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ORIGINAL ARTICLES

FUZZY TIME SERIES FORECASTING WITH BELIEF MEASURE

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Abstract: The profit of investment does not lie solely in the accuracy of prediction, but in the degree of belief as well. The greater the degree of belief is, the more capital the investors might venture, which results in more profit returns. On the contrary, under the condition of an accurate prediction, if the degree of belief is little, investors will not put in too much capital, which leads to limited profit. This study attempts to apply belief functions in explaining the prediction results of multivariate fuzzy time series, i.e. the degree of belief that the prediction model has for the prediction result. By utilizing multivariate fuzzy time series model, combining with two variables of closing price and volume of transaction in weighted stock price index, the author tries to predict the Taiwan weighted stock price index and estimate the degree of belief, which are trusted to be of great meaning for risk control and a better rate of return.

Keywords: *Belief measure, Fuzzy time series forecastinf, Fuzzy rule base, Average rank accuracy.*

1. INTRODUCTION

When we make a decision, we need to evaluate the possibility of the future outcome and do action according to them. Accordingly, in constructing a model of prediction, not only should we consider the accurate of the prediction but also to establish a proper fuzzy mathematical model on the degree of belief in the prediction results.

In analyzing time series, the tendency of data can be the basis of determining the occurrence of incidents, such as progressive increase or progressive decrease and seasonal circulation or sudden boost. Hence, based upon the characteristics observed, we can choose the best fitness model from the already-tested model groups, like ARIMA model group, ARCH model group or threshold model group. Yet, due to the inaccuracy of data collection, time lag, or the interwork among variables, the numerical value of a single measurement may seem like an accurate one superficially, however, in reality, a possible one within a range. For example, which is the exact number of registered students every year--at the beginning of the semester, in the middle of the semester, at the end of the semester or an average of the above numbers? Different sampling time often results in different value. Further, which is the weighted stock price index--the opening quotation, the closing quotation, or an average of the highest and the lowest stock price? The result varies a lot. Under the circumstance, if we use the traditional way of model construction and analysis to find the best fitness model and to explain the tendency of time series data, the danger of finding the best fitness of model to an undue extent might occur.

Ever since Zadeh's (1965) proposition of Fuzzy Set Theory, the theory has incorporated the property of linguistic variables, which is capable of reducing possible trouble in dealing with uncertain problems as well as providing a more reliable way in processing complex, diverse and uncertain phenomena. Lately, the application of fuzzy set theory in time series has been increasing gradually, e.g. Song and Chissom (1993a, b, 1994), Chen (1996), Song (1997), Huarng (2001). Wu and Hung (1999) proposed fuzzy identification rule to be the criterion in determining ARCH model group and Bilinear Model Group. Wu and Chen (1999) utilized fuzzy clustering method to check the data structure's transitional span in time series. Tseng and Tzeng, etc. (2001) proposed a fuzzy ARIMA model to predict the NT dollar exchange rate to US dollar by combining traditional time series ARIMA model with fuzzy regression model. Tseng and Tzeng (2002) also combined fuzzy theory with seasonal ARIMA to establish a prediction model.

While in the process of applying fuzzy set to time series analysis, the first step is to figure out the way to integrate analyzing methods of linguistic variables in order to solve the indefiniteness in data. Aiming at solving this, Tong (1978) suggested the logical examination method and used a strategy-making chart to describe fuzzy model, but it is difficult to extend the method to multivariate systems. Thus, in order to obtain a more accurate fuzzy model, Graham and Newell (1989) and Xu and Lee (1987) proposed a method with learning ability to modify fuzzy model, while Chiang, etc. (2000) recommended a fuzzy linguistic summary system to collect time series data to find useful information. Besides, there is a rather inconvenient method that tries to choose the proper weighted factors with trial-and-error procedure. As a matter of fact, a more comprehensible and applicable way than the use of a strategy-making chart is to access the problem with fuzzy formulas. Seeing that, most scholars often try to find solution through fuzzy formulas, like Song and Chissom

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(1993a, b) using them to present a detailed construction process of fuzzy time series and model theory. Song and Chissom (1993a, 1994) also applied the method in predicting the number of registered freshmen students for University of Alabama. Furthermore, Lee, etc. (1994) suggested a two-phase verification process, combined with linguistic methods and fuzzy formulas, to verify a fuzzy model.

Most of the past bibliographies have been focused on single-variate fuzzy time series rather than on multivariate fuzzy time series, and none of them has ever mentioned the degree of belief toward the prediction results made by the model. In view of this, this study attempts to incorporate belief functions into multivariate fuzzy time series, and use the belief functions to describe the degree of belief toward the prediction results made by prediction model. At last, the study takes two factors into consideration, i.e. the closing price and volume of transaction in weighted stock price index, and tries to make predictions with the establishment of a multivariate fuzzy time series model. Furthermore, the author uses average prediction rank of accuracy to verify the prediction and adds the belief functions to describe the degree of belief toward the prediction results, which will be of great meaning to monetary finance in predicting future market trend as well as forming policies to deal with changes.

2. FUZZY TIME SERIES ANALYSIS AND PREDICTION

2.1 Human thought and decision making under fuzzy environment

Fuzzy set theory is a kind of new trend in processing human thinking with a quantified way. The term does not imply carelessness or inaccuracy: rather, confronted with various sorts of uncertainty of life, it aims to analyze and to manage control in order to obtain more efficient, humanized and intelligent results. The construction of study subjects for modern science is becoming more and more complicated and obscure, resulting from factors of subjectivity, time discrepancy, circumstance changes and perspectives. This has made it hard for scientists to clearly study the true essence of subjects and to properly establish hypothetical mathematic model, which makes the fuzzy theory to come about.

The fuzzy theory can be referred to as including fuzzy set theory generated from general set theory, fuzzy estimate theory with extended meaning of probability, and fuzzy logic originated from general logic with fuzziness notion. Fuzzy estimate theory does not deal with semantic uncertainty, but the subjectivity uncertainty in making judgment. The real human society is full of fuzziness. A lot of fuzzy uncertainty lies in our thinking, judgment, communication, etc. Formerly, duality set regulates an element either belonging to a set (represented as 1) or not belonging to a set (represented as 0). In fuzzy theory, we use the numbers between 0 and 1 to indicate its degree of belongingness.

The application of fuzzy logic claims that personal preference does not have to be very clear or orderly, which is in contrary to the concepts in Boolean logic. For example, according to Boolean logic, the comparison between a and b results in three possible results: (1) b$>a$ (2) $a<b$ (3) $a=b$. Yet, the operation of human thinking is far more complex than the Boolean logic, especially with a great deal of uncertain preferences. Therefore, many researchers have long been trying to grasp the real situation in a comprehensive way. As far as social science problems are concerned, it is more apprehensible to present things by fuzzy model than to assign a specific value to a single object, which is more suitable for evaluating the relative characteristics between two objects. Also, since other characteristics often may be helpful in evaluating some opinion, we have to explain the so-call "other characteristics" so that we can convert people's preferences into more convenient utility functions.

The main feature of fuzzy set theory is its preference for limitless possibility, thus, providing limitless explanations.

In traditional set theory, an element either belongs to a set or not belongs to it. However, the element in fuzzy set, its membership grade may be that only part of it belongs to the set. For example, the phrase "young people." Exactly over what age can be consider being "not young"? There seems to be no definite demarcation line to separate "young" and "not young." According to the definition of fuzzy set, a 25-year-old man is "80% young," while a 70-year-old man is only "20% young." In fuzzy set, the whole range of membership is set to be between 0 and 1. Each linguistic variable, like "young," represents a possible distribution. The mean value of the distribution is used to express the value that people adopted to determine what "young" is. As to the distribution, it depends and doesn't need to be normality.

Membership function is the foundation of fuzzy theory. It originates from characteristic function and is used to indicate the membership grade to sets, whose range is between 0 and 1. The greater the membership of an element is, the closer to 1 the membership grade is. While the less the membership of an element is, the smaller the membership grade value is. It is not an easy job to properly establish a membership function that can fully express the notion of fuzziness. Although membership grade is objective in its nature, it usually exists within human subjective conscience. There is no general theorem or formula to measure it, but we usually verify it through experience or statistics.

