

MONETARY CONFIDENCE AND ASSET PRICES

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ABSTRACT

Three issues are studied in this paper: the existence of sunspot equilibria; the excessive volatility of asset prices; and the possibility that assets may be undervalued relative to their market fundamentals. We show that (i) stationary sunspot equilibria exist in a very general environment; (ii) asset prices may fluctuate despite a constant stream of dividends; and (iii) assets may be undervalued. *JEL: 021, 023, 131.*

I. INTRODUCTION

This paper is concerned with topics that are discussed in three different sets of literature. The first of these is the literature on the existence of stationary sunspot equilibria (Azariades, 1981; Azariades & Guesnerie, 1986; Woodford, 1986). The second is the literature on the theoretical explanation of the excessive volatility of stock prices at the aggregate level (Tirole, 1985; Aiyagari, 1988). The third is the literature that has to do with whether stocks could be undervalued relative to the market fundamentals (Diba & Grossman, 1987, 1988; West, 1988).

Consider first the literature on when stationary sunspot equilibria can exist. In overlapping-generations economies, Azariades and Guesnerie have demonstrated that stationary sunspot equilibria are present only if the aggregate savings function is decreasing in the

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International Review of Economics and Finance, 5(4): 363-376
ISSN: 1059-0560

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rate of interest in the vicinity of the monetary steady state. In a cash-in-advance economy, Woodford has also shown that stationary sunspot equilibria exist only if the labor supply is backward bending in the vicinity of the stationary monetary equilibrium. On empirical grounds these may be regarded as limiting appeal of such equilibria.¹

For the second literature on excessive volatility of asset prices, Tirole has demonstrated that, in an overlapping-generations economy, the volatility tests of Shiller (1981) and LeRoy and Porter (1981) may be invalid if there are aggregate bubbles, i.e., assets not priced according to the market fundamentals. Second, the excessive volatility may be due to multiple nonstationary equilibria coexisting with the (unique) stationary equilibrium. Such a demonstration, based on the theory of endogenous business cycles developed by Grandmont (1985), can be found in Aiyagari. These developments have not been fully satisfactory. First, Tirole's interpretation requires that the prices of a subset of assets be negatively correlated with those of its complimentary subset of assets. Such a claim may not be empirically verifiable since the number of total assets is arbitrarily large. Second, in order to apply Aiyagari's reasoning, it is necessary for the interest elasticity of saving be negative; or alternatively, the wealth effect must dominate the substitution effect. Third, the Tirole-Aiyagari's argument is based on a bubbly view of the economy. The main difference between "bubbles" and "sunspots" is that the former asserts that the expectation is formed on an ad-hoc basis, while in the latter the expectation is conditional on certain state variables. As Huo (1995, pp.839-840) has demonstrated that these state variables might be interpreted as extrinsic uncertainties, or as some economic relevant variables which are not directly observable by economists.

As for the third literature, Diba-Grossman and West have argued that stock prices can only fluctuate above their market fundamentals, and one should never see an "undervalued" asset. While their arguments seemed intuitive, the approach was based on a partial-equilibrium arbitrage equation. Hence, their claim must be verified in a full general equilibrium model.

In this paper we consider a generalized version of the type of extrinsic uncertainty due to Blanchard and Watson (1979, 1982). The appeal of this type of uncertainty is its realism as well as consistency with the general market psychology. We extend the method developed in Huo (1992, 1995) to establish a class of stationary sunspot equilibria (or, "stochastic rational bubbles" in Blanchard's terminology) which are immune to previous criticisms.² It is shown that their existence does not hinge on any (strong) presumptions on preferences.³ For these equilibria stock prices may fluctuate even though the dividend stream remains perpetually constant. Also, contrary to the claims of Diba and Grossman, we demonstrate that stock prices may fall below their market fundamentals.³

This paper is organized as follows. Section II describes an exchange economy with extrinsic beliefs. The existence as well as the uniqueness of a stationary sunspot equilibrium is established in section III. In section IV we examine the implication for stock prices. The final conclusion is presented in section V.

