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MODELLING BUSINESS CYCLES IN TAIWAN WITH TIME–VARYING MARKOV–SWITCHING MODELS

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ABSTRACT

This paper employs Hamilton's (1989) original Markov-switching model and the time–varying Markov–switching model developed by Filardo (1994), respectively, to investigate the business cycle and evaluate the usefulness of the coincident and leading indexes in dating the business cycle and in predicting future GDP in Taiwan. The empirical results suggest that these two indexes help date the business cycle in Taiwan and improve precision in predicting turning points. As for forecasting future GDP, the coincident index is useful whereas the leading index is not.

Keywords: Markov–switching model, Time–varying transition probability, Taiwan business cycle, Leading index, Coincident index

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1. INTRODUCTION

Leading and coincident indexes have been regularly compiled and published by the Council for Economic Planning and Development (CEPD) in Taiwan. These are two particularly important statistical indexes because the Taiwan government actually uses them to monitor the economy and changes her economic policy accordingly. An important question then arises: how useful are these two statistics in dating business cycles and forecasting the future growth rate of GDP? It would be unwise to devise a discretionary policy rule based on statistics that have no predictive power. The primary purpose of this paper is to model the business cycle and investigate the usefulness of leading and coincident indexes in dating and predicting the business cycle in Taiwan.

The traditional multiple linear regression model, vector autoregression model and error correction model are all candidate models for our analysis. However, these methods share a common possible shortcoming: they all assume that the growth rate of GDP is linear and stationary. The nonlinear and asymmetric behavior, i.e., expansion, peak, contraction and trough, during different phases of the business cycle have long been recognized; see Burns and Mitchell (1946). Linear models cannot adequately explain these nonlinear and asymmetric phenomena and we need to turn to nonlinear models. As advocated by Hamilton (1989), the Markov-switching model maintains the assumption that time series data may display frequent changes in terms of observed behavior and accounts for such changes through switches in states, where the data-generating process and average duration of each state are allowed to differ. Importantly, the statistical features and identification of the states are not imposed exogenously on the data, but are rather determined endogenously by the estimation procedure. Previous empirical results have witnessed the success of the Markov-switching model in capturing the observed nonlinearity.

Up to the present time, numerous theoretical studies and empirical applications of this approach have been published in the literature. In particular, Hamilton (1990) suggested using the expectation maximization (hereafter EM) algorithm in the Markov–switching model. Diebold, Lee and Weinbach (1994) extended the EM algorithm to the Markov–switching model with transition probabilities varying with fundamentals. Kim (1994) extended the Markov–switching model to a general state–space model and proposed a simple approximate smoothing algorithm. Psaradakis and Sola (1998) examined the finite–sample properties of the maximum likelihood estimator in autoregressive models with switching means and variances. Various authors extended Hamilton's original model to allow for time–varying or duration–dependent transition probabilities and applied such models to investigate the U.S. business cycle. For examples, Filardo (1994), Filardo and Gorden (1998) model the transition probabilities as functions of exogenous information, Durland and McCurdy (1994) analyze the case of duration-dependent transition, and Ghysels (1993) introduces seasonals as determinant factors.

Hamilton (1996), Hansen (1992, 1996) and Garcia (1998) investigated the issue of nuisance parameters in hypothesis testing within Markov–switching models. Other applications related to the U.S. business cycle include Goodwin (1993), Lam (1990), Layton (1996, 1998), Lahiri and Wang (1994), Diebold and Rudebusch (1996), Hamilton and Lin (1996), Hamilton and Perez–Quiros (1996), Kim and Nelson (1998) and Kim and Yoo (1995).[1]

Huang, Kuan and Lin (1998) first applied the Markov–switching model to analyze the business cycle in Taiwan. They modeled real GNP by means of an autoregression process of order four with intercept following a Markov chain in contrast to Hamilton's (1989) original model where the mean switches from one state to another. Huang (1999) used a three-sate Markov-switching model to examine whether Taiwan's business cycle exhibits the particular asymmetric pattern described as the 'plucking model' proposed by Friedman (1969, 1993).

This paper studies business cycle characteristics in Taiwan. In addition to modeling the observed nonlinearity and asymmetry of Taiwan's business cycle, we are particularly interested in the role of leading and coincident indicators in modeling real GDP. By allowing the transition probability to depend upon economic predic-

[1] See chapter 15 of the book entitled "Unit Roots, Cointegration and Structural Changes," written by Maddala and Kim (1998), for a survey of the literature in this area.

tors, the time–varying Markov–switching model provides a convenient framework for investigating the usefulness of these two indicators in dating business cycles and forecasting future GDP. More specifically, we can examine the impacts of these two indicators on filtered and smoothed probability estimates and then compare the model–defined chronologies with the CEPD–defined chronologies. Our empirical results conclude that including coincident and leading indexes in transition probabilities does help in forecasting and dating turning points.

In addition to this introduction, this paper is organized as follows. Section 2 gives the model specifications, while the data sources and empirical discussions are explained in Sections 3 and 4, respectively. Section 5 concludes.

2. MODEL SPECIFICATION

2.1 Markov–Switching Model with Fixed Transition Probability

Let $\tilde{y}_t = \log(\text{GDP}_t)$ and $y_{4,t} = (1-B^4)\tilde{y}_t$ denote the logarithmic transformation of real GDP and its annual growth rate, respectively. It is commonly believed that most macroeconomic time series contain unit roots or seasonal unit roots. The Hylleberg et al. (1990) (hereafter HEGY) seasonal unit roots test in the next section confirms this belief and indicates that seasonal differencing is more appropriate for our data. We consider an unobserved latent variable s_t which takes on the value 1 when the economy is in expansion and 2 when the economy is in contraction. Hamilton's (1989) fixed transition probability (hereafter FTP) Markov–switching model is as follows:

$$y_{4,t} - \mu_{s_t} = a_1(y_{4,t-1} - \mu_{s_{t-1}}) + \ldots + a_p(y_{4,t-p} - \mu_{s_{t-p}}) + \epsilon_t \tag{1}$$

where $\epsilon_t \sim$ i.i.d. $\text{N}(0, \sigma^2)$ and mean, μ_t, switches between two states. For ease of exposition, the variance of the disturbance term, σ^2, is assumed to be the same across regimes. The state variable s_t is assumed to be governed by a first-order Markov chain:

$$\begin{aligned} \mathrm{p}(s_t = 1|s_{t-1} = 1) &= p_{11} \\ \mathrm{p}(s_t = 2|s_{t-1} = 1) &= p_{12} \\ \mathrm{p}(s_t = 1|s_{t-1} = 2) &= p_{21} \\ \mathrm{p}(s_t = 2|s_{t-1} = 2) &= p_{22} \end{aligned} \tag{2}$$

where $p_{11} + p_{12} = p_{21} + p_{22} = 1$ and $0 \leq p_{ij} \leq 1$, $i, j = 1, 2$. Equations (1) and (2) constitute our first empirical model, which is denoted as Model(1).