Traditional social and economic researches have long been devoted to the analysis of human thought and decision making. Uncertain examples are often encountered in a typical model construction. For example, whether the exact number of annual registered students is sampled at the beginning of the year or the middle or the end, the value usually differs. Again, the New Taiwan Dollar exchange rate against US Dollar should be the one at the opening market, the closing market or the average of the highest and the lowest price? Hendershot and Placek (1981) have done an extensive survey regarding the bibliographies in the field. In studies of social science and economics, the answer to a question is seldom a definite true or false. If we try to analyze human ideas, we will find that it is inevitable to face the uncertainty of behavior. While, in fuzzy set, the continuous range value is capable of dealing with true-as-well-as-false situations. Thus, using the fuzzy characteristic of the continuous range value in analysis can facilitate researchers a lot in handling uncertainty, and is a more realistic measurement tool in practical application.

2.2 Fuzzy Time Series Analysis

The first step before we go deeper into fuzzy time series is to convert common data into fuzzy data. In this study, we use membership function to convert the observed data into fuzzy set. Membership function is the most basic notion of fuzzy theory, and is able to describe the nature of fuzzy set. The only way to quantify fuzzy set is through membership function, which enables us to further analyze and process fuzzy data with accurate mathematic method. Thus, in order to set the observation value of a fuzzy model or to estimate the fuzzy output value of a fuzzy model, the first step is to convert the observation value into fuzzy set, a process called fuzzification. Consequently, membership function plays a rather important role in establishing a fuzzy model.

Because the subject of the study is about the establishment of multivariate fuzzy time series, it is necessary to explain the meaning of fuzzy time series. The so-called fuzzy time series is a method of applying fuzzy logic in the analyzing procedure of time series, combined with the analyzing procedure of linguistic variable, to solve the fuzziness of data. Hence, before establishing multivariate fuzzy time series and making predictions, we must provide definitions for some terms regarding fuzzy time series.

Definition 2.1 Fuzzy time series

Let $\{X_t \in R,\ t = 1,2,\dots,n\}$ be a time series, Ω be the range of $\{X_t \in R,\ t = 1,2,\dots,n\}$ and $\{P_i;\ i=1,2,\dots,r, \bigcup_{i=1}^{r} P_i = \Omega\}$ be an ordered partition on Ω. Let $\{L_i, i = 1,2,\dots,r\}$ denote linguistic variables with respect to the ordered partition set. For $t = 1,2,\dots,n$, if $\mu_i(X_t)$, the grade of membership of $\{X_t\}$ belongs to L_i, satisfies $\mu_i : R \to [0,1]$ and $\sum_{i=1}^{r} \mu_i(X_t) = 1$, then $\{FX_t\}$ is said to be a fuzzy time series of $\{X_t\}$ and written as

$$FX_t = \mu_1(X_t)/L_1 + \mu_2(X_t)/L_2 + \cdots + \mu_r(X_t)/L_r,$$

where / is employed to link the linguistic variables with their memberships in FX_t, and the + indicates, rather than any sort of algebraic addition, that the listed pairs of linguistic variables and memberships collectively.

For the sake of convenience, we simplify FX_t as $FX_t = (\mu_1, \mu_2, \dots, \mu_r)$, and use triangle membership grade function to proceed with the conversion.

In time series analysis, the determination of the value of autocorrelation is very important, because it reflects the long-term autocorrelation of a time series. Thus, we attempt to analyze the degree of autocorrelation of fuzzy time series with fuzzy correlation.

Definition 2.2 Fuzzy relation

Let $\{P_i, i = 1,2,\dots,r\}$ be an ordered partition set in Ω. Assume that $G = (\mu_1,\dots,\mu_r)$ and $H = (v_1,\dots,v_r)$ are the fuzzy sets in Ω, where μ_i and v_j are the membership functions over Ω for $i, j=1,2,3\dots,r$. Then we denote the fuzzy relation between G and H as $R = G^t \circ H = [R_{ij}]_{r\times r}$, where "$\circ$" is the fuzzy composition (we use the max-min here), "t" is the transpose and R_{ij} is membership function between G and H.

As far as multivariate fuzzy time series is considered, and suppose the tendency of each variable of time series data is steady, this study focuses upon multivariate fuzzy time series models with the Markov property. The definition of Fuzzy-Markov-relative-matrix R is as following.

Definition 2.3 Fuzzy-Markov-relative matrix

Assume that $\{FX_t,\ t = 1,2,\dots,n\}$ is a FAR (1) (fuzzy autoregression of order one) fuzzy time series, i.e. for any time t, FX_t depends only on FX_{t-1}. If the fuzzy set FX_t consists of finite membership functions $\mu_i(X_t), i = 1,2,\dots,r$, then

$$\boldsymbol{R} = [R_{ij}]_{r\times r} = \max_{2\le t\le n} [\min(\mu_i(X_{t-1}), \mu_j(X_t))]_{r\times r}$$

is called a fuzzy-Markov-relative matrix.

On Multivariate Fuzzy Time Series Models

In the process of establishing Multivariate Fuzzy Time Series Model, we need to pay attention to many conditions, which are stated as following:

Because the type of data we collected may be numerical, characteristic, or of linguistic values (e.g. data acquired from food sampling, etc.), it is difficult to use traditional time series procedure to analyze these sorts of data. Therefore, if we make use of fuzzy set procedure, we will be able to establish an adequate model without being confined by the type of data we collect.

There is no common agreement about the number of divided sets that is considered appropriate in founding a fuzzy universe of discourse set. In general, the more the sets being divided, the more accurate the set is. However, the complexity of operation will increase as well. Thus, the choice between accuracy and complexity depends on individual need.

As to the fuzzification of data, the authors use the standard membership function to accomplish the process. Since there is no fixed way of acquiring the typical value, we adopt a way of using median, mean, or classified community center of the elements in each set as the typical value. Yet, if the community center is deemed as the typical value, this might result in a single set with multiple typical values, causing a trapezoid membership function and making the conversion process more complicated. On the contrary, if median or mean is chosen as the typical value, this will lead to each single set with only one typical value, causing a triangle membership function and making the conversion easier.

The key point in the process of model establishment lies in the stability of data structure. Because the data in this study is stable to a certain extent, we can move on directly to the step of model establishment. Nevertheless, in general situations, if the data collected are converted from numerical ones into fuzzy ones, the tendency may be increasing or decreasing. We have to make a difference for the original data and make them stable so as to continue the above step and to establish the model. Contrarily, if the data collected possess fuzzy structure and are verified to have fuzzy tendency, there is no solution for it at present.

The order identification plays a very important role in multivariate fuzzy time series analysis. If we can find the right order, we can grasp factors that affects data tendency and establish a practical mathematic model. In practical application, especially stock, exchange rate, and futures data, the time series tendency often display nonlinear forms of either the high one getting higher or the low one getting lower. Most of the operation in financial market conforms to Markov nature. For this reason, we can choose the order according to real situations.

After choosing the order of multivariate fuzzy time series, we can utilize the calculating method defined in Definition 2.2, combined with multiple factors that might affect data, and generate Fuzzy-Markov-relative-matrix R. Based up the above interpretation, we analyze multivariate fuzzy time series model. Here, the author only discusses multivariate first order of auto-regression, defined as following:

Definition 2.4 Multivariate first order of auto-regressive fuzzy time series model

If a multivariate fuzzy time series, $\{(FX_{1,t}, FX_{2,t},\cdots, FX_{k,t})\}$ for any t, can be written as

$$(FX_{1,t}, FX_{2,t},\cdots, FX_{k,t}) = (FX_{1,t-1}, FX_{2,t-1},\cdots, FX_{k,t-1})\begin{pmatrix} R_{11} & \cdots & \cdots & R_{1k} \\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ R_{k1} & \cdots & \cdots & R_{kk} \end{pmatrix},$$

where R_{ij} is the fuzzy-Markov-relative matrix for i^{th} variable relative to j^{th} variable, $i,j = 1,2\ldots,k$, then we call the multivariate fuzzy time series $\{(FX_{1,t}, FX_{2,t},\cdots, FX_{k,t})\}$ a multivariate first order of auto-regressive fuzzy time series model and denote it as VFAR(1). In the model, $(FX_{1,t}, FX_{2,t},\cdots, FX_{k,t})$, is only depends on $(FX_{1,t-1}, FX_{2,t-1},\cdots, FX_{k,t-1})$, thus the model can be referred as the Markov Process.