II. AN EXCHANGE ECONOMY

Consider a monetary model a la Brock (1974, 1975). Time is discrete and runs from zero to infinity. There is a finite number of identical firms. Each firm produces a deterministic

output y of non-storable goods in every period. Without loss of generality, the number of firms is normalized to be equal to the number of households, so that the number of firms per household is equal to one. The representative household is endowed with k shares of a firm's stock and M dollars of (intrinsically useless) money at the beginning of period 0. Each share yields a fixed rate of return of r units of the consumption good per period, so that the total return to stock holdings is $rk(= y)$. In each period a household receives its share of dividends, rk_t , and purchases $k_{t+1} - k_t$ stock shares. The budget constraint faced by the representative household is

$$c_t + p_t M_t + q_t(k_{t+1} - k_t) = rk_t + p_t M_{t-1}, \quad (1)$$

where M_t is the money holdings, p_t is the (commodity) price of money, q_t is the (commodity) price per share of the stock in period t .

The expected present discounted value of a representative household's utility stream is represented by

$$E \left\{ \sum_{t=0}^{\infty} \beta^t [u(c_t, m_t)] \right\}, \quad 1 > \beta > 0, \quad (2)$$

where E is the expectation operator, β is the subjective discount factor, c_t is the consumption in period t , and m_t is the level of real money balances in period t . The instantaneous utility function u is assumed to satisfy

Assumption 1. $u: R_+^2 \rightarrow R$ is bounded, continuously differentiable, strictly concave, strictly increasing, and

$$\lim_{m \rightarrow \infty} \frac{u_2(c, m)}{u_1(c, m)} = 0, \quad \lim_{m \rightarrow 0} \frac{u_2(c, m)}{u_1(c, m)} = \infty$$

Although the environment of the economy described above is deterministic, we nonetheless hypothesize that there are "extrinsic" shocks indexed by the state $i \in N$. These shocks may be thought as "sunspots" as in Cass and Shell (1983). The evolution of these states is described by a first-order, time-invariant Markov process whose transition matrix is represented by $[\pi_{ij}]$, where π_{ij} denotes the conditional probability of realization of state j , given state i .

Assumption 2. N is a finite set, i.e., $N \equiv \{1, \dots, n\}$.

Following the customary practice, we will motivate a definition of stationary equilibrium in which prices and quantities are independent of time, and are fixed functions of the state of the economy. To do so, we first express the budget constraint (1) as

$$c^i + p^i M' + q^i(k' - k) \leq rk + p^i M, \quad (3)$$

where M' is the money balances, k' is the share holdings at the beginning of the next period. Also, in order to rule out "Ponzi game," we assume

$$k' \in K = [-\kappa, \kappa] \quad \text{for some} \quad 0 < \kappa < \infty \quad (4)$$

Given p^i and q^i , let $\omega(i, M, k)$ denote the set of (c^i, M', k') -values satisfying (3) and (4). For $p^i, q^i \geq 0$, the correspondence $\omega(i, M, k)$ is compact, convex, continuous in M and k , and for fixed i , $\omega(i, M, k)$ is convex in M and k . Let $U(i, M, k)$ be the value of the maximized objective function for the representative household beginning the period with M and k , when the economy is in state i . Then $U(i, M, k)$ must satisfy

$$U(i, M, k) = \max_{(c^i, M', k') \in \omega(i, M, k)} \left\{ u(c^i) + \beta \sum_{j=1}^n \pi_{ij} U(j, M', k') \right\}. \quad (5)$$

Denote \mathcal{U} as the space of bounded, real-valued functions $U(i, M, k)$ that are continuous in the second and the third argument, defined on $N \times R_+ \times K$ with the norm $\|U\| = \max_{i, M, k} |U(i, M, k)|$. We are ready to prove the following

Lemma: Under Assumptions 1 and 2, given nonnegative, bounded vectors (p_1, \dots, p^n) , (q^1, \dots, q^n) in R^n , there exists a unique value function $U \in \mathcal{U}$ satisfying (5). U is strictly concave, strictly increasing, and continuously differentiable in M and k . For fixed i , the maximum in (5) is attained by a unique value $\psi(i, M, k)$, and the function ψ is continuous in M and k .

