# Switching ARCH Models of Stock Market Volatility in Taiwan

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**Abstract**

This paper examines the volatility of Taiwan's stock market by means of the GARCH and the SWARCH models. Our empirical results conclude that the SWARCH models do a better job in forecasting than the GARCH models. In addition, for Taiwan stock market there exists a positive and significant leverage effect such that a stock price decrease has a greater effect on subsequent volatility than would a stock price increase of the same magnitude. We have identified every episode causing the high-volatility state in Taiwan stock market. Our estimates attribute most of the persistence in stock price volatility to the persistence of low-, medium- and high-volatility regimes. The high-volatility regime is associated with the business recession at the beginning of 1990s.

**JEL classification:** C22; C52; G12
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# 1 Introduction

This paper is concerned with the econometric modeling of volatility of the stock market in Taiwan. The stock market in Taiwan was established in 1962, the valued-weighted stock index (Taiex) culminated to highest 12000 in February 1990, then dove downward and hit the bottom of 2900 in October, 1990. Political events, financial crises and the Gulf War all contributed to the collapse of the market. From there, Taiex rebound up and down to the current 6000. By examining the figure for Taiwan stock prices, one can easily find out there are periods for low-volatility and periods for high-volatility. If we observe the state more closely, there maybe even exist periods of low-volatility, medium-volatility and high-volatility in Taiwan stock market, respectively. Understanding the way in which the stock market volatility changes is crucial to our understanding of Taiwan economy as both are closely intertwined.

The most commonly used methods to characterize the volatility clustering of the stock returns are the univariate GARCH models developed by Engle (1982) and Bollerslev (1986). See Bollerslev, Chou and Kroner (1992) or Bollerslev, Engle and Nelson (1994) and papers therein. But the high persistence in the GARCH model is difficult to reconcile with the poor forecasting performance. Diebold (1986) and Lamoureux and Lastrapes (1990) argued that the high persistence may reflect structural change in the variance process.

Following this line of thought, Hamilton and Susmel (1994) employed switching-regime ARCH model (SWARCH) to model the the high persistence of variance. The idea of their SWARCH model is to model changes in regimes as changes in the scale of the ARCH process. Cai (1994) also parameterized a similar model to analyze the volatility in Treasury bill yield in the US. See Turner, Startz, and Nelson (1989), Dueker (1997) and Schaller and Norden (1997) for similar applications to the stock market analysis. Ramchond and Susmel (1998) also applied bivariate SWARCH model to investigate correlations among major stock market in the world. Another application of the SWARCH models including Gomez-Puig and Montalvo (1997) and Susmel and Thompson (1998).

Markov-switching models have been successfully used to model level changes for many economic and financial time series, including aggregate output (Hamilton, 1989; Lam, 1990, 1996; Diebold and Rudebusch, 1996; Hamilton and Lin, 1996; Huang et al., 1998) leading and coincident indicators (Hamilton and Perez-Quiros, 1996; Lin and Chen, 1998), exchange rate (Engle and Hamilton, 1990; Engle, 1994), interest rate (Hamilton, 1988; Sola and Drifill, 1994; Garcia and Perron, 1996; Gray, 1996), unemployment rate (Bianchi and Zoega, 1998; Montgomery et al., 1998), future markets (Chow, 1998), among others. Before concluding the relevant literature review, it is worth noting that except for Gray (1996) the Markov switching mechanism is added to ARCH models but not to GARCH model. This is due to the fact for GARCH models, the path dependence of states make the maximum likelihood estimate impossible. To be more specific, let us turn to the simple GARCH(1,1) model. Let $h_t$ and $\epsilon_t$ denote the conditional variance and disturbance term respectively. Then, for GARCH(1,1), $h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta h_{t-1}$. Since $h_t$ depends upon $h_{t-1}$, which latter depends upon $h_{t-2}$ and so on. Thus, $h_t$ depends upon the regimes at time $t, t-1, \cdots, 1$. To obtain maximum likelihood estimate, one needs to evaluate the likelihood for $2^T$ cases where $T$ is the sample size. This is infeasible even for moderate $T$. Gray (1996) cleverly removed the path dependence by aggregating the conditional variance for past $h_t$. Thus, $h_t$ only depends only on regime at time $t$ but not $t-1$ and further past. However, doing so destroys the AR representation for $\epsilon_t^2$. How successful is Gray model is yet to be seen.

The primary purpose of this paper is to construct an econometric model which can adequately account for the volatility of the stock market in Taiwan. We employ the Markov-switching ARCH model, developed by Hamilton and Susmel (1994), and GARCH-type models introduced by Engle (1982), Bollerslev (1986) and Nelson (1991). In particular, we are interested in the following issues: Is there evidence of nonlinearity of volatility in Taiwan stock market? If yes, could the nonlinearity be characterized by a Markov-switching ARCH model and identified regimes explained by relevant market factors? Does the forecasting performance of the SWARCH models perform better than the GARCH models? Does asymmetric leverage effect exist in Taiwan stock market?

In addition to this introduction, the rest of this paper is organized as follows. Section 2 gives the models specifications, while the data sources and empirical discussions are explained in Section 3. Section 4 concludes.

# 2 Model Specification

Let $y_t$ denotes the daily stock return measured in percent. We estimate the following model

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + u_t \tag{1}$$

$$u_t = \sqrt{g_{s_t}}\tilde{u}_t \tag{2}$$

$$\tilde{u}_t = \sqrt{h_t}v_t, \quad v_t \sim \text{Gaussian or Student } t \text{ distribution} \tag{3}$$

$$h_t = \beta_0 + \sum_{i=1}^{q} \beta_i \tilde{u}_{t-i}^2 + \xi d_{t-1}\tilde{u}_{t-1}^2 \tag{4}$$

$$d_{t-1} = \begin{cases} 0 & \tilde{u}_{t-1} > 0 \\ 1 & \tilde{u}_{t-1} \leq 0 \end{cases}$$

$s_t$ denotes an unobserved random variable that can take values $1, 2, ..., k$ and is assumed to be governed by a first order Markov chain with transition probability, $p_{i,j}$. For example, $k = 2$, $p_{i,j}$, the transition probability from state $i$, at time $t-1$ to state $j$ at time $t$ is defined as:

$$\begin{aligned} \text{p}(s_t = 1|s_{t-1} = 1) &= p_{11}, \\ \text{p}(s_t = 2|s_{t-1} = 1) &= p_{12}, \\ \text{p}(s_t = 1|s_{t-1} = 2) &= p_{21}, \\ \text{p}(s_t = 2|s_{t-1} = 2) &= p_{22} \end{aligned} \tag{5}$$

with $p_{11} + p_{12} = p_{21} + p_{22} = 1$.

In the absence of a leverage effect ($\xi = 0$), equation (2)–(4) are called $k$-state, $q$th-order Markov-switching ARCH process, denoted $u_t \sim$ SWARCH($k$,$q$). In the presence of leverage effects ($\xi \neq 0$), equation (2)–(4) are denoted $u_t \sim$ SWARCH-L($k$,$q$). The leverage effect predicts that $\xi > 0$. Both Gaussian and Student $t$ distributions are investigated.