2.2 Time–Varying Markov-Switching Model

Following the specification of Filardo (1994), we modify the constant transition probability to be a logistic function of economic–indicator variables so that the probabilities are always between 0 and 1. These variables–dependent probabilities are:

$$\begin{aligned} \mathrm{p}(s_t = 1|s_{t-1} = 1,\ \mathbf{z}_t) &= p_{11}(\mathbf{z}_t) \\ \mathrm{p}(s_t = 2|s_{t-1} = 1,\ \mathbf{z}_t) &= p_{12}(\mathbf{z}_t) \\ \mathrm{p}(s_t = 1|s_{t-1} = 2,\ \mathbf{z}_t) &= p_{21}(\mathbf{z}_t) \\ \mathrm{p}(s_t = 2|s_{t-1} = 2,\ \mathbf{z}_t) &= p_{22}(\mathbf{z}_t) \end{aligned} \tag{3}$$

and

$$\begin{aligned} p_{11}(\mathbf{z}_t) &= \frac{\exp(\theta_{p0} + \sum_{j=1}^{J_1} \theta_{pj} z_{t-j})}{1 + \exp(\theta_{p0} + \sum_{j=1}^{J_1} \theta_{pj} z_{t-j})} \\ p_{22}(\mathbf{z}_t) &= \frac{\exp(\theta_{q0} + \sum_{j=1}^{J_2} \theta_{qj} z_{t-j})}{1 + \exp(\theta_{q0} + \sum_{j=1}^{J_2} \theta_{qj} z_{t-j})} \end{aligned} \tag{4}$$

where $p_{11}(\mathbf{z}_t) + p_{12}(\mathbf{z}_t) = p_{21}(\mathbf{z}_t) + p_{22}(\mathbf{z}_t) = 1$, and J_1 and J_2 are of the order of lag terms of indicators. $\mathbf{z}_t = \{z_t, z_{t-1}, ...\}$ denotes the history of the economic-indicator variables. In this paper, the $\mathbf{z}_t$ are leading and coincident indicators.[2] The merits of this specification, as emphasized by Filardo (1994), are that the time–varying Markov–switching (hereafter TVMS) model may capture more complicated temporal persistence than the FTP model, and allow the transition probabilities to rise just before a contraction or an expansion begins. More specifically, the leading and coincident indicators, complied by the CEPD, contain information that is useful in dating the business cycle. The TVMS model provides a convenient framework to incoporate this useful information through the variation in transition probabilities. This feature implies that the expected business cycle duration will also vary throughout the cycles with $\mathbf{z}_t$ giving information about where the economy is proceeding. Filardo and Gordon (1998) found that the expected durations of a recession tend to fall off quickly near the end of the recession, hence signaling an upcoming change in the state of expansion. Finally, it is worth noting that the information index variables should be stationary.

2.3 Estimation and Algorithm

There are several methods used to estimate the unknown parameters, i.e., the EM algorithm proposed by Hamilton (1990), the Gibbs sampling approach of Albert and Chib (1993), and the maximum likelihood estimation (hereafter MLE) of Hamilton (1989). Generally speaking, the Gibbs sampling approach requires heavy computation, while the EM algorithm is difficult to implement in the presence of AR lags as pointed out by Filardo (1994). Hence, we adopt the MLE numerical method in this paper.

According to the specification of (1) and (2), the economy at time t depends upon $s_t, s_{t-1}, \ldots, s_{t-p}$. To account for this, we construct a new state variable S_t^* defined as:

$$S_t^* = 1 + (s_t - 1)2^0 + (s_{t-1} - 1)2^1 + \ldots + (s_{t-p} - 1)2^p$$

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[2] Strictly speaking, the information–predictor, z_t, should be defined as $z_t = z_{4,t} = (1 - B^4)\tilde{z}_t$, where $\tilde{z}_t = \log(\mathrm{CI}_t)$ or $\tilde{z}_t = \log(\mathrm{LEAD}_t)$. As no confusion can be caused, we still use the notation $z_t (= z_{4,t})$ in the text for simplicity.

S_t^* takes a value from 1 to $N = 2^{p+1}$ and the resulting transition probability P^* is

$$P^* = P_{ij}^* = \text{prob}(S_t^* = j | S_{t-1}^* = i)$$

By letting $\xi_{t|s}$ be the probability of S_t given information up to time s, it can be shown that $y_{4,t}$ constitutes a stationary process provided that there exists a stationary distribution for P^* which is assumed throughout the paper. The filtered probabilities, $p(s_t|\mathbf{y}_t)$, denote the conditional probability for the state at date t as s_t, based upon information contained in $\mathbf{y}$ and observed up to date t. The smoothed probabilities, $p(s_t|\mathbf{y}_T)$, on the other hand, represent the conditional probability based on the data available throughout the whole sample at the future date T.

The likelihood function can be found by summing up the joint likelihood $f(y_t, S_t^*|\mathbf{y}_{t-1}, \theta)$ over S_t^* which in turn can be easily derived using the conditional likelihood function $f(y_t|\mathbf{y}_{t-1}, S_t^*, \theta)$ as in (1) and (2). To be specific, we summarize the estimation algorithm proposed by Hamilton (1989, 1994) as follows.