2.3 How to determine the attribute through fuzzy rule base?

In multivariate fuzzy time series analyzing, one of the significant fields worth studying is how to convert fuzzy values (membership functions) into linguistic variables. Generally, it is determined by the location of maximum membership function. If there is more than one maximum membership function, which should we choose to decide its attribute? Up to now, there is no fixed rule. On account of this, the study defines a linguistic vector index function to cope with those conditions that have more than one maximum membership function.

Definition 2.5 Linguistic vector index function

Let $L = \{(L_{11},\cdots,L_{1r}),\cdots,(L_{k1},\cdots,L_{kr}) \mid L_{ij}$ is linguistic variable for $i=1,2,\ldots k$ and $j=1,2,\ldots r\}$ and FX_t be a multivariate fuzzy time series with respect to the membership function in L. Assume that $F\overline{X}_t$ is a linguistic vector index function. Let

$$F\overline{X}_t = \{(I_{11},\cdots,I_{1r_t}),\cdots,(I_{k1},\cdots,I_{kr_t});I_{ij_t} = 1 \text{ or } 0\}, \text{ and}$$

$$I_{i,j}=\begin{cases} 1, \text{ if } \mu_{L_{ij}}(F\hat{X}_t)=\max\left[\mu_{L_{ij}}(F\hat{X}_t)\right] \\ 0, \text{ if } \mu_{L_{ij}}(F\hat{X}_t)<\max\left[\mu_{L_{ij}}(F\hat{X}_t)\right] \end{cases},$$

where $\mu_{L_{ij}}(FX_t)$ is the membership function in linguistic variable L_{ij}.

Example 2.1 *Let $L = \{(L_{11},L_{12},L_{13},L_{14},L_{15})(L_{21},L_{22},L_{23},L_{24},L_{25})$; L_{11} = plunge, L_{12} = down, L_{13} = unchanged, L_{14} = up, L_{15} = soar, L_{21} = very low, L_{22} = low, L_{23} = medium, L_{24} = high, and L_{25} = very high\}. If one time series of two variables is computed by fuzzy-Markov-relative matrix R and the* *membership function with respect to L is*

$FX_t = \{(0.59,0.58,1.1,1.1,1.1),(0.81,1.36,1.20,1.36,1.21)\}$ *i.e.*

The membership function belonging to L_{11} is 0.59;
The membership function belonging to L_{12} is 0.85;
The membership function belonging to L_{13} is 1.1;
The membership function belonging to L_{14} is 1.1;
The membership function belonging to L_{15} is 1.1;
The membership function belonging to L_{21} is 0.81;
The membership function belonging to L_{22} is 1.36;
The membership function belonging to L_{23} is 1.20;
The membership function belonging to L_{24} is 1.36;
The membership function belonging to L_{25} is 1.21.

By Definition 2.5, we get $F\overline{X}_t = \{(0,0,1,1,1),(0,1,0,1,0)\}$

According to definition 2.5, we can convert the fuzzy values predicted by multivariate fuzzy time series model into linguistic vector index function. Yet, how should we determine the linguistic variables through linguistic vector index function? Here, the author regrets to say that there is no fixed rule for reference up to now. So, based on the above definition, we can make use of fuzzy inference to construct a set fuzzy rule base to analyze the linguistic variables it outputs.

How can we obtain such a fuzzy rule base? As the name implies, what exists inside the base are some inference rules, i.e. a corpus of linguistic rules describing reasoning strategy. A fuzzy rule base is an expert system (rules) constructed upon the foundation of everyday fuzzy phenomena or knowledge. As to fuzzy time series, there is no definite measure for us to follow. Hence, since the rules in this study are statistics related, we use autocorrelation function (ACF) and partial autocorrelation function (PACF) to find the coefficient of time series model. As to the nonlinear time series, because its ACF and PACF are not clear and definite, we can obtain the best fitness model, following the traditional Autoregressive Integrated Moving Average (ARIMA) model, with the help of a three-step construction, i.e. (1) Order identification, (2) Parameter estimation, and (3) Diagnostic checking. As a result, the fuzzy rules in this study are established from the above notions, accumulation of experience and fuzzy inference; so, the methodology is intuitive and subjective.

In this study, the universe of discourse of the two factors, degree of price fluctuation and transaction volume, are set to be {plunge, down, unchanged, up, soar} and {very low, low, medium, high, very high} respectively. Thus, n is set to be 5. Also, we choose $(I_{k1_t},\cdots,I_{k5_t})$ as fuzzy inference index, and $I_{kj_t} = 0$ or 1 and $j = 1,2,\cdots,5$; upon which we obtain 32 linguistic vectors. Vector (0,0,0,0,0) must be excluded, because it cannot represent any linguistic variable. However, it is not easy to classify 31 vectors into proper linguistic variables. For if linguistic vector has only a 1 and the rest are 0, the output is the linguistic variable represented by the location of 1. For instance, (0,0,0,1,0) indicates the membership function of up is 1, so we output the linguistic variable "up."

However, how should we decide if the linguistic vector has more than one 1? If we start each linguistic vector of fuzzy time series I_{kj_t}, $j = 1,2,\cdots,5$ from $I_{k1_t} = 0$ or 1 to $I_{k5_t} = 0$ or 1 and then judge from our accumulated knowledge and experience, this would be a very time-consuming job. Or if we judge by the whole linguistic vector, we can easily determine the linguistic variable it represents based on law of experience. For instance, in (0,0,0,1,1), the membership function of up and soar is 1 respectively. According to law of experience, we can determine that the output linguistic variable is "soar." Accordingly, the output linguistic variable for (1,1,0,0,0) is "plunge." The following is the fuzzy rule base established in this study.

From example 2.1, we derive $F\overline{X}_t = (0,0,1,1,1)(0,1,0,1,0)$. Thus, from the above fuzzy rule base we know that, as far as degree of price fluctuation and transaction volume difference are concerned, the output linguistic variables are "soar" and "medium."

2.4 Prediction of Multivariate Fuzzy Time Series

People usually hope to have certain understanding for future situations; therefore, prediction results have become essential information in the decision-making process. Besides, accurate predictions will help decision-makers to

Fuzzy Rule Base

Linguistic vector index $F\overline{X}_t$	The output linguistic variable
(1,0,0,0,0), (1,1,0,0,0), (1,0,1,0,0), (1,1,1,0,0)	plunge (very low)
(0,1,0,0,0), (1,1,0,1,0), (1,1,1,0,1), (1,1,0,0,1), (1,0,0,1,0), (1,1,1,1,0), (0,1,1,0,0), (1,0,1,1,0)	down (low)
(0,0,1,0,0), (1,0,1,0,1), (1,0,0,0,1), (1,1,1,1,1), (0,1,0,1,0), (1,1,0,1,1), (0,1,1,1,0)	unchanged (medium)
(0,0,0,1,0), (0,1,0,1,1), (1,0,1,1,1), (1,0,0,1,1), (0,1,0,0,1), (0,1,1,1,1), (0,0,1,1,0), (0,1,1,0,1)	up (high)
(0,0,0,0,1), (0,0,0,1,1), (0,0,1,0,1), (0,0,1,1,1)	soar (very high)

make correct decisions and to react properly. That is why we make use of multivariate fuzzy time series model to conduct prediction. The definition of prediction with multivariate fuzzy time series model is as following.