Following Lo and MacKinlay (1990) and Hamiltion and Susmel (1994), we use AR(1) specification for the mean return equation.[1] Cai (1994) proposed a similar model specification except that he also allowed the constant in the mean equation governed by the unobserved state $s_t$. The reason we still follow the specification of Hamilton and Susmel (1994) is that, first, as Hamilton and Lin (1996) pointed out, given the limited predictability of stock returns, it is surely a mistake to over-parameterize the mean of $y_t$. Second, with the specification of equations (2)–(4), the scaled $\tilde{u}_t$ follows a standard ARCH($q$) process, the process is therefore multiplied by the constant $\sqrt{g_{s_1}}$ when the process is in the regime represented by $s_1$, multiplied by the constant $\sqrt{g_{s_2}}$ when the process is in the regime represented by $s_2$, and so on. And it is easy to interpret the results for this specification. When we normalize $g_1 = 1$, in which case $g_2$ has the interpretation as the ratio of the average variance of stock returns when $s_t = 2$ compared to that observed when $s_t = 1$. Third, the GAUSS code for estimating the SWARCH models are kindly supported from Hamilton.

It should be noted that the economy depends upon $s_t, s_{t-1}, \ldots, s_{t-q}$. To account for this, construct a new state variable $S_t^*$ defined as:

$$S_t^* = 1 + (s_t - 1)2^0 + (s_{t-1} - 1)2^1 + \ldots + (s_{t-q} - 1)2^q.$$

$S_t^*$ takes the value from 1 to $N = 2^{q+1}$ and the resulting transition probability $P^*$ is:

$$P^* = P_{ij}^* = \text{prob } (S_t^* = j | S_{t-1}^* = i).$$

[1]The other reason is that the estimate of the parameter $\alpha_2$ of $y_{t-2}$ is not significant in all models we investigate.

By letting $\Xi_{t|s}$ be the probability of $S_t^*$ given information up to time $s$, it can be shown that $y_t$ constitute a stationary process provided there exists a stationary distribution for $P^*$ which is assumed throughout the paper. As a result, the maximum likelihood estimator amounts to explicitly spelling out the likelihood function. The likelihood function can be found by summing up the joint likelihood $f(y_t, S_t^*|Y_{t-1}, \theta)$ over $S_t^*$ which in turn can be easily derived using the conditional likelihood function $f(y_t|Y_{t-1}, S_t^*, \theta)$ as in (2)–(4). To sum up, the estimation algorithm as proposed by Hamilton (1989, 1994), is as shown below.

First of all, we solve the ergodic probability $\pi$ and set $\Xi_{1|0} = \pi$ to start the algorithm. We then compute the filtering probability by

$$\Xi_{t|t} = \frac{(\Xi_{t|t-1} \odot \eta_t)}{1'(\Xi_{t|t-1} \odot \eta_t)},$$

where $\eta_t$ is the $N \times 1$ vector whose $j$-th element is the conditional density of

$$f(y_t|y_{t-1}, \ldots, y_1, S_t^* = j)$$

and $\odot$ denotes the element by element multiplication. Next, we compute the prediction probability by

$$\Xi_{t+1|t} = P^* \Xi_{t|t}.$$

As a side product, the likelihood function can be calculated as:

$$L(\theta) = \sum_{t=1}^{T} \log f(y_t|Y_{t-1}, \theta),$$
$$f(y_t|Y_{t-1}; \theta) = 1'(\Xi_{t|t-1} \odot \eta_t).$$

Finally, the smoothing probability can be obtained by

$$\Xi_{t|T} = \Xi_{t|t} \odot \{P' \Xi_{t+1|T} \oslash \Xi_{t+1|t}]\}.$$

where $\oslash$ denotes the element by element division. We start the algorithm from $t = T - 1$, and then proceed backward until $t = 1$.

The persistence of the ARCH component of a SWARCH process can be obtained from the largest eigenvalue of the following matrix

$$\begin{bmatrix} (\beta_0 + \frac{\xi}{2}) & \beta_1 & \ldots & \beta_{q-1} & \beta_q \\ 1 & 0 & \ldots & 0 & 0 \\ 0 & 1 & \ldots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \ldots & 1 & 0 \end{bmatrix}$$

For detailed derivation of the persistence and $m$-period-ahead forecast of $u^2_{t_m}$, see Hamilton and Susmel (1994). It is worth noting that $y_t$ is nonlinear but nevertheless stationary processes and hence conventional asymptotic results apply.

# 3 Empirical Results

Taiex is taken from the AREMOS data bank. The original data are daily from January 4, 1990 to October 2, 1998, which amounts to 2491 observations. We take logarithm transformation to get the daily stock index return (denoted by $y_t$) for analysis. The scatter plot of $y_t$ is presented in the top panel of the Figure 1.

## 3.1 Empirical Results of GARCH

To compare with the performance of the SWARCH models, we first estimate the traditional GARCH family models. The basic GARCH-L($p$,$q$) model is as follow:

$$\begin{aligned} y_t &= \alpha_0 + \alpha_1 y_{t-1} + u_t \\ u_t &= \sqrt{h_t} v_t, \quad v_t \sim \text{Gaussian or Student } t \text{ distribution} \\ h_t &= \beta_0 + \sum_{i=1}^{q} \beta_i u^2_{t-i} + \sum_{i=1}^{p} \delta_i h_{t-i} + \xi d_{t-1} u^2_{t-1} \end{aligned} \tag{6}$$

$$d_{t-1} = \begin{cases} 0 & u_{t-1} > 0 \\ 1 & u_{t-1} \leq 0 \end{cases}$$

The parameter $\xi$ is accounted for the leverage effect, that is, a stock price decrease has a greater effect on subsequent volatility than would a stock price increase of the same magnitude. This parameterizations of the leverage effect was first proposed by Glosten, Jagannathan and Runkle (1993). We also estimate the EGARCH model, which was introduced by Nelson (1991), to take the leverage effect into account.

Since most empirical implementations of GARCH($p$,$q$) models adopt low orders for the lag length $p$ and $q$ and such small numbers of parameters seem sufficient to model the variance over very long sample periods, we set $p = q = 1$ for the GARCH models and set $q = 2$ for the ARCH-L model. And $v_t$ is set to be Gaussian or Student $t$ distribution, respectively. The empirical results are summarized in Table 1 and Table 2. Like previous empirical works, see Bollerslev, Chou and Kroner (1992) and papers therein, we get high persistence in the Gaussian or the Student $t$ GARCH(1,1), the GARCH-L(1,1) and the EGARCH(1,1) models. For example, any change in the stock market today will continue to have nonnegligible consequence of $(0.99)^{182} = 0.16$ a half year later and $(0.99)^{365} = 0.03$ a full year later. The leverage effect is also positive and significant in the Gaussian or the Student $t$ EGARCH(1,1) models. The results also confirm the findings of early works that stock price increase lead to a smaller increase in volatility than would a stock price decrease of the same magnitude, The residual diagnostics for the standardized residuals, including of the coefficient of skewness, kurtosis and the Ljung-Box test for the 24 serial correlations, Jarque-Bera Normality test and LM test for no ARCH(4) effect, are summarized in Table 5. It is noteworthy that the Gaussian and Student $t$ ARCH-L(2), GARCH(1,1) and Student $t$ EGARCH(1,1) models for the conditional variance does not provide a sufficient fit in that the null hypothesis of no ARCH effect in the standardized residuals are rejected at 5% significant level. The sample kurtosis coefficient of the standardized residuals of all models are still greater than 3. The rejection of the conditional normality assumption is frequently encountered in applications of the ARCH model, see Bera and Higgins (1993). We postpone the discussion of forecast performance in later section.