First of all, we solve the Markov chain equation to obtain the ergodic probability π and set $\xi_{1|0} = \pi$ to start the algorithm. We then compute the filtering probability by means of

$$\xi_{t|t} = \frac{(\xi_{t|t-1} \odot \eta_t)}{1'(\xi_{t|t-1} \odot \eta_t)}$$

for $t = 1, 2, \ldots, T$, where η_t is the $N \times 1$ vector whose j–th element is the conditional density of

$$f(y_t|y_{t-1}, \ldots, y_1, S_t^* = j)$$

and $\odot$ denotes the element by element multiplication. Next, we compute the prediction probability by

$$\xi_{t+1|t} = P^* \xi_{t|t}$$

As a side product, the likelihood function can be calculated as:

$$L(\theta) = \sum_{t=1}^{T} \log f(y_t|\mathbf{y}_{t-1}, \theta)$$

$$f(y_t|\mathbf{y}_{t-1}; \theta) = 1'(\xi_{t|t-1} \odot \eta_t)$$

Finally, the smoothed probability can be obtained by means of the following recursive equation proposed by Kim (1994)

$$\xi_{t|T} = \xi_{t|t} \odot \{P'[\xi_{t+1|T} \oslash \xi_{t+1|t}]\}$$

where $\oslash$ denotes the element by element division. We start the algorithm from $t = T-1$, and then proceed backward until $t = 1$.

This estimation and algorithm can be applied to the case where there is time–varying transition probability with minor revision. For details, see Hamilton (1989, 1994, 1996) and Hamilton and Perez–Quiros (1996). Note that $y_{4,t}$ is nonlinear, but nevertheless stationary, and hence conventional asymptotic theorems apply.

3. DATA DESCRIPTION

The major variable used in this paper is seasonally–unadjusted real gross domestic product (GDP) which is taken from the Statistical Abstract of National Income, Taiwan Area. The candidate series for the information variables are the leading (LEAD) and coincident (CI) indicators, which are considered to be useful business–cycle predictors. All series are quarterly data from 1961:Q1 to 1998:Q3 which amount to 151 observations. The reason seasonally-unadjusted data is used is as follows. The simulation results of Franses and Paap (1999) pointed out that a two-sided seasonal filter such as X-11 tends to smooth out business cycle fluctuations. Lin and Chen (1999) also found similar results for Taiwan data.

The GDP, LEAD and CI indicators and their corresponding annual growth rates

Table 1 Index Components

Coincident index	Leading index
Industrial production index	MFGs' new orders (12-month span)
MFG production index (12-month span)	Average monthly working hours, MFG
Manufacturing sales	Exports by customs (12-month span)
Average monthly wages and salaries	Money supply M1B
Bank clearing (12-month span)	Wholesale price index (6-month span)
Quantum of domestic traffic	Stock price index (12-month span)
	Floor area permitted for building construction in Taiwan area

are plotted in Figure 1. The shaded areas are the contraction periods determined by the CEPD. It should be noted that the average growth rate over the whole sample period is 8.24%, and except for the three quarters from 1974:Q3 to 1975:Q1, the growth rates of GDP are well above zero. Hence, contraction here means low growth rather than recession.

Table 1 summarizes the detailed index components of leading and coincident indicators. The procedure for compiling the indexes is as follows. First, all individual monthly component series are grouped into three categories: leading, coincident and lagged series. Each component of them is first seasonally adjusted using the X–11 method and then growth rates are computed. Furthermore, each individual growth rate is standardized by using the 60–month moving average of the absolute value of the growth rates. Next, the standardized growth rates are used to compute the composite leading, coincident and lagged indexes. In turn, the three indexes are standardized by the moving average of the absolute values as in the case of the component series. Next, the base year is selected, cumulative indexes and de–trending is performed and the resulting indexes are re–weighted to produce the composite index. Finally, the Trough–Peak algorithm is then used to date the peak and the trough. Occasionally, upon the recommendation of 'experts', minor adjustments with regard to dating the cycle are made. The period between the trough and the peak is classified as a period of expansion and the period between the peak and the trough is considered to be a period of contraction. CEPD has

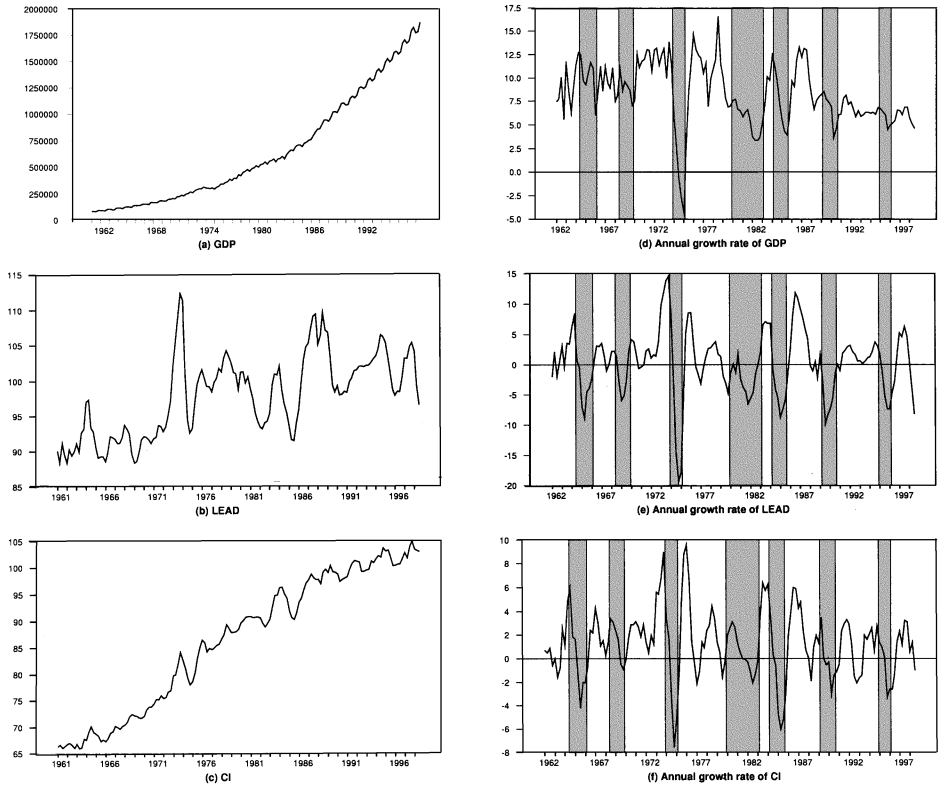

Figure 1 Plots of the Seasonally–Unadjusted GDP, Leading Index and Coincident Index and Their Annual Growth Rates. The Shaded Areas Are the Contraction Periods Determined by the CEPD

identified seven cycles over the whole sample period.

