  $d\bar{k}/d\mu < 0$ .

(iv) The cash-in-advance constraint is applied to both  $c_2$  and i jointly:

$$p_1(t)i(t) + p_2(t)c_2(t) \le M(t)$$
.

In this case, a permanent inflation produces positive relative price effect and negative Stockman effect. In general, the net effect on the steady-state capital stock is uncertain.

(v) The cash-in-advance constraint is applied to  $c_1$ ,  $c_2$ , and *i* jointly:

$$p_1(t)[c_1(t) + i(t)] + p_2(t)c_2(t) \le M(t)$$
.

In this case, as in (ii), a permanent inflation does not change the relative price  $p_1/p_2$ , yet the Stockman effect still operates. It then follows that  $d\bar{k}/d\mu < 0$ .

In conclusion, this paper makes two contributions. First, we show how the Tobin effect can be reestablished in an economy with a cash-in-advance constraint. Second, we show how Stockman's result can be reproduced without imposing the cash-in-advance constraint to the purchase of capital. The distinguishing feature of the model is that there are more than one sector,

and factor inputs are subject to the influence of inflation. Specifically, inflation alters the relative price, and expands one sector at the expense of the other sector. Given the assumed structure of production, this leads to a change in the accumulation of capital in the long run.

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### Appendix

To study the dynamic behavior of the system in the neighborhood of the steady state, we linearize (1), (16) and (22) around the steady state to obtain

$$\begin{bmatrix} \dot{c}_1 \\ \dot{l} \\ \dot{k} \end{bmatrix} = \begin{bmatrix} 0 & -F_{12}u'/u'' & -F_{22}u'/u'' \\ A^{-1}B & A^{-1}C & A-1D \\ -1 & F_1 & F_2 \end{bmatrix} \begin{bmatrix} \tilde{c}_1 \\ \tilde{l} \\ \tilde{k} \end{bmatrix},$$
(30)

and

$$\begin{split} \tilde{c}_{1} &= c_{1} - \bar{c}_{1} , \\ \tilde{l} &= l - \bar{l} , \\ \tilde{k} &= k - \bar{k} , \\ B &= \frac{F_{12}}{F_{1}} + \frac{u''v'F_{1}G_{1}}{\left(u'F_{1}\right)^{2}} \end{split}$$

$$\begin{split} C \ &= \ \frac{u'v'F_1G_{11} \ + \ (G_1)^2v'' \ + \ u'v'G_1F_{11}}{(u'F_1)^2} < 0 \ , \\ D \ &= \ F_{22} \ - \ \frac{F_{12}F_2}{F_1} \ + \ \frac{u'v'G_1F_{12}}{(u'F_1)^2} \ , \end{split}$$

where all derivatives are evaluated at the steady state.

In order for the steady state to be a saddle point, it is necessary that the matrix in (30) has a unique negative characteristic root. The product of the characteristic roots of the system is given by the determinant

$$A^{-1} \left( \frac{u'}{u''} \right) \left\{ \frac{-F_{22}[u'v'F_1G_{11} + v''(G_1)^2] + u''v'F_1G_1(F_2F_{12} - F_1F_{22})}{(u'F_1)^2} \right\} < 0 .$$

This establishes that there are either three negative roots or one. To establish that there is only one negative root, we proceed to examine the trace of the matrix, which is given by

$$A^{-1}C + F_2 > 0 \; .$$

Since the trace is positive, this implies that there is at least one positive root. But since we know it has either zero or two positive roots, it has two. There is therefore a unique negative characteristic root, and a unique perfect foresight path satisfying (1), (16) and (22) that converges to the steady state.

When the monetary growth rate is raised from  $\mu_0$  to  $\mu_1$ , the local dynamic system is characterized by the following equations:

$$k(t) = \bar{k}(\mu_1) - [\bar{k}(\mu_1) - k_0]e^{\lambda t}, \qquad (31)$$

$$l(t) = \bar{l}(\mu_1) + \left(\frac{F_{22}}{F_{12}}\right) [\bar{k}(\mu_1) - k_0] e^{\lambda t} , \qquad (32)$$

$$c_1(t) = \bar{c}_1(\mu_1) + \left(\frac{E}{B}\right) [\bar{k}(\mu_1) - k_0] e^{\lambda t} , \qquad (33)$$

where

$$E \equiv F_{22} - \frac{F_2 F_{12}}{F_1} - \frac{u'v' F_1 G_{11} F_{22} + (G_1)^2 v'' F_{22}}{(u'F_1)^2 F_{12}} < 0 ,$$

 $\lambda < 0$  is the unique negative characteristic root of the matrix in (30), and all

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derivatives are evaluated at the new steady state. If the initial capital is equal to the steady-state capital under  $\mu_0$ , i.e.,  $k(0) = \bar{k}(\mu_0)$ , (31) implies that the capital stock monotonically increases to the new steady-state level  $\bar{k}(\mu_1)$ . Equation (32) implies that the fraction of labor supply used in the credit-good sector monotonically increases, and the fraction of labor supply used in the cash-good sector monotonically decreases. Equation (33) implies that the consumption of credit goods is monotonically increasing (decreasing) if B is positive (negative). But the consumption of cash goods is monotonically decreasing. Finally,  $p_1/p_2 = G_1(1 - l)/F_1(l, k)$  jumps up initially, and reaches a higher steady-state value.