## 3.2 Empirical Results of SWARCH

The estimation results for equations (2)–(4) described in Section 2 are summarized in Tables 3 and 4. For example, the estimated Student $t$ SWARCH-L(2,2) and SWARCH-L(3,2) equations with standard errors in parentheses are as below:

SWARCH-L(2,2) with Student $t$ distribution:

$$
\begin{aligned}
y_t &= \underset{(0.014)}{0.003} + \underset{(0.023)}{0.045}\ y_{t-1},\\
u_t &= \sqrt{g_{s_t}}\tilde{u}_t,\\
\tilde{u}_t &= \sqrt{h_t}v_t,\\
v_t &\sim \text{Student } t \text{ unit variance and } \underset{(1.300)}{7.648} \text{ d.f.},\\
h_t &= \underset{(0.076)}{1.143} + \underset{(0.038)}{0.010}\ \tilde{u}_{t-1}^2 + \underset{(0.037)}{0.198}\ \tilde{u}_{t-2}^2 + \underset{(0.058)}{0.085}\ d_{t-1}\tilde{u}_{t-1}^2,\\
g_1 &= 1,\ \ \hat{g}_2 = \underset{(0.694)}{6.619}\ ,\\
\hat{P} &= \begin{bmatrix} \underset{(0.002)}{0.994} & \underset{(0.005)}{0.013} \\ \underset{(0.002)}{0.006} & \underset{(0.005)}{0.987} \end{bmatrix}.
\end{aligned}
$$

SWARCH-L(3,2) with Student $t$ distribution:

$$
\begin{aligned}
y_t &= \underset{(0.020)}{0.011} + \underset{(0.019)}{0.042}\ y_{t-1},\\
u_t &= \sqrt{g_{s_t}}\tilde{u}_t,\\
\tilde{u}_t &= \sqrt{h_t}v_t,\\
v_t &\sim \text{Student } t \text{ unit variance and } \underset{(1.689)}{8.666} \text{ d.f.},\\
h_t &= \underset{(0.095)}{0.476} + \underset{(0.013)}{0.000}\ \tilde{u}_{t-1}^2 + \underset{(0.036)}{0.182}\ \tilde{u}_{t-2}^2 + \underset{(0.035)}{0.078}\ d_{t-1}\tilde{u}_{t-1}^2,\\
g_1 &= 1,\ \ \hat{g}_2 = \underset{(0.535)}{2.650},\ \ \hat{g}_3 = \underset{(3.663)}{16.653},
\end{aligned}
$$

$$\hat{P} = \begin{bmatrix} 0.991 & 0 & 0.002 \\ (0.010) & & (0.002) \\ 0.009 & 0.993 & 0.014 \\ (0.010) & (0.003) & (0.006) \\ 0 & 0.007 & 0.984 \\ & (0.003) & (0.008) \end{bmatrix}.$$

We estimate models with $k = 2, 3$ states and $q = 2, 3$ ARCH terms. When $k = 3$, we initially impose no constraints on any of the transition probabilities $p_{ij}$ other than the conditions that $0 \leq p_{ij} \leq 1$ and $\sum_{j=1}^{k} p_{ij} = 1$. We also encounter the same problem as in Hamilton and Susmel (1994) that several of these unrestricted MLE's fell on the boundary $p_{ij} = 0$, which is another violation of the regularity condition. In particular, $p_{21} = 1.256e{-}05$ and $p_{13} = 1.566e - 05$ for the Student $t$ SWARCH-L(3,2) model, respectively. So we impose $p_{21} = 0$ and $p_{13} = 0$ and treat this parameter as a known constant for purpose of calculating the second derivatives of the log-likelihood and obtain the standard errors.

When $k = 2$, $s_t = 1$ denotes the low-volatility state and $s_t = 2$ denotes the high-volatility state. If $k = 3$, $s_t = 1, 2, 3$ represent low-, medium-, and high-volatility state, respectively. Note that $\hat{g}_2$ is about 6.6, for all cases with Gaussian and Student $t$ innovations, and with or without the leverage effect $\xi$. The same results are also appeared in $k = 3$, which $\hat{g}_2$ is about 2.8 and $\hat{g}_3 = 17.7$ for Gaussian innovation and $\hat{g}_3 = 16.4$ for Student $t$ innovation. The coefficient estimate of $\hat{g}_2$ suggests that, when $k = 2$, variance in the high-volatility state is more than seven times that in the low-volatility state. When we divide the state finer, i.e., $k = 3$, the variance in the medium-volatility state is more than three times that in the low-volatility state, while variance in the high-volatility state is more than eighteen times that in the low-volatility state. The Figures 1 and 2 show the smoothed probabilities for Student $t$ SWARCH-L(2,2) and Student $t$ SWARCH-L(3,2), respectively.[2]

As can be seen from Figure 1, SWARCH models captures the volatility very well if

[2] We plot smoothed probabilities for all cases and find the same pattern for $k = 2$ and $k = 3$ SWARCH models. So we choose the one based on minimum one-period-ahead forecast which is summarized in the Table 9.

we divide the state into low and high-volatility state. The shaded areas are contraction periods determined by the Council for Economic Planning and Development (CEPD) in our empirical length. Basically, the Student $t$ SWARCH-L(2,2) identifies seven high-volatility states, which are, 90:1–91:8, 92:9–92:10, 93:1–93:4, 93:12–94:1, 95:7–95:8, 96:4 and 97:10–97:11. It appears that high stock market volatility is related with business recession at the beginning in 1990s. From Figure 2, we note that low-, medium-, and high-volatility states Switching ARCH models also perform as well as 2-state SWARCH models in modeling the volatility. Similarly, high-volatility match the business contraction at the beginning in 1990s. It is apparent that most observations come from the medium-volatility, while the low-volatility state describes the quiet period from August 1996 to February 1997. The Student $t$ SWARCH-L(3,2) identifies nine high-volatility as follows: 90:1–91:5, 91:7–91:10, 92:9–92:10, 93:1–93:4, 93:12–94:1, 94:10, 95:7–95:8, 96:4, 97:10–97:11.[3] By closely examining the history of political and economical events in Taiwan, we find that every high-volatility state in Student $t$ SWARCH-L(3,2) is accompanied by some important policy changes or financial crises in Taiwan or other critical events in the international economy. For example, The financial event, "Horsen Event", which occurred in September and October 1992, was responsible for the high-volatility in Taiwan stock market. The collapses in the Taiwan stock market in July 1995, August 1995 and March 8–15, 1996 respectively resulted from the three missile crises exerted by China. The currency crisis of Southeast Asia in early July 1997 was responsible for the collapse in Taiwan stock market in October, November 1997. We summarize every critical events in Table 6. Basically, Taiwan stock market is very sensitive to exogenous shocks.