4. EMPIRICAL RESULTS

We adopt the HEGY procedure to test for the seasonal unit root and report the results in Table 2. To detect the seasonality of the time series, $\tilde{x}_t$, we perform the following regression.

$$\Phi_q^*(B)x_{4,t} = \pi_1 x_{1,t-1} + \pi_2 x_{2,t-1} + \pi_3 x_{3,t-2} + \pi_4 x_{3,t-1} + \varepsilon_t \tag{5}$$

Table 2 Seasonal Unit Root Test Results

GDP	Lags	$\pi_1 = 0$	$\pi_2 = 0$	$\pi_3 = 0$	$\pi_4 = 0$	$\pi_3 \cap \pi_4 = 0$
None	4	4.503	−2.088	−1.440	−2.088	3.225
I only	4	−3.267	−2.034	−1.494	−1.865	2.861
I,SD	4	−3.239	−1.949	−1.314	−2.439	3.849
I,Tr	4	−0.569	−2.021	−1.492	−1.855	2.838
I,SD,Tr	4	−0.534	−1.942	−1.315	−2.424	3.814
CI	Lags	$\pi_1 = 0$	$\pi_2 = 0$	$\pi_3 = 0$	$\pi_4 = 0$	$\pi_3 \cap \pi_4 = 0$
None	4	4.132	−6.432	−4.644	−5.546	32.720
I only	4	−1.844	−6.317	−4.697	−5.441	32.314
I,SD	4	−1.840	−6.236	−4.654	−5.445	32.107
I,Tr	4	−0.463	−6.288	−4.677	−5.421	32.022
I,SD,Tr	4	−0.463	−6.207	−4.633	−5.425	31.814
LEAD	Lags	$\pi_1 = 0$	$\pi_2 = 0$	$\pi_3 = 0$	$\pi_4 = 0$	$\pi_3 \cap \pi_4 = 0$
None	4	0.425	−5.300	−2.432	−6.493	26.415
I only	4	−2.387	−5.367	−2.672	−6.352	26.442
I,SD	4	−2.373	−5.502	−2.582	−6.256	25.402
I,Tr	4	−3.057	−5.488	−2.903	−6.171	26.327
I,SD,Tr	4	−3.045	−5.624	−2.814	−6.075	25.273
Critical Value (5%)		$\pi_1 = 0$	$\pi_2 = 0$	$\pi_3 = 0$	$\pi_4 = 0$	$\pi_3 \cap \pi_4 = 0$
None		−1.94	−1.95	−1.92	−1.65	3.16
I only		−2.87	−1.92	−1.90	−1.66	3.12
I,SD		−2.91	−2.89	−3.38	−1.96	6.61
I,Tr		−3.44	−1.95	−1.92	−1.66	3.07
I,SD,Tr		−3.49	−2.91	−3.41	−1.92	6.57

Note. I=Intercept. SD=Seasonal Dummy. Tr=Trend.

where $\Phi_q^*(B) = 1 - a_1 B - a_2 B^2 - ... - a_q B^q$ with $q = 4$, $x_{1,t} = (1 + B + B^2 + B^3)\tilde{x}_t$, $x_{2,t} = -(1 - B + B^2 - B^3)\tilde{x}_t$, $x_{3,t} = -(1 - B^2)\tilde{x}_t$ and $x_{4,t} = (1 - B^4)\tilde{x}_t$, $\tilde{x}_t$ are log(GDP), log(CI) or log(LEAD). Testing for root 1 (zero frequency) and root −1 (2 cycles per year) amount to a simple test for $\pi_1 = 0$ and $\pi_2 = 0$, respectively. For the complex root, this is equivalent to testing $\pi_3 = 0$ and $\pi_4 = 0$, or a joint test of $\pi_3 \cap \pi_4 = 0$. Table 2 shows that seasonally–unadjusted GDP appears to have a seasonal unit root, while the LEAD and CI have only regular unit roots. This is not surprising since the LEAD and CI have already been seasonally adjusted. However, for consistency we apply seasonal differencing to all variables.

For ease of reading, the empirical models are reproduced as follows:

$$y_{4,t} - \mu_{s_t} = a_1(y_{4,t-1} - \mu_{s_{t-1}}) + \ldots + a_p(y_{4,t-p} - \mu_{s_{t-p}}) + \epsilon_t$$

$$P(s_t = j|s_{t-1} = i, \mathbf{z}_t) = \begin{bmatrix} p_{11}(\mathbf{z}_t) & p_{21}(\mathbf{z}_t) \\ p_{12}(\mathbf{z}_t) & p_{22}(\mathbf{z}_t) \end{bmatrix}$$

$$p_{11}(\mathbf{z}_t) = \frac{\exp(\theta_{p0} + \sum_{j=1}^{J_1} \theta_{pj} z_{t-j})}{1 + \exp(\theta_{p0} + \sum_{j=1}^{J_1} \theta_{pj} z_{t-j})}$$

$$p_{22}(\mathbf{z}_t) = \frac{\exp(\theta_{q0} + \sum_{j=1}^{J_2} \theta_{qj} z_{t-j})}{1 + \exp(\theta_{q0} + \sum_{j=1}^{J_2} \theta_{qj} z_{t-j})}$$