Definition 2.6 The prediction of multivariate first order of auto-regression fuzzy time series model.
Assume that a multivariate first order of auto-regression fuzzy time series model is

$$(FX_{1,t}, FX_{2,t},\cdots, FX_{k,t}) = (FX_{1,t-1}, FX_{2,t-1},\cdots, FX_{k,t-1})\begin{pmatrix} R_{11} & \cdots & \cdots & R_{1k} \\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ R_{k1} & \cdots & \cdots & R_{kk} \end{pmatrix}$$

and the known observation is $(FX_{1,t}, FX_{2,t},\cdots, FX_{k,t})$, $t = 1,2,\ldots,n$ *then the prediction value forward* l *period is as following :*

(1) When $l = 1$,

$$(FX_{1,n}(1), FX_{2,n}(1),\cdots, FX_{k,n}(1)) = (FX_{1,n}, FX_{2,n},\cdots, FX_{k,n})\begin{pmatrix} R_{11} & \cdots & \cdots & R_{1k} \\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ R_{k1} & \cdots & \cdots & R_{kk} \end{pmatrix}.$$

(2) When $l = 2$,

$$(FX_{1,n}(2), FX_{2,n}(2),\cdots, FX_{k,n}(2)) = (FX_{1,n}, FX_{2,n},\cdots, FX_{k,n})\begin{pmatrix} R_{11} & \cdots & \cdots & R_{1k} \\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ R_{k1} & \cdots & \cdots & R_{kk} \end{pmatrix}^2.$$

(3) The prediction value forward l *period is*

$$(FX_{1,n}(l), FX_{2,n}(l),\cdots, FX_{k,n}(l)) = (FX_{1,n}, FX_{2,n},\cdots, FX_{k,n})\begin{pmatrix} R_{11} & \cdots & \cdots & R_{1k} \\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ R_{k1} & \cdots & \cdots & R_{kk} \end{pmatrix}^l.$$

2.5 Average Rank-forecasting Accuracy

After the establishment of multivariate fuzzy time series and fuzzy rule base, we are capable of making predictions successfully. In order to compare the discrepancy between linguistic variables predicted by our model and those in real situation, we set up a measuring index: average rank accuracy. It utilizes the notion of non-parameter rank and assigns each linguistic variable a representing value, like plunge (very low) as –2, down (low) as –1, unchanged (medium) as 0, up (high) as 1, and soar (very high) as 2.

Definition 2.7 Average rank-forecasting accuracy
Let $\{RL_t, t = 1,\cdots,n\}$ *be the linguistic variable of a time series, and* $\{FL_t, t = 1,\cdots,n\}$ *be the linguistic variable obtained from the prediction of fuzzy time series model. Assume that* $L = \{(L_1,L_2,\cdots,L_r) = (-(r-1)/2, -(r-3)/2, \cdots,-(r-1)/2,) : L_j$ *is the linguistic variable for* $j = 1,\cdots,r\}$, *then*

$$P=1-\frac{\sum_{t=1}^{n}\frac{|FL_t-RL_t|}{r-1}}{n}$$

is called average rank-forecasting accuracy, where r *denotes the number of linguistic variables.*

Example 2.2 *Consider the universe of discourse* Ω *with five parts: {plunge, down, unchanged, up, soar}. For a time series, if its corresponding linguistic value is { up=1, soar=2, down=-1, up=1, unchanged=0, plunge=-2}; while the linguistic value for the realization is {0, 2, 0, 2, -1, -1}. By definition 2.7, we know r=5 and n=6; therefore, the average rank-forecasting accuracy is*

$$P=1-\frac{\frac{|1-0|+|2-2|+|-1-0|+|1-2|+|0+1|+|-2+1|}{4}}{6}$$

$$=1-\frac{5}{24}=0.8.$$

3. MEASURING BELIEFS IN THE FORECASTING PROCESS

3.1 The use of belief functions

As people attempt to make decision for future affairs, they usually refer to past experience. Multivariate fuzzy time series forecasting model is just like human decision-making model. Fuzzy relative matrix derives from previous quantity-price relationship and is just like former experience imprinted in human brain. Thus, by means of fuzzy relative matrix, we can further achieve prediction results. Nonetheless, just like people's judgments, people are not always right about the decisions they make. The same goes to the results of prediction made by forecasting models. Thereupon comes the question: How should we regard the prediction results forecasted by the model each time? Should we totally, partially or hardly accept them?

Try to recall the reaction we had when we had to make decisions for the future. Obviously, there is more than just the decision itself. The degree of belief towards the decision is also there. In other words, people not only make decisions but also make decisions with "degree of belief" in them. They tend to execute decisions more bravely for confident ones. By contrast, they act more carefully for less confident decisions. Accordingly, in the case of trying to make an investment, if we have faith in future market, we can make a large sum of investment; whereas, if we have little faith in future market, we can reduce the sum of money to be invested. In this regard, we can avoid loss due to misjudgment, increase investment returns for right judgment, and enhance our risk-control ability for capital.

So, this study not only makes predictions on multivariate fuzzy time series, but also establishes belief functions. Belief functions are used to describe the degree of belief in the predictions we made. Since the theoretically mathematical foundation is constructed on the basis of the notion of belief function, some definitions and properties of belief function are provided, under limited conditions, as following:

Definition 3.1 *Let Ω be a finite set. The set function $G: 2^{\Omega} \rightarrow [0,1]$ is a belief function, if*

(*i*) $G(\phi) = 0$, $G(\Omega) = 1$.

(*ii*) *For any $n \geq 1$ and $A_i \subseteq \Omega$, $i = 1,\ldots,n$,*

$$G\left(\bigcup_{i=1}^{n} A_i\right) \geq \sum_{\phi \neq I \subseteq \{1,\ldots,n\}} (-1)^{|I|+1} G\left(\bigcap_{j \in I} A_j\right).$$

Theorem 3.1 (*i*) *Let G be a belief function in Ω. Then the function g defined on 2^{Ω},*

$g(A) = \sum_{B \subseteq A} (-1)^{|A-B|} G(B)$, is nonnegative.

(*ii*) *Let G and g be two functions from 2^{Ω} to R.*

$G(A) = \sum_{B \subseteq A} g(B)$ if and only if $g(A) = \sum_{B \subseteq A} (-1)^{|A-B|} G(B)$.

Proof. (i) Let $A = \{w_1, w_2, \ldots, w_n\} \subseteq \Omega$ and $A_i = A - \{w_i\}$. Then

$$G(A) \geq \sum_{i=1}^{n} G(A_i) - \sum_{i<j} G(A_i \cap A_j) + \ldots + (-1)^n \sum_{i=1}^{n} G\left(\bigcap_{j \neq 1} A_j\right),$$

Note: $\bigcap_{i=1}^{n} A_i = \phi$, so $g(A) \geq 0$.

(ii) "$\Rightarrow$" Assume $G(A) = \sum_{B \subseteq A} g(B)$. Then

$$\sum_{B \subseteq A} (-1)^{|A-B|} G(B) = \sum_{B \subseteq A} (-1)^{|A-B|} \sum_{D \subseteq B} g(D) = \sum_{D \subseteq B \subseteq A} (-1)^{|A-B|} g(D).$$

If $D = A$, then the last form is $g(A)$. If $D \neq A$, $A - D$ has $2^{|A-D|}$ subsets. Therefore, there exist even number of subsets B, $D \subseteq B \subseteq A$, exactly half of the sets have even number of elements. Thus the number of +1 is $(-1)^{|A-B|}$, half of the total number and the other is –1. For each D, $D \neq A$,

$$\sum_{D \subseteq B \subseteq A} (-1)^{|A-B|} g(D) = 0. \text{ So } \sum_{B \subseteq A} (-1)^{|A-B|} G(B) = g(A).$$

"$\Leftarrow$" The way is the same as above.