Proof: To prove the existence and uniqueness of $U \in \mathcal{U}$, it is sufficient to show that the operator T on \mathcal{U} defined by

$$T(U(i, M, k)) = \max_{(c^i, M', k') \in \omega(i, M, k)} \left\{ u(c^i, m_i) + \beta \sum_{j=1}^n \pi_{ij} U(j, M', k') \right\} \quad (6)$$

maps \mathcal{U} into itself and is a contraction.

Under Assumption 1, $T(U)$ is bounded. Since $\omega(i, M, k)$ is compact, so T maximizes a continuous function over a compact set, and therefore, is well defined. Also, the right-hand-side of (6) is continuous in (c^i, M', k') , and $\omega(i, M, k)$ is continuous in M and k , it follows from Berge (1963, pp. 115–116) that $T(U(i, M, k))$ is continuous in M and k . Therefore, $T: \mathcal{U} \rightarrow \mathcal{U}$. By the contraction theorem of Blackwell (1965, Theorem 5), T has a unique fixed-point $U \in \mathcal{U}$.

Since $u(\cdot, \cdot)$ is strictly concave and strictly increasing, $\omega(i, M, k)$ is convex in M and k , T maps functions that are increasing and concave in M and k into functions that are strictly increasing and strictly concave in M and k . Hence, $U(i, M, k)$ is strictly increasing and strictly concave in M and k for each i .

Also from Berge (1963, p.116), the correspondence $\psi: N \times R \times K \rightarrow R_+^3$ consisting of the maximizing (c^i, M', k') is upper-semi-continuous in M and k . Since $u(\cdot, \cdot)$ is strictly concave, $\omega(i, M, k)$ is unique, so that ψ is continuous in M and k .

Finally, the theorem of Benveniste and Scheinkman (1979) applies, so that U is continuously differentiable in M and k .

Q.E.D.

With the existence of the value function established, we can define a stationary rational-expectations equilibrium as follows.

Definition: Given the stock of money supply M and the stock shares k , a stationary rational-expectations equilibrium consists of nonnegative prices $\{p^i\}$ and $\{q^i\}$, a value function $U(i, M, k) \in \mathcal{U}$, and a choice function $\psi(i, M, k)$ such that

- (i) $U, \{p^i\}, \{q^i\}$ satisfy (5);
- (ii) the associated choice function ψ satisfies $\psi(i, M, k) = (c^i, M', k')$; and
- (iii) (c^i, M', k') satisfies

$$c^i = y, \quad (7)$$

$$k = k', \quad (8)$$

$$M = M'. \quad (9)$$

In a stationary rational-expectations equilibrium, the first-order necessary and sufficient conditions for an interior optimal path of the household's problem are

$$U_2(i, M, k) = \beta \sum_{j=1}^n \pi_{ij} [u_1(c^j, p^j M') + u_2(c^j, p^j M')] p^j, \quad (10)$$

$$U_3(i, M, k) = \beta \sum_{j=1}^n \pi_{ij} u_1(c^j, p^j M') (r + q^j). \quad (11)$$

By the envelope theorem,

$$U_2(i, M, k) = u_1(c^i, p^i M) p^i, \quad (12)$$

$$U_3(i, M, k) = u_1(c^i, p^i M) q^i. \quad (13)$$

Combining (7)–(13), the equilibrium conditions can be summarized as

$$u_1(y, m^i) m^i = \beta \sum_{j=1}^n \pi_{ij} [u_1(y, m^j) + u_2(y, m^j)] m^j, \quad (14)$$

$$u_1(y, m^i) q^i = \beta \sum_{j=1}^n \pi_{ij} u_1(y, m^j) (r + q^j). \quad (15)$$

where $m^i = p^i M$. To demonstrate the existence of a stationary rational-expectations equilibrium, it suffices to establish a solution to (14) and to (15). Focusing on (14) first, we impose the following