In this paper, Model(1) denotes Hamilton's (1989) original Markov–switching model, and Model(2) and Model(3) use leading and coincident indicators as economic–indicator variables, respectively. Model(4) uses both leading and coincident indexes as predictors for the transition probability. When the economic predictors are not informative about the evolution of the state of the economy, the TVMS model becomes a Hamilton (1989) model. The maximum likelihood estimation of the unknown parameters was performed using the OPTIMUM module of GAUSS 3.2 with a combination of the Broyden–Fletcher–Goldfarb–Shanno (BFGS) numerical algorithm. We impose no constraints on any of the transition probabilities p_{ij} other than the conditions that $0 \leq p_{ij}(\mathbf{z}_t) \leq 1$ and $\sum_{h=1}^{2} p_{ih}(\mathbf{z}_t) = 1, i, j = 1, 2$. The estimation results of various specifications are summarized in Table 3. According to the Akaike (AIC) and Hannan–Quinn (HQ) criteria, AR(4) with mean–switching specification is adopted. The first question is whether the economic predictors statistically significantly influence the probability of a phase shift in GDP. Under the null hypothesis of no dependence, the likelihood ratio test statistic is known to be distributed as a chi–square distribution with degrees of freedom equal to the number of additional parameters estimated. From the results of Table 3, we

Table 3 Empirical Results of Various Specifications

	AR(4)	Model(1) FTP	Model(2) $\{\text{LEAD}_{t-1}\}$	Model(3) $\{\text{CI}_{t-1}\}$	Model(4) $\{\text{LEAD}_{t-1}, \text{CI}_{t-1}\}$
μ_1	8.271	11.214	11.172	11.227	11.188
	(0.478)	(0.581)	(0.594)	(0.579)	(0.534)
μ_2		7.093	7.071	7.076	7.279
		(0.477)	(0.456)	(0.490)	(0.412)
σ^2	1.882	1.368	1.360	1.372	1.417
	(0.111)	(0.106)	(0.106)	(0.108)	(0.096)
ϕ_1	0.840	0.945	0.937	0.948	1.003
	(0.079)	(0.111)	(0.125)	(0.110)	(0.145)
ϕ_2	0.027	−0.057	−0.058	−0.065	−0.179
	(0.100)	(0.153)	(0.155)	(0.147)	(0.210)
ϕ_3	0.057	0.006	0.019	0.004	0.167
	(0.101)	(0.112)	(0.131)	(0.064)	(0.143)
ϕ_4	−0.253	−0.238	−0.249	−0.233	−0.341
	(0.079)	(0.113)	(0.131)	(0.101)	(0.098)
θ_{p0}		1.496	1.656	1.538	1.172
		(0.078)	(0.548)	(0.596)	(0.532)
θ_{q0}		2.588	2.552	2.663	3.551
		(0.032)	(0.507)	(0.539)	(0.918)
θ_{p1}			−0.062		−0.144
			(0.089)		(0.150)
θ_{q1}			−0.057		−0.559
			(0.107)		(0.259)
θ_{p2}				−0.008	0.553
				(0.097)	(0.360)
θ_{q2}				0.079	0.738
				(0.167)	(0.400)
# param.	6	9	11	11	13
Log-Like	−293.304	−285.050	−284.620	−284.940	−283.225
BIC	−299.770	−294.749	−296.747	−296.794	−297.235

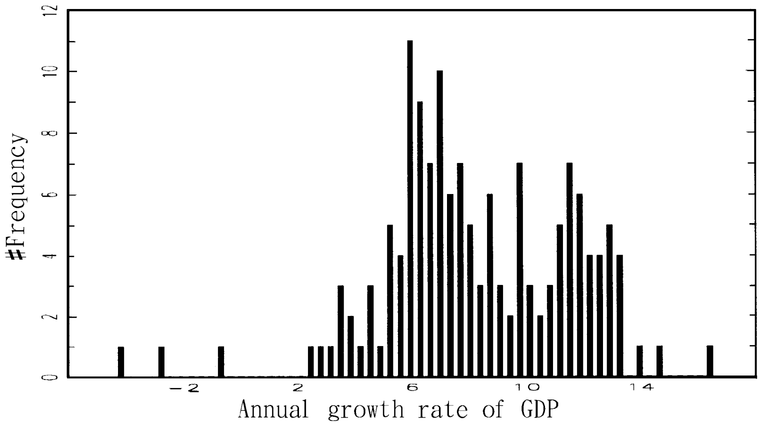

Figure 2 The Bimodal Appearance of the Annual Growth Rate of GDP

observe that the null hypothesis of no dependence cannot be rejected. That is, the economic–indicator variables of leading and coincident indexes seem to provide no further information in dating the business cycle in Taiwan. However, these two variables might still be helpful for prediction in TVMS models and this conjecture is confirmed by the mean square error (MSE) analysis presented in the next section.

From the histogram of the annual growth rates of GDP in Figure 2, we found that the economic growth rates in Taiwan might follow a bimodal distribution. While a linear model with normal errors cannot generate a bimodal distribution, a mixed distribution model such as Markov–switching model can. Note from Table 3 that our estimates of μ_1 and μ_2 are about 11.2% and 7.1%, respectively. The estimation results suggest that the economy can be dichotomized into high–growth and low–growth states. Using the criterion proposed by Hamilton (1989) and Hamilton and Perez–Quiros (1996), we classify the sample periods with $p(s_t = 1|\mathbf{y}_t) \geq 0.5$ as the high–growth state, and those with $p(s_t = 2|\mathbf{y}_t) \geq 0.5$ as the low–growth state. In particular, from the estimation results that state that $p_{11} < p_{22}$, we conclude that, over our samples, the durations of the contraction periods are longer than those of the expansion periods in Taiwan.