To explain Theorem 3.1, we discover that any belief function G in Ω can be written as $g: 2^{\Omega} \rightarrow [0,1]$, $\sum_{A \subseteq \Omega} g(A) = 1$, and $g(\phi) = 0$. Hence, formally g is a probability density function of some random set S in Ω, i.e. P(S=A)=g(A), and $G(A) = P(S \subseteq A)$. Refer to the Theorem 3.2 below. Besides G plays the role of the distribution function of the random set S. In the theory of evidence, the value $g(A)$ is interpreted as the weight of evidence in support of A.

Theorem 3.2 *Let $g: 2^{\Omega} \rightarrow [0,1]$, $g(\phi)=0$ and $\sum_{A \subseteq \Omega} g(A) = 1$. Define $G(A) = \sum_{B \subseteq A} g(B)$, then the set function G is a belief function.*

Proof. *Since $G(\phi)=0$ and $G(\Omega)=1$, we have to show that G is infinitely monotone. Let $I = \{1, 2, \ldots, n\}$ and $A_i \subseteq \Omega$, $i \in I$. We have*

$$G\left(\bigcup_i A_i\right) = \sum_{B \subseteq \bigcup_i A_i} g(B) \geq \sum_{B \in \Gamma} g(B),$$

where at least one A_i contains Γ which is a subcollection of $\bigcup_i A_i$.

$$\sum_{\phi \neq J \subseteq I} (-1)^{|J|+1} G\left(\bigcap_j A_j\right) = \sum_{\phi \neq J \subseteq I} (-1)^{|J|+1} \sum_{B \subseteq \bigcap_j A_j} g(B)$$
$$= \sum_{B \in \Gamma} \sum_{\phi \neq J \subseteq I} (-1)^{|J|+1} g(B) = \sum_{B \in \Gamma} g(B).$$

Since $\sum_{\phi \neq J \subseteq I} (-1)^{|J|+1} = 1$, so the theorem is proved.

3.2 How to establish and calculate the degree of belief?

This study uses the maximum membership grade to convert fuzzy forecasting value into linguistic vector index, and then obtain the predicted attributes of linguistic variables upon the fuzzy rule base. Just as when people are not sure to the very same extent if they make the right decision for the future, multivariate time series model doesn't have the same degree of belief in every prediction it makes. Thus, it is necessary to form a function to evaluate the degree of belief of the forecasting model. In order to form and calculate degree of belief, the following definitions must be given:

Definition 3.2 Generalized membership rank and maximum membership

Let $L = \{(L_{11},\cdots L_{15}),\cdots,(L_{k1},\cdots L_{k5}) : L_{ij}$ is a linguistic varriable$\}$, and FX_t be the membership function of multivariate fuzzy time series with respect to L. Let $F\overline{X}_t$ be the linguistic vector index by converting FX_t. Thus

$$F\overline{X}_t = \{(I_{11},\cdots I_{15}),\cdots,(I_{k1},\cdots I_{k5}), I_{ij} = 1 \text{ or } 0\}.$$

For each FX_A, assume that FA_{ti} is the generalized membership rank of FA_{ti} and FC_{ti} is the maximum membership of $F\hat{X}_{ti}$, where

$$FA_{ti} = \left[\sum_{j=1}^{5} (j-3) I_{ij_t}\right] / \left[\sum_{j=1}^{5} I_{ij_t}\right],$$
$$FC_{ti} = \max\left[\mu_{L_{i1}}(FX_t),\cdots,\mu_{L_{i5}}(FX)\right],$$

in which $\mu_{L_{ij}}(FX_t)$ is the membership function of FX_{ti} in the linguistic variable L_{ij}.

Example 3.1 *Assume that there is a membership function of multivariate fuzzy time series with respect to L, say FX_t = {(0.56,1.15,1.38,1.38),(0.96,1.32,1.40,0.74,0.28)}. Then $F\overline{X}_t$ = {(0,0,0,1,1),(0,0,1,0,0)}. According to definition 3.2, we have*

FA_{t1} = (–2x0–1x0+0x0+1x1+2x1)/(0+0+0+1+1) = 1.5

FA_{t2} = (–2x0–1x0+0x1+1x0+2x0)/(0+0+1+0+0) = 0.

Now, we computed the maximum membership, FC_{t1} = 1.38, FC_{t2} = 1.40.

Definition 3.3 Confidence interference grade

For each $F\hat{X}_t$, let FI_{tij} be the confidence interference grade *of each element for generalized membership rank in $\hat{FX}_{ti}$. Then*

$$FI_{tij}=\frac{(1-I_{ij})\mu_{L_{ij}}(FX_t)}{FC_{ti}},\ j = 1,2,\cdots 5,$$ *where $\mu_{L_{ij}}(FX_t)$ is the membership function of FX_t in the linguistic variable L_{ij}.*

Example 3.2 Assume that $\hat{FX}_t$ = {(0.56,0.45,1.15,1.38, 1.38), (0.96,1.32,1.40,0.74,0.28)}. Then the Confidence interference grade of each element is as following:

$FI_{t11} = 0.41$, $FI_{t12} = 0.33$, $FI_{t13} = 0.83$, $FI_{t14} = 0$, $FI_{t15} = 0$,
$FI_{t21} = 0.66$, $FI_{t22} = 0.94$, $FI_{t23} = 0$, $FI_{t24} = 0.53$, $FI_{t25} = 0.2$.

Definition 3.4 The weight of confidence interference grade

For the confidence interference grade of each element in $\hat{FX}_{ti}$, let FW_{tij} be the weight of confidence interference grade of FI_{tij}, then

$$FW_{tij}=\frac{|(j-3)-FA_{ti}|}{4},\ j=1,2,\cdots,5.$$

Example 3.3 *Assume* FX_t = {(0.56,1.15,1.38,1.38)(0.96, 1.32,1.40,0.74,0.28)}, *then the weight of confidence interference grade of each element is as following respectively*

$FW_{t11} = 0.875$, $FW_{t12} = 0.625$, $FW_{t13} = 0.375$, $FW_{t14} = 0.125$, $FW_{t15} = 0.125$.
$FW_{t21} = 0.5$, $FW_{t22} = 0.25$, $FW_{t23} = 0$, $FW_{t24} = 0.25$, $FW_{t25} = 0.5$.

Definition 3.5 Belief function

Let $L = \{(L_{11},\cdots L_{15}),\ldots,(L_{k1},\cdots L_{k5})$, L_{ij} is a linguistic variable$\}$ and FX_t be a membership function of multivariate fuzzy time series with respect to L. Let C_{ti} be the belief function of FX_{ti}, then

$$C_{ti}=1-\frac{\sum_{j=1}^{5}FI_{tij}\times FW_{tij}}{\sum_{j=1}^{5}(1-I_{ij})},\ where\ FX_t=\{FX_{t1},FX_{t2},\cdots FX_{tk}\}.$$

Example 3.4 *Assume that* FX_t = {(0.56,0.45,1.15,1.38, 1.38)(0.96,1.32,1.40,0.74,0.28)}. *Then the belief functions are*

$$C_{t1}=1-\frac{0.41\times0.875+0.33\times0.625+0.83\times0.375+0.125\times0+0.125\times0}{(1+1+1+0+0)}=0.71$$

and

$$C_{t2}=1-\frac{0.66\times0.5+0.94\times0.25+0\times0+0.53\times0.25+0.2\times0.5}{(1+1+0+1+1)}=0.80.$$

3.3 Some heuristic properties for the measuring belief in the time series analysis

Property 3.1 *Let C_{ti} be the belief function of fuzzy forecasting value FX_{ti}. For generalized rank FA_{ti} and maximum membership FC_{ti}, if the confidence interference grade and the weight coefficient in each element get smaller, then the confidence function C_{ti} gets higher.*

Property 3.2 *If the distribution pattern of membership in FX_{ti} is of single kurtosis, then the larger the maximum membership value FC_{ti} is, the higher the belief function value will be.*

Property 3.3 *If the distribution pattern of membership is approximately uniform distribution, then the belief function value C_{ti} will be lower.*

Property 3.4 *If the forecasting property is unchanged (medium), then the belief function value C_{ti} will be higher. However, if the forecasting property is soar (very high) or plunge (very low), then the belief function value C_{ti} will be lower.*

Property 3.5 *Assume that $C_{(t-1)i}$ and C_{ti} are the belief functions of fuzzy forecasting values $FX_{(t-1)i}$ and FX_{ti} respectively. Then $C_{(t-1)i}$ will not affect C_{ti}. In other words, the forecasting value of belief function of a certain day does not affect the one in the next day.*

4. EMPIRICAL STUDY

4.1 Data Analysis

The data in this study are the daily weighted stock price index fluctuation and trade volume high/low difference information, taken from Taiwan Stock Exchange Corporation from 2003 January 3 to 2003 March 11, as illustrated in Figure 4.1 and 4.2.