Note that our maximum likelihood estimate is that the low-volatility state is never pre-

[3]The smoothed probabilities in Figure 2 are under the constraint of $p_{21} = 0$ and $p_{13} = 0$. In fact, the unconstrained Student $t$ SWARCH-L(3,2) model has the similar pattern of smoothed probabilities as in Figure 2.

ceded by the medium-volatility state ($\hat{p}_{21} = 0$) and the low-volatility state is never followed by the high-volatility state ($\hat{p}_{13} = 0$). Hamilton and Susmel (1994) also found similar results. The market was in the quiet state 1 for only a single episode from August 1996 to February 1997 in the sample, which episode was preceded by state 3 and followed by state 2. One reason for this period belongs to low-volatility state is that the level of domestic interest rate in the market is low in that period, most of the fund are running into the bond market, causing the trade volumes in the security market shrinks. Comparing the Figure 1 with Figure 2, we observe that the low-volatility state of 2-state is the combination of low and medium-volatility state of 3-state.

The expected duration for each state are summarized in Table 7. For the case of two states ($k = 2$), the low-volatility is expected to last for $(1 - \hat{p}_{11})^{-1} = 167$ days, while high-volatility typically $(1 - \hat{p}_{22})^{-1} = 77$ days for the Student $t$ SWARCH-L(2,2). For the case of three states ($k = 3$), the low-, medium- and high-volatility are expected to last for 111, 143 and 63 days, respectively, for the Student $t$ SWARCH-L(3,2). Although the states are highly persistent, the underlying fundamental ARCH-L(2) process for $\tilde{u}_t$ is much less so, with decay parameter estimated to 0.472 for 2-state and 0.447 for 3-state. Note that $(0.472)^8 = 0.002$, $(0.447)^7 = 0.004$, meaning that the volatility effects captured by $\tilde{u}_t$ die out almost completely after 8 days and 7 days for 2-state and 3-state, respectively. The residual diagnostics are summarized in Table 5. It is clear that only Ljung-Box statistics is significant at 5% level for Student $t$ SWARCH models, but is insignificant at 1% level.

### 3.3 Comparison between GARCH and SWARCH

Table 8 reports the model selection statistics proposed by Akaike (1976) and Schwarz (1978). Hamilton and Susmel (1994) suggested that the average squared forecast error such as MSE, MAE is probably an unfair standard for judging the specification, since it is based on the fourth moments of the actual data $y_t$. The unconditional fourth moment would fail to exist if (6) were the data-generating process. So we evaluate the one-period-ahead forecast performance based on $[\mathrm{LE}]^2$ and $|\mathrm{LE}|$ summarized in the Table 9.[4] According to the AIC and the Schwarz criteria, it is apparent that the Student $t$ GARCH-L(1,1) perform best, which also has the minimum one-period-ahead forecast among the GARCH models. Among the SWARCH models, The AIC and the Schwarz criteria also approximately lead to same chosen models. Since our ultimate criterion is based on the forecast performance, we select the Student $t$ GARCH-L(1,1) among the GARCH models, the Student $t$ SWARCH-L(2,2) among 2-state SWARCH models and the Student $t$ SWARCH-L(3,2) among 3-state models. By examining the one-period-ahead forecast in Table 9, we make the following observations.

- The Student $t$ SWARCH models perform better than the corresponding Gaussian SWARCH models in forecasting.
- Leverage factor improves forecasting performance in the SWARCH models.
- The three states SWARCH model perform better than two states SWARCH, and the Student $t$ SWARCH-L(3,2) performs the best among the other models, and the Student $t$ SWARCH-L(2,2) performs the best among 2-state SWARCH models.
- The SWARCH models do a better job in forecasting than the GARCH models.

Finally we plot the estimated conditional variance of the Student $t$ GARCH-L(1,1), SWARCH-L(2,2) and SWARCH-L(3,2) in Figure 3, respectively. All three panels show the similar pattern, and it is clear that each clustering of large deviations, of either sign, in the returns is associated with a rise in the conditional variance. By closely examining the

[4] The loss functions are defined as follow:

$$[\mathrm{LE}]^2 = T^{-1}\sum_{t=1}^{T}(\ln(\tilde{u}_t^2) - \ln(h_t))^2, \quad |\mathrm{LE}| = T^{-1}\sum_{t=1}^{T}|\ln(\tilde{u}_t^2) - \ln(h_t)|$$

panel for Student $t$ SWARCH-L(3,2) and the Student $t$ SWARCH-L(2,2) we find that the conditional variance in the low-volatility state are relative smooth but are relative volatile in high-volatility state.

### 3.4 Testing of No Regime-Switching

Does stock index return in Taiwan really follows a SWARCH model? Note that Student $t$ ARCH-L(2) is the nested model of the Student $t$ SWARCH-L($K$,2) with $K = 1$. The usual likelihood ratio is $-2(4690.689 - 4782.779) = 184.18$. The conventional likelihood ratio test would suggest that the null hypothesis of no regime switching be rejected since the critical value for $\chi^2_{0.05}(3)$ is 7.815. In other words, the Student $t$ SWARCH-L(2,2) is supported. Unfortunately, the usual asymptotic distribution theory does not hold for this case since the nuisance parameters $p_{11}$ and $p_{22}$ are unidentified under the null hypothesis that $g_1 = g_2$. For these reasons, standard likelihood ratios are inappropriate and can only be used as a rough approximation. Hansen (1992, 1996) had proposed asymptotically valid test by the standardized likelihood statistic, but their application here would be quite difficult numerically and time-consuming. Following Hamilton and Susmel (1994), we still regard the result as a useful descriptive summary of the fit of alternative models. As for the Student $t$ ARCH-L(2) and SWARCH-L(3,2) models, the associate log-likelihood values are $-4782.779$ and $-4684.174$, respectively. The likelihood ratio statistic is $-2(4684.174 - 4782.779) = 197.21$, standard likelihood ratio test also reject the ARCH-L(2) model.

Finally, comparing the log-likelihood function of the Student $t$ SWARCH-L(2,2) and SWARCH-L(3,2) models, standard likelihood ratio tests is $2(-4684.174+4690.689) = 13.03$, compare it with usual $\chi^2_{0.05}(1) = 3.841$, which implies that the three-regime specification is supported. Again, we should point out the usual asymptotic distribution theory does not hold for this case, because under the null hypothesis of $k-1$ states, the parameters that describe the $k$th state are unidentified. Since the improvement in the likelihood value is significant, so we still take it as a strong indication of a three-state model.