Another important question is whether the annual growth rate of GDP can be properly characterized by a two–regime AR(4) model. To answer this question, note that the two–regime AR(4) model reduces to a single–regime AR(4) model under the restriction

$$H_0 : \mu_1 = \mu_2 \tag{6}$$

The likelihood ratio (LR) test statistic is $-2(-293.304 + 284.620) = 17.368$. Note that the usual critical value for the χ^2 distribution is $\chi^2_{0.05}(1) = 3.841$. If the asymptotic distribution results applied we would reject the null hypothesis that y_t follows the linear autoregressive model of order 4. However, this is not the case. Standard asymptotic distribution results cannot be used here. The problem comes from two sources: under the null hypothesis, some parameters are not identified and corresponding scores are identically zero. Hansen (1992, 1996) proposed a bound test to circumvent these problems, but the computational difficulty has limited its applicability. A closely–related approach, suggested by Garcia (1998), is much easier to compute. By treating the transition probabilities as nuisance parameters, one can rule out the problematic boundary values 0 and 1. Then, Garcia (1998) exploited the fact that the likelihood ratio test statistic for the null hypothesis of one state only is the supremum over all admissible values of the nuisance parameters (the transition probabilities), and derived analytically the asymptotic distribution of the test statistics. It should be noted that no critical values currently exist for the Markov–switching model with more than two states. We therefore adopt Garcia's test procedure in this paper. From the simulated critical values tables in Garcia (1998), we found the appropriate critical value to be 8.59 (12.24) at the 5% (1%) significance level. It is apparent that our LR results still reject the AR(4) model under the null hypothesis.[3]

Figure 3 shows the filtered and smoothed probabilities of the low–growth state for the Model(1) to Model(4), respectively. CEPD has identified seven contraction periods, denoted by the shaded area in Figure 3, throughout the whole samples. We make the following observations from Figure 3. First, our models identify more than seven contraction periods and these periods have no corresponding CEPD–defined

[3] Note that it is less obvious if we apply Garcia's test to models with time–varying probabilities since the Markov process in the case of the state variable varies from one observation to the other.

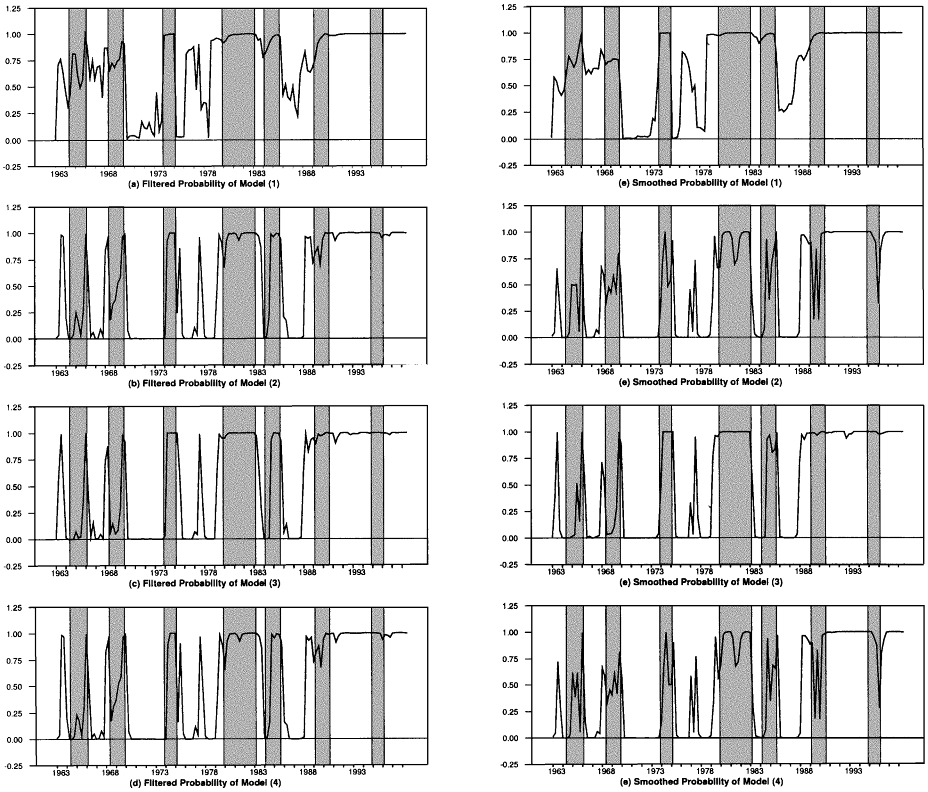

Figure 3 Plots of Filtered Probabilities and Smoothed Probabilities of Model(1) to Model(4). The Shaded Areas Are the Contraction Periods Determined by the CEPD

peaks or troughs (see Table 4). In particular, except for the seven contraction periods identified by the CEPD, all our four empirical models identify 63:Q3 as the contraction period when the actual growth rate of GDP is as low as 6.31%. Model(1) also identifies 76:Q2 to 77:Q3 as contraction periods and models (2) to (4) only pick up 77:Q3 as the contraction period. Secondly, all sample periods from 88:Q1 up to the present are identified as low-growth states by all four models while the CEPD has identified two cycles, 89:Q2–90:Q3 and 95:Q1–96:Q1, during these periods. These different results are reasonable if we closely examine the actual annual growth rates of GDP. Starting from 1988, the economic growth rates of Taiwan never exceed 8.5%, and remain at about 6% from 1992 onwards. It even reaches its trough of 4.55% in 1998. The intrinsic built–in mechanism in the

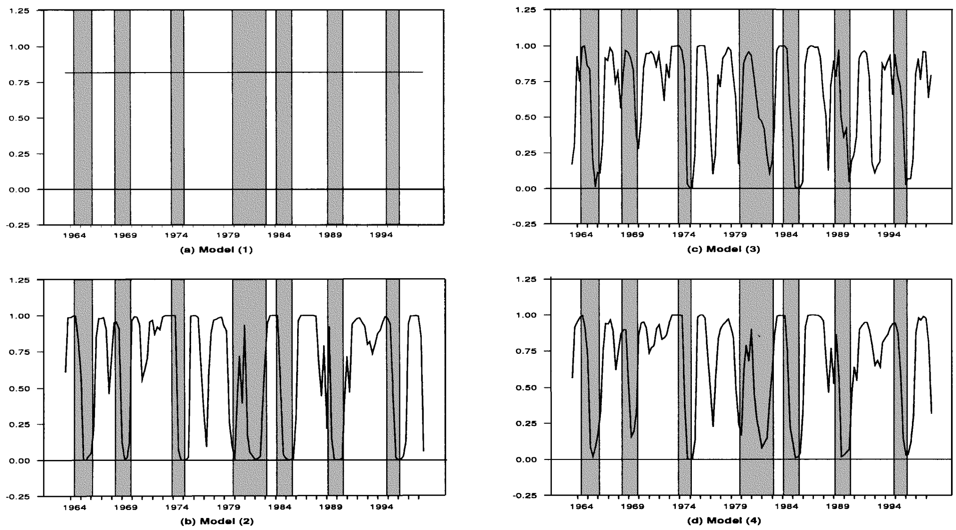

Figure 4 The Transitional Probability of $p(s_t = 1|s_{t-1} = 1, \mathbf{z}_t)$ for Model(1) to Model(4), Respectively. The Shaded Areas Are the Contraction Periods Determined by the CEPD