Assumption 3: $u_{12}(c, m) \geq 0$, all $c, m \geq 0$.⁴

Two constant solutions of (14) are immediate: $m^i = 0$ and $m^i = m^*$ all $i \in N$, where m^* is the unique solution, ensured by Assumptions 1 and 3, to $u_1(y, m) = \beta[u_1(y, m) + u_2(y, m)]$. The first solution corresponds to an autarky equilibrium.⁵ The second one corresponds to a deterministic monetary equilibrium. Except for these two, other solutions, if exist, are not constant. Henceforth, in the spirit of Cass and Shell, one may call those "sunspot" solutions, and the two constant solutions "fundamental" solutions of (14). According to this definition, it is easy to construct a sunspot solution consisting of a randomization of $\{m^*, 0\}$, with the associated transition probability matrix $\pi_{11} = 1$, $\pi_{12} = 0$, $\pi_{21} = 0$, and $\pi_{22} = 1$. In addition to this particular sunspot solution, we establish, in the next section, that equation (14) has many non-trivial sunspot solutions.

III. SUNSPOT EQUILIBRIA

Theorem 1: Suppose Assumptions 1–3 hold. If $\{\pi_{ij}\}$ satisfies the following property:

Assumption 4:⁶ $\pi_{in} < 1$, $i \in \{1, \dots, n-1\}$ and $\pi_{nn} = 1$.

Then, equation (14) has a solution such that $m^i > 0$ all $i \in \{1, \dots, n-1\}$ and $m^n = 0$.

Proof:⁷ First define, for future references, $m(\phi)$ as the solution to $u_1(y, m)m = \phi \geq 0$. By Assumption 3, $m(\phi)$ is a strictly increasing function. Denote ϕ^* as the solution to $m(\phi) = m^*$. By Assumption 3 and that $m(\phi)$ is strictly increasing, $\beta[u_1(y, m(\phi)) + u_2(y, m(\phi))] < u_1(y, m(\phi))$ all $\phi > \phi^*$.

Define an operator $H = (h^1, \dots, h^{n-1})$ mapping from $S \equiv \{s = (s^1, \dots, s^{n-1}): 0 < s^i \leq \tilde{\phi}\}$ into R^{n-1} by

$$h^i(s) = \beta \sum_{j=1}^{n-1} \pi_{ij} [u_1(y, m(s^j)) + u_2(y, m(s^j))] m(s^j),$$

where $\tilde{\phi} \equiv \max_{m^* \geq m \geq 0} \beta[u_1(y, m) + u_2(y, m)]m$.⁸ Clearly, if H has a fixed point s , then $(s, 0)$ corresponds to a solution to (14). Two observations of H are noted: (i) h^i is continuous; and (ii) for each i ,

$$\begin{aligned} h^i(\phi) &\leq \beta \max_j \{ [u_1(y, m(\phi^j)) + (y, m(\phi^j))] m(\phi^j) \} \\ &\equiv \beta [u_1(y, m(\phi^\sigma)) + u_2(y, m(\phi^\sigma))] (m(\phi^\sigma)) \end{aligned}$$

If $\phi^* \geq \phi^\sigma > 0$, then,

$$\beta [u_1(y, m(\phi^\sigma)) + u_2(y, m(\phi^\sigma))] m(\phi^\sigma) \leq \tilde{\phi}$$

If $\phi^* < \phi^\sigma$, then $m^* < m(\phi^\sigma)$ and

3. If

$$\bar{\phi} \geq f^\alpha \geq \phi^*$$

then

$$\beta\gamma \left[1 + \frac{u_2(y, m(f^\alpha))}{u_1(y, m(f^\alpha))} \right] f^\alpha > \beta\gamma f^\alpha \geq \beta\gamma \phi^* \geq \psi^*.$$

In all three cases, $\bar{\phi} \geq h^i(f) \geq \psi^*$ for all $i = 1, \dots, n-1$, hence, H maps $S(\psi^*, \bar{\phi})$ into itself.

Q.E.D.

Remark: The sunspot solutions established in Theorem 1 are the stochastic counterpart of the deterministic hyperinflationary equilibria in Obstfeld and Rogoff (1983). While their hyperinflationary equilibria are nonstationary, the sunspot equilibria here are stationary. These equilibria are also different in that the value of money does not necessarily become zero in a finite number of periods. It all depends on the configuration of $[\pi_{ij}]$. For some $[\pi_{ij}]$, it is possible that the economy may cycle for a long period of time before it falls into the absorbing states eventually. Thus, although these sunspot equilibria are transient, the transient stage could be arbitrarily long. From the empirical point of view, these equilibria are hard to distinguish from those described in Matsuyama (1991).