# 4 Conclusions

This paper employs the GARCH and SWARCH models to analyze the stock market volatility in Taiwan. The empirical findings are as follows. First, the stock returns in Taiwan can be adequately characterized by SWARCH model. Secondly, we have identified episodes responsible for causing the high-volatility in stock market. Thirdly, our estimates attribute most of the persistence in stock price volatility to the persistence of low-, medium- and high-volatility regimes. The high-volatility regime is associated with the business recession at the beginning of 1990s. Fourth, there exists a positive and significant leverage effect such that a stock price decrease has a greater effect on subsequent volatility than would a stock price increase of the same magnitude. Finally, the SWARCH models perform better than the GARCH models in forecasting.

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Table 1: Empirical Results of Gaussian GARCH-Family

| | Gaussian ARCH-L(2) | Gaussian GARCH(1,1) | Gaussian GARCH-L(1,1) | Gaussian EGARCH(1,1) |
|---|---|---|---|---|
| $\alpha_0$ | $-0.009$ | 0.018 | $-0.008$ | $-0.014$ |
| | (0.031) | (0.027) | (0.028) | (0.027) |
| $\alpha_1$ | 0.056 | 0.047 | 0.052 | 0.050 |
| | (0.020) | (0.022) | (0.022) | (0.021) |
| $\beta_0$ | 1.654 | 0.044 | 0.052 | 0.027 |
| | (0.053) | (0.008) | (0.008) | (0.004) |
| $\beta_1$ | 0.224 | 0.079 | 0.052 | 0.181 |
| | (0.032) | (0.007) | (0.008) | (0.013) |
| $\beta_2$ | 0.329 | | | |
| | (0.031) | | | |
| $\delta_1$ | | 0.906 | 0.900 | 0.979 |
| | | (0.007) | (0.007) | (0.003) |
| $\xi$ | 0.024 | | 0.062 | 0.252 |
| | (0.046) | | (0.012) | (0.046) |
| $\kappa$ | 0.553 | 0.985 | 0.983 | 0.979 |

Table 2: Empirical Results of Student $t$ GARCH-Family

| | Student $t$ ARCH-L(2) | Student $t$ GARCH(1,1) | Student $t$ GARCH-L(1,1) | Student $t$ EGARCH(1,1) |
|---|---|---|---|---|
| $\alpha_0$ | 0.008 | 0.026 | 0.006 | 0.004 |
| | (0.027) | (0.026) | (0.026) | (0.026) |
| $\alpha_1$ | 0.043 | 0.039 | 0.044 | 0.041 |
| | (0.020) | (0.021) | (0.021) | (0.020) |
| $\beta_0$ | 1.583 | 0.038 | 0.047 | 0.020 |
| | (0.131) | (0.012) | (0.013) | (0.024) |
| $\beta_1$ | 0.258 | 0.099 | 0.067 | 0.199 |
| | (0.057) | (0.014) | (0.015) | (0.022) |
| $\beta_2$ | 0.445 | | | |
| | (0.066) | | | |
| $\delta_1$ | | 0.894 | 0.885 | 0.986 |
| | | (0.013) | (0.014) | (0.004) |
| $\xi$ | 0.054 | | 0.075 | 0.227 |
| | (0.078) | | (0.020) | (0.061) |
| DF | 4.159 | 6.497 | 6.688 | 6.507 |
| | (0.448) | (0.879) | (0.924) | (0.906) |
| $\kappa$ | 0.703 | 0.993 | 0.990 | 0.986 |

Table 3: Empirical Results of Gaussian SWARCH

| | Gaussian SWARCH(2,2) | Gaussian SWARCH(3,2) | Gaussian SWARCH-L(2,2) | Gaussian SWARCH-L(3,2) |
|---|---|---|---|---|
| $\alpha_0$ | 0.015 | 0.021 | 0.012 | 0.020 |
| | (0.132) | (0.019) | (0.070) | (0.056) |
| $\alpha_1$ | 0.052 | 0.051 | 0.051 | 0.050 |
| | (0.040) | (0.020) | (0.020) | (0.022) |
| $\beta_0$ | 1.052 | 0.418 | 1.050 | 0.417 |
| | (0.132) | (0.067) | (0.083) | (0.072) |
| $\beta_1$ | 0.027 | 0.011 | 0.006 | 0.000 |
| | (0.101) | (0.024) | (0.045) | (0.020) |
| $\beta_2$ | 0.165 | 0.155 | 0.164 | 0.155 |
| | (0.039) | (0.031) | (0.033) | (0.033) |
| $\xi$ | | | 0.050 | 0.040 |
| | | | (0.032) | (0.040) |
| $p_{11}$ | 0.985 | 0.985 | 0.986 | 0.986 |
| | (0.005) | (0.013) | (0.004) | (0.012) |
| $p_{22}$ | 0.972 | 0.983 | 0.972 | 0.983 |
| | (0.010) | (0.005) | (0.009) | (0.005) |
| $p_{33}$ | | 0.968 | | 0.969 |
| | | (0.012) | | (0.011) |
| $g_2$ | 6.671 | 2.825 | 6.689 | 2.806 |
| | (1.021) | (0.480) | (0.604) | (0.502) |
| $g_3$ | | 17.976 | | 17.742 |
| | | (3.248) | | (3.306) |
| $\kappa$ | | | 0.421 | 0.404 |

Table 4: Empirical Results of Student $t$ SWARCH

| | Student $t$ SWARCH(2,2) | Student $t$ SWARCH(3,2) | Student $t$ SWARCH-L(2,2) | Student $t$ SWARCH-L(3,2) |
|---|---|---|---|---|
| $\alpha_0$ | 0.008 | 0.016 | 0.003 | 0.011 |
| | (0.012) | (0.022) | (0.014) | (0.020) |
| $\alpha_1$ | 0.046 | 0.044 | 0.045 | 0.042 |
| | (0.022) | (0.023) | (0.023) | (0.019) |
| $\beta_0$ | 1.147 | 0.475 | 1.143 | 0.476 |
| | (0.079) | (0.101) | (0.076) | (0.095) |
| $\beta_1$ | 0.054 | 0.042 | 0.010 | 0.000 |
| | (0.026) | (0.031) | (0.038) | (0.013) |
| $\beta_2$ | 0.200 | 0.184 | 0.198 | 0.182 |
| | (0.037) | (0.037) | (0.037) | (0.036) |
| $\xi$ | | | 0.085 | 0.078 |
| | | | (0.058) | (0.035) |
| $p_{11}$ | 0.994 | 0.991 | 0.994 | 0.991 |
| | (0.002) | (0.007) | (0.002) | (0.010) |
| $p_{22}$ | 0.987 | 0.993 | 0.987 | 0.993 |
| | (0.005) | (0.003) | (0.005) | (0.003) |
| $p_{33}$ | | 0.984 | | 0.984 |
| | | (0.008) | | (0.008) |
| $g_2$ | 6.489 | 2.667 | 6.619 | 2.650 |
| | (0.651) | (0.558) | (0.694) | (0.535) |
| $g_3$ | | 16.388 | | 16.653 |
| | | (3.677) | | (3.663) |
| DF | 7.550 | 8.552 | 7.648 | 8.666 |
| | (1.306) | (1.717) | (1.300) | (1.689) |
| $\kappa$ | | | 0.472 | 0.447 |