As shown in the data, the daily maximum value of stock price index fluctuation is 211.09 and the daily minimum

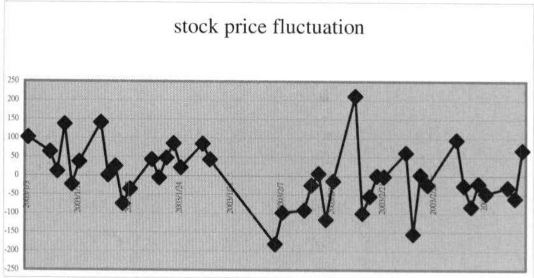

Figure 4.1: TAIEX Fluctuation Summary (2003/January/3~2003/March/11)

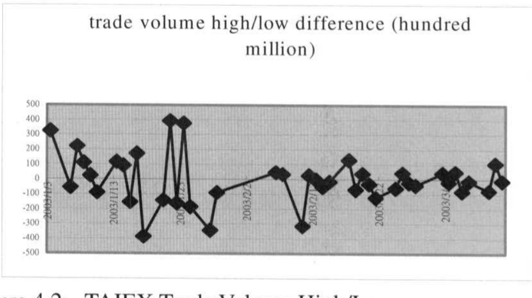

Figure 4.2: TAIEX Trade Volume High/Low (2003/January/3~2003/March/11)

one, −181.58; the maximum value of daily trade volume high/low difference is 393(hundred million), and the minimum one, −387(hundred million). Generally speaking, universe of discourse should include the maximum and the minimum values; hence, we choose set (−181.58, 211.09) and (−387, 393) as the universe of discourse of everyday stock price index fluctuation and daily trade volume high/low difference respectively. Because this study is based on fuzzy theory, we need to first fuzzify the data and, then, proceed to establish the model. Thus, (−181.58, 211.09) and (−387, 393) are divided 5 intervals as follows: let E_1=1/8 quantile, E_2=3/8 quantile, E_3=5/8 quantile, E_4=7/8 quantile.

I_{11} = *(minimum, E_1)=(−181.58,−91.5), and its representative value is −181.58 ;*

I_{12} =*(E_1, E_2)=(−91.5,−24.23), and its representative value is -46.58 ;*

I_{13} =*(E_2, E_3)=(−24.23,11.22), and its representative value is −2.35 ;*

I_{14} =*(E_3, E_4)= (11.22,85.53), and its representative value is 43.25 ;*

I_{15} =*(E_4, maximum)=(85.53,211.09), and its representative value is 211.09.*

I_{21} =*(minimum, E_1)=(−387,−154), and its representative value is -387 ;*

I_{22} =*(E_1, E_2)=(−154,−52), and its representative value is −81 ;*

I_{23} =*(E_2, E_3)=(−52, 34), and its representative value is −16 ;*

I_{24} =*(E_3, E_4)=(34,129), and its representative value is 48 ;*

I_{25} =*(E_4, maximum)=(129,393), and its representative value is 393*

Among which, $\{I_{11}, I_{12}, I_{13}, I_{14}, I_{15}\}$ and $\{I_{21}, I_{22}, I_{23}, I_{24}, I_{25}\}$ are the five intervals of (−181.58, 211.09) and (−387, 393) respectively. Then, we define five linguistic variables within (−181.58, 211.09) and (−387, 393), that is, L_{11} = *plunge* ; L_{12} = *down* ; L_{13} = *unchanged* ; L_{14} = *up* ; L_{15} = *soar.*

L_{21} = very low ; L_{22} = low ; L_{23} = medium ; L_{24} = high ; L_{25} = very high. Each of the linguistic variables stands for a fuzzy set, and the components of each fuzzy set are I_{ij} (i = 1,2; j = 1,2,...,5) and the corresponding membership function.

4.2 Establishment of Fuzzy Time Series Analysis Model

Before constructing the model, we have to fuzzify both daily weighted stock price index fluctuation and trade volume high/low difference. By applying the procedure of Definition 2.1, for each fuzzy set L_{ij} (i = 1,2; j = 1,2,...,5), we gain daily weighted stock price index fluctuation and trade volume high/low difference as well as the corresponding membership function of each linguistic variable, as shown in Table 4.1 and 4.2. For the brief reason we only illustrated the first ten data

The daily weighted stock price index fluctuation and trade volume high/low difference from 2003/January/3 to 2003/March/11 are shown in Table 4.1 and 4.2. Suppose the maximum membership grade of some day is located at L_{1j} (j = 1,2,...,5), its linguistic variable will be regarded as L_{1j} (j = 1,2,...,5). Take 2003/January/3 as an example. The maximum membership grade is located at L_{14} and L_{25}. Thus, the weighted stock price index fluctuation of 2003/January/3 is L_{14} and the trade volume difference is L_{25}. Or we can call the fluctuation value of the day as "up" and the trade volume difference as "very high." The fuzzy relationship among data can be located, based on past fuzzy data, and furthermore, fuzzy-Markov-relative matrix is obtained as well.

$$R=\begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix}=$$

$$\begin{bmatrix}
0.38 & 0.62 & 0.80 & 0.10 & 0.00 & 0.38 & 0.26 & 0.80 & 0.78 & 0.00 \\
0.33 & 0.75 & 0.94 & 0.75 & 0.30 & 0.80 & 0.94 & 0.77 & 0.64 & 0.23 \\
0.52 & 0.59 & 0.95 & 0.94 & 0.74 & 0.48 & 0.86 & 0.63 & 0.74 & 0.36 \\
0.99 & 0.61 & 0.94 & 0.75 & 0.58 & 0.52 & 0.75 & 0.70 & 0.95 & 1.00 \\
0.39 & 0.61 & 0.58 & 0.35 & 0.12 & 0.25 & 0.83 & 0.38 & 0.58 & 0.13 \\
0.20 & 0.51 & 0.49 & 1.00 & 0.25 & 0.34 & 0.86 & 0.36 & 0.64 & 0.26 \\
0.97 & 1.00 & 0.80 & 0.86 & 0.58 & 0.66 & 0.68 & 0.95 & 0.95 & 0.80 \\
0.52 & 0.74 & 0.77 & 0.63 & 1.00 & 0.22 & 0.94 & 0.63 & 0.95 & 0.38 \\
0.39 & 0.67 & 0.90 & 0.63 & 0.50 & 0.77 & 1.00 & 0.89 & 0.70 & 0.18 \\
0.23 & 0.36 & 0.48 & 0.98 & 0.50 & 0.36 & 0.74 & 0.38 & 0.50 & 0.18
\end{bmatrix}$$

R_{11} is the fuzzy-Markov-relative matrix of the stock price fluctuation of (a certain day) and the stock price fluctuation of (the next day). R_{12} is the fuzzy-Markov-relative matrix of the stock price fluctuation of (a certain day) and the trade volume difference of (the next day). R_{21} is the fuzzy-Markov-relative matrix of the trade volume difference of (a certain day) and the stock price fluctuation of (the next day). R_{22} is the fuzzy-Markov-relative matrix of the trade volume difference of (a certain day) and the trade volume difference of (the next day). Thus, the multivariate first order of auto-regression model is $(FX_{1,t}, FX_{2,t}) = (FX_{1,t-1}, FX_{2,t-1})R$. $(FX_{1,t-1}, FX_{2,t-1})$ and $(FX_{1,t}, FX_{2,t})$ each indicates the membership function of the linguistic variable of multivariate fuzzy set of Taiwan weighted stock price index fluctuation and trade volume high/low difference in Day t-1 and Day t. At last, Table 4.3 and 4.4 present the membership functions output by the model and the membership functions converted from linguistic vector index functions.