Theorem 1 establishes the conditions under which a sunspot solution of (14) exists. For later use, it would be convenient if a uniqueness result can be established for any given $[\pi_{ij}]$ satisfying Assumption 4. Before the study of such an issue, it is noted that the proof of Theorem 1 implies that for any $0 < \psi \leq \psi^*$ and $\bar{\phi} \leq \phi < \infty$, the mapping h also maps $S(\psi, \bar{\phi})$ into itself. This observation will be useful to our next theorem.

Theorem 2: Suppose Assumptions 1–4 hold and the following condition is satisfied:

Assumption 5: For some $\rho \in [0, 1)$,

$$1 - \rho \leq -\frac{m[u_{12}(c, m) + u_{22}(c, m)]}{u_1(c, m) + u_2(c, m)} \leq 1 + \rho, \quad \forall c, m > 0.$$

Then the solution to (14) is unique within the class of solutions such that $m^i > 0$, $i = 1, \dots, n-1$, and $m^n = 0$.

Proof:⁹ Denote \mathcal{L} a bounded subspace of R^{n-1} with the max norm. Define an operator $G = (g^1, \dots, g^{n-1})$ mapping \mathcal{L} into \mathcal{L} by

$$g^i(f) = \ln \left\{ \beta \sum_{j=1}^{n-1} \pi_{ij} [u_1(y, m(\exp f^j)) + u_2(y, m(\exp f^j))] m(\exp f^j) \right\}$$

$$\beta[u_1(y, m(\phi^\sigma)) + u_2(y, m(\phi^\sigma))]m(\phi^\sigma) < u_1(y, m(\phi^\sigma))m(\phi^\sigma) = \phi^\sigma.$$

Again, $h^i(\phi) \leq \tilde{\phi}$ all $\phi \leq \tilde{\phi}$.

In the following we show that there exists a positive number ψ^* such that $H(S(\psi^*, \tilde{\phi})) \subset S(\psi^*, \tilde{\phi})$ where $S(\psi^*, \tilde{\phi}) \equiv \{s \in S: \Psi^* \leq s_i \leq \tilde{\phi}\}$. That is, H maps a convex, compact subset of R^{n-1} into itself. By the Brouwer fixed point theorem, H has a fixed point.

To find such a ψ^* , consider

$$\psi^* = \max \left\{ \psi > 0: \frac{1}{\gamma\beta} \leq 1 + \frac{u_2(y, m(\psi/\gamma\beta))}{u_1(y, m(\psi/\gamma\beta))} \right\}$$

where $1 \geq \gamma = \min_i (1 - \pi_{in}) > 0$, $i = 1, \dots, n-1$. Since $1/\gamma\beta \geq 1/\beta$, $\psi^* < \psi^*/\beta\gamma \leq \phi^* \leq \tilde{\phi}$. For any $f \in S(\psi^*, \tilde{\phi})$,

$$\begin{aligned} h^i(f) &= \beta \sum_{j=1}^{n-1} \pi_{ij} [u_1(y, m(f^j)) + u_2(y, m(f^j))] m(f^j) \\ &\geq \beta(1 - \pi_{in}) \min_j \{ [u_1(y, m(f^j)) + u_2(y, m(f^j))] m(f^j) \} \\ &\equiv \beta(1 - \pi_{in}) [u_1(y, m(f^\alpha)) + u_2(y, m(f^\alpha))] m(f^\alpha) \\ &\geq \beta\gamma \left[1 + \frac{u_2(y, m(f^\alpha))}{u_1(y, m(f^\alpha))} \right] f^\alpha. \end{aligned}$$

Three cases need separate consideration.