Table 5: Residual diagnostics for various specifications

| Model | Skewness | Kurtosis | LB(24) | Normality | ARCH(4) |
|---|---|---|---|---|---|
| Gaussian ARCH-L(2) | −0.347 | 4.999 | 28.928 | 464.147 | 9.221 |
| | | | (0.182) | (0.000) | (0.000) |
| Gaussian GARCH(1,1) | −0.287 | 4.470 | 36.866 | 258.383 | 3.578 |
| | | | (0.033) | (0.000) | (0.006) |
| Gaussian GARCH-L(1,1) | −0.271 | 4.467 | 38.727 | 253.930 | 2.055 |
| | | | (0.021) | (0.000) | (0.084) |
| Gaussian EGARCH(1,1) | −0.266 | 4.397 | 39.951 | 231.526 | 4.652 |
| | | | (0.015) | (0.000) | (0.001) |
| Student $t$ ARCH-L(2) | −0.361 | 5.215 | 28.317 | 562.682 | 7.957 |
| | | | (0.204) | (0.000) | (0.000) |
| Student $t$ GARCH(1,1) | −0.302 | 4.632 | 37.403 | 314.248 | 2.721 |
| | | | (0.029) | (0.000) | (0.028) |
| Student $t$ GARCH-L(1,1) | −0.286 | 4.639 | 38.944 | 312.397 | 1.746 |
| | | | (0.020) | (0.000) | (0.137) |
| Student $t$ EGARCH(1,1) | −0.272 | 4.544 | 40.979 | 278.066 | 4.277 |
| | | | (0.012) | (0.000) | (0.002) |
| Gaussian SWARCH(2,2) | −0.388 | 4.883 | 35.437 | 429.867 | 1.390 |
| | | | (0.062) | (0.000) | (0.235) |
| Gaussian SWARCH(3,2) | −0.353 | 4.585 | 36.127 | 312.136 | 1.232 |
| | | | (0.053) | (0.000) | (0.295) |
| Gaussian SWARCH-L(2,2) | −0.381 | 4.870 | 35.155 | 421.244 | 1.492 |
| | | | (0.066) | (0.000) | (0.202) |
| Gaussian SWARCH-L(3,2) | −0.349 | 4.585 | 35.720 | 310.892 | 1.406 |
| | | | (0.058) | (0.000) | (0.229) |
| Student $t$ SWARCH(2,2) | −0.382 | 4.931 | 40.241 | 446.825 | 1.716 |
| | | | (0.020) | (0.000) | (0.143) |
| Student $t$ SWARCH(3,2) | −0.356 | 4.638 | 39.392 | 330.506 | 1.553 |
| | | | (0.025) | (0.000) | (0.184) |
| Student $t$ SWARCH-L(2,2) | −0.370 | 4.900 | 40.134 | 430.610 | 2.032 |
| | | | (0.021) | (0.000) | (0.087) |
| Student $t$ SWARCH-L(3,2) | −0.370 | 4.900 | 39.265 | 320.064 | 1.854 |
| | | | (0.026) | (0.000) | (0.116) |

Table 6: Some Critical Events Occurred in 1990–1997

| Date | Critical Events |
|---|---|
| 90:1–91:5 | Political instability occurred from February to April, 1990 in Taiwan. |
| | Gulf War occurred in August 1990. |
| | Gulf War ended and the recovery of Taiwan economy. |
| 91:7–91:8 | A large number of new banks was founded in the late 1991. |
| 92:9–92:10 | The financial event, “Horsen Event”, occurred in November 1992. |
| 93:1–93:4 | The stock transaction tax was reduced from 0.6% to 0.3% in January 1993. |
| 93:12–94:1 | The Central Bank released the monetary base. |
| | The Central Bank dismissed the limitations of foreign direct investment in Taiwan security market. |
| 94:10 | The financial event, “Honfu Event”, occurred in October 5–7,1994. |
| 95:7–95:8 | The first missile crisis occurred in July 1995. |
| | The second missile crisis occurred in August 1995. |
| 96:4 | The third missile crisis occurred in March 8–15, 1996. |
| | The Morgan Stanley company declared that it would include 75 preferred stock of Taiwan in its new free market index since November 2, 1996. |
| 97:10–97:11 | The currency crisis of the Southeast Asia occurred in July 1997. |

Table 7: Expected duration

| Model | $(1-\hat{p}_{11})^{-1}$ | $(1-\hat{p}_{22})^{-1}$ | $(1-\hat{p}_{33})^{-1}$ |
|---|---|---|---|
| Gaussian SWARCH(2,2) | 66.667 | 35.714 | |
| Gaussian SWARCH(3,2) | 66.667 | 58.826 | 31.250 |
| Gaussian SWARCH-L(2,2) | 71.428 | 35.714 | |
| Gaussian SWARCH-L(3,2) | 71.428 | 58.823 | 35.258 |
| Student $t$ SWARCH(2,2) | 166.667 | 76.923 | |
| Student $t$ SWARCH(3,2) | 111.111 | 142.857 | 62.500 |
| Student $t$ SWARCH-L(2,2) | 166.667 | 76.923 | |
| Student $t$ SWARCH-L(3,2) | 111.111 | 142.857 | 62.500 |

Table 8: Summary statistics for various specifications

| Model | Param. | $L(\theta)$ | AIC | Schwarz |
|---|---|---|---|---|
| Gaussian ARCH-L(2) | 6 | −4884.263 | −4878.263 | −4907.719 |
| Gaussian GARCH(1,1) | 5 | −4716.292 | −4711.292 | −4735.840 |
| Gaussian GARCH-L(1,1) | 6 | −4705.902 | −4699.902 | −4729.360 |
| Gaussian EGARCH(1,1) | 6 | −4709.341 | −4703.341 | −4732.799 |
| Student $t$ ARCH-L(2) | 7 | −4782.779 | −4775.779 | −4810.145 |
| Student $t$ GARCH(1,1) | 6 | −4666.618 | −4660.618 | −4690.075 |
| Student $t$ GARCH-L(1,1) | 7 | −4658.834 | −4651.834 | −4686.201 |
| Student $t$ EGARCH(1,1) | 7 | −4662.182 | −4655.182 | −4689.549 |
| Gaussian SWARCH(2,2) | 8 | −4708.240 | −4700.240 | −4739.514 |
| Gaussian SWARCH(3,2) | 11 | −4696.562 | −4684.562 | −4743.472 |
| Gaussian SWARCH-L(2,2) | 9 | −4707.331 | −4698.331 | −4742.514 |
| Gaussian SWARCH-L(3,2) | 12 | −4695.840 | −4683.840 | −4742.750 |
| Student $t$ SWARCH(2,2) | 9 | −4692.631 | −4683.631 | −4727.813 |
| Student $t$ SWARCH(3,2) | 12 | −4686.010 | −4674.010 | −4732.920 |
| Student $t$ SWARCH-L(2,2) | 10 | −4690.689 | −4680.689 | −4729.781 |
| Student $t$ SWARCH-L(3,2) | 13 | −4684.174 | −4671.174 | −4734.994 |