4.3 Comparisons and Analyses of Prediction Results

Since this study focuses mainly on the property tendency of time series, we make use of the fuzzy rule base discussed in section 3.2 to verify the converted membership functions and output the properties of the prediction results. Furthermore, we have introduced the meaning and definition of belief function in detailed in section 3.5. The properties of the prediction results and the belief functions are listed in Table 4.5 and 4.6.

As shown in Table 4.5 and 4.6, the multivariate fuzzy time series forecast model established in this study has proved itself to be a very effective predicting device. Because the predictions are made through the five divided sections, the matching average should be about 0.20, regarding either stock price fluctuation or trade volume difference. However, if we made predictions through the model in this study, the matching averages of stock price fluctuation and trade volume difference are 0.53 and 0.55, and the average forecasting accuracy are 0.81 and 0.86 respectively. Belief function has also been helpful in deciding the amount of capital to be invested and controlling risks. It is clearly manifested that if the belief function is higher, the properties we predict are in conformity with the properties of true value. On the contrary, when the predicted properties and the true ones are distinct, the belief functions of the prediction are usually lower. As shown in the following Table 4.7, the authors present the true value properties as well as the predicted value properties of TAIEX fluctuation from March 12 to April 23, generated by the model we propose in this study.

Owing to the *Second Gulf War* during March, 20 to May, 2, 2003, the first SARS case identified on March, 8 in Taiwan, the seal-off of Taipei Municipal Hoping Hospital on April, 24, and the fast spread of SARS, Taiwan stock market had suffered great impact. Nonetheless, under the severe market unrest, the matching average for the next 31 days, as shown in Table 4.7, manages to reach 0.27 and the average forecasting accuracy, 0.72. If we exclude the ten days prior and post Gulf War and the days after SARS outbreaks, we find that there are six days with matching average as high as 0.5 and average forecasting accuracy, 0.79 as shown in Table 4.8. This has proved that the multivariate fuzzy time series forecast model we establish is, to a certain extent, credible in predicting future conditions. The reason of some prediction values' failing to reach the true values is that we only take the maximum membership function into consideration and neglect the secondary membership functions in the conversion process.

To sum up, with the help of a reasonable forecast model and belief functions for evaluating the prediction results, we can make an appropriate investment strategy and won't feel at a loss for future investment and can reduce the risks as well.

5. CONCLUSIONS

In order to increase profit gains, simply enhancing the accuracy of forecast is not enough. A more substantial factor is to have great belief in accurate predictions, so that investors can place more capital and gain maximum returns. On the contrary, if investors have little belief in the predictions, they will act with care and fear and place less capital in the investment, causing less profits, despite the prediction results being accurate. Again, no forecast model can guarantee ever accuracy. According to the belief function proposed in the study, accurate predictions generally tend to have greater degree of belief, while inaccurate ones will lead to less degree of belief. As a consequence, when predictions have lower degree of belief, investors should strictly control investment rate so as to prevent loss caused by wrong predictions. On the other hand, when predictions have greater degree of belief, investors might as well considering larger sum of investment so as to obtain more profits. Thus, investors can try utilizing belief function to formulate financial management and investment strategies to better rate of return and risk management.

The study uses the defined fuzzy time series, fuzzy relation and fuzzy-markov-relative matrix to establish fuzzy multivariate time series forecast model through fuzzy formula's inference, and makes use of fuzzy rule base in judging the property of forecast value, which is further evaluated by belief functions to verify its degree of belief. With the above methodology, we propose a model for prediction, incorporating two factors of TAIEX fluctuation and trade volume difference, and establish an appropriate multivariate fuzzy time series model, based on historical data of weighted stock price index fluctuation and trade volume difference from 2003/January/3 to 2003/March/11. We have measured the accuracy of the prediction value generated by the multivariate fuzzy time series with average predict rank accuracy, and have evaluated the degree of belief in each of the prediction with belief function values. As shown in the practical analysis, the forecast model is plausible. With the help of belief functions in each prediction, investors not only can obtain more accurate predictions, but also can refer to the value of belief functions to adjust their investment strategy and to increase their ability of risk control as well as rate of returns, which is of great financial significance.

The following are some issues to be solved and are worthy of further investigation in the future.

1. This study only takes two factors into consideration, i.e. closing price and trade volume difference; however,

there are still many other factors that may affect weighted stock price index, like a company's financial structure, exchange rate, interest rate, and government policies, etc. Thus, to gain more accurate forecasting, other variables may be incorporated into the establishment of forecast models.

2. Take Taiwan weighted stock price index as an example. A great many quoted companies distribute annual profits at the 2nd quarter, i.e. ex rights and ex dividend. Yet, the peak period for electronic industry is the 4th quarter, which may inspire us to include the seasonal factor in our prediction.
3. The study adopts five-rank taxonomy and converts time series data into fuzzy values through membership functions. Another common classification method in social science is seven-rank taxonomy, which is an alternative way worthy of trying. The more ranks the classification has, the more complicated the calculation will be. Nonetheless, finer classification will definitely enhance the predicting capability to a certain extent.
4. Different constructing technique of membership functions will lead to different predictions. Hence, if we make an effort to improve the technique, it is possible to strengthen the predicting and analyzing potency of the model towards time series data.
5. The belief function we proposed in the study is created to conduct the conversion procedure through maximum membership grade. If is suggested that we may develop alternative and more realistic belief measure process in the future study.

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Table 4.1: Membership Grade of Daily TAIEX Fluctuation

Date	Daily TAIEX Fluctuation	L_{11}	L_{12}	L_{13}	L_{14}	L_{15}
2003/1/3	101.45	0.00	0.00	0.00	0.65	0.35
2003/1/6	63.54	0.00	0.00	0.00	0.88	0.12
2003/1/7	11.22	0.00	0.00	0.70	0.30	0.00
2003/1/8	135.85	0.00	0.00	0.00	0.45	0.55
2003/1/9	-23.2	0.00	0.47	0.53	0.00	0.00
2003/1/10	37.07	0.00	0.00	0.14	0.86	0.00
2003/1/13	140.46	0.00	0.00	0.00	0.42	0.58
2003/1/14	1.16	0.00	0.00	0.92	0.08	0.00
2003/1/15	25.28	0.00	0.00	0.39	0.61	0.00
2003/1/16	-74.41	0.20	0.80	0.00	0.00	0.00

Table 4.2: Membership Grade of Trade Volume Difference

Date	Trade Volume Difference (hundred million)	L_{21}	L_{22}	L_{23}	L_{24}	L_{25}
2003/1/3	325	0.00	0.00	0.00	0.20	0.80
2003/1/6	-56	0.00	0.62	0.38	0.00	0.00
2003/1/7	221	0.00	0.00	0.00	0.50	0.50
2003/1/8	110	0.00	0.00	0.00	0.82	0.18
2003/1/9	24	0.00	0.00	0.38	0.63	0.00
2003/1/10	-89	0.03	0.97	0.00	0.00	0.00
2003/1/13	116	0.00	0.00	0.00	0.80	0.20
2003/1/14	92	0.00	0.00	0.00	0.87	0.13
2003/1/15	-154	0.24	0.76	0.00	0.00	0.00
2003/1/16	171	0.00	0.00	0.00	0.64	0.36

Table 4.3: Membership functions output by the model regarding Taiwan weighted stock price index fluctuation and trade volume high/low difference