1. If

$$\frac{\psi^*}{\beta\gamma} \geq f^\alpha \geq \psi^*,$$

then

$$\beta\gamma \left[1 + \frac{u_2(y, m(f^\alpha))}{u_1(y, m(f^\alpha))} \right] f^\alpha \geq \beta\gamma \left[1 + \frac{u_2(y, m(\psi^*/\beta\gamma))}{u_1(y, m(\psi^*/\beta\gamma))} \right] f^\alpha = f^\alpha \geq \psi^*$$

2. If

$$\phi^* \geq f^\alpha \geq \frac{\psi^*}{\beta\gamma}$$

then

$$\beta\gamma \left[1 + \frac{u_2(y, m(f^\alpha))}{u_1(y, m(f^\alpha))} \right] f^\alpha \geq \beta\gamma \left[1 + \frac{u_2(y, m(\psi^*))}{u_1(y, m(\psi^*))} \right] f^\alpha = \gamma f^\alpha \geq \psi^*.$$

where $m_{\eta} = m(\exp \eta^i)$ and $a^i \leq \eta^i \leq b^i$. By Assumption 5, the second term is no greater than ρ . In addition, the third term is also no greater than one. Therefore,

$$\|Ga - Gb\| \leq \rho \|a - b\|,$$

and G is a contraction with modulus ρ . It follows by the contraction mapping theorem that G has a unique fixed point in $\mathcal{L}(\psi^*, \Phi)$. Since any $-\infty < \psi \leq \psi^* \leq \Phi \leq \phi < \infty$ can be used to define the set $\mathcal{L}(\psi, \phi)$ and $\mathcal{L}(\psi^*, \Phi)$ is a closed, convex subset of $\mathcal{L}(\psi, \phi)$, by Corollary 1 of Stokey and Lucas (p. 52), it follows that the fixed point is unique in $\mathcal{L}(\psi, \phi)$, the equation (14) has a unique fixed point in \mathcal{L} .

Q.E.D.

IV. SUNSPOTS AND ASSET PRICING

The model developed in the previous section is particularly relevant with regard to the issue of asset pricing. Since sunspots matter and introduce uncertainty into money prices, a nonzero cross partial of the utility function injects uncertainty into commodity rates of return.¹⁰ In the following, we will show how sunspots affect asset prices in a certain direction.

Let us define the market fundamentals of stocks as

$$q^* \equiv \frac{\beta r}{1 - \beta}.$$

Clearly, it corresponds to the price of the productive asset when $m^i = 0$ and $m^i = m^*$. Our next theorem shows that, if $u_{12} > 0$, then asset prices may be undervalued relative to q^* .

Theorem 3: Suppose Assumptions 1–5 hold. If, in addition,

Assumption 6: $[u_1(y, m) + u_2(y, m)]m$ is increasing in m ;

and

Assumption 7:

$$\frac{\partial \ln u_{12}(y, m)}{\partial m} \leq 2 \left[\frac{\partial \ln u_1(y, m)}{\partial m} \right]$$

also hold, then $q^i \leq q^*$, all $i = 1, \dots, n-1$, and $q^n = q^*$.

Proof: That $q^n = q^*$ is already noted. To show $q^i \leq q^*$, $i = 1, \dots, n-1$, first, recall from (15), this suggests an operator $J = (j^1, \dots, j^n)$ on \mathcal{L} :

$$j^i(q) = \beta \sum_{j=1}^n \pi_{ij} \left[\frac{u_1(y, m^j)}{u_1(y, m^i)} \right] (r + q^j)$$

G defined here and H defined in Theorem 1 are related by $G(s) = \ln H(\exp s)$. Since H maps $S(\Psi^*, \tilde{\Phi})$ into itself, G maps $\mathcal{L}(\Psi^*, \tilde{\Phi})$ into itself, where $\Psi^* = \ln \Psi^*$, $\tilde{\Phi} = \ln \tilde{\Phi}$ and $\mathcal{L}(\Psi^*, \tilde{\Phi}) \subset \mathcal{L}$ is the subset of vectors $f = (f^1, \dots, f^{n-1})$ satisfying $\Psi^* \leq f^i \leq \tilde{\Phi}$. Here, we will show that G is a contraction.