Markov-switching model classifies each sample point into different states based on the level of the observed growth rate. Note that as the estimated mean growth rate in contraction is 7.1%, it is natural for all models in our paper to classify the periods 88:Q1–98:Q3 as the contraction periods. Third, if we identify the periods with economic growth rates above 10% as high-growth state periods, there will be 46 high–growth state periods and 101 low-growth state periods. These are consistent with the empirical result that $p_{11} < p_{22}$, which implies that the duration of the expansion state is shorter than the duration of the contraction state.

It is of interest to compare our empirical results with those of Huang, Kuan and Lin (1998) and Huang (1999). The fifth and sixth columns of Tables 4 summarize the business cycle chronologies identified in these two papers. From the tables, we observe different chronology identification which can be explained by the fact that the model setups in the three papers are somewhat different. First, our paper is more similar to that of Huang (1999) in that the univariate Markov model is mean–switching, while the one in Huang et al. (1998) is intercept–switching. Second, our paper and that of Huang (1999) both used seasonally–unadjusted real GDP as a summative measure of the business cycle, while Huang et al. (1998)

Table 4 Taiwan Business Cycle Chronology

CEPD Peak/Trough	Model(2) Peak/Trough	Model(3) Peak/Trough	Model(4) Peak/Trough	HKL (1998) Trough	Huang (1999) Peak/Trough
	63:Q3/63:Q3	63:Q3/63:Q3	63:Q3/63:Q3		63:Q2/63:Q4
64:Q3/66:Q1	65:Q1/66:Q1	65:Q3/66:Q2	65:Q1/66:Q1	66:Q1	64:Q3/66:Q2
68:Q3/69:Q4	68:Q1/70:Q1	68:Q1/68:Q2	68:Q1/68:Q2		68:Q1/70:Q1
		69:Q4/70:Q1	69:Q4/70:Q1		
74:Q1/75:Q1	74:Q2/75:Q2	74:Q1/75:Q2	74:Q2/75:Q2	75:Q1	74:Q1/75:Q1
	77:Q3/77:Q3	77:Q3/77:Q3	77:Q1/77:Q3		76:Q2/77:Q1
					77:Q3
80:Q1/83:Q1	79:Q3/83:Q1	79:Q2/83:Q2	79:Q3/83:Q1	82:Q2	79:Q2/NA
				83:Q4	
84:Q2/85:Q3	84:Q4/85:Q4	84:Q4/85:Q4	84:Q4/85:Q4	85:Q3	NA/85:Q4
89:Q2/90:Q3	88:Q2/NA	88:Q1/NA	88:Q2/NA	90:Q4	87:Q4/NA
95:Q1/96:Q1	NA/98:Q3	NA/98:Q3	NA/98:Q3		NA/96:Q4

Note. NA = not available.

used seasonally–unadjusted GNP. Third, our model allows the transition probability to vary with some economic–predictors, while Huang et al. (1998) and Huang (1999) employed a constant transition probability model. It is not surprising that, since our model setups are more similar to that of Huang (1999), our business cycle chronologies are also more similar. However, the univariate Markov–switching models used in three papers all share a common shortcoming: they do not identify the turning points for years in the 1990s.

The advantage of the TVMS model lies in its ability to capture the information regarding the economic-indicator variables in dating the business cycle. This additional ability can be seen in Figure 4 which summarizes the transition probability of $p_{11}(\mathbf{z}_t) = p(s_t = 1|s_{t-1} = 1, \mathbf{z}_t)$ of various specifications. Note that $s_t = 1$ denotes that the economy is in an expansion state. In the case of the FTP model, the transition probability remains constant at about 0.82 in the expansion state. In other words, the transition probability is constant before, during and after the turning points in the FTP model. However, it is apparent from Figure 4 that there is a tendency towards a dramatic increase in $p_{11}(\mathbf{z}_t)$ immediately following the troughs for Models (2) to (4). The shaded areas (contraction chronologies) identified by the CEPD correspond to low probability values of $p_{11}(\mathbf{z}_t)$ in models (2) to (4). The TVMS models have the flexibility to identify systematic variations in the transition probabilities both before and after the turning points. The existence of this phenomenon indicates that the leading and coincident indicators do reflect some information in dating the business cycle in Taiwan. Figure 5 presents the out-of-sample smoothed probabilities of $p(s_t = 2|\mathbf{y}_T, \mathbf{z}_t)$ over periods from 1983:Q1 to 1998:Q3 of the various models. The corresponding mean square error (MSE) is calculated as follows:

$$\text{MSE} = T^{-1} \sum_{t=1}^{T} (y_{4,t} - \hat{y}_{4,t|t-1})^2 \tag{7}$$

where $\hat{y}_{4,t|t-1}$ denotes the one–period–ahead forecast of y_t based upon information up to time $t-1$ for out–of–sample comparisons. Following Hamilton and Perez–Quiros (1996), the turning point (TP) criterion is defined as

$$\text{TP} = K^{-1} \sum_{t=1}^{K} \{\text{prob}(s_t = 2|\mathbf{y}_T) - d_t\}^2 \tag{8}$$

where $d_t = 1$ if dated as a period of contraction by the CEPD.[4] In all cases, the last $K = 63$ observations are reserved for making forecasts. The in–sample and out–of–sample forecasting results are reported in Table 5. We will focus on the out–of–sample forecasting performance analysis. The out–of–sample MSE for Models(1) to (4) are 0.827, 1.035, 0.793 and 1.102, respectively. It is apparent that Model (3), i.e., where CI is added into the transition probabilities, performs the best in terms of out–of–sample forecasting. The forecasting improvement of Model(3)

[4] In fact, the turning point criterion defined in equation (8) was first proposed by Diebold and Rudebusch (1989) and alternatively named the quadratic probability score (QPS). Diebold and Rudebusch (1989) proposed the QPS as a measure of correspondence between turning point probabilities and actual turning points. By contrast, Filardo (1994) and Hamilton and Perez–Quiros (1996) used it in connection with the actual NBER phase dates and the model–generated regime probabilities for each data point in the series.