Table 9: One-period-ahead forecasts

| Model | $[LE]^2$ | $\|LE\|$ |
|---|---|---|
| Gaussian ARCH-L(2) | 9.497 | 2.211 |
| Gaussian GARCH(1,1) | 8.075 | 2.022 |
| Gaussian GARCH-L(1,1) | 8.080 | 2.007 |
| Gaussian EGARCH(1,1) | 8.689 | 2.032 |
| Student $t$ ARCH-L(2) | 9.116 | 2.221 |
| Student $t$ GARCH(1,1) | 8.448 | 2.035 |
| Student $t$ GARCH-L(1,1) | 7.918 | 1.996 |
| Student $t$ EGARCH(1,1) | 7.929 | 1.998 |
| Gaussian SWARCH(2,2) | 8.617 | 2.076 |
| Gaussian SWARCH(3,2) | 7.890 | 2.025 |
| Gaussian SWARCH-L(2,2) | 8.343 | 2.066 |
| Gaussian SWARCH-L(3,2) | 7.905 | 2.022 |
| Student $t$ SWARCH(2,2) | 8.230 | 2.061 |
| Student $t$ SWARCH(3,2) | 8.220 | 2.036 |
| Student $t$ SWARCH-L(2,2) | 7.971 | 2.046 |
| Student $t$ SWARCH-L(3,2) | 7.774 | 2.016 |

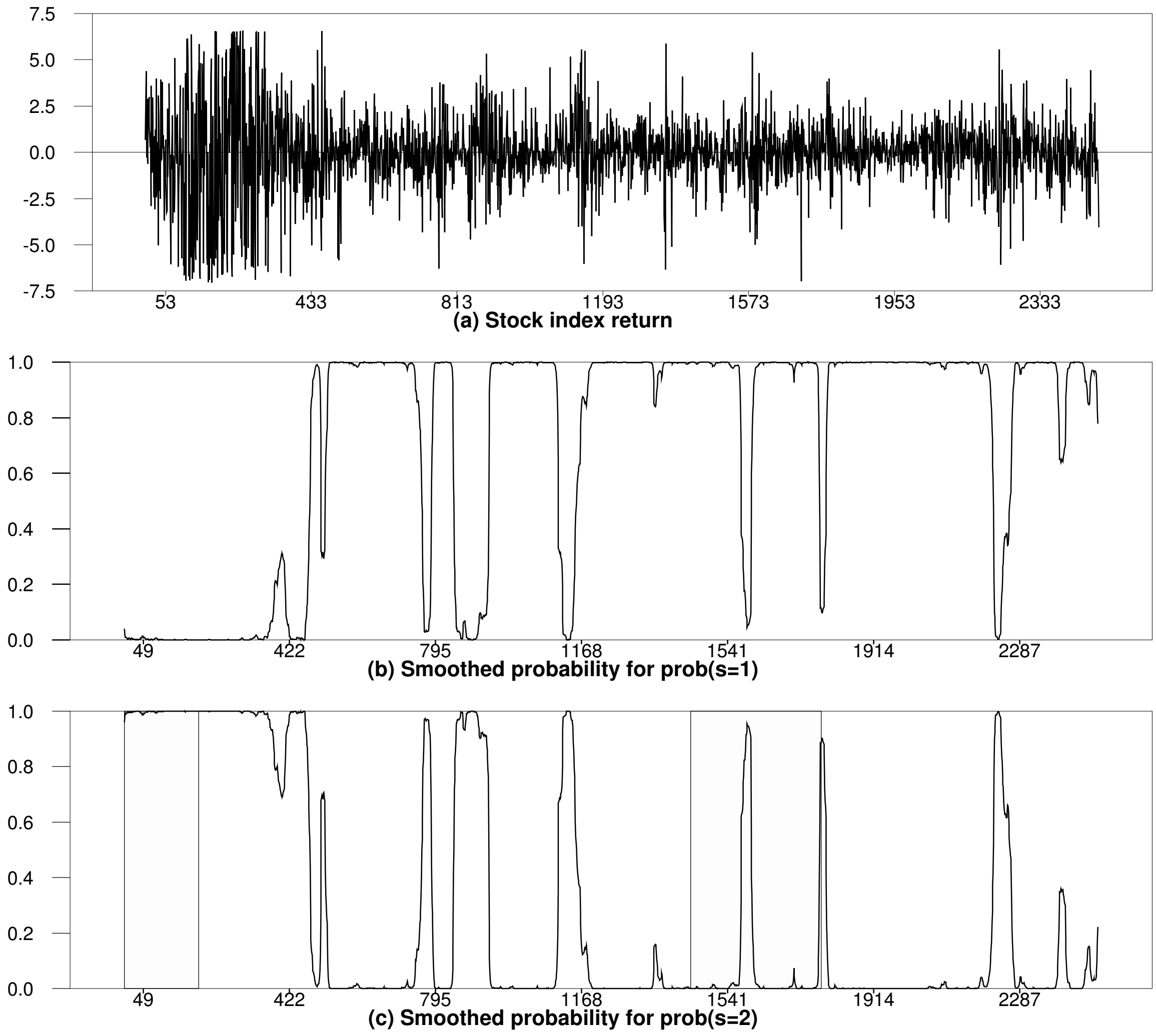


Figure 1: The top panel denotes the scatter plot of the stock index return. The second and third panels denote the smoothed probabilities of low-, and high-volatility states, respectively, of the Student $t$ SWARCH-L(2,2) model. The shaded areas are the contraction periods determined by the CEPD.