To verify the claim, note that from the definition of $m(\phi)$,

$$m' = \frac{1}{u_1(y, m) + m u_{12}(y, m)}. \quad (16)$$

To show that G is a contraction mapping, that is, for any $a, b \in \mathcal{L}(\Psi^*, \tilde{\Phi})$, $\|Ga - Gb\| \leq \rho \|a - b\|$ for some $\rho \in [0, 1)$, where $\|\cdot\|$ is the Euclidean norm, note that

$$\begin{aligned} \|Ga - Gb\| &= \max_i \left| \ln \sum_{j=1}^{n-1} \pi_{ij} m(\exp a^j) [u_1(y, m(\exp a^j)) + u_2(y, m(\exp a^j))] \right. \\ &\quad \left. - \ln \sum_{j=1}^{n-1} \pi_{ij} m(\exp b^j) [u_1(y, m(\exp b^j)) + u_2(y, m(\exp b^j))] \right| \\ &= \max_i \left| \ln \sum_{j=1}^{n-1} \pi_{ij} \left[\frac{m(\exp b^j) [u_1(y, m(\exp b^j)) + u_2(y, m(\exp b^j))]}{\sum_{k=1}^{n-1} \pi_{ik} m(\exp b^k) [u_1(y, m(\exp b^k)) + u_2(y, m(\exp b^k))]} \right] \right. \\ &\quad \left. \frac{[m(\exp a^j) [u_1(y, m(\exp a^j)) + u_2(y, m(\exp a^j))]]}{[m(\exp b^j) [u_1(y, m(\exp b^j)) + u_2(y, m(\exp b^j))]]} \right| \\ &\leq \max_i \left| \ln \frac{m(\exp a^i) [u_1(y, m(\exp a^i)) + u_2(y, m(\exp a^i))]}{m(\exp b^i) [u_1(y, m(\exp b^i)) + u_2(y, m(\exp b^i))]} \right| \\ &= \max_i \left| \ln m(\exp a^i) [u_1(y, m(\exp a^i)) + u_2(y, m(\exp a^i))] \right. \\ &\quad \left. - \ln m(\exp b^i) [u_1(y, m(\exp b^i)) + u_2(y, m(\exp b^i))] \right| \end{aligned}$$

By the mean value theorem,

$$\begin{aligned} &\max_i \left| \ln m(\exp a^i) [u_1(y, m(\exp a^i)) + u_2(y, m(\exp a^i))] \right. \\ &\quad \left. - \ln m(\exp b^i) [u_1(y, m(\exp b^i)) + u_2(y, m(\exp b^i))] \right| \\ &= \max_i \left| [a^i - b^i] \left[\frac{\partial \ln m(y, m) + u_2(y, m)}{\partial m} \right] m' \exp \eta^i \right| \\ &\quad (\text{by (16)}) \\ &= \max_i |a^i - b^i| \left| 1 + \frac{m_\eta [u_{12}(y, m_\eta) + u_{22}(y, m_\eta)]}{u_1(y, m_\eta) + u_2(y, m_\eta)} \right| \left| \frac{u_1(y, m_\eta)}{u_1(y, m_\eta) + m_\eta u_{12}(y, m_\eta)} \right| \end{aligned}$$

Standard argument can be used to show that, given $\{m^i\}$, J is a contraction and has a unique fixed point, i.e., a solution to (15).

Next, note that if

$$u_1(y, m^i) \geq \sum_{j=1}^n \pi_{ij} u_1(y, m^j) \quad (17)$$

then

$$j^i(q^*) = \beta \sum_{j=1}^n \pi_{ij} \left[\frac{u_1(y, m^j)}{u_1(y, m^i)} \right] (r + q^*) = q^* \sum_{j=1}^n \pi_{ij} \left[\frac{u_1(y, m^j)}{u_1(y, m^i)} \right] \leq q^*.$$

By induction, $q^* \geq \lim_{n \rightarrow \infty} (j^i)^n q^* = q^i$. Hence, it suffices to show (17).