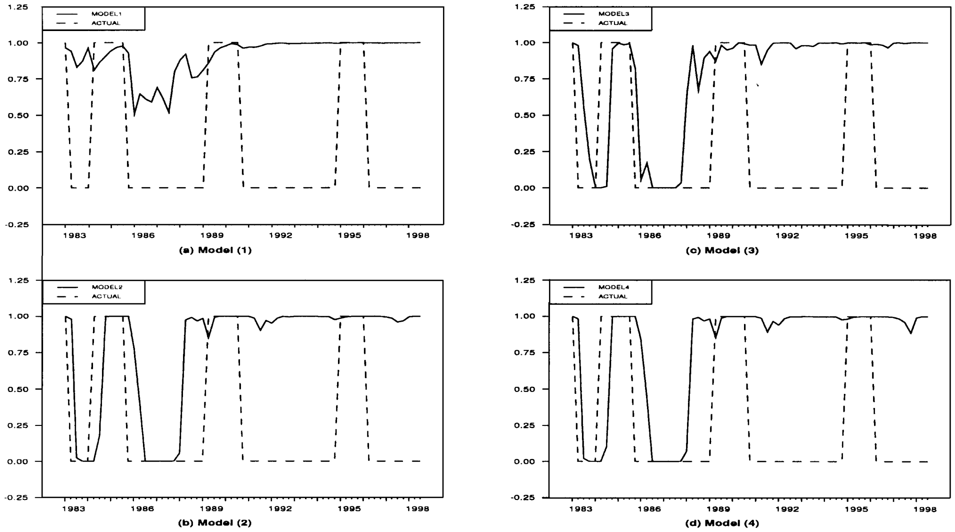

Figure 5 The Out–of–Sample Smoothed Probabilities of Model (1) to Model(4), Respectively

relative to Model (1) is 4.111%, while adding LEAD to the transition probabilities does not improve the forecasting precision but worsens the forecasting improvement by 25.151% relative to Model (1). In other words, CI is useful in forecasting future GDP while LEAD is useless in forecasting future GDP. However, by adding both CI and LEAD into the transition probabilities, i.e., Model (4), the model performs the worst in terms of forecasting improvements. The solid line and dashed line in Figure 5 denote the estimated and actual probabilities of the low–growth state since 1983, respectively. The out–of–sample TP are 0.593, 0.549, 0.534 and 0.551 for the models (1) to (4), respectively. Model(2) and Model(3) improve the turning points prediction by 7.420% and 9.949% relative to Model(1), respectively. Both the leading and coincident indexes are helpful in predicting turning points as shown in Figure 5 and Table 5. It is interesting to compare our results with those of Lin and Chen (1999). By using a bivariate Markov–switching model, Lin and Chen (1999) derives the conclusions that CI (LEAD) is useful (useless) in forecasting future GDP, and that CI is futile when it comes to predicting turning points, whereas LEAD helps predict turning points. Our empirical results are consistent with theirs except with regard to the performance of CI in predicting the turning points.

Table 5 Forecasting Performance

Model	MSE In-sample	MSE Out-sample	TP In-sample	TP Out-sample
1	3.772	0.827	0.351	0.593
2	3.794	1.035	0.327	0.549
3	3.802	0.793	0.356	0.534
4	3.910	1.102	0.329	0.551
Forecasting Improvement (%)				
1–2	−0.583	−25.151	6.838	7.420
1–3	−0.795	4.111	−1.425	9.949
1–4	−3.659	−33.252	6.268	7.083
2–4	−3.057	−6.473	−0.612	−0.362
3–4	−2.841	−38.966	7.584	−3.184

Note. In–sample Period: 63:Q1–98:Q3. Out–sample Period: 83:Q1–98:Q3.

5. CONCLUSIONS

Hamilton's (1989) Markov–switching model has been widely and successfully applied for many macroeconomic and financial time series variables. The time–varying Markov–switching model allows the transition probabilities to vary systematically with information–predictors which reflect the future course of the economy.

This paper has employed the fixed and time–varying Markov–switching models to evaluate the usefulness of the coincident and leading indexes in dating the business cycle and in predicting future GDP in Taiwan. The TVMS transition probabilities are set in a logistic function form. Our results provide evidence in support of asymmetry between contractions and expansions of GDP. The identification problem of the state is investigated by the Garcia test, which provides some evidence in support of our two–state Markov–switching specification. The duration of the contraction periods is longer than that of the expansion periods in the FTP model. Furthermore, the expected business cycle duration varies over time in the TVMS model, which implies that the coincident and leading indexes give some information in dating business cycles in Taiwan. By using the mean square error and turning point criterion the empirical results provide additional evidence that the leading and coincident indexes help date the business cycle and improve forecasting performance. In particular, our empirical analysis finds that the leading and coincident indexes are all helpful in terms of predicting turning points. The coincident index is helpful in forecasting while the leading index is of no use in forecasting. Finally, our time–varying Markov–switching models give business cycle chronologies that are much more similar to those defined by the CEPD as compared with the original Markov–switching model.

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臺灣景氣循環之探討：變動移轉機率馬可夫轉換模型之應用

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摘　要

本文應用變動移轉機率馬可夫轉換模型，以分析同時指標及領先指標是否有助於台灣經濟景氣循環轉折點之認定及經濟成長之預測。變動移轉機率模型較固定移轉機率模型更具有彈性，可以處理景氣轉折前後移轉機率的變動。實證結果發現同時指標與領先指標有助於景氣循環轉折點之預測，而且同時指標有助於經濟成長的預測而領先指標則無此效果。

關鍵詞：馬可夫轉換模型、景氣循環、領先指標、同時指標