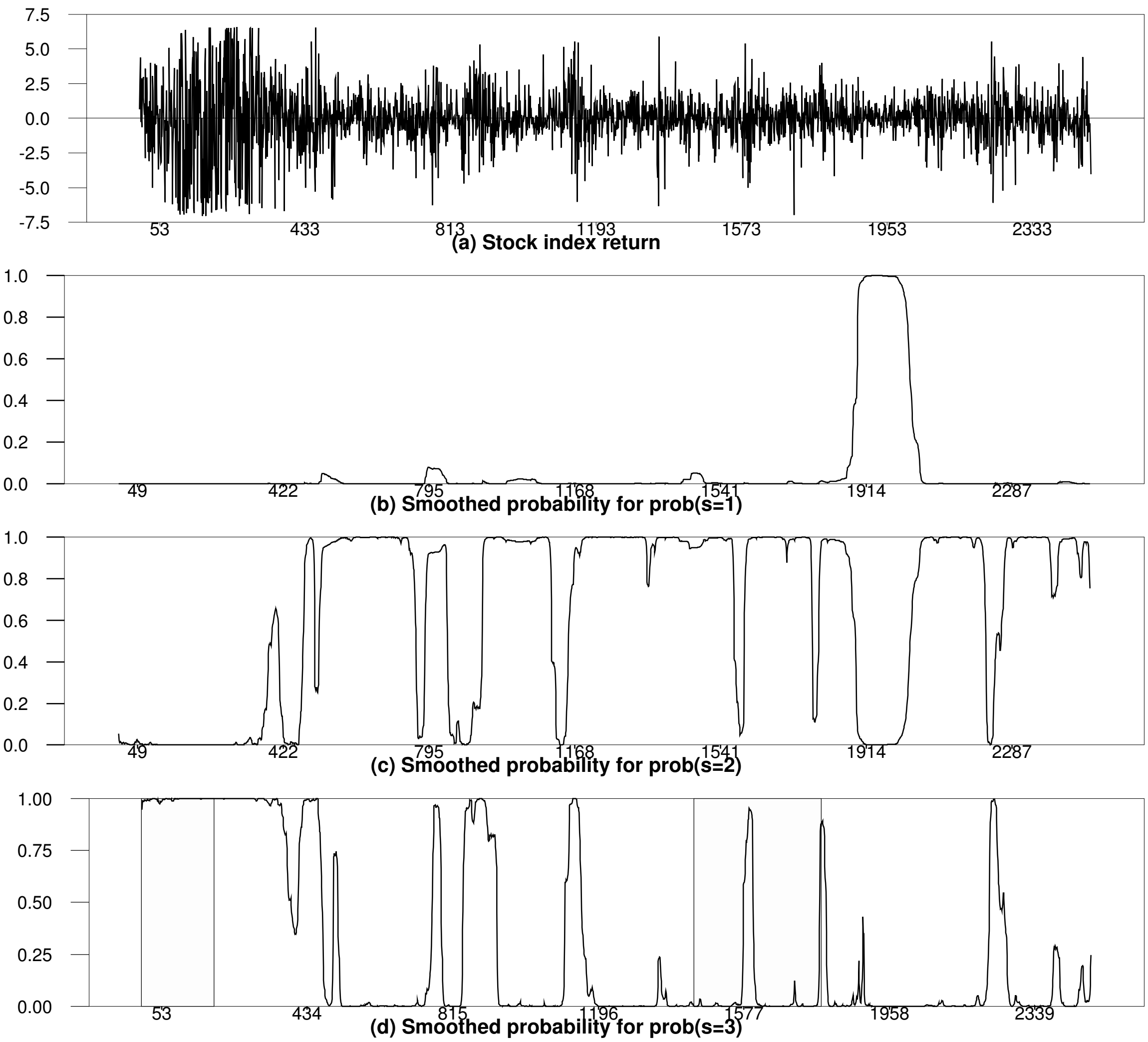


Figure 2: The top panel denotes the scatter plot of the stock index return. The second to fourth panels denote the smoothed probabilities of the low-, medium- and high-volatility states, respectively, of the Student $t$ SWARCH-L(3,2) model. The shaded areas are the contraction periods determined by the CEPD.

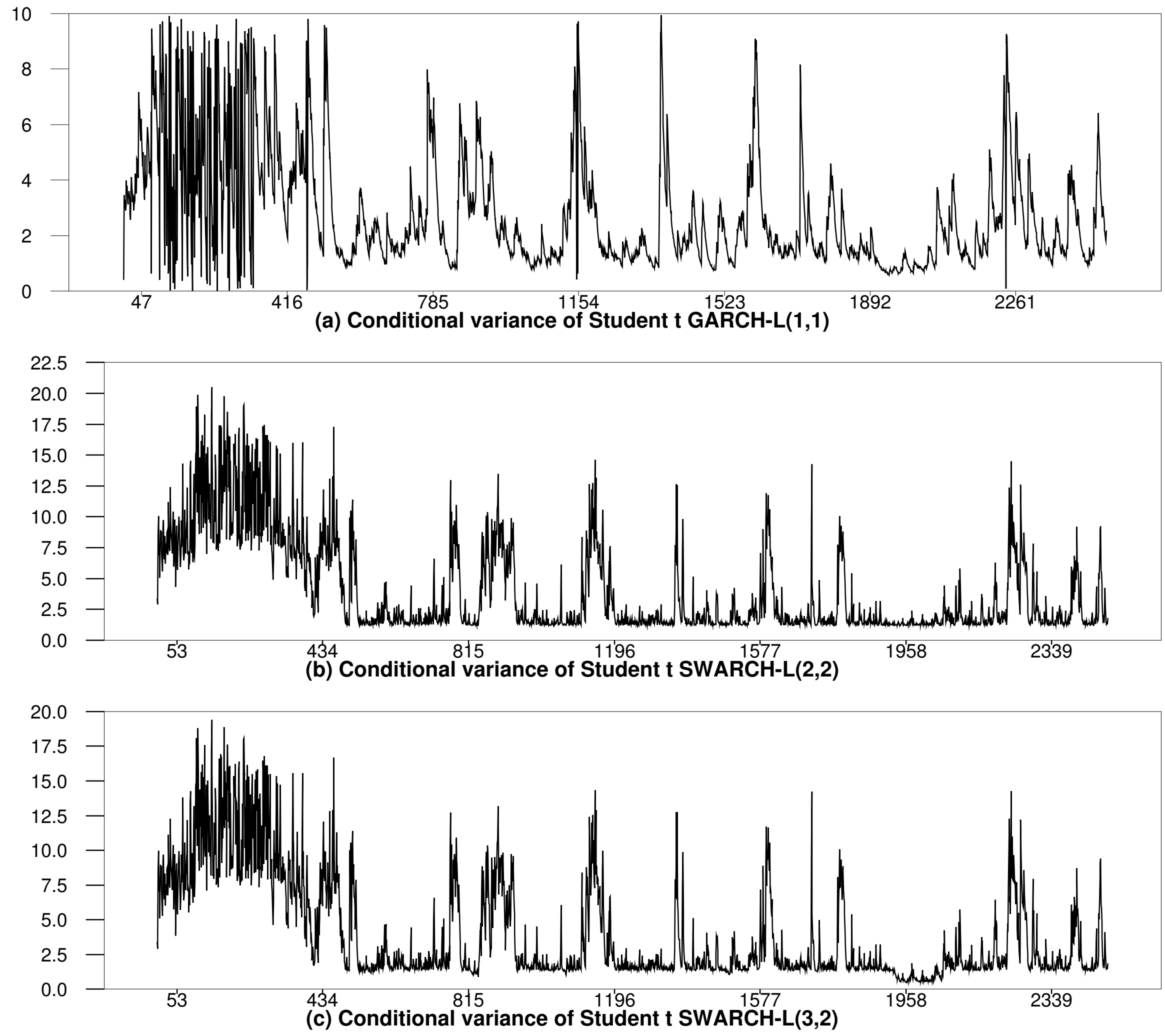


Figure 3: The first to third panels denote the estimated conditional variance of Student $t$ GARCH-L(1,1), Student $t$ SWARCH-L(2,2) and Student $t$ SWARCH-L(3,2) models, respectively.