With Assumption 6 we can set ϕ in the proof of Theorems 1 and 2 equal to ϕ^* , hence, the unique fixed point $s = (s^1, \dots, s^{n-1})$ established in Theorem 2 has the following property:

$$s^i \leq \phi \quad \text{and} \quad m(s^i) \leq m^*.$$

Therefore,

$$(1 - \beta)u_1(y, m(s^i)) \leq (1 - \beta)u_1(y, m^*) = \beta u_2(y, m^*) \leq \beta u_2(y, m(s^i)). \quad (18)$$

Equations (14) and (18) together imply

$$\begin{aligned} s^i &= u_1(y, m(s^i))(m(s^i)) = \beta \sum_{j=1}^n \pi_{ij} [u_1(y, m(s^j)) + u_2(y, m(s^j))](m(s^j)) \\ &\geq \sum_{j=1}^n \pi_{ij} u_1(y, m(s^j)) m(s^j) = \sum_{j=1}^n \pi_{ij} s^j \end{aligned} \quad (19)$$

From Assumptions 3 and 7, $u_1(y, m(s))$ is an increasing and concave function of s . Hence, (17) follows from (19) by the Jensen's inequality.

Q.E.D.

Remark: The following class of utility functions,

$$u(c, m) = \frac{[g(c)m]^{-1-\eta}}{-1-\eta}, \quad \eta > 0,$$

has been used by Matsuyama (1991) to show the existence of deterministic equilibrium with endogenous price fluctuations. Assumptions 6 and 7 above are satisfied when $-1 < \eta \leq 0$. Thus, the kind of sunspot equilibria in Theorem 1 and 2 would have never arisen in his context.

The intuition underlying the undervaluation of stocks is straightforward. The value of money depends on agents' expectations. The extrinsic uncertainty will affect the marginal rate of substitution of current for future consumption. Under the assumptions stated above, the corresponding marginal rate of substitution in a sunspot equilibrium is

$$\frac{u_1(y, m^i)}{\beta \sum_{j=1}^n \pi_{ij} u_1(y, m^j)} \geq \frac{1}{\beta}$$

That is, the extrinsic uncertainty adds a pure risk premium to the market returns on stocks, and henceforth depressed the value of stocks relative to the market fundamentals.

V. CONCLUSION

In conclusion, we have demonstrate, on the one hand, the existence of a class of stationary sunspot equilibrium in a stylized monetary economy. These equilibria exist without stringent assumptions placed on preferences. On the other hand, we have shown that stock prices may fluctuate despite a constant stream of dividends. The volatility tests of stock prices such as Shiller and Leroy-Porter may be invalid. Moreover, the asset prices may fall below their market fundamentals, a result contrary to the previous literature.

ACKNOWLEDGEMENT

I thank the comment from an anonymous referee. Any errors remain mine alone.

NOTES

1. The scope for the existence of nonstationary sunspot equilibria is much less stringent than the stationary ones. For a very general existence result of such equilibria, please see Peck (1988), and the recent survey by Chiaporri and Guesnerie (1991).
2. In contrast, Matsuyama (1991) has shown that endogenous price fluctuations unrelated to economic fundamentals may arise in a monetary economy identical to ours. However, a number of conditions on preferences Matsuyama assumed are violated here, hence, the deterministic sunspot equilibria he described do not occur in the present model.
3. In a recent paper, Weil (1990) constructed an example in an overlapping-generations economy in which the price of a productive asset may fluctuate around its fundamentals in a cyclical manner.
4. As it will become clear later, this assumption is imposed rather for the sake of convenience.
5. The boundedness of u in Assumption 1 guarantees the feasibility of this equilibrium.

6. Process of Blanchard (1979). The probability $1 - \pi_{in}$ has the interpretation *monetary confidence* (Weil, 1987).
7. An alternative proof of the existence of a monetary equilibrium in the Brock model has been demonstrated in Danthine and Donaldson (1986). However, as it was pointed out in Huo (1992), the assumptions Danthine and Donaldson used were inconsistent, hence, their proof was vacuous. The proof given below can be modified to suit their purposes.
8. Since $u(\cdot, \cdot)$ is bounded by Assumption 1, it follows that $\lim_{m \rightarrow 0} \mu_2(c, m) = 0$, a result due to Obstfeld and Rogoff (1983). Therefore, $\bar{\phi}$ is strictly positive and finite.
9. The proof is strictly analogous to one in Lucas (1972).
10. If $u_{12}(c, m) = 0$, equation (15) implies that asset prices are independent of $[\pi_{ij}]$.

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