Date	Membership functions output by the model
2003/1/6	(0.89, 0.96, 1.13, 1.46, 1.08) (0.88, 1.39, 1.04, 1.15, 0.83)
2003/1/7	(1.49, 1.22, 1.49, 1.37, 1.16) (1.14, 1.37, 1.31, 1.49, 1.49)
2003/1/8	(0.91, 1.09, 1.20, 1.20, 1.20) (0.98, 1.20, 1.13, 1.20, 0.54)
2003/1/9	(0.84, 1.22, 1.37, 1.07, 0.95) (1.22, 1.37, 1.27, 1.35, 0.63)
2003/1/10	(0.91, 1.16, 1.16, 1.16, 1.03) (1.10, 1.16, 1.16, 1.16, 0.73)
2003/1/13	(1.84, 1.58, 1.67, 1.62, 1.16) (1.18, 1.43, 1.65, 1.82, 1.67)
2003/1/14	(0.81, 1.25, 1.38, 1.05, 0.92) (1.19, 1.38, 1.22, 1.38, 0.60)
2003/1/15	(0.91, 1.26, 1.80, 1.55, 1.24) (1.25, 1.74, 1.50, 1.54, 0.54)
2003/1/16	(1.37, 1.37, 1.37, 1.37, 1.16) (1.18, 1.28, 1.37, 1.37, 1.37)

Table 4.4: Membership functions converted from linguistic vector index functions regarding Taiwan weighted stock price index fluctuation and trade volume high/low difference

Date	Membership Functions After Transformation
2003/1/6	(0, 0, 0, 1, 0) (0, 1, 0, 0, 0)
2003/1/7	(1, 0, 1, 0, 0) (0, 0, 0, 1, 1)
2003/1/8	(0, 0, 0, 1, 1) (0, 1, 0, 1, 0)
2003/1/9	(0, 0, 1, 0, 0) (0, 1, 0, 0, 0)
2003/1/10	(0, 1, 1, 1, 0) (0, 1, 1, 1, 0)
2003/1/13	(1, 0, 0, 0, 0) (0, 0, 0, 1, 0)
2003/1/14	(0, 0, 1, 0, 0) (0, 1, 0, 1, 0)
2003/1/15	(0, 0, 1, 0, 0) (0, 1, 0, 0, 0)
2003/1/16	(1, 1, 1, 1, 0) (0, 0, 1, 1, 1)

Table 4.5: Comparison of fitness value for TAIEX fluctuation

Date	True Value	Predicted Value of Multivariate Fuzzy Time Series	Belief Function of the Predicted Value
2003/1/6	up	up	0.71
2003/1/7	unchanged	plunge	0.65
2003/1/8	soar	soar	0.47
2003/1/9	unchanged	unchanged	0.73
2003/1/10	up	unchanged	0.58
2003/1/13	soar	plunge	0.51
2003/1/14	unchanged	unchanged	0.74
2003/1/15	up	unchanged	0.75
2003/1/16	down	down	0.47
2003/1/17	down	unchanged	0.75
2003/1/20	up	up	0.79
2003/1/21	unchanged	plunge	0.50
2003/1/22	up	up	0.78
2003/1/23	up	plunge	0.50
2003/1/24	up	up	0.74
2003/1/27	up	down	0.42
2003/1/28	up	up	0.72
2003/2/6	plunge	plunge	0.54
2003/2/7	down	unchanged	0.83
2003/2/10	down	unchanged	0.75
2003/2/11	down	up	0.75
2003/2/12	unchanged	down	0.60
2003/2/13	plunge	unchanged	0.69
2003/2/14	unchanged	down	0.60
2003/2/17	soar	soar	0.55
2003/2/18	down	unchanged	0.76
2003/2/19	down	unchanged	0.76
2003/2/20	unchanged	unchanged	0.80
2003/2/21	unchanged	unchanged	0.71
2003/2/24	up	up	0.65
2003/2/25	plunge	plunge	0.66
2003/2/26	unchanged	unchanged	0.81
2003/2/27	unchanged	unchanged	0.69
2003/3/3	up	up	0.44
2003/3/4	down	unchanged	0.73
2003/3/5	down	soar	0.48
2003/3/6	unchanged	unchanged	0.78
2003/3/7	down	down	0.63
2003/3/10	down	unchanged	0.74
2003/3/11	down	down	0.66
Matching Average: 0.53			
Average Rank-Forecasting Accuracy: 0.81			

Table 4.6: Comparison of fitness value for TAIEX trade volume high/low difference

Date	True Value(after realization)	Predicted Value of Multivariate Fuzzy Time Series	Belief Function of the Predicted Value
2003/1/6	low	low	0.70
2003/1/7	very high	very high	0.48
2003/1/8	high	medium	0.79
2003/1/9	high	low	0.68
2003/1/10	low	medium	0.60
2003/1/13	high	high	0.67
2003/1/14	high	medium	0.78
2003/1/15	low	low	0.73
2003/1/16	high	very high	0.44
2003/1/17	very low	very low	0.61
2003/1/20	low	low	0.75
2003/1/21	very high	very high	0.52
2003/1/22	low	low	0.77
2003/1/23	very high	very high	0.50
2003/1/24	low	low	0.69
2003/1/27	very low	medium	tie-undecided
2003/1/28	low	low	0.70
2003/2/6	high	high	0.68
2003/2/7	high	medium	0.80
2003/2/10	very low	medium	0.67
2003/2/11	high	low	0.73
2003/2/12	medium	low	0.61
2003/2/13	low	low	0.72
2003/2/14	medium	high	0.59
2003/2/17	high	high	0.73
2003/2/18	low	low	0.77
2003/2/19	high	high	0.61
2003/2/20	medium	low	0.74
2003/2/21	low	low	0.72
2003/2/24	low	high	0.64
2003/2/25	high	very high	0.48
2003/2/26	medium	medium	0.79
2003/2/27	medium	low	0.73
2003/3/3	high	medium	0.65
2003/3/4	medium	low	0.67
2003/3/5	high	high	0.70
2003/3/6	low	low	0.74
2003/3/7	medium	high	0.65
2003/3/10	low	low	0.75
2003/3/11	high	high	0.63
Matching Average: 0.55			
Average Rank-Forecasting Accuracy: 0.86			

Table 4.7: True value, predicted value and belief function of TAIEX fluctuation

Date	True Value	Predicted Value of Multivariate Fuzzy Time Series	Belief Function of the Predicted Value
2003/3/12	up	unchanged	0.79
2003/3/13	up	unchanged	0.67
2003/3/14	up	unchanged	0.74
2003/3/17	plunge	up	0.58
2003/3/18	soar	up	0.74
2003/3/19	down	up	0.72
2003/3/20	up	unchanged	0.53
2003/3/21	unchanged	unchanged	0.66
2003/3/24	unchanged	up	0.65
2003/3/25	down	up	0.60
2003/3/26	unchanged	unchanged	0.73
2003/3/27	unchanged	unchanged	0.70
2003/3/28	down	unchanged	0.73
2003/3/31	plunge	unchanged	0.73
2003/4/1	unchanged	unchanged	0.81
2003/4/2	down	up	0.59
2003/4/3	up	down	0.59
2003/4/4	soar	unchanged	0.74
2003/4/7	up	unchanged	0.73
2003/4/8	unchanged	unchanged	0.71
2003/4/9	unchanged	unchanged	0.54
2003/4/10	unchanged	up	0.61
2003/4/11	unchanged	unchanged	0.71
2003/4/12	down	up	0.61
2003/4/15	up	down	0.68
2003/4/16	up	soar	0.45
2003/4/17	down	up	0.73
2003/4/18	up	unchanged	0.74
2003/4/21	unchanged	up	0.73
2003/4/22	down	up	0.62
2003/4/23	unchanged	unchanged	0.66
Matching Average: 0.27			
Average Rank-Forecasting Accuracy: 0.72			

Table 4.8: True value, predicted value and belief function of TAIEX fluctuation

Date	True Value	Predicted Value of Multivariate Fuzzy Time Series	Belief Function of the Predicted Value
2003/4/8	unchanged	unchanged	0.71
2003/4/9	unchanged	unchanged	0.54
2003/4/10	unchanged	up	0.61
2003/4/11	unchanged	unchanged	0.71
2003/4/12	down	up	0.61
2003/4/15	up	down	0.68
Matching Average: 0.5			
Average Rank-Forecasting Accuracy: 0.